

# Robust IMC Controllers with Optimal Setpoints Tracking and Disturbance Rejection for Industrial Boiler

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*Robust controllers based on Internal Model Control (IMC) theory are developed in this paper to improve the robust performance of industrial boiler system against uncertainties and disturbances. A simplified model of a boiler's drum unit is developed and transfer matrix realization of its dynamics is obtained for a nominal operational condition. Controllers parameters are selected in accordance with the frequency domain optimization method based on  $\mu$ -optimality frameworks. The proposed controllers are robust for reference signals and/or for disturbances. Finally, a comparison between the performances of the closed-loop system with designed IMC controllers is obtained.*

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**Keywords: industrial boiler, robust control, internal model control**

## 0 INTRODUCTION

The main aim of this work is to present the problem and to devise a method for designing robust IMC controllers of industrial boiler subsystem using the frequency domain optimization method based on  $\mu$ -optimality framework. Various control techniques have been applied to boiler or boiler-turbine controller design, e.g., inverse Nyquist array [1], linear quadratic Gaussian (LQG) [2], LQG/loop transfer recovery (LTR) [3], mixed-sensitivity approach [4], loop-shaping approach [5], and predicative control [6]. The  $\mu$ -optimality framework takes into account the system's nominal plant model, incorporating real and complex uncertainties, which describe interested parameter variation range to ensure that the closed-loop with the controller is stable with a certain degree of performance over all possible plants. A brief introduction to the robust control theory and its application on the distillation column, dc/dc converters and solid-fuel boiler are given in [7] to [15].

The boiler studied is an industrial boiler system with a normal steam production of 8.7 kg/s and with an outlet steam pressure and a steam temperature of 18.105 Pa and 400°C. Set of nonlinear equations for describing the boiler's subsystem (steam-water part) dynamics is presented. The Boiler model, in the form of transfer functions matrixes is developed by

linearization around the operating point. For this multivariable model, IMC controllers (IMCr,0, IMC0,d, IMCr,d) are designed. The controllers are proposed for three opposite goals. The first controller (IMCr,0) is designed for optimal setpoint tracking, the second (IMC0,d) for optimal disturbance rejection and the third (IMCr,d) for optimal overcome of the trade-off between these opposite demands. The final goal is to compare the robustness of closed-loop systems with IMCr,0, IMC0,d, IMCr,d controllers using frequency analysis and to verify the results using transient analysis.

## 1 THE PROCESS AND ITS MODEL

In the literature, modeling of boilers has been treated in many different ways, [16] to [20]. The boiler process consists of water heater, steam drum, downcomers tubes, mud drum, riser tubes, and superheater (Fig. 1). However, in this paper only the steam-water part (i.e. steam drum, downcomers and risers) is taken into account (Fig. 2), because the water heater and the super heater system are weakly coupled to the steam-water system and it is natural to treat the three systems separately. The input variables are the powder coal flow rate, the feedwater flow rate and the steam flow rate. The output variables are the drum level and the drum pressure.

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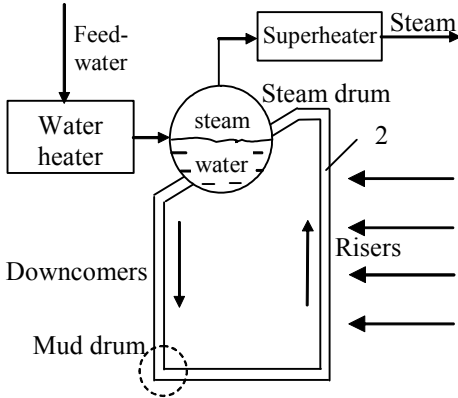


Fig. 1. Industrial boiler system

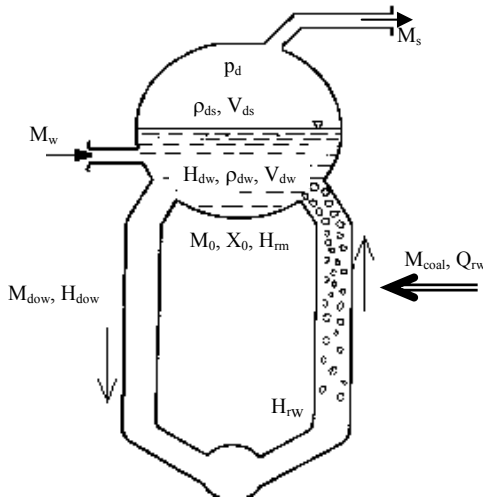


Fig. 2. A simplified description of the steam-water part of the boiler system

The mass balance of the water in the drum determines the dynamics of the water mass:

$$\frac{d}{dt} V_{dw} = \frac{1}{\rho_{dw}} ((1 - X_0)M_0 + M_w - M_{dow}) \quad (1)$$

The drum water level is given by:

$$V_d - V_{dw} = A_d h_d \quad (2)$$

where  $V_d$  is the reference volume of water in the drum at nominal point.

The mass flow rate of steam condensing in the drum is neglected, that is:

$$\frac{d}{dt} (V_{dw} \rho_{dw}) \approx \rho_{dw} \frac{d}{dt} V_{dw} \quad (3)$$

The dynamics of the steam density is taken from the mass balance in the drum:

$$\frac{d}{dt} (V_{ds} \rho_{ds}) \approx X_0 M_0 - M_s \quad (4)$$

As the volume of the drum is constant, an increase of steam volume results in a decrease of water volume and vice versa:

$$\frac{d}{dt} V_{ds} + \frac{d}{dt} V_{dw} = 0 \quad (5)$$

A combination of Eqs. (1) and (4) gives:

$$\frac{d}{dt} \rho_{ds} = \frac{1}{V_{dt} - V_{dw}} [(M_0 + M_w - M_{dow} - M_s) + \frac{(\rho_{ds} - \rho_{dw})}{\rho_{dw}} \frac{d}{dt} V_{dw}] \quad (6)$$

From the steam table, for known  $\rho_{ds}$ , it is possible to find a corresponding drum pressure.

The water in the drum is not in the saturation state, and the energy balance is:

$$\frac{d}{dt} (\rho_{dw} V_{dw} H_{dw}) = M_0 (1 - X_0) H_{rw} - M_{dow} H_{dow} + M_w H_{ewo} \quad (7)$$

It is presumed that the feedwater temperature is constant, i.e.  $H_{ewo} = \text{const}$ . This is a rational proposition as the water heater has its own control system. The combination with Eq. (1) gives the dynamics of the drum water enthalpy:

$$\rho_{dw} V_{dw} \frac{d}{dt} H_{dw} = M_0 (1 - X_0) (H_{rw} - H_{dw}) + M_w (H_{ewo} - H_{dw}) \quad (8)$$

Water density,  $\rho_{dw}$  is determined with an assumption of a saturation condition and  $\rho_{dw}$  is then the function of drum pressure.

The energy balance of the steam-water mixture in the risers is:

$$\frac{d}{dt} (\rho_{rm} H_{rm}) V_r = M_{dow} H_{dow} + Q_{rw} - M_0 (H_{rw} + X_0 r) \quad (9)$$

$H_{rw}$  and  $r$  (evaporation heat) are functions of drum pressure. Heat flow to the risers is assumed to be a function of the powder coal flow rate:

$$Q_{rw} = k_{rw} M_{coal} \quad (10)$$

The enthalpy of steam-water mixture is a function of the steam quality:

$$H = H_{rw} + (H_{rs} - H_{rw}) X \quad (11)$$

$H_{rs}$  is a function of the drum pressure and if steam quality is considered as linearly distributed along the raisers, the average enthalpy in raisers is:

$$H_{rm} = H_{rw} + \frac{rX_0}{2}. \quad (12)$$

Eq. (9) can now be written as:

$$\frac{d}{dt} H_{rm} = \frac{1}{V_r \rho_{rm}} [Q_{rw} - M_{dow}(H_{rm} - H_{dow}) - M_0 r \frac{X_0}{2}]. \quad (13)$$

The density of the steam-water mixture is given by:

$$\frac{1}{\rho_{rm}} = \frac{1}{\rho_{rw}} + X \left( \frac{1}{\rho_{rs}} - \frac{1}{\rho_{rw}} \right) \quad (14)$$

Mass flow rate in downcomers is shown by the Bernouli's equation:

$$M_{dow} = k_c \sqrt{\rho_{dw} - \rho_{rm}}, \quad (15)$$

where  $k_c$  represents the inverse of the circulation losses.

Steam-water flow at the top of the risers ( $M_0$ ) can be obtained from mass and energy balance of the risers. To simplify, it is assumed that it can be approximated by the empiric expression:

$$M_0 = M_{dow} - \Delta M_0 + k_1 M_{oil} + k_3 M_s, \quad (16)$$

where  $\Delta M_0$  is transient contribution to  $M_0$ :

$$\frac{d}{dt} \Delta M_0 = \frac{1}{T} [k_1 M_{oil} + k_2 M_s - \Delta M_0], \quad (17)$$

where time ( $T$ ) and gain ( $k_1, k_2$ ) factors are load dependent, and can be estimated from plant recordings.

The set of nonlinear Eqs. (1), (6), (8), (13) and (17) the present state space description of the industrial boiler subsystem. The state vector's elements are:  $h_d$  – drum water level;  $H_{dw}$  – drum water enthalpy;  $\rho_{ds}$  – drum steam density;  $H_{rm}$  – average enthalpy of steam-water mixture in risers;  $\Delta M_0$  – transient contribution to the steam-water flow rate in risers.

Transfer matrix, as a description of boiler subsystem dynamics is obtained by linearization for nominal working conditions, given in Table 1.

$$y = \tilde{P}(s)u + P_d d, \quad (18a)$$

$$\tilde{P}(s) = \begin{bmatrix} \frac{0.048s}{\Delta(s)} & \frac{1.119s^2 - 0.8247s}{1000 \cdot \Delta(s)} \\ -0.386s & \frac{1.92s^2 - 1.71s + 0.2007}{1000 \cdot \Delta(s)} \end{bmatrix}, \quad (18b)$$

$$\Delta(s) = 100s^3 + 10.8s^2 + 0.08s, \quad (18c)$$

Table 1. Boiler's working conditions

| Steam drum            |                                  |                    | Downcomers             |                         |           |
|-----------------------|----------------------------------|--------------------|------------------------|-------------------------|-----------|
| Input water flow rate | $M_w$ [kg/s]                     | 8,7                | Water enthalpy         | $H_{dow}$ [kJ/kg]       | 2100      |
| Input water enthalpy  | $H_{ewo}$ [kJ/kg]                | 463                | Water flow rate        | $M_{dow}$ [kg/s]        | 0,18      |
| Water level           | $h_d$ [m]                        | 0,75               |                        |                         |           |
| Volume                | $V_d$ [m <sup>3</sup> ]          | 9,54               | Risers                 |                         |           |
| Water surface size    | $A_d$ [m <sup>2</sup> ]          | 8,10               | Water enthalpy         | $H_{rw}$ [kJ/kg]        | 2100      |
| Water volume          | $V_{dw}$ [m <sup>3</sup> ]       | 4,76               | Steam quality          | $X_0$ [kg/kg]           | 0,75      |
| Pressure              | $p_d$ [Pa]                       | 20·10 <sup>5</sup> | Volume                 | $V_r$ [m <sup>3</sup> ] | 0,03×300  |
| Water density         | $\rho_{dw}$ [kg/m <sup>3</sup> ] | 849,90             | Mixture enthalpy       | $H_{rm}$ [kJ/kg]        | 3125,00   |
| Water temperature     | $T_{dw}$ [°C]                    | 212,37             | Mixture flow rate      | $M_0$ [kg/s]            | 0,18      |
| Water enthalpy        | $H_{dw}$ [kJ/kg]                 | 908,5              | Transient contribution | $\Delta M_0$ [kg/s]     | 0,11      |
| Steam density         | $\rho_{ds}$ [kg/m <sup>3</sup> ] | 1,00               | Heat flow              | $Q_{rw}$ [kW]           | 16.447,35 |
| Steam flow rate       | $M_s$ [kg/s]                     | 8,70               | Coal flow rate         | $M_{coal}$ [kg/s]       | 1,866     |

$$P_d(s) = \left[ \begin{array}{c} \frac{-0.0002437}{s+0.008} \\ \frac{0.0409(s-0.006)}{s(s+0.1)} \end{array} \right], \quad (18d)$$

The following notations are used:

**Output vector**  $y = [y_1 \ y_2]^T$

$y_1 - p_d$ , drum pressure;  $y_2 - h_d$ , drum-water level

**Input vector**  $u = [u_1 \ u_2]^T$

$u_1 - M_{coal}$ , fuel flow rate;  $u_2 - M_w$ , water flow rate

**Disturbance**

$d - M_s$ , steam flow rate

The goal of the control law is to generate  $u_1$  and  $u_2$  so that it maintains  $y_1$  and  $y_2$  close to setpoint  $r = [y_{1ref} \ y_{2ref}]^T$  and insensitive to disturbance  $d$ .

## 2 THE MODEL UNCERTAINTY

The description of the model uncertainty arises from the fact that process plant operates with certain flow rates (coal and water) on its inputs. Changes on inputs are effected by servo-controlled valves, which rely on the measurement of the flow. Existing of 1% error in flow measure produces 10% error in required variation [8]. Thus, our plant model, which describes changes about some operating point is subject to errors of up to 10% on each input channel. Since the error on each input channel is independent of the others, the suitable representation of the disagreement between the real plant  $P$  and model  $\tilde{P}$  is described by a multiplicative input perturbation  $L$  and structured model uncertainty description of multivariable model  $\tilde{P}$ .

$$\begin{aligned} P &= \tilde{P}(I + L) \\ &= \tilde{P}(I + l_u \Delta_u), \quad \bar{\sigma}(\Delta_u) \leq 1. \end{aligned} \quad (19)$$

$l_u = \text{diag}(l, l)$ ,  $l = 0.1$ , represents the uncertainty weighted operator (the frequency dependent magnitude bound of  $\Delta_u$ ) and  $\Delta_u = \text{diag}(\Delta_1, \Delta_2)$  is unknown unity norm bounded block diagonal perturbation matrix that reflects the structure of the uncertainty. Also, such description of uncertainty covers the neglected heat capacity of the riser metal, i.e. it was included in the uncertainty of fuel flow.

## 3 THE PERFORMANCE OBJECTIVE

The sensitivity weighting operator  $W_p$  is selected by a designer to give a preferred shape to the sensitivity operator  $E$ . The feedback system satisfies robust performance if the  $\infty$ -norm of the weighted sensitivity operator is unity bounded for any perturbation  $\Delta_u$  of the plant:

$$\|W_p E\|_{\infty} = \sup_{\omega} \bar{\sigma}(W_p E) < 1, \text{ for any } \Delta_u \quad (20)$$

The required limiting values of the closed-loop time constant of the closed-loop system give the following term of weighted sensitivity operator:

$$W_p = 0.25 \frac{50s+1}{50s} I \quad (21)$$

The weight (21) implies that we require an integral action ( $W_p(0) = \infty$ ) and allow an amplification of disturbances at high frequencies by a factor four at most ( $W_p(\infty) = 0.25$ ). A particular sensitivity function, which matches the performance bound (21) exactly for low frequencies, is

$$E = \frac{200s}{200s+1} I. \quad (22)$$

This corresponds to a first order response with time constant 200 s.

## 4 IMC IN THE $\mu$ -OPTIMALITY FRAMEWORK

The IMC structure was developed in [7] as an alternative to the classic feedback structure. Its main advantage is that closed-loop stability is guaranteed simply by choosing a stable IMC controller. This concept is based on an equivalent transformation of the standard feedback structure into IMC structure shown in Fig. 3a.

The synthesis and analysis of robust IMC controllers, based on the structured singular value approach, impose a forming interconnection matrix by rearrangement of the block diagram of the IMC control structure shown in Fig. 3a in general  $G-\Delta$  form (Fig. 3b) necessary for  $\mu$ -analysis.

The interconnection matrix  $G$  in Fig. 3b is partitioned into four blocks consistent with the dimensions of the two input and the two output vectors:

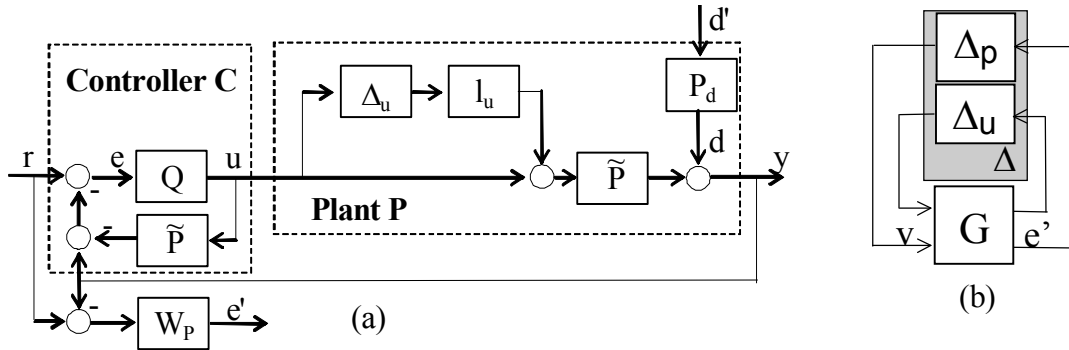


Fig. 3. The block diagram of IMC control structure a) and  $G$ - $\Delta$  form; b) in accordance with  $\mu$ -optimality framework

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}. \quad (23)$$

The input vector of  $G$  consists of the outputs from the uncertainty block  $\Delta_u$  and the desired external input  $v$  (setpoints, disturbance or both). The output vector of  $G$  is formed by the inputs to the uncertainty block  $\Delta_u$  and the weighted error  $e'$ .

Form of matrix  $\Delta$  is:

$$\Delta = \begin{bmatrix} \Delta_u & 0 \\ 0 & \Delta_p \end{bmatrix}, \quad (24)$$

where  $\Delta_p$  is a full unity norm bounded matrix  $\bar{\sigma}(\Delta_p) \leq 1$ .

The  $\mu$ -optimality framework adopts measures of robust stability (RS) and robust performance (RP) as suitable objectives [7] and [10], which define the performance of the multivariable feedback system in the presence of structured uncertainty:

$$RS = \|rs(\omega)\|_\infty = \|G_{11}\|_{\mu_\Delta} = \sup_\omega \mu_\Delta(G_{11}) < 1, \quad (25)$$

$$RP = \|rp(\omega)\|_\infty = \|G\|_{\mu_\Delta} = \sup_\omega \mu_\Delta(G) < 1. \quad (26)$$

The operator  $\mu_\Delta$  is a structured singular value ( $\mu$ -norm) computed according to the block-diagonal structure of  $\Delta$  [7], [8] and [10]  $\mu$ -norm is the natural extension of  $\infty$ -norm when both the bound and the structure of model uncertainty are known. The upper bound of  $\mu_\Delta(G)$  is defined as

$$\mu_\Delta(G) \leq \inf_D \bar{\sigma}(DGD^{-1}), \quad (27)$$

where  $D$  is any real positive diagonal matrix with the structure  $\text{diag}(d_i I_i)$  where each block (i.e., the size of  $I_i$ ) is equal to the size of the blocks  $\Delta_i$ .

Ideally, the goal is to find a controller  $C$  that satisfies (25) and (26). These objectives ensure stability and performance of the closed-loop system in the presence of all expected uncertainties. The demand is not always reachable, especially with controllers of simple structure. The convenient objective for the synthesis of the robust controller  $C$  using  $\mu$ -norm is

$$\min_C \|G\|_{\mu_\Delta}, \quad (28)$$

with constrain:

$$\|G_{11}\|_{\mu_\Delta} < 1. \quad (29)$$

Thus, the optimal controller minimizes the performance index RP, i.e. the  $\infty$ -norm of the weighted sensitivity operator for all possible plants.

## 5 THE OPTIMAL PERFORMANCE

The design of the robust IMC controller is a two-step design procedure. In the first step it is assumed that the model is perfect ( $P = \tilde{P}$ ) and an optimal controller  $\tilde{Q}$  that minimizes a performance objective is designed. It is implied that an  $H_2$ -optimal (minimum variance) controller should be selected, which minimizes the nominal performance, i.e. the 2-norm of the weighted nominal sensitivity operator  $\tilde{E}$ .

$$\min_{\tilde{Q}} \|W_p \tilde{E}\|_2 = \min_{\tilde{Q}} \|W_p (I - \tilde{P}\tilde{Q})\|_2. \quad (30)$$

As the nominal plant  $\tilde{P}$  is stable but the **nonminimum phase transfer function**, it has to be factored into a stable all pass portion  $P_A$  and a minimum phase portion  $P_M$  such that:

$$P = P_A P_M. \quad (31)$$

The controller  $\tilde{Q}$  that solves (30) satisfies [7]:

$$\tilde{Q} = P_M^{-1} W_p^{-1} \{W_p P_A^{-1}\}. \quad (32)$$

Here the operator  $\{\}$  denotes that after a partial fraction expansion of the operand all terms involving the poles of  $P_A^{-1}$  are omitted.

In the second step of the algorithm,  $\tilde{Q}$  has to be augmented by a low-pass filter  $F$  for robustness:

$$Q = \tilde{Q}F. \quad (33)$$

The structure of  $F$  is predetermined and the parameters should be selected in order to minimize the  $RP$  measure.

In this approach the following definitions of the nominal sensitivity operator  $\tilde{E}$  are adopted, according to choices of appropriate inputs of the system. The transfer function of nominal sensitivity operator  $\tilde{E}_{r,0}$  referring to setpoints ( $v=r$ ),  $\tilde{E}_{0,d}$  referring to disturbance ( $v=d$ ), and  $\tilde{E}_{r,d}$  referring to all inputs  $v=(r^T, d^T)^T$  are defined in the following terms:

$$\tilde{E}_{r,0} = (I - \tilde{P}Q) = \tilde{E}, \quad (34)$$

$$\tilde{E}_{0,d} = -(I - \tilde{P}Q)P_d = -\tilde{E}P_d, \quad (35)$$

$$\tilde{E}_{r,d} = (I - \tilde{P}Q)[I - P_d] = -\tilde{E}[I - P_d]. \quad (36)$$

$\tilde{E}_{r,0}$  is adopted for optimal setpoint tracking,  $\tilde{E}_{0,d}$  for optimal disturbance rejection  $\tilde{E}_{r,d}$  is adopted to optimal overcome of the trade-off between setpoint tracking and disturbance rejection.

The corresponding interconnection matrix  $G_{r,0}$ ,  $G_{0,d}$  and  $G_{r,d}$  referring to sensitivity operators (34), (35) and (36) are obtained as follows:

$$G_{r,0} = \begin{bmatrix} -Q\tilde{P}l_u & Q \\ -W_p\tilde{E}\tilde{P}l_u & W_p\tilde{E} \end{bmatrix}, \quad (37)$$

$$G_{0,d} = \begin{bmatrix} -Q\tilde{P}l_u & -QP_d \\ -W_p\tilde{E}\tilde{P}l_u & W_p\tilde{E}P_d \end{bmatrix}, \quad (38)$$

$$G_{r,d} = \begin{bmatrix} -Q\tilde{P}l_u & Q & -QP_d \\ -W_p\tilde{E}\tilde{P}l_u & W_p\tilde{E} & W_p\tilde{E}P_d \end{bmatrix}. \quad (39)$$

The optimal design problem is:

$$\min_Q \|G\|_{\mu_\Delta}, \quad (40)$$

with constraint on robust stability:

$$\|Q\tilde{P}l_u\|_{\mu_\Delta} < 1. \quad (41)$$

## 6 THE IMC FILTER

The structure of filter  $F$  is fixed and with a few tuning parameters adjusted to obtain desired robustness properties (40) and (41). If the zero steady-state error for step inputs is required, then it is necessary that  $F(s=0)=I$ . The simple filter with a unity steady-state gain and with four tuning parameters is:

$$F = \begin{bmatrix} \frac{(k_1s+1)^n}{(k_2s+1)^{n+2}} & 0 \\ 0 & \frac{(k_3s+1)^n}{(k_4s+1)^{n+2}} \end{bmatrix}, \quad (42)$$

where  $k_1$ ,  $k_2$ ,  $k_3$  and  $k_4$  are the filter's tuning parameters and the  $n$  selected is large enough to make  $Q$  proper. In this case  $n = 1$  is selected and the corresponding controller  $Q$  is:

$$Q = \tilde{Q} \begin{bmatrix} \frac{k_1s+1}{(k_2s+1)^3} & 0 \\ 0 & \frac{k_3s+1}{(k_4s+1)^3} \end{bmatrix}. \quad (43)$$

## 7 IMC CONTROLLER DESIGN

The controllers (43) are designed using [21] and [22]. The measures of performance of the closed-loop system  $rp$  with all here designed controllers, using  $G_{r,0}$ ,  $G_{0,d}$  and  $G_{r,d}$  respectively are computed and shown in Fig. 4. The values of robust stability ( $RS$ ) and robust performance ( $RP$ ) are given in Table 2.

Table 2. Performance and stability measures of closed-loop system with IMC controllers

| Controller         | $k_1$  | $k_2$ | $k_3$  | $k_4$ | RS   | $RP(G_{r,0})$ | $RP(G_{0,d})$ | $RP(G_{r,d})$ |
|--------------------|--------|-------|--------|-------|------|---------------|---------------|---------------|
| IMC <sub>r,0</sub> | 12.10  | 4.33  | 16.64  | 3.64  | 0.19 | 0.97          | 9.60          | 6.81          |
| IMC <sub>0,d</sub> | 300.86 | 27.87 | 213.23 | 44.48 | 0.50 | 2.74          | 0.61          | 2.74          |
| IMC <sub>r,d</sub> | 82.28  | 72.76 | 195.10 | 40.37 | 0.25 | 1.28          | 2.24          | 1.34          |

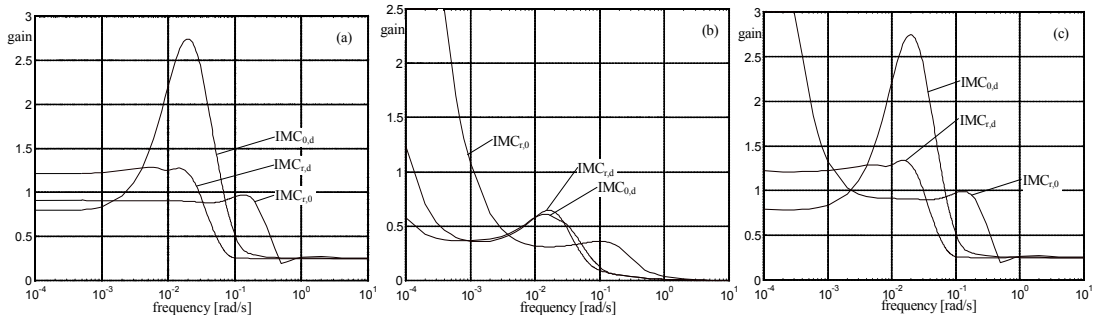


Fig. 4. Frequency responses of  $rp(\omega)$  evaluating for a)  $G_{r,0}$ ; b)  $G_{0,d}$ ; c)  $G_{r,d}$

It is evident that robust stability condition is satisfied for all the proposed controllers. According to the interconnection matrix used for robust performance determination, optimal results are obtained with different controllers, i.e. IMC<sub>r,0</sub> is optimal for  $RP(G_{r,0})$ , IMC<sub>0,d</sub> for  $RP(G_{0,d})$  and IMC<sub>r,d</sub> for  $RP(G_{r,d})$  measure. Such results were expected, but it is interesting to analyze the change of  $RP$  measure over the frequency range of interest. High values of  $RP$  for IMC<sub>r,0</sub> controller with an inclusion of disturbance in the interconnection matrix are characteristic for a very low frequency range, so the consequence could be visible in a steady state. Over the higher frequency range the  $RP$  characteristic of this controller is quite similar to the IMC<sub>r,d</sub> controller. The IMC<sub>0,d</sub> controller has the maximum of  $RP$  measure in the range 0.01 to 0.1 rad/s, for interconnection matrixes with included reference signals, so the consequence could be expected in a transient response. The IMC<sub>r,d</sub> controller has overall optimal values for all the frequency range and for all interconnection matrix.

### 8 THE TRANSIENT ANALYSIS

Computer simulations are performed to evaluate the performance of the proposed robust controllers. Time responses of closed-loop system, using IMC<sub>r,0</sub>, IMC<sub>0,d</sub>, IMC<sub>r,d</sub> are compared in order to observe how the system tracks setpoint changes and rejects external disturbance. The most typical time responses of

outputs of nominal and perturbed plant are shown in Figs. 5 to 7. For a satisfactory confirmation of evaluated robust performances, time responses for the worst-case assumption are computed and shown for the following cases:

$$P(s) = \tilde{P}(s) \begin{bmatrix} 1.1 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad (44)$$

$$P(s) = \tilde{P}(s) \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix}. \quad (45)$$

As maximal perturbations are used, simulations only have a theoretical importance, because a reasonably large variation which shifts the operation of the system from its nominal point exists in those cases. However, such time domain simulations are the best tools for a remarkably good confirmation of computed robust performances. In real cases, uncertainties will be smaller, so that the corresponding time responses will have similar though less distinctive features.

### 9 DISCUSSION

Time responses of nominal plant are shown in Fig. 5. The achievement of diagonal dominance in the reference tracking due to a specific structure, Eqs. (32) and (33), of IMC controllers is evident. Nominal system behavior is optimal for reference tracking with IMC<sub>r,0</sub> controller, i.e. with IMC<sub>0,d</sub> controller for disturbance rejection. The behavior of the system with IMC<sub>r,d</sub> is somewhere between them, but for

the nominal plant all proposed controllers are acceptable.

Time responses of perturbed plants are shown in Figs. 6 and 7. Again, the  $IMC_{r,0}$  controller is optimal for reference tracking and  $IMC_{0,d}$  controller for disturbance rejection, but the problem with those two controllers is their unsatisfactory characteristics for the opposite mission. The  $IMC_{r,0}$  controller used to reject disturbance is not able to provide zero steady

state errors and a transient response in reference tracking is with significant overshoot with  $IMC_{0,d}$ . Physically this means that the system with the  $IMC_{r,0}$  controller will not be able to achieve insensitivity of drum water level to change in the steam flow rate and that the system with  $IMC_{0,d}$  controller will have a significant (and possibly unacceptable) variation in drum pressure and drum water level, especially with a change of fuel flow rate.

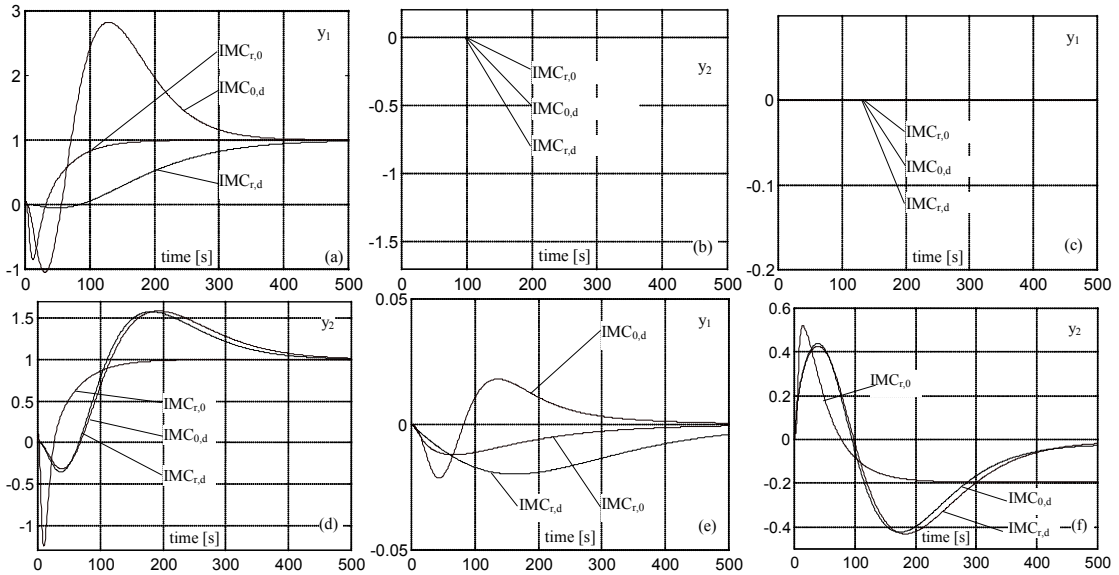


Fig. 5. Time responses of nominal plant to unity step signal: a,b) in input  $u_1$ ; c,d) in input  $u_2$ ; e,f) in input  $d$

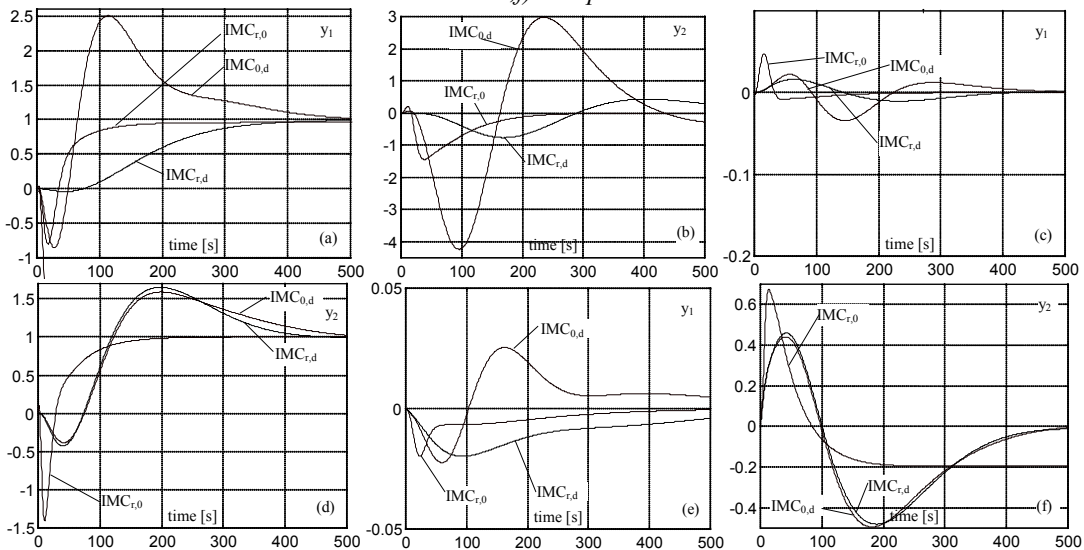


Fig. 6. Time responses of perturbed plant (44) to unity step signal:(a-b) in input  $u_1$ ; (c-d) in input  $u_2$ ; (e-f) in input  $d$



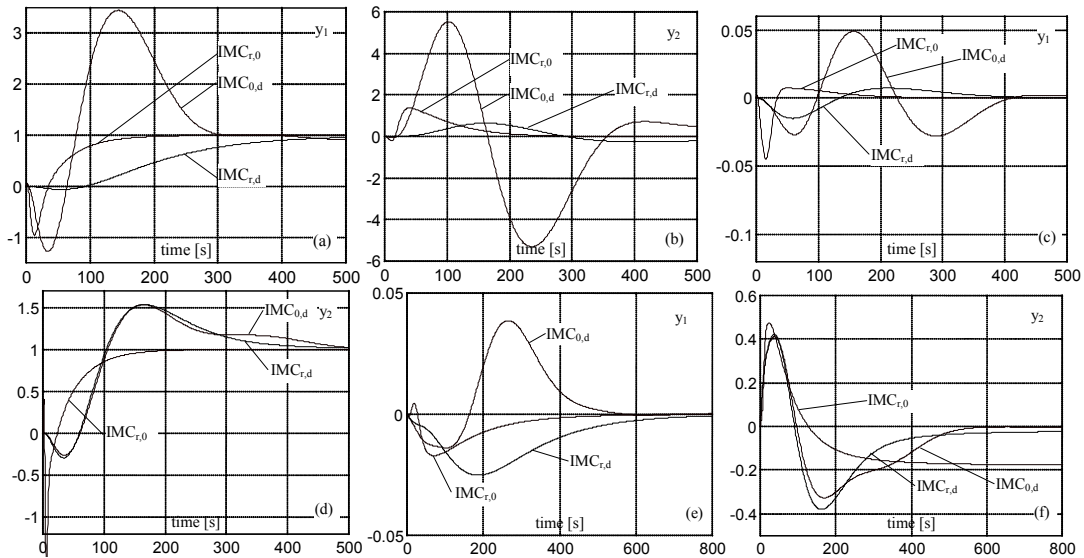


Fig. 7. Time responses of perturbed plant (45) to unity step signal: a,b) in input  $u_1$ ; c,d) in input  $u_2$ ; e,f) in input  $d$

The correlation of perturbed plant time response with robust performance measures is evident. The highest  $rp$  measures of the  $IMC_{0,d}$  controller with  $G_{r,0}$  and  $G_{r,d}$  have the obvious influence on time response in reference tracking. Also, the worst  $rp$  measure with  $G_{0,d}$  at low frequencies is confirmed in problems with disturbance rejection.

The  $IMC_{r,d}$  controller is the best solution for overcoming the trade-off between setpoint tracking and disturbance rejection. Time responses of perturbed plant with that controller confirm such a conclusion. It is obvious that  $IMC_{r,d}$  is somewhat slower than  $IMC_{r,0}$  for setpoint tracking, i.e. has longer settling time than  $IMC_{0,d}$  for disturbance rejection, but without their disadvantage in initial variations and steady-state error.

## 10 CONCLUSIONS

In this paper the design of the multivariable robust controllers for an industrial boiler subsystem has been investigated. Steam-water part (i.e. a steam drum, downcomers and risers) of an industrial boiler system has been modeled and the input model uncertainty is defined. The controllers are designed using  $\mu$ -analysis control theory and they achieve robustness against uncertainties and disturbance. The designed controllers are robust for setpoints

and/or disturbance. The tuning parameters are selected so that they minimize the robust performance objective. The performances of the closed-loop system with the designed controllers are evaluated by simulations and compared. The achieved results have shown remarkable robustness of the proposed controllers i.e. the closed-loop stability and a satisfactory degree of performance over all the possible plants. Using IMC framework, optimal controller switches effectively overcome the trade-off between setpoint tracking and disturbance rejection over a wide range of possible plants that are investigated.

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- 18.105 or 18,105. Pls check the use of decimals as wrongly used above. It's impossible for me to know which one is correct in this case.
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