Mile Dželalija [1]*
Nikola Rausavljević [1]
Bojan Jošt [2]

RELATIONSHIP BETWEEN JUMP LENGTH AND THE POSITION ANGLE IN SKI JUMPING

RAZMERJE MED DOLŽINO SKOKA IN KOTOM SKAKALCA PRI SMUČARSKIH SKOKIH

Abstract

Using a simple physical model of ski jumping the current study investigates the relationship between a ski jumper's position angle during the flight phase and the jump length as well as other physics variables. The physical parameters of the jump (velocity, current drag, jump length) were calculated taking into account height of the jumping hill, inclination of the outrun, and position of the system jumper-skis during the flight phase. The analysis of the jump curve was made on the basis of the angle between the flight trajectory of the common centre of gravity and the x-axis (φ). For the analysis of the jumper's position during the flight phase the angle between the upper part of the body and the flight trajectory of the common centre of gravity of the body (ϑ) was chosen. In the computer simulation of the jump length two variants of the course of the angle ϑ were used. The flight velocity was very similar in both variants. Immediately after take-off, the resultant velocity v(t) and its horizontal component $v_x(t)$ slightly decreased. After that the resultant velocity increased constantly due to the constant increase in the vertical component $v_y(t)$. Due to the current drag the horizontal flight component $v_x(t)$ decreased constantly during the entire flight phase. The current drag $F_D(t)$ and lift components $F_L(t)$ rose to a greater extent in the changeable ski jumper's position angle ϑ.

Key words: ski jumping, physical model, computer simulation, position angle

Izvleček

Cilj raziskave je bil z enostavnim fizikalnim modelom smučarskega skoka ugotoviti povezanost med skakalčevim kotom med fazo leta in dolžino skoka ter nekaterimi drugimi fizikalnimi parametri. Fizikalni parametri skoka (hitrost, sila upora, dolžina skoka) so bili izračunani z upoštevanjem višine skakalnice, nagiba pristajalnega dela skakalnice in pozicije sistemska skakalec - smuči med fazo leta. Analiza krivulje skoka je bila narejena na osnovi kota med linijo leta glede na skupno težišče in x-osjo (φ). Za analizo pozicije skakalca med letom je bil izbran kot med zgornjim delom telesa in linijo leta glede na skupno težišče telesa (ϑ). V računalniški simulaciji dolžine skoka smo uporabili dve varianti kota ϑ. Hitrost leta je bila je bila v obeh različicah podobna. Takoj po vzletu sta se hitrost v(t) in njena horizontalna komponenta $v_x(t)$ nekoliko zmanjšali. Temu sledi stalno naraščanje hitrosti, ki gre na račun stalnega povečevanja njene vertikalne komponente $v_y(t)$. Horizontalna komponenta leta $v_x(t)$ je zaradi sile upora skozi celoten let stalno upadala. Komponenti sil upora $F_D(t)$ in dviga $F_L(t)$ s spreminjanjem kota pozicije skakalca ϑ naraščata.

Ključne besede: smučarski skoki, fizikalen model, računalniška simulacija, kot skakalca

1 Faculty of Natural Sciences, Mathematics and Education, University of Split, Croatia
2 Faculty of Sport, University of Ljubljana, Slovenia

* Corresponding author: Faculty of Natural Sciences, Mathematics and Education, University of Split, Nikole Tesle 12, 21000 Split, Croatia
Tel.: +385 (0) 21 385 009, fax: +385 (0) 21 385 431
E-mail: mile@pmfst.hr

INTRODUCTION

Ski jumping is a very complex skill involving several phases such as: inrun, take-off, flight and landing. The main purpose of the inrun and take-off phases is to give the jumper optimal values of the velocity and to produce a favourable body position angle. The main purpose of the flight phase is to achieve the optimal flight velocity, maximise the horizontal component and minimise the vertical component of the flight velocity (Vaverka, 1987). In the last several decades, biomechanics of skiing has demonstrated considerable growth, evolving from an exercise in the filming of human movement to an applied science with a vast array of measurements and modelling techniques. The earliest analytical model of ski jumping was made by Straumann (1927), who used a model for the wind tunnel measurements of drag forces. Since then, many approaches to studying the physics of ski jumping have been adopted, but only one model of ski jumping that takes into account the changes in the athlete's position during the flight phase has been presented in literature (Jin, Shimizu, Watanuki, & Kobayashi, 1995; Muller, Platzer, & Schmolzer, 1996; Schmolzer, & Muller, 2002). There are only a few reliable studies of the effects of the athlete's position angle during the flight phase (Muller, Platzer, & Schmolzer, 1996; Schmolzer & Muller, 2002).

When a ski jumper moves through the air, as shown in Figure 1, he is exposed to the gravitational force and the dynamic fluid force in the air. The magnitude of the dynamic fluid force is proportional to the air density ρ, the surface area *A* of the jumper in the air, and the square of the ski jumper's velocity **v** relative to air (Halliday, Resnick, & Walker, 2001).

$$F \propto \rho A v^2 \tag{1}$$

For practical reasons, the dynamic fluid force that results from motion in the air is commonly divided into two components: the drag force and the lift force. The drag force F_D is the component of the resultant dynamic fluid force that acts in opposition to the relative motion of the jumper with respect to air. This force tends to decrease the relative velocity. The drag force is produced by two different means: surface drag and form drag. Surface drag is the sum of the friction forces acting between air molecules and the surface of the jumper. Form drag is the sum of the impact forces resulting from the collisions between the air and the jumper. The lift force F_L is the dynamic fluid force component that acts perpendicularly to the relative motion of the jumper, and causes a change in the direction of the relative motion of the jumper.

If during the flight phase an athlete's body is oriented at the angle ϑ towards the direction of motion, the magnitude of the drag force can be expressed as follows

$$F_D = \frac{1}{2} C_D \rho v^2 \left[A_\perp + A_\parallel \sin^2(\vartheta) \sin(\vartheta) \right], \tag{2}$$

where $A_\perp$ and $A_{||}$ are surfaces of the frontal and the longitudinal areas of the jumper. C_D is the coefficient of drag. In addition, the magnitude of the lift force F_L is

$$F_L = \frac{1}{2} C_L \rho v^2 A_\parallel \sin^2(\vartheta) \cos(\vartheta) \tag{3}$$

where C_L is the coefficient of lift. The horizontal component F_x of the resultant force that acts on the ski jumper during the flight phase is

$$F_x = \frac{1}{2} C_L \rho v^2 A_\parallel \sin^2(\vartheta)\cos(\vartheta)\sin(\varphi) - \frac{1}{2} C_D \rho v^2 \left[A_\perp + A_\parallel \sin^2(\vartheta)\sin(\vartheta)\right]\cos(\varphi) \qquad (4)$$

where φ is the current angle of the velocity with respect to the x-axis (see Figure 1). The vertical component F_y of the resultant force is

$$F_y = -mg + \frac{1}{2} C_L \rho v^2 A_\parallel \sin^2(\vartheta)\cos(\vartheta)\cos(\varphi) + \frac{1}{2} C_D \rho v^2 \left[A_\perp + A_\parallel \sin^2(\vartheta)\sin(\vartheta)\right]\sin(\varphi), \qquad (5)$$

where m is the mass of the ski jumper, and $g = 9.8\ \mathrm{ms}^{-2}$ is the acceleration caused by the Earth's gravitational force.

On the basis of vertical and horizontal components F_x and F_y of the resultant force and the inrun and take-off velocities, it is possible to calculate the components of the instantaneous jumper's velocities (v_x, v_y) and positions (x, y) by using the following differential equations:

$$\frac{dv_x}{dt} = \frac{F_x}{m}, \qquad \frac{dv_y}{dt} = \frac{F_y}{m}, \qquad \frac{dx}{dt} = v_x, \qquad \frac{dy}{dt} = v_y. \qquad (6)$$

The resultant force influences the athlete, his velocity, position and the position angle change during the flight phase.

METHOD

Using a simple physical model of ski jumping the current study investigates the relationship between a ski jumper's position angle during the flight phase and the jump length as well as other physics variables (Figure 1).

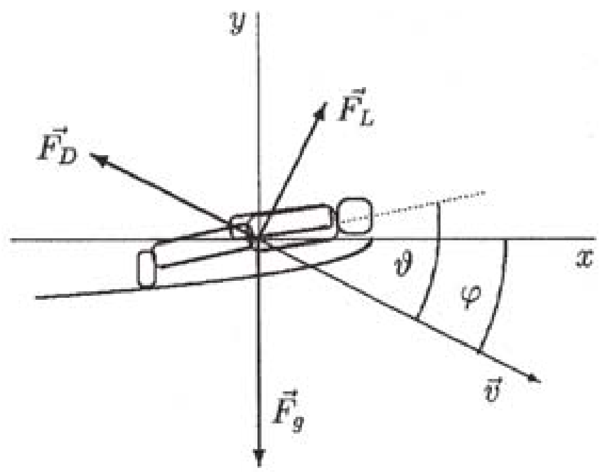

Figure 1: The gravitational force F_g and the components of the dynamic fluid force (drag F_D and lift F_L forces) acting on a ski jumper. v is the ski jumper's velocity relative to air, φ is the angle of the velocity with respect to the x-axis and ϑ is the angle between the upper part of the body and the flight trajectory.

In the computer simulation series $A_\perp = 0.15\ m^2$, $A_{||} = 1.2\ m^2$ have been taken as the surfaces of the frontal and longitudinal areas of the ski jumper, and the equipment $m = 65$ kg has been taken as his mass. According to the International Civil Aviation Organisation (Dubs, 1987) the air density ρ in normal atmosphere conditions equals 1.225 kgm^{-3} at mean sea level, 1.15 kgm^{-3} at an elevation of 650 m, and 1.0 kgm^{-3} at 2000 m. Since jumping hills are located at the elevation of about 2000 m, the air density of ρ = 1.0 kgm^{-3} has been applied. For the approach and take-off velocities (due to the athlete's jumping force) $v_{x0} = 27\ ms^{-1}$ and $v_{y0} = 2.5\ ms^{-1}$ have been used (Virmavirta, Kiveskas, & Komi, 2001). The profile of a typical K = 120 m jumping hill with a maximum gradient of $\alpha = 37.5^\circ$ has been used (H(K) = 60 m, N(K) = 103.9 m). For the drag and lift coefficients $C_D = 1$ and $C_L = 1$ have been applied as in [3]. A fixed time step integration of $\Delta t = 0.001$ s has been used.

Figure 2 shows the profile hill used in the simulation and the flight trajectories. The solid line corresponds to the simulation with a changeable jumper's position angle (the longest simulated jump), and the dashed line correspond to the fixed one, $\vartheta = 35^0$. For the longest jump (133.4 m) the flight time obtained by simulation is 4.72 s, and for the flight with a fixed position angle of $\vartheta = 35^0$ the simulated flight time is 4.51 s.

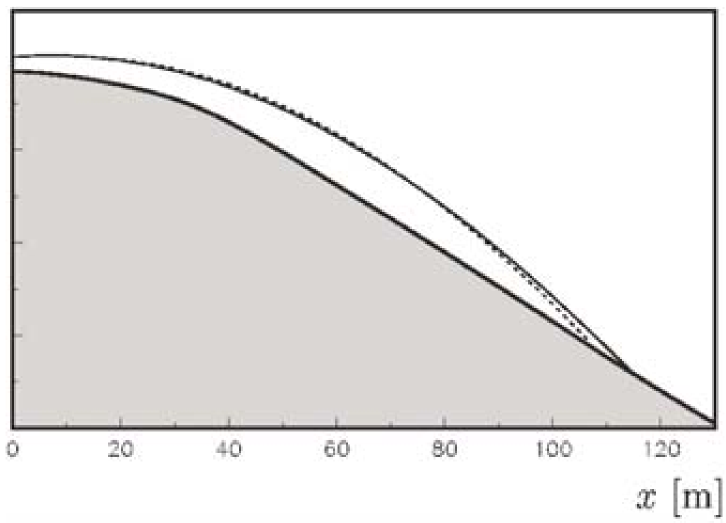

Figure 2: The jumper's positions during the flight phase. The solid line corresponds to the simulation with a changeable jumper's position angle (the longest simulated jump), and the dashed line corresponds to the fixed one, $\vartheta = 35^o$.

Figure 3 presents the jump length d calculated for the variable jumper's position angle ϑ.

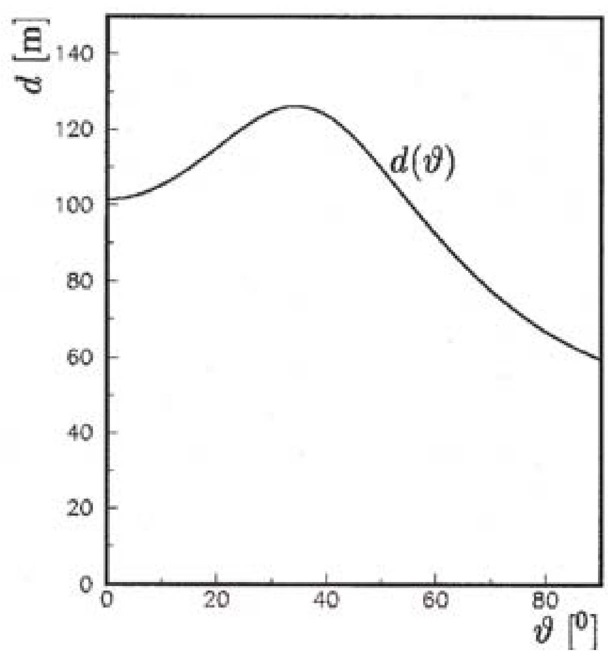

Figure 3: The jump length d as a function of the jumper's position angle ϑ with respect to the flight path.

If the position angle ϑ is fixed relative to the flight path during the whole flight phase, the simulated jump length varies from d = 59.6 m to d = 126.2 m, and reaches the peak at a fixed jumper's position angle of $\vartheta = 35^o$. However, even a longer jump length d can be reached, if the position angle ϑ changes properly during the flight phase. The longest jump length d = 133.4 m has been simulated when the jumper's position angle varied with time nearly linearly, from 18º to 54º, as indicated by the solid line in Figure 4. Optimal position angle as a function of time has been obtained by using the iteration procedure, starting with the constant value of the position angle, and then in the next iteration step changing it as a function of time, choosing the optimal values. These values have been then used as starting values for the next iteration step. The last iteration step has been made when the optimal position angle as a function of time resulted in the jump length close enough to the value recorded in the previous iteration step. The dashed line in the same figure represents the current angle *č* of the flight path with respect to the *x*-axis under equal simulation conditions.

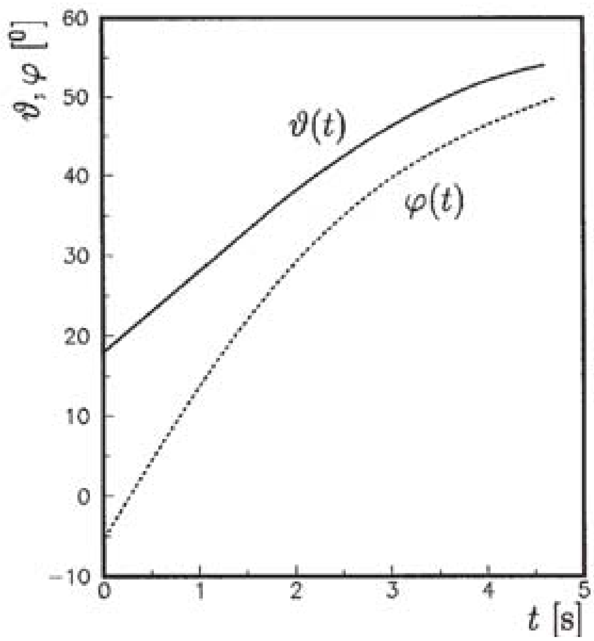

Figure 4: The athlete's position angle ϑ as a function of time t at which the longest jump length of d = 133.4 m has been obtained. The dashed line represents the current angle č of the flight path with respect to the x-axis.

The difference between the variant of the computer simulation for establishing the parameters of the ski jumper's jump carried out at a fixed angle ϑ = 35ºand the variant carried out at the changeable angle arises in the first phase of the flight. About 0.3 second after the take-off the angle is ϑ = 35º (Jošt, Vaverka, Kugovnik, & Čoh, 1998). The angle increases slightly and amounts to ϑ = 45º approximately in the second third of the flight. On landing the angle ϑ can equal 55º.

RESULTS AND DISCUSSION

We have used differential equations (6) together with the expressions (4) and (5) for the horizontal and vertical components of the resultant force that acts on a ski jumper.

The jumper's current velocity $v(t)$ and its horizontal $v_x(t)$ and vertical components $v_y(t)$ are shown in Figure 5. The solid lines represent the values that have been simulated for the variable jumper's position angle ϑ that corresponds to the longest simulated jump and the dashed lines correspond to the fixed position angle of ϑ = 35º.

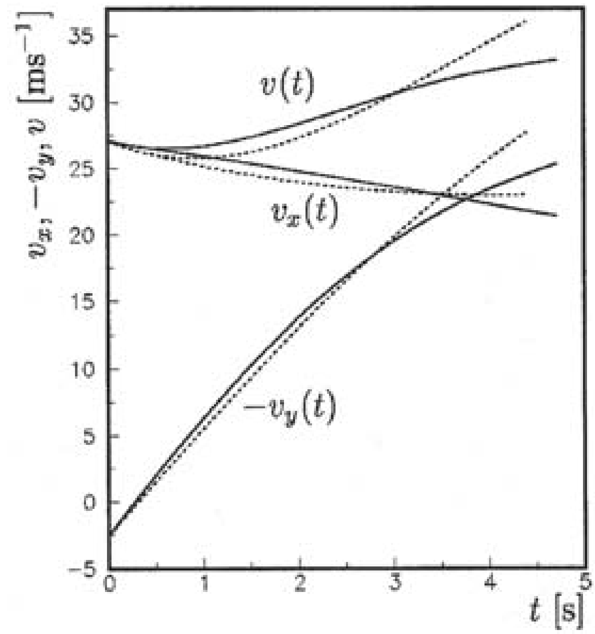

Figure 5: The current athlete's velocity v(t) and its components $v_x(t)$, $v_y(t)$ when the position angle ϑ varies with time as in Figure 4. The solid lines represent the values that have been simulated for the variable jumper's position angle ϑ that corresponds to the longest simulated jump and the dashed lines correspond to the fixed position angle of $\vartheta = 35^0$.

As presented in Figure 5, the basic velocity of motion of the common centre of gravity of the ski jumper v(t) and its horizontal component $v_x(t)$ decreases immediately upon take-off. In the ultimate stage of the take-off the vertical component of the resultant velocity $v_y(t)$ reaches the trough. After that an upward trend is present in $v_y(t)$ during the entire phase of the flight up until landing. In the final stage of the flight the vertical component of the velocity $v_y(t)$ is greater than the horizontal component $v_x(t)$. When the jumper's position angle ϑ changes with time as indicated in Figure 3, the velocity of motion v(t) shows a minimum at the flight time t = 0.68 s, whereas its horizontal component $v_x(t)$ decreases evenly. A slight downward movement is recorded in the horizontal component of velocity $v_x(t)$ throughout the flight, which is in agreement with the empirical findings of the research carried out by Jošt and colleagues (1998) in 1996 on the Innsbruck hill. The difference between the 8 m and 74 m marks beyond the edge of the take-off was approximately 2 ms^{-1}, which meant slightly less than 10% of the absolute value measured at the first mark. In this section of the Innsbruck hill the vertical component $v_y(t)$ rose by about 10 ms^{-1}. The resultant velocity v(t) grew by about 2 ms^{-1} and in both cases exceeded the horizontal and vertical components. The landing velocity v(t) is 33.1 ms^{-1} and its perpendicular component to the hill is 7.0 ms^{-1}. It corresponds to the landing velocity of a jump onto a horizontal plane from the height of 2.5 m.

The current drag and lift components of the dynamic fluid force are presented in Figure 6.

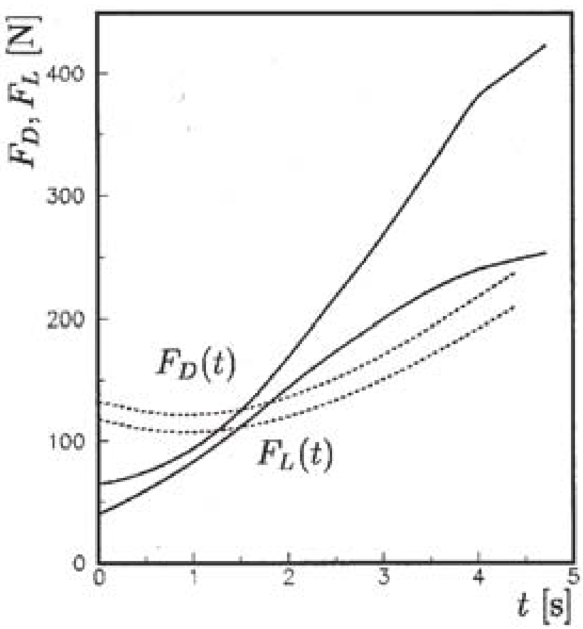

Figure 6: The current drag $F_D(t)$ (solid lines) and lift components $F_L(t)$ (dashed lines) of the dynamic fluid force influencing the athlete. Zero time (0) represents the start of the flight phase.

The current drag reaches the minimum in the final stage of the take-off or in the first phase of the flight. Later both forces increase evenly in the first second of the flight but no so rapidly. During the first and the second seconds of the flight, when the flight angle changes significantly, both forces grew more. The current drag $F_D(t)$ increases almost entirely linearly up until landing. The rise in lift components $F_L(t)$ is almost linear all the way to the point when the ski jumper prepares for landing. During landing the lift component $F_L(t)$ increases minimally.

The horizontal and vertical components of the resultant force that acts on the ski jumper are presented in Figure 7. The solid lines correspond to the simulation with a changeable athlete's position angle, as shown in Figure 3 (the longest jump), and the dashed lines correspond to the fixed one, i.e. $\vartheta = 35^0$.

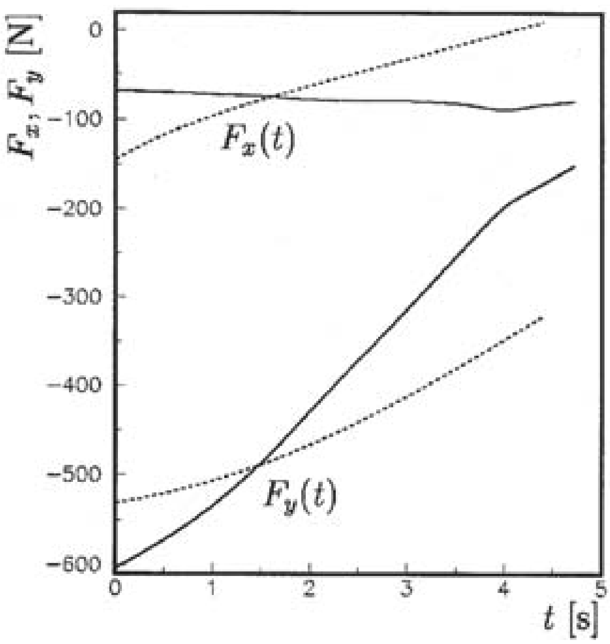

Figure 7: The current horizontal $F_x(t)$ and vertical components $F_y(t)$ of the resultant force that acts on the athlete. The solid lines correspond to the simulation with a changeable athlete's position angle, as shown in Figure 3 (the longest jump), and the dashed lines correspond to the fixed one, i.e. $\vartheta = 35^0$.

Figure 7 shows that the resultant force $F_x(t)$ stays nearly constant. The vertical component of the resultant force $F_y(t)$ decreases quite evenly during the entire flight with a linear trend, which is represented inversely in Figure 7. The vertical component of the resultant force is much greater than the horizontal component $F_x(t)$. This difference is particularly marked in the first and middle stages of the flight of a ski jumper.

This model is somewhat limited. Detailed studies of other parameters dependencies are required - such as the jumper's mass and its surfaces of the frontal and the longitudinal areas - which we plan to carry out in the near future.

REFERENCES

Dubs, F. (1987). *Aerodynamik der reinen Unterschallströmung* [Aerodynamics of Pure Subsonic Currents]. Basel: Birkhduser Verlag.

Halliday, D., Resnick, R., & Walker, J. (2001). *Fundamentals of Physics*. New York: John Wiley & Sons.

Jin, H., Shimizu, S., Watanuki, H. K., & Kobayashi, K. (1995). Desirable gliding styles and techniques in ski jumping. *Journal of Applied Biomechanics 11*, 460-474.

Jošt, B., Vaverka, F., Kugovnik, O., & Čoh, M. (1998). Differences in selected Kinematic flight parameters of the most and the least successful ski jumpers of the 1996 World Cup *Competition in Innsbruck. Biology of Sport*, Vol. 15, 4, 245 - 251.

Muller, W., Platzer, D., & Schmolzer, B. (1996). Dynamics of human flight on skis: improvements on safety and fairness in ski jumping. *Journal of Biomechanics 29*, 1061-1068.

Schmolzer, B., & Muller, W. (2002). The importance of being light: aerodynamic forces and weight in ski jumping. *Journal of Biomechanics 35*, 1059-1069.

Straumann, R. (1927). *Vom Skiweitsprung und seiner Mechanik: Jahrbuch des Schweizerischen Ski Verbandes* [*From Ski-jumping and its Mechanics: Yearbook of the Swiss Ski Union*]. *Berri: Selbstverlag des SSV*.

Vaverka, F. (1987). *Biomechanika skoku na lyžich* [*Biomechanics of ski-jumping*]. Olomouc: UP.

Virmavirta, M., Kivekas, J., & Komi, P.V. (2001). Take-off aerodynamics in ski jumping. J*ournal of Biomechanics 34*, 465-470.

Acknowledgments

The authors gratefully acknowledge the financial support of the Croatian Science Foundation.

Received: 25 September 2002 - Accepted: 20 May 2003