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Motion and distinguishing number two

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Abstract

A group A acting faithfully on a finite set X is said to have distinguishing number two if there is a proper subset Y whose (setwise) stabilizer is trivial. The motion of A acting on X is defined as the largest integer k such that all non-trivial elements of A move at least k elements of X. The Motion Lemma of Russell and Sundaram states that if the motion is at least 2 $\log_2|A|$, then the action has distinguishing number two. When X is a vector space, group, or map, the Motion Lemma and elementary estimates of the motion together show that in all but finitely many cases, the action of $\mathrm{Aut}(X)$ on X has distinguishing number two. A new lower bound for the motion of any transitive action gives similar results for transitive actions with restricted point-stabilizers. As an instance of what can happen with intransitive actions, it is shown that if X is a set of points on a closed surface of genus g, and $|X|$ is sufficiently large with respect to g, then any action on X by a finite group of surface homeomorphisms has distinguishing number two.

Keywords: Distinguishing number, group action, stabilizer, motion.

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Gibanje in 2-razločljivost

Povzetek

Grupa *A*, zvesto delujoča na končni množici *X*, je 2-razločljiva, če obstaja prava podmnožica *Y*, katere stabilizator (množic) je trivialen. *Gibanje* grupe *A*, ki deluje na *X*, je po definiciji največje tako celo število *k*, da vsi netrivialni elementi grupe *A* premaknejo vsaj *k* elementov množice *X*. "Lema o gibanju" (Russell in Sundaram) pravi, da če je gibanje vsaj $2 \log_2|A|$, potem je delovanje 2-razločljivo. Za primer, ko je *X* vektorski prostor, grupa ali zemljevid, nam Lema o gibanju v kombinaciji z elementarnimi ocenami gibanja pokaže, da je delovanje grupe Aut(*X*) na *X* 2-razločljivo v vseh, razen v končno mnogo, primerih. Nova spodnja meja za gibanje poljubnega tranzitivnega delovanja da podobne rezultate za tranzitivna delovanja z omejenimi stabilizatorji točk. Kot primer tega, kaj se lahko zgodi pri netranzitivnih delovanjih, pokažemo, da če je *X* množica točk na sklenjeni ploskvi rodu *g*, in če je njena moč |*X*| dovolj velika v primerjavi z *g*, potem je za vsako končno grupo homeomorfizmov ploskve njeno delovanje na *X* 2-razločljivo.

Ključne besede: Razločljivost (razlikovalno število), delovanje grupe, stabilizator, gibanje.