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Editorial

Problem Solving and Problem Posing: From Conceptualisation to Implementation in the Mathematics Classroom

Problem solving and problem posing are leading mathematical activities that stimulate mathematical thinking. From the theoretical point of view, these activities are very complex, partly due to the various issues that describe/define problem solving and problem posing and their role in the process of teaching and learning mathematics. Problem solving and problem posing are interrelated activities; we could say that they are in an interdependent relationship: we solve the problems we pose, we pose the problems in a way that we can solve them. However, the two processes are not equally present in every situation.

Research into problem solving focuses mainly on the following areas: the basic characteristics of a mathematical problem; the nature (conceptual, procedural) and role of representation (interplay between internal and external) of a mathematical problem; mental schemas for problem solving; heuristics as principles, methods and (cognitive) tools for solving problems; types of generalisations and reasoning (abductive, narrative, naïve, arithmetic, algebraic); problem solving as a challenging activity for mathematically gifted students; and the role of the teacher in guiding problem solving as a way of implementing student problem solving in the classroom. Regarding problem posing, there are also some critical questions: How can the existing definitions of problem posing be categorised? How is problem posing conceived by the research community in relation to other mathematical constructs? What are the possible ways of implementing problem posing in research and teaching settings? Regarding problem solving, problem posing is formulated/used in research findings for generating (formulating, finding, creating) new problems; reformulating existing problems; creating and/or reformulating problems; raising questions and viewing old questions from a new angle; and an act of modelling.

Research has demonstrated and frequently confirmed that (mathematical) problem posing and solving possess great potential for learners, but the reality in terms of teaching practice, external examinations, teaching material, and mathematics curricula seems out of alignment with the research findings. In this focus issue, we have considered two aspects of problem posing and problem solving: conceptualisation and implementation in the mathematics classroom.

This issue contains five articles that address the issues of problem posing and problem-solving. The authors come from different backgrounds (Greece, Croatia, Hungary, Germany), which means that diverse perspectives and research findings in this field are confronted. Furthermore, each school system responds to the implementation of problem solving in different ways, and the research presented is related to this. Problem solving in mathematics education as a conceptual premise is not new; it has its roots in 1945, in the book *How to Solve It* (Polya), yet we are constantly faced with new challenges in implementing the findings of researchers on problem solving in mathematics education, such as how to create an environment for problem solving in mathematics education, how to place problem solving in certain mathematical content, how to establish an appropriate teaching role for the teacher (we are beyond believing that the student will become a good problem solver or problem poser on his/her own, without the teacher's intervention), how to assess problem solving, where to get problems that are appropriate for different age groups of students and their abilities, pre-knowledge and similar. Unfortunately, the latter (where to get mathematical problems) has a vital connection with the authors of the textbook materials used by teachers, which regulate the proportion of problem solving in mathematics lessons. It is true that problem solving in the sense presented here cannot be the main topic of instruction (problem solving cannot replace the learning of fundamental mathematical concepts and content), but the inclusion of selected problems and related strategies and heuristics in the classroom certainly makes sense from at least two points of view: the deepening and application of mathematical knowledge, and the acquisition of the generic problem-solving skills expected of us in an ever-changing society.

The introductory article in this focus issue, entitled Multiple Approaches to Problem Posing: Theoretical Considerations Regarding its Definition, Conceptualisation, and Implementation, by Ioannis Papadopoulos, Nafsika Patsiala, Lukas Baumanns, and Benjamin Rott answers the question of the conceptualisation of problem-setting.

In their theoretical research paper, they attempt to capture different meanings and aspects of problem posing by approaching it from three different levels: (1) by comparing definitions, (2) by relating it to other constructs, and (3) by referring to research and teaching settings. Their analysis of the documents and research findings considering the first level shows no consensus regarding the conceptualisations of problem posing, which certainly causes much uncertainty in terms of implementation in the classroom and in research. In the second level, they examine how problem posing is conceived by the research community compared to other mathematical constructs, such as problem solving, mathematical creativity, or modelling. In empirical research on the connection of problem posing to these constructs, it is noticeable that the focus is mainly on the products, meaning the problems posed. Their discussion emphasises that there is a lack of research to evaluate the process of problem posing when investigating connections to problem solving or creative mathematical thinking.

Furthermore, they argue that the products can only reflect one component of the activity of problem posing. In practice, the descriptions in this article can be helpful in understanding the enormous spectrum of conceptualisations of problem posing. This may enable a targeted selection and assessment of appropriate problem-posing activities for educational purposes to be achieved. The third level (research and teacher settings) summarises possible ways of implementing problem posing in research and teaching settings as depicted in the relevant literature. The authors do not offer definitive answers; their intentions are to stimulate discussion on this far-reaching and complex topic; a future systematic literature review may provide insights of greater validity into definitions, conceptualisations, and implementations of problem posing in research and practice.

In the second paper, Reading Mathematical Texts as a Problem-Solving Activity: The Case of the Principle of Mathematical Induction by Ioannis Papadopoulos and Paraskevi Kyriakopoulou, we are faced with the possibility of implementing problem solving in mathematics lessons in conjunction with the reading of mathematical texts. Reading complex mathematical texts is closely related to the effort of the reader to understand its content; therefore, it is reasonable to consider such reading as a problem-solving activity. In this paper, the principle of mathematical induction was introduced to secondary education students through mathematical text; their efforts to comprehend the text were examined to identify whether significant elements of problem solving are involved. The findings show that while negotiating the content of the text, the students went through Polya's four phases of problem solving. Moreover, this approach of reading the principle of mathematical induction in the sense of a problem that must be solved seems a promising idea for the conceptual understanding of the notion of mathematical induction. The article opens the door to a new understanding of problem solving by showing that reading mathematical texts also might have characteristics of problem solving in terms of the process experienced by the problem solver in this activity.

There is less research on geometry problems than on arithmetic problems, which is understandable (given that less attention is given to school geometry in comparison to other topics), as solving geometric problems requires complex geometric knowledge, which, due to the nature of concepts at higher levels of schooling, becomes much more abstract because it is based on a good understanding of definitions and the hierarchy between concepts. The paper entitled Factors Affecting Success in Solving a Stand-Alone Geometrical Problem by Students aged 14 to 15, by Branka Antunović-Piton and Nives Baranović, investigates and considers factors that affect success in solving a stand-alone geometrical problem by students of the 7th and 8th grades of elementary school. The starting point for consideration is a geometrical task from the National Secondary School Leaving Exam in Croatia (State Matura), utilising elementary-level geometry concepts. The task was presented as a textual problem with an appropriate drawing and a task within a given mathematical context. After data processing, the key factors affecting the process of problem solving were singled out: visualisation skills, detection and connection of concepts, symbolic notations, and problem-solving culture. The obtained results are the basis of suggestions for changes in the geometry teaching-learning process. They conclude that the selected sample of students lacked fully developed problem-solving skills, understanding certain geometrical concepts, and the skill to identify and connect conceptual properties, resulting in students' inability to find a systematic way to the required solution. The underdeveloped visualisation skills were observed as a particular issue, as fully-developed visualisation skills are required for the problem-solving process of geometrical tasks. All the aforementioned difficulties experienced throughout the problem-solving process indicate that the learning and teaching of geometry should emphasise the visualisation skills (drawing, interpretation, formation of connections among different notations, etc.) and systematic notetaking. The authors conclude that this skill set can be learned and developed by solving geometry problems of different cognitive requirements, and the role of the teacher should not be underestimated, as we mentioned in the introduction of this editorial.

It has been repeatedly shown that involving readers in research can make a difference to a teacher's teaching. Of course, the question remains to what extent these changes remain in the teaching after the project or research activity is over. In an optimistic scenario, at least 'traces' of the research or changes in teachers' teaching remain, as well as the possibility for a qualitative upgrading of professional-didactic knowledge in the area of integrating problem solving into mathematics teaching. The paper Management of Problem Solving in a Classroom Context by Eszter Kónya and Zoltán Kovács addresses the role of the professional development of teachers in implementing problem solving in the mathematics classroom. The authors discuss the results of a professional development programme involving four Hungarian teachers of mathematics. The programme aims to support teachers in integrating problem solving into their classes. The basic principle of the programme, as well as its novelty (at least compared to Hungarian practice), is that the development takes place in the teacher's classroom, adjusted to the teacher's curriculum and in close cooperation between the teacher and researchers. The teachers included in the programme were supported by the researchers with lesson plans, practical teaching advice and lesson analyses. The progression of the teachers was assessed after the one-year programme based on a self-designed trial lesson, focusing particularly on how the teachers plan and implement problem-solving activities in lessons, as well as on their behaviour in the classroom during problem-solving activities. The findings of this qualitative research are based on video recordings of the lessons and on the teachers' reflections. The authors conclude that the lesson plans and the self-reflection habits of the teachers contribute to the successful management of problem-solving activities.

The last paper in this focus issue, titled MERIA – Conflict Lines: Experience with Two Innovative Teaching Materials by Željka Milin Šipuš, Matija Bašić, Michiel Doorman, Eva Špalj and Sanja Antoliš, presents and evaluates the implementation of two tasks as part of didactic scenarios for inquiry-based mathematics teaching, examining teachers' classroom orchestration supported by these scenarios. The context of the study is the Erasmus+ project MERIA – Mathematics Education: Relevant, Interesting and Applicable, which aims to encourage learning activities that are meaningful and inspiring for students by promoting the reinvention of target mathematical concepts. As innovative teaching materials for mathematics education in secondary schools, MERIA scenarios cover specific curriculum topics and were created based on two well-founded theories in mathematics education: realistic mathematics education and the theory of didactical situations. With the common name Conflict Lines (Conflict Lines – Introduction and Conflict Set – Parabola), the scenarios aim to support students' inquiry about sets in the plane that are equidistant from given geometrical figures: a perpendicular bisector as a line equidistant from two points, and a parabola as a curve equidistant from a point and a line. They examine the results from field trials in the classroom regarding students' formulation and validation of the new knowledge and describe the rich situations teachers may face that encourage them to proceed by building on students' work. This is a crucial and creative moment for the teacher, creating opportunities and moving between students' discoveries and the intended target knowledge. These situations indicate the numerous creative moments when teachers make decisions, create opportunities, and challenge the situation to support a productive exchange of mathematical ideas, which could be of interest to any practising teacher. Such studies are expected to contribute to a better understanding of how to support teachers in the crucial and creative moments when they try to recognise and use opportunities for moving between students' discoveries and intended target knowledge. Once again, the teacher is in the role of using his/her professional-didactic knowledge to identify the potential of the problem, to understand the student's solution and to guide him/

her towards the achievement of the goal, if this is the purpose of the problem situation in mathematics education. It is certainly possible to design scenarios for teaching problems, but the classroom situation is alive, always new and, to a certain extent, unpredictable and requires a skilled teacher who improves his or her teaching by constantly questioning his or her performance in the classroom.

This issue also includes a varia section that covers different topics: the history of foreign language values in Sweden from the seventeenth century to the present, relationships between statistics anxiety (SA), trait anxiety, attitudes towards mathematics and statistics, and academic achievement among university students, a survey on outdoor lessons of the subject Science and Technology in education, effects of parental supervision and school climate on the relationship between exosystem variables (time spent with media and perceived neighbourhood dangerousness) and peer aggression problems and effect of altruism in the relationship between empathic tendencies, the nature relatedness and environmental consciousness.

Another valuable contribution to this issue is the reviews of the two books: *Inclusion in Education: Reconsidering Limits, Identifying Possibilities* (editor Pavel Zgaga) by Melina Tinnacher and *Visible Learning for Mathematics: Grades K-12* (authors John Hattie, Douglas Fisher, Nancy Frey) by Monika Zupančič.

Here we present only a few highlights from the reviews, inviting the reader to read books that address contemporary issues, stimulate deep reflection and open up space for finding solutions to raise the quality and breadth of teaching. Inclusion is a major issue in every place and time, and the teaching of mathematics has never been subject to more 'innovation' than it is today.

'What distinguishes this book from other volumes on inclusion is the broader context in which it situates its discussion, namely society in general rather than the more restricted frame of reference of the school system. It also provides an unprecedented insight into the inclusion debate in Slovenia, comparing it with other countries.' (from the review of the book Inclusion in Education)

'One theme that we believe makes an essential contribution to the value of this book is the importance of using mathematical language in the process of teaching and learning mathematics. The content of this book, through teachers' reflection, represents a great opportunity to strengthen teachers' didactic knowledge and contribute to the quality of their teaching.' (from the review of the book Visible Learning for Mathematics)

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