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Parameterization of megavoltage transmission curves used in shielding calculations

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An analytical expression based on six fitting parameters is proposed to model the transmission of megavoltage photon beams through shielding barriers made of standard density concrete, iron, and lead. The model reproduces published transmission curves within ±0.7%, and can be used with excellent accuracy for any beam with nominal energy in the clinical range (5–45 MV). Extrapolation of the model beyond this energy range or to other shielding materials, however, is not recommended.

Key words: radiotherapy, high-energy; radiation protection

Introduction

The design or modification of a radiotherapy department is a complex process beginning with the approval of a budget and progressing to the selection and purchase of treatment units. Once the general layout for an installation has been decided, it is usually the responsibility of a medical physicist to design the shielding necessary to limit the radiation exposure outside the radiation therapy rooms to acceptable levels.

A detailed analysis of the formalism used in shielding calculations can be found in the literature.[1] There are several variables which collectively define the thickness of a shielding barrier that will adequately protect individuals in the vicinity of a radiation therapy treatment machine. These include the maximum permissible dose equivalent incurred annually by any individual, the workload for the therapy machine, the use factor for a given machine orientation, the occupancy factor of the location to be shielded, and the distance between the source of radiation and the shielded area. Once the values of these parameters have been decided, a barrier transmission factor representing the level of transmission through a shield resulting in the maximum permissible dose equivalent in the shielded area can be calculated using a straightforward formalism.[1] The thickness of a shielding barrier corresponding to this transmission factor is then determined using published transmission curves.[1, 2] These curves are a function of the shielding material and the energy of the radiation beam to be shielded. Unfortunately, the original transmission curves are small and not very clear. Furthermore, imterpolation betwen energies, which is often necessary to obtain information for beam energies not specifically plotted, is very difficult and imprecise.

In this paper we present an analytical expression which can be used to relate the thickness

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of a shielding barrier to its transmission of radiation. The formalism is valid for radiation beams with a nominal energy in the range most commonly produced by contemporary external beam therapy accelerators (1 MV to 25 MV). The expression reproduces published transmission curves for the standard shielding materials (concrete, iron, and lead) with good accuracy and simplifies interpolation of transmission data between energies.

Materials and methods

A data set containing barrier thickness as a function of radiation transmission was derived from the transmission curves for standard density concrete, iron, and lead given in NCRP Reports #49 and #51,[1,2] to be known below as NCRP #49 and NCRP #51, respectively. The transmission curves in NCRP #49 are plotted for beam energies up to 10 MV. For higher energy beams, one must refer to NCRP #51, where transmission curves are given for beams produced by monoenergetic electrons with energies up to 176 MeV interacting with a target. For this work it was assumed that the nominal energy of the resulting x-ray beam is equal to the energy of the electrons which produce the beam. In other words, the transmission properties of an x-ray beam resulting from, for example, 1.0 MeV electrons hitting a target are identical to those of a 1MV x-ray beam.

The following function was fitted to the data set derived from each transmission curve:

$$t = a \log B, \tag{1}$$

where t and B are the tickness (in cm) and transmission, respectively, of a shielding barrier and α is a fitting parameter. The fitting was done using a commercially available graphics software package (KaleidaGraph, Abelbeck Software, Reading, PA) which incorporates an algorithm for non-linear curve fitting. The program was implemented on a Macintosh personal computer (Apple Computers, Cupertino, CA) and allows the user to define single variable functions with up to nine parameters to be fit to a data set. The efficiency of the fitting procedure is enhanced with the use of partial derivatives which effectively steer the algorithm towards optimal fits. In Eq. (1) the dependence of shield thickness and transmission is reversed compared to the transmission curves in NCRP #49 and #51. This is because shielding calculations usually require the determination of barrier thickness for a calculated transmission, thus it is more intuitive to analyze the curves with transmission as the independent variable and shield thickness as the dependent variable. Should one require the inverse information, *i.e.*, the transmission of a given thickness of shielding material, the equations can always be inverted with little loss of accuracy.

To facilitate interpolation of transmission curves between energies, an analytical expression was fitted to the relationship between a for a given shielding material and the energy of the beam. The form of the equation is

$$a = c_1 E^{c_2} exp\left(-\frac{E}{c_3}\right) + c_4 E + c_5 \log(c_6 E), \tag{2}$$

where $c_{1\text{-}6}$ are the fitted parameters and E is the energy of the beam. The functional dependence of Eq. (2) does not arise from any theoretical basis and was chosen only because of its ability to fit the data points for all three shielding materials with minimal error.

Results and discussion

In fitting Eq. (1) to the transmission curves in NCRP #49 and #51, it was assumed that the curves are mono-exponential. Based solely on qualitative shape, there is no question that this assumption is valid for the megavoltage transmission curves given in NCRP #49. However, several transmission curves given in NCRP #51 show a definite shoulder at small shield thicknesses. This shoulder is most pronounced at high beam energy (E > 38 MV) and is a result of selective attenuation by the pair production interaction of higher energy photons in the first few centimeters of shielding material. The inaccuracy introduced by fitting Eq. (1) to a transmission curve with a shoulder is acceptable since the effect occurs primarily for beam energies higher than those currently used clinically. It is possible to achieve a better fit to these high energy transmission curves by introducing extra terms into Eq. (1) to account for the shoulder region. The point of this analysis, however, is to devise an analytic expression which can be used to interpolate between energies. If more than one parameter were used to describe the transmission curves, the simple analytic expression given by Eq. (2) relating the slope of a transmission curve to beam energy would not be valid.

Table 1 gives a listing of the value of a for photon beams of several nominal energies interacting in concrete, iron, and lead. The values of a determined for energies duplicated in NCRP #49 and #51 are slightly different; those shown in Table 1 are the higher of the two values as determined from either report. The difference is most likely a result of the definition of the photon beam energy used in the two reports. In any case the discrepancies are small and can be ignored. Figures 1 (a–c) are plots of a as a function of beam energy for concrete, iron, and lead, respectively. As expected the value of a decreases with increasing energy for concrete and iron, reflecting the more penetrating power of higher energy photon beams. For lead, however, a first decreases, reaches a minimum around 6 MV, and then increases for higher energies. This behavior is a result of the increase in the probability for

Table 1. Values of the parameter a obtained from a fit of Eq. (1) to the megavoltage transmission curves in NCRP #49 and #51. The error in the parameter resulting from the fit is approximately constant at 0.5%.

Beam Energy (MV)	Concrete (cm)	Iron (cm)	Lead (cm)
1.0	−16.0340	−5.0740	−2.5003
2.0	−20.7205	−7.1715	−3.7617
3.0	−24.5509	−8.2177	−4.6826
4.0	−29.1533	−9.0735	−5.2512
6.0	−34.6469	−9.8341	−5.5817
8.0	−38.0341	−10.3140	–
10.0	−40.0698	−10.5116	−5.2341
20.0	−45.5178	–	−4.8200
31.0	–	−11.1280	–
38.0	−47.9993	–	–
86.0	–	–	−4.2154

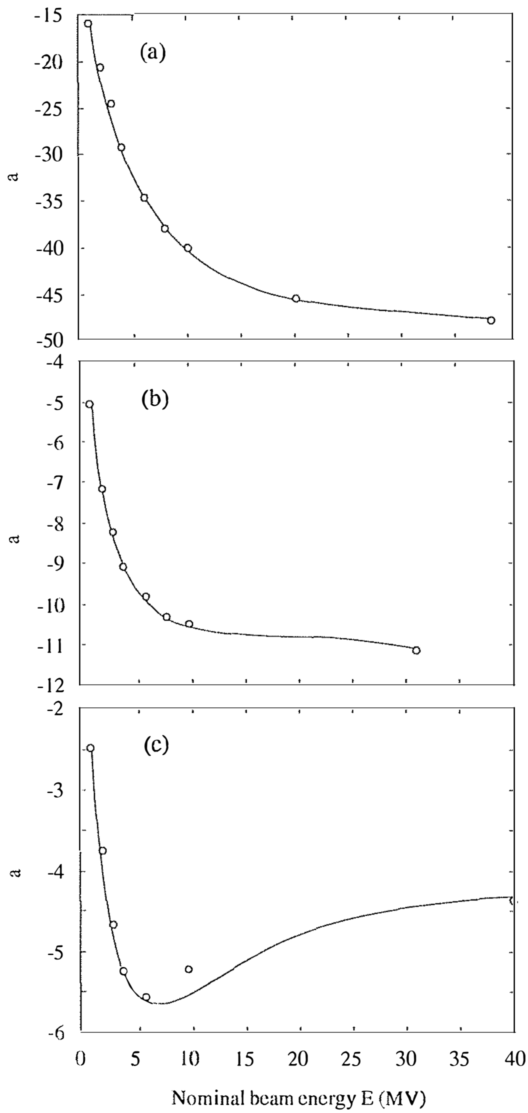

Figure 1. Plots of the parameter a, determined by fitting Eq. (1) to the megavoltage x-ray beam transmission curves for (a) standard density concrete, (b) iron, and (c) lead, as a function of nominal beam energy. The transmission curves used in the determination of a are found in NCRP #49 and #51.[1,2]

pair production and the concurrent dicrease in the Compton effect with increasing beam energy.

The parameters c_{1-6} of Eq. (2) fit to the data points in Figure 1 are given in Table 2. The solid lines in Figure 1 (a–c) represent plots of Eq. (2) with these parameters. It was found that four parameters are sufficient to describe the behavior of concrete, while for iron and lead all six parameters are significant. The correlation coefficient for each fit is 0.995, 0.998 and 0.900 for concrete, iron, and lead, respectively. As demonstrated in Figure 1 and by the value of the correlation coefficients, the fits to the data points are excellent with the exception of the curve for lead, which overestimates by 4 % the value of a at 10 MV. In the energy range used most often clinically (4–25 MV), the average error for all sheilding materials is 0.7 % with a range of 0.04 % to 4 %. At lower beam enery (1 MV to 4 MV), the formalism can also be used to provide estimates of transmission, however the accuracy is only ±7 %.

Equations (1) and (2) can be combined as follows to determine the shielding thicknes necessary to produce a given transmission of x rays in the energy range (1 MV–25 MV) analyzed in this work:

$$t = \left[c_1 E^{c_2} \exp\left(-\frac{E}{c_3}\right) + c_4 E + c_5 \log(c_6 E) \right] \log B, \quad (3)$$

where t is the shielding barrier thickness (in cm) that will allow a transmision of B and c_{1-6} are given in Table 2. To illustrate the usefulness of this formalism, say, for example, that one must determine the thickness of a shielding barrier that will allow a transmission of 10^{-6} of 18 MV x rays. This photon beam in currently very commonplace in modern radiotherapy departments, yet transmission curves for this energy are not specifically plotted in either NCRP #49 or #51. Equation (3) facilitates interpolation of the curves that are published in the reports to obtain transmission information for an 18 MV beam. Using Eq. (3) with $B = 10^{-6}$ and the values of c_{1-6} found in Table 2, we find that a 270 cm thick concrete barrier will be sufficient to limit the transmission of an 18 MV x-ray beam to 10^{-6}. If the space for shielding is limited to, say 250 cm, then a concrete wall alone cannot be used. In this situation a shielding barrier comprised of a combination of concrete and iron or lead would most likely be designed. As mentioned above, Eq. (3) can be used inversely to calculate the transmission for a given thickness of shielding material. This inverse functional relationship is useful when determining the individual thicknesses of combined layers of two or more shielding materials that will provide adequate shielding yet not violate a given total thickness constraint. For the situation described above, one finds using Eq. (3) that a barrier comprised of 240 cm of concrete and 7.2 cm of iron will result in a transmission of 10^{-6} and also remain within the thickness constraint.

Conslusions

In this work we have presented an analytical expression based on published transmission data which can be used to determine the shielding barrier thicknesses that will result in a given transmission of radiation. The expression

Table 2. Values of the constants c_{1-6} obtained from a fit of Eq. (2) to the data in Figure 1. The errors in the constants were on the order of 1 %. The correlation coefficient of each fit is also shown.

Shielding Material	c_1	c_2	c_3	c_4	c_5	c_6	R^2
Concrete	−15.920	0.490	25.833	−0.684	0	–	0.995
Iron	−4.760	0.290	13.579	−0.0541	−5.110	1.244	0.998
Lead	−2.770	0.470	7.706	0.0408	−3.570	1.082	0.900

can also be used inversely to determine the transmission of a given thickness of shielding material. The materials analyzed are those used most often for shielding purposes: standard density concrete, iron, and lead. The equations were derived using transmission data for photon energies in the range of 1 MV to 38 MV, however, they are most accurate in the clinical energy range, *i.e.,* 4 MV to 24 MV. Extrapolations of the curves to energies below 1 MV or above 38 MV is not recommended as this will not provide accurate transmission information. Furthermore, the formalism is valid only for the standard shielding materials: concrete, iron, and lead.

References

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