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Low-lying states of the Y-string in two dimensions ⋆

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Abstract. We use $SU(2)\times SU(2)$ algebraic methods to calculate the energy-splitting pattern of the K=2,3 excited states of the Y-string in two dimensions. To this purpose we use the dynamical O(2) symmetry of the Y-string in the shape space of triangles and compare our results with known results in three dimensions and find qualitative agreement.

1 Introduction

The three-quark confinement problem has been attacked in many ways: 1) by way of the harmonic oscillator models with some non-harmonic two-body potential components [1–3]; 2) by way of Y-string three-body potentials, Refs. [4–15]; 3) by way of the hyperspherical formalism applied to two-quark potentials, Refs. [16, 17] and 4) by way of dynamical symmetry Lie-algebraic methods, Refs. [18–23], with some success for the low-lying bands of states (up to $K \leq 3$). The higher-lying bands ($K \leq 4$) have generally not been studied systematically (to our knowledge), only individual states with highest values of the orbital angular momentum, for purposes of Regge analyses, with one significant exception ($K = 4$), the Ref. [11].

QCD seems to demand a genuine three-body confining potential: the so-called Y-junction string three-quark potential, defined by

$$V_Y = \sigma \min_{\mathbf{x_0}} \sum_{i=1}^{3} |\mathbf{x_i} - \mathbf{x_0}|, \tag{1}$$

or, explicitly

$$V_{\text{string}} = V_Y = \sigma\sqrt{\frac{3}{2}(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2 + 2|\boldsymbol{\rho}\times\boldsymbol{\lambda}|)}. \tag{2}$$

The $|\boldsymbol{\rho}\times\boldsymbol{\lambda}|$ term is proportional to the area of the triangle subtended by the three quarks. The Y-string potential was proposed as early as 1975, see Refs. [4, 5] and the first schematic calculation (using perturbation theory) of the baryon spectrum up to $K\leq 2$ followed soon thereafter, Ref. [6]. Refs. [7–9], elaborated on this. The first non-perturbative calculations (variational approximation) of the K=3 band with the Y-string potential were published in the early 1990's, Ref. [10] and extended to the K=4 band later in that decade, Ref. [11]. Yet, some of the most basic

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properties of this potential, such as the ordering of the low-lying states in the spectrum, without the "QCD hyperfine interaction" and/or relativistic kinematics remain unknown.

The first systematic attempt to solve the Y-string spectrum, albeit only up to the K=2 band, can be found in Ref. [12]. That paper used the hyperspherical harmonics formalism, which led to the discovery of a new dynamical O(2) symmetry in the Y-string potential, with the permutation group $S_3 \subset O(2)$ as the subgroup of the dynamical O(2) symmetry, see Ref. [13]. That symmetry was further elaborated in Ref. [15]. The present work is a continuation of that line, which has also been represented in this series of workshops [14]. The three-body sum of two-body potentials has only the permutation group S_3 as its symmetry.

2 O(4) algebraic method

The existence of an additional dynamical symmetry strongly suggests an algebraic approach, such as those used in Refs. [18–23]. A careful perusal of Ref. [18,19] shows, however, that an O(2) group had been used as an enveloping structure for the (discrete) permutation group $S_3 \subset O(2)$, but was not interpreted as a (possible) dynamical symmetry. Refs. [20–23] did not use this symmetry, however. We start an algebraic study of Y-string-like potentials with this in mind. For the sake of technical simplicity we confine ourselves to the two-dimensional case here. We say here "Y-string-like potentials", rather than the Y-string potential, because the complete Y-string potential contains "additional" two-body terms that are valid only in certain parts of the tree-particle configuration space (a.k.a. triangle "shape space") and that do not have the O(2) dynamical symmetry. This wider class of three-body potentials has the same dynamical O(2) symmetry in shape space as the Y-string potential, thus making them equivalent in the algebraic sense. We must therefore first establish the basic properties of the dynamical symmetry of the Y-string potential.

In two dimensions (2D) the non-relativistic three-body kinetic energy is a quadratic form of the two Jacobi two-vector velocities, $\dot{\boldsymbol{\rho}}, \dot{\boldsymbol{\lambda}}$, so its "hyper-spherical symmetry" is O(4), and the residual dynamical symmetry of the Y-string potential is $O(2) \otimes O_L(2) \subset O(4)$, where $O_L(2)$ is the (orbital) angular momentum. As the $O(4)$ Lie group can be "factored" in two mutually commuting O(3) Lie groups: $O(4) \simeq O(3) \otimes O(3)$, one may use for our purposes many of the O(3) group results, such as the Clebsch-Gordan coefficients. The 3D case is substantially more complicated than the 2D one: the three-body "hyper-spherical symmetry" is O(6), and the residual dynamical symmetry of the Y-string potential is $O(2) \otimes O_L(3) \subset O(6)$. The $O(6)$ Lie group cannot be "factored" in two mutually commuting O(3) Lie groups and one cannot simply reduce this problem to one in the O(3) group. For these reasons we limit ourselves to the two-dimensional case in this paper.

Thus we are looking for the "chain" of symmetries $O(2) \otimes O_L(2) \subset O(3) \otimes O_L(2) \subset O(4)$. Rather than parametrize the energy E as a function of corresponding Casimir operators, and thus calculate the spectrum, as was done in Refs. [20–23], we reformulate the problem in terms of $O(4)$ variables and then

bring the potential into a form that can be (exactly) solved, i.e. we expand it in O(4) hyperspherical harmonics. As the potential must be spherically symmetrical, this imposes and additional constraint on the allowed hyperspherical harmonics and one ends up with only a few (leading) terms: 1) the area-term containing the O(4) hyperspherical harmonic $\mathcal{Y}^{22}_{00}$, which, in turn is related to the O(3) spherical harmonic $Y_{20}(\alpha, \phi)$ of the shape space (hyper)spherical angles (α, ϕ), i.e., the V_4 term in the notation of Richard and Taxil [16]), that is present in both the two-body and the Y-string potentials; and 2) the O(2) symmetry-breaking term containing $\mathcal{Y}^{33}_{0\pm3} \simeq Y_{3\pm3}(\alpha, \phi)$, i.e., the V_6 term in the notation of Richard and Taxil [16], that is important in the two-body potential, and less so in the "complete" Y-string potential and not at all in Eq. (2). The energy spectrum is a function of the O(4) hyperspherical expansion coefficients for the potential, and of the O(4) Clebsch-Gordan coefficients, that are products of the ordinary O(3) Clebsch-Gordan coefficients.

3 Results

Next we proceed to evaluate the K=2,3 bands' splittings and compare them with the 3D case:

1) At the K=2 level, there are four SU(6) multiplets (other than the hyper-radial excitation $[56, 0^+]'$ of the K=0 state): $[70, 0^+]$, $[56, 2^+]$, $[70, 2^+]$, $[20, 1^+]$ in 3D. The main difference between the 2D and 3D is that the $[20, 0^+]$ state has vanishing orbital angular momentum in 2D, rather than unity, as in the 3D state $[20, 1^+]$.

The only difference between the 2D and 3D K=2 states' splittings is that the $[70, 0^+]$ and $[56, 2^+]$ states are degenerate in 2D, whereas in 3D they are split by one half of the energy difference between $[70, 2^+]$ and $[70, 0^+]$. This shows that the 2D case does relate fairly closely to the 3D one.

2) The energy splittings in the K=3 band, for the Y-string potential in 3D has not been worked out analytically, as yet, to our knowledge. Therefore, we compare our 2D Y-string potential K=3 results with the 3D K=3 two-body potential results of Ref. [16] and find certain similarities, and a few distinctions. There are six SU(6) multiplets in the K=3 sector (other than the hyper-radial excitation $[70, 1^-]''$ of the K=1 state): $[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$, $[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$ in 3D. The main difference between the 2D and 3D is that the $[70, 2^-]$ state disappears in 2D.

In 3D two-body potential the energy splittings have been divided in two parts in Ref. [16]: a) those due to the V_4 perturbation; and b) due to the V_6 perturbation. This corresponds to our Y_{20} and $Y_{3\pm3}$ terms, respectively.

a) In the $V_4 \neq 0$, $V_6 \to 0$ limit, the states are roughly divided in two groups: the $[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$ which are pushed down, and the $[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$ which are pushed up by the V_4 perturbation. Two pairs of states are left degenerate: ($[20, 1^-]$, $[56, 1^-]$) in the lower set and ($[56, 3^-]$, $[20, 3^-]$) in the upper set. In this limit in 2D we find complete degeneracy of all three members of the lower- ($[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$) and upper levels ($[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$).

b) In the $V_4 \neq 0$, $V_6 \neq 0$ case, the remaining degeneracy of states is removed in 3D: the $[20, 1^-]$ and the $[56, 1^-]$ are split in the "lower set" and the $[56, 3^-]$ and

the $[20, 3^-]$ in the "upper set". In 2D we find the same sort of splitting, and in almost the same ratio of strengths.

So, in the K=2,3 bands, one sees similarities of dynamical symmetry-breaking patterns in 2D and 3D. This lends credence to the belief that this similarity may persist at higher values of K, where there are no known 3D results, at present.

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Exotic molecules of heavy quark hadrons

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Abstract. We discuss hadronic molecules containing both heavy and light quarks. The interactions are provided by meson exchanges between light quarks in the constituent hadrons. The tensor force in the one-pion exchange potential mixes states of different spins and angular momenta. This provides attraction and generates rich structure in exotic channels in the heavy quark sectors. The method has been applied to exotic baryons with a $\bar{c}$ or $\bar{b}$ quark, and exotic mesons containing $b\bar{b}$ including the recently found Z_b's.

Recent interest in hadron physics has been largely motivated by the observations of candidates for exotic multi-quark states which are not (easily) explained by the conventional quark model [1–4]. Many of them appear near the threshold region of their possible decay channels. The finding of the twin Z_b's is perhaps the most striking in that they appear very close to the BB^* and B^*B^* thresholds [4–6].

Strictly, multiquarks does not make much sense for light flavors especially for u and d quarks when the quark number is not a conserved quantity. In fact, they interact strongly at the energy scale of Λ_{QCD}, creating $q\bar{q}$ pairs and generating massive constituent quarks. It is known that it is a consequence of spontaneous breaking of chiral symmetry. In the low energy region we expect that such constituent quarks become active degrees of freedom as almost on-shell particles, forming exotic multi-quark states. Contrary to the light flavor sector heavy quarks such as c and b with mass $M \gg \Lambda_{QCD}$ conserve their quark number. Thus we can treat them as almost on shell particles with non-relativistic kinematics at low energies of typical hadron resonances.

Starting from the conventional quark model picture for orbitally excited states, multiquark configurations can mix with them because the typical excitation energy of about 0.5-1 GeV is sufficient to create a (constituent) $q\bar{q}$ pair. A color singlet multiquark system of more than the minimal number ($\bar{q}q$ or qqq) may form color singlet sub-systems (clusters) of hadrons. Clustering phenomena of multiparticle systems have been extensively studied in nuclear physics for many years [7]. Alpha particles saturate the dominant component of spin and isospin dependent nuclear force. The spin-isospin neutral alpha particles interact rather weakly and can form loosely bound states near the threshold regions of alpha decay.

In QCD, the state corresponding to alpha particle is a hadron which saturates the strong color dependent force. If these hadrons have sufficient amount of attraction (but weak as compared to the color force), they may form a bound or

resonant state, which is the hadronic molecule. it must be a rather loosely bound state having an extending spatial structure to retain the identity of hadronic constituents. We expect that the relevant energy scale of binding and resonant states should be sufficiently small as compared to Λ_{QCD} of some hundreds MeV.

To establish exotic states is interesting not only for its own sake, but also because it is expected to reveal important aspects of non-perturbative dynamics of QCD. In this respect, as experimental observations imply, hadrons of light and heavy quarks are interesting, where more candidates of exotic states are observed. There, heavy quark symmetry and chiral symmetry play simultaneously. The former suppresses the spin dependent interactions, leading to degeneracy of different spin states. On the other hand, the latter is responsible for the pion coupling to the light quarks, which provides the source of the strong one pion exchange potential between heavy flavor hadrons. When these two conditions are satisfied, we expect the formation of exotic hadronic molecules. The spin and isospin dependent nature of the pion exchange potential as well as its orientation dependence of the tensor structure are the cause of the rich structure of hadron spectrum.

Based on these ideas, we have studied hadronic molecular states for exotic heavy baryons in Refs. [8–10], and for exotic heavy mesons in Ref. [11–13]. They are exotic not only due to hadronic molecular structure but also due to their exotic quantum numbers which are not accessible by the minimal number of quarks. In forming the hadronic molecular state, the following three points are important; (1) heavy mass which suppresses kinetic energy of constituent hadrons, (2) one pion exchange force of tensor nature which mixes the 0^- and 1^- states (DD^* and BB^*), and (3) degeneracy of 0^- and 1^- states which makes the wider space of coupled channels more effective to gain more attraction.

Hadronic molecules have been also studied for DN systems of ordinary quantum numbers [14,15]. These channels allow even more attraction leading to deeply bound states of a binding energy of order a few hundred MeV with much spatially compact configuration. Here $q\bar{q}$ annihilation is also possible, the treatment of which is more difficult than in the case of exotic channel without $q\bar{q}$ annihilation.

Turning to the exotic channels, employing an interactions between heavy flavor hadrons in a boson exchange model including one pion exchange potential, we find several bound and resonant states near the threshold regions. Many of them with small binding energy of order ten MeV or less have a rather extended size compatible to hadronic molecules. For baryons, we have found bound states of $J^P = 1/2^-$ states of exotic quark content $\bar{c}q$-qqq and $\bar{b}q$-qqq just below the threshold of $\bar{D}N$ and BN, respectively. Other resonant sates are also found for $J^P = 3/2^-, 1/2^+, 3/2^+, 5/2^+$ with similar structure of mass spectrum for c and b quark sectors [9,10].

For mesons, in the hidden bottom sector, we have found ten $B\bar{B}, B\bar{B}^*, B^*\bar{B}^*$ molecules for low lying spin $J \leq 2$. In particular, the hidden bottom exotic mesons Z_b's are well predicted [11]. Further exotic states of double heavy flavor (charm and bottom) mesons are also found [12]. In Ref. [13], we have estimated the decay and production rates of various states in the limit of heavy quarks which are

characteristic to the hadronic molecular structure. These theoretical predictions for rich structure of hadronic molecules can be studied in the facilities such as Belle, JPARC and LHC.

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