

# ANALYSIS OF PIPELINE VIBRATION

## ANALIZA VIBRACIJ V CEVOVODIH

Jurij Avsec<sup>1</sup><sup>✉</sup>, Urška Novosel<sup>1</sup>

**Keywords:** pipeline vibration, pipeline fluid flow, continuous vibration

### **Abstract**

Vibrations occur in almost all energy systems. This article presents an analysis of vibrations in pipelines under the influence of fluxes of displaced persons with a pressure difference or with the help of electromagnetic forces. For this purpose, we analysed pipelines of different diameters and the flow of crude oil within the pipelines.

### **Povzetek**

Vibracije se pojavljajo v skoraj vseh energetskih sistemih. V članku je prikazana analiza vibracij v cevovodih pod vplivom tokov gnanih s tlačno razliko ali s pomočjo elektromagnetnih sil. V ta namen smo analizirali cevovode različnih premerov. V cevovodih smo obravnavali tok surove nafte.

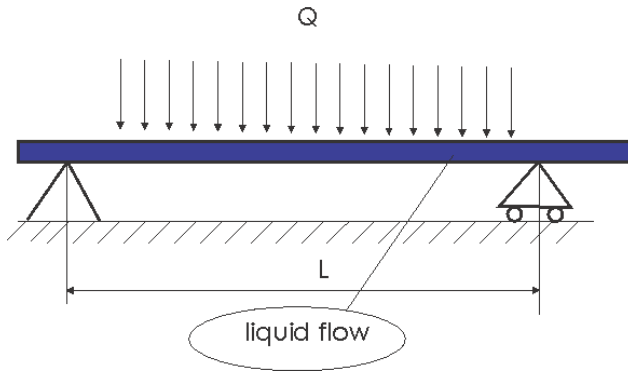
## **1 INTRODUCTION**

Energy devices are affected by many mechanical influences, including mechanical fluctuations. The analysis (analytical and experimental) of mechanical oscillations of energy devices is in many cases of paramount importance. In the present article, we have studied pipeline vibrations for a simply supported pipeline with the help of continuous vibration theory, [1-4]. Fluid flow in pipelines and channels is driven due to the presence of the electric field, magnetic field, or pressure-driven flow, and some other effects, [1] (Fig. 1). Electrohydrodynamics (EHD), known as electrokinetics, is the theory of the fluid dynamics of electrically charged fluids. It is the study of the motions of ionized

<sup>✉</sup> Corresponding author: Corresponding author: Prof. Jurij Avsec, Ph. D., Tel.: +386-7-620-2217, Fax: +386-2-620-2222, Mailing address: Hočevarjev trg 1, 8270 Krško, Slovenia  
E-mail address: [jurij.avsec@um.si](mailto:jurij.avsec@um.si), [urska.novosel@um.si](mailto:urska.novosel@um.si)

<sup>1</sup> University of Maribor, Faculty of Energy Technology, Laboratory for Thermomechanics, Applied Thermal Energy Technologies and Nanotechnologies, Hočevarjev trg 1, SI-8270 Krško, Slovenia

particles or molecules and their interactions with electric fields and the surrounding fluid. The fundamental concept for magnetohydrodynamics (MHD) is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and changes the magnetic field itself. This paper develops a formulation of crude oil motion, in macro channels and mini-channels with pipeline fluid flow. A mathematical model is developed for channels with rectangular and circular cross-sections.



**Figure 1:** Thermal influence on pipeline vibration and fluid flow

## 2 FLUID FLOW DUE TO PRESSURE AND MAGNETIC EFFECTS

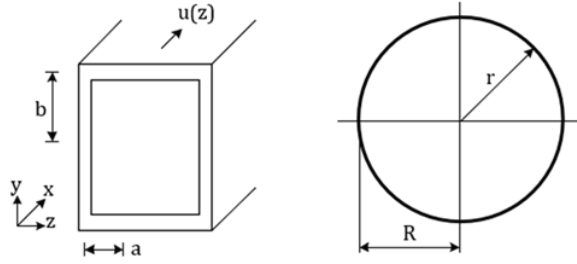
Consider electromagnetic flow in rectangular and circular channels (Fig. 2). The charged surface of a channel wall may attract ions of the opposite charge in the surrounding fluid. The general form of the momentum equation for electrohydrodynamic flow is:

$$0 = -\frac{dp}{dx} + \mu \frac{d^2u}{dz^2} + i_y B_z,$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \nabla \vec{v} = -\nabla p + \nabla(\mu \nabla \vec{v}) + \vec{i} \times \vec{B}, \tag{2.1}$$

where the last term presents the electromagnetic force, and  $i$  and  $B$  refer to the current density and magnetic field strength, respectively. For steady-state flow in a channel at small Reynolds numbers, the transient and inertia terms can be neglected, so Eq. (2. 1) is simplified in the next equation:

$$0 = -\nabla p + \nabla(\mu \nabla \vec{v}) + \vec{i} \times \vec{B}. \tag{2.2}$$



**Figure 2:** Rectangular and circular micro channels

### Laminar flow

During electrokinetic flow in channels, the charged surface of a channel wall may attract ions of the opposite charge in the surrounding fluid. Assuming the fluid velocity, magnetic field and current density are orthogonal, the reduced momentum equation becomes:

$$0 = -\frac{dp}{dx} + \mu \frac{d^2 u}{dz^2} + i_y B_z. \quad (2.3)$$

The terms represent pressure, viscous and electromagnetic forces in the liquid. Using Ohm's Law to express the current density in terms of fluid velocity:

$$\mu \frac{d^2 u}{dz^2} + \sigma_e B_z^2 u = \frac{dp}{dx}, \quad (2.4)$$

where:  $\sigma_e$  and  $B_z$  refer to the electrical conductivity and magnetic field strength. For fully developed flow in a channel, the pressure gradient becomes constant and independent of the magnetic field strength. In terms of the Hartman number,  $M_H$  ( $M_H = aB_z \sqrt{\sigma_e / \eta}$ ),

$$\mu \frac{d^2 u}{dz^2} - \left( \frac{M_H^2 \eta}{a^2} \right) u = \frac{dp}{dx}. \quad (2.5)$$

Applying the no-slip boundary conditions at  $z=0$  and  $z=-2a$ , the analytical solution of Eq. (2.5) becomes:

$$u = -\frac{a^2 \left(\frac{dp}{dx}\right)}{M_H^2 \mu} + \frac{a^2 \left(\frac{dp}{dx}\right)}{(1 + 2e^{2M_H}) M_H^2 \mu} e^{\frac{z M_H}{a}} + \frac{a^2 \left(\frac{dp}{dx}\right) e^{2M_H}}{(1 + 2e^{2M_H}) M_H^2 \mu} e^{-\frac{z M_H}{a}}.$$

The mean velocity within the channel becomes:

$$u_b = \frac{1}{2a} \int_0^{2a} u(z) dz \qquad u_b = \frac{a^2 \left(\frac{dp}{dx}\right) (-M_H + \text{Tanh}(M_H))}{M_H^3 \mu} \tag{2.6}$$

Non-dimensionlizing this result ( $z^* = z/a, u^* = u/u_b$ ), we obtain the following equation:

$$u^* = \frac{M_H \left( -1 + \frac{1}{(1 + 2e^{2M_H})} e^{z^* M_H} + \frac{e^{2M_H}}{(1 + 2e^{2M_H})} e^{-z^* M_H} \right)}{(-M_H + \text{Tanh}(M_H))}. \tag{2.7}$$

Without electromagnetic effects, equations (5-6) transform into the following expressions:

$$u(z) = \frac{\left(\frac{dp}{dx}\right)}{2\mu} (-2az + z^2), \quad u_b = \frac{1}{2a} \int_{-a}^a u(z) dz = \frac{a^2 \left(\frac{dp}{dx}\right)}{3\mu}, \quad u^* = \frac{3}{2} (2 - z^*) z^*, \tag{2.8}$$

For the circular channel without electromagnetic forces, the governing equation is:

$$\frac{dp}{dx} = \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \tag{2.9}$$

Solving the differential equation subject to boundary conditions,

$$u = \frac{\left(\frac{dp}{dx}\right)}{4\mu} (r^2 - R^2), \quad u_b = \frac{1}{\pi R^2} \int_0^R u 2\pi r dr = -\frac{\left(\frac{dp}{dx}\right) R^2}{8\mu}$$

$$u^* = 2 - 2r^{*2}. \tag{2.10}$$

If we wish to calculate the velocity profile for MHD flow in a circular channel, we have to solve the next differential equation:

$$\frac{dp}{dx} = \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \mu \frac{M_H^2}{R^2} u. \quad (2.11)$$

The analytical solution of equation (2.12) is slightly more complicated. We have obtained the next solution of a differential equation with the boundary conditions ( $u(R)=0$ ,  $u'(0)=0$ ):

$$u[r] = \frac{R^2 \left( \frac{dp}{dx} \right) \left( -\text{BesselI} \left[ 0, M \right] + \text{BesselI} \left[ 0, \frac{Mr}{R} \right] \right)}{M^2 \mu \text{BesselI} \left[ 0, M \right]}. \quad (2.12)$$

$$u_b = \frac{1}{\pi R^2} \int_0^R u 2\pi r dr = \frac{R^2 \left( \frac{dp}{dx} \right) \text{BesselI} \left[ 2, M \right]}{M^2 \mu \text{BesselI} \left[ 0, M \right]}. \quad (2.13)$$

### **Turbulent flow**

In the case of turbulent flow, the logarithmic distribution of velocity that is the most suitable is, according to experimental work, the following:

$$\frac{u}{u^*} = \frac{1}{\kappa} \ln \left( \frac{(R-r) \rho u^*}{\mu} \right) + B. \quad (2.14)$$

With the upper equation, we could calculate the mean velocity in the circular channel for turbulent flow [7,8]:

$$u_b = \frac{1}{\pi R^2} \int_0^R \left( \frac{1}{\kappa} \ln \left( \frac{(R-r) \rho u^*}{\mu} \right) + B \right) 2\pi r dr =$$

$$\frac{1}{2} u^* \left( \frac{2}{\kappa} \ln \frac{R \rho u^*}{\mu} + 2B - \frac{3}{\kappa} \right). \quad (2.15)$$

In the presented model, we used the following values:  $\kappa = 0.41$  and  $B=5$ .

### 3 VIBRATION OF PIPELINES

The equation of beam vibration containing flowing fluid could be written with the next equation, [9-13]:

$$EI \frac{\partial^4 z}{\partial x^4} = -\rho v^2 \frac{\partial^2 z}{\partial x^2} - 2\rho v \frac{\partial^2 z}{\partial x \partial t} - m \frac{\partial^2 z}{\partial t^2}. \quad (3.1)$$

Equation (3.1) is obtained with continuous vibration theory. In equation (3.1) the  $v$  means the average fluid velocity,  $\rho$  the mass of fluid per unit length,  $E$  the modulus of elasticity, and  $I$  the moment of inertia, [10-12]. The first term is the force caused by the change in direction of the velocity of the fluid due to the curvature of the pipe. The second term is the force associated with the Coriolis acceleration, and the last term represents the force that is related to the vertical acceleration of the pipe.

For the simply supported pipeline, we obtain the next solution of the differential equation, [9-12]:

$$\frac{32\rho v\omega}{3l \left[ EI \left( \frac{\pi}{l} \right)^4 - \rho v^2 \left( \frac{\pi}{l} \right)^2 - m\omega^2 \right]}{3l \left[ EI \left( \frac{2\pi}{l} \right)^4 - \rho v^2 \left( \frac{2\pi}{l} \right)^2 - m\omega^2 \right]} = \frac{8\rho v\omega}{8\rho v\omega}. \quad (3.2)$$

In equation (3.2),  $\omega$  means the angular velocity of the pipe relative to its length, and  $l$  the length between supports. With equation (3.2), the relation between  $v$  and  $\omega$  is expressed.

If we limit  $\omega \Rightarrow 0$ , equation (3.3) calculates the critical velocity at which static divergence occurs:

$$v_c = \sqrt{\frac{EI\pi^2}{\rho l^2}}. \quad (3.3)$$

### 4 RESULTS AND VIBRATIONAL ANALYSIS

Figure 3 shows analytical results for crude oil SAE 15W-40 flow at 200C through the pipeline with the next data:  $D=0.2\text{m}$ ,  $\mu=0.287\text{ Pas}$ ,  $\rho=878.7\text{ kg/m}^3$ . The fluid flow is also turbulent at small pressure differences. Figure 4 shows analytical results for crude oil flow through the pipeline with the following data:  $D=0.002\text{m}$ ,  $\mu=0.287\text{ Pas}$ ,  $\rho=878.7\text{ kg/m}^3$ . The fluid flow is also laminar at high-pressure differences.

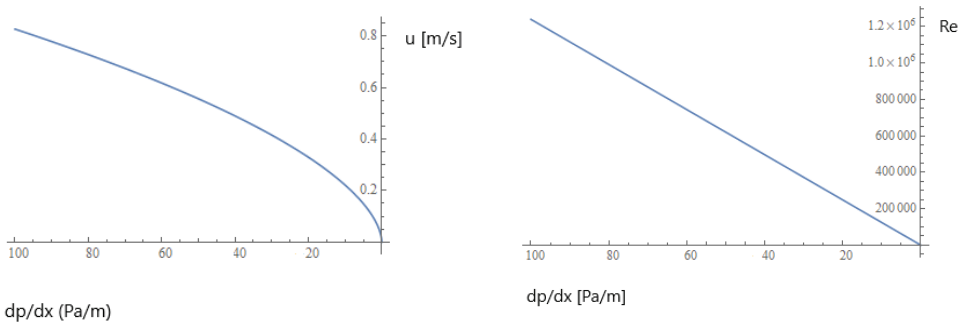


Figure 3: Mean pipe velocity and Reynolds number developed by the turbulent model

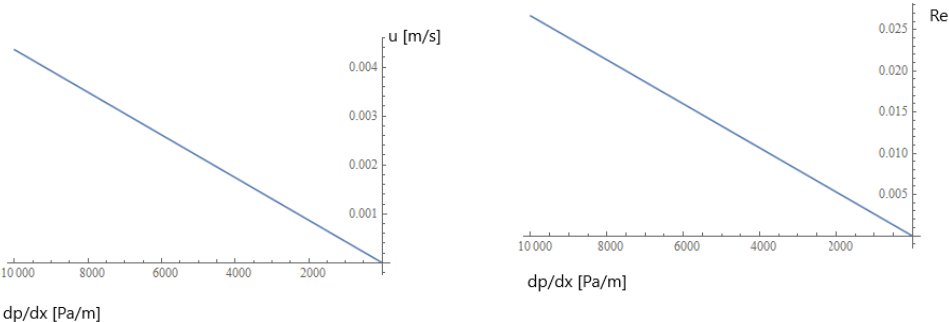


Figure 4: Mean pipe velocity and Reynolds number developed by laminar fluid model

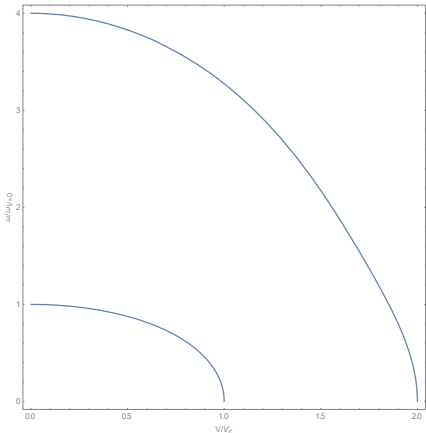
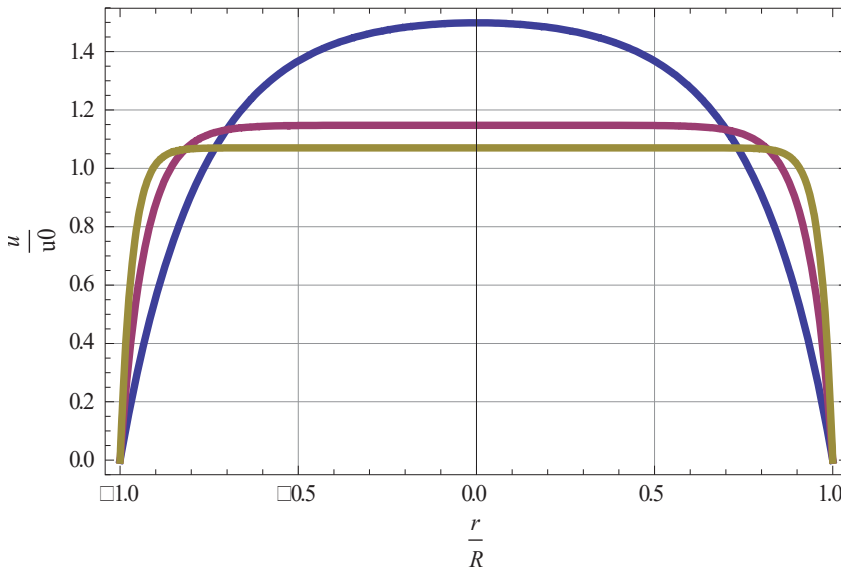


Figure 5: Vibrational characteristics for steel pipeline containing crude oil ( $D=0.2m$ ,  $\mu=0.287$  Pas,  $\rho=878.7$  kg/m<sup>3</sup>,  $\delta=0.02m$ ,  $L=10$  m)



**Figure 6:** Velocity profile with slip ( $\beta^*=0.5$ ) at  $M_H=5$  (blue line),  $M_H=10$  (red line) and  $M_H=30$  (green line) in circular minichannel without slip.

The analysis shows the velocity profile of crude oil in the pipeline. The inner diameter of the pipeline is in the first case 0.2 m in the second case 0.002 m. In the pipeline with diameter 0.2 m, the fluid flow is turbulent; in the pipeline with the diameter of 0.002 m, the fluid flow is laminar. Even more interesting are results for the calculation of natural frequencies for the first and second mode vibration regarding pipelines with crude oil. As we see from Figure 5, the natural frequencies of the pipe decrease as the fluid flow increases; from the figures, the influence of the average velocity profile in macro and mini-pipelines is also evident. Figure 6 shows velocity profile for MHD flow in the circular channel.



## 5 CONCLUSION AND OUTLOOK

The present article shows the vibrational analysis of pipes conveying crude oil SAE 15W-40 flow. The fluid flow was calculated for laminar and turbulent flow. The vibrational characteristics have been calculated with vibration continuous pipeline theory.

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