ASTROGEODETIC NETWORK OF SLOVENIA AND GEOID

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Abstract

The astrogeodetic geoid was computed for the area covered by the astrogeodetic network of Slovenia. With deflections of the vertical and with geoidal heights, the observed directions and distances can be correctly reduced onto the reference ellipsoid. The goal of this paper was to establish how reduced observations change the positions of points in the network and how they improve network accuracy.
Keywords: *astrogeodetic network, geoid, overall accuracy of geodetic network*

1 SHAPE AND SIZE OF THE ASTROGEODETIC NETWORK OF SLOVENIA

The astrogeodetic network of Slovenia covers the territory of the Republic of Slovenia. As regards its shape, it is a standard trigonometric network (Figure 1). Due to the requirements of standard geodesy, trigonometric points are located on hills and stabilised by short or long concrete beams. When discussing the astrogeodetic network of Slovenia, the first series of trigonometric points located in the territory of the Republic of Croatia is often also taken into account. This network covers an area of approximately 260 km x 180 km. Together with points in the territory of the Republic of Croatia, the astrogeodetic network of Slovenia comprises 46 points which make 66 triangles. Due to the fact that at present the territory of the network also comprises the state boundary between the Republics of Slovenia and Croatia, only the points located in the territory of Slovenia were kept in one version of the network. The network in the territory of Slovenia comprises 34 first order trigonometric points. For the needs of this paper, it was assumed that the 375 Gorjanci trigonometric point is also part of the national geodetic network of Slovenia. For this reason, the astrogeodetic network of Slovenia is discussed in this paper as a network of 35 points which make 46 triangles. The network covers and area of 230 km x 140 km.

2 RENOVATION OF THE ASTROGEODETIC NETWORK OF SLOVENIA

For historical reasons, the position of the astrogeodetic network on the reference ellipsoid is incorrect; the network has large scale deformations and its accuracy is not homogeneous (Jenko, 1986). Work for the renovation of the Slovenian part of the astrogeodetic network of the former Yugoslavia began after 1974. Renovation was performed on the astrogeodetic network of Slovenia, with the first series of points in Croatia. The greatest emphasis was put on the measurement of lengths in the network and determination of the scale of the official national geodetic network. In addition to length measurements, the heights above sea level were determined anew for many points, such that they have been determined for all points (Jenko, 1986).

Renovation resulted in the final trial adjustment of the national geodetic network in the local coordinate system on the Gauss-Krueger projection plane without the introduction of any conditions or links into adjustment. This adjustment included data from observations of direction from the period 1963 - 1966 and newly measured lengths. In order to determine the positions of 46 points, 222 directions and 49 lengths were used. The values of reference standard deviations determined a priori, i.e. σ_{0s} = 0,45" for directions and σ_{0d} = 0,038 m for lengths, were used in the adjustment. The value of the reference standard deviation determined a posteriori is $\hat{\sigma}_0 = 1{,}0106$ (Jenko, 1986). On the basis of this adjustment, the scale of the official network was determined by comparing the official point coordinates with point coordinates estimated in the trial adjustment. The result of these comparisons confirmed the hypothesis that the network, in addition to being displaced and rotated, also exhibits strong scale deformations. Linear scale deformations range within the interval of –43,7 mm/km to 11,0 mm/km, which means that the official network is quite nonhomogeneous with respect to scale (Jenko, 1986).

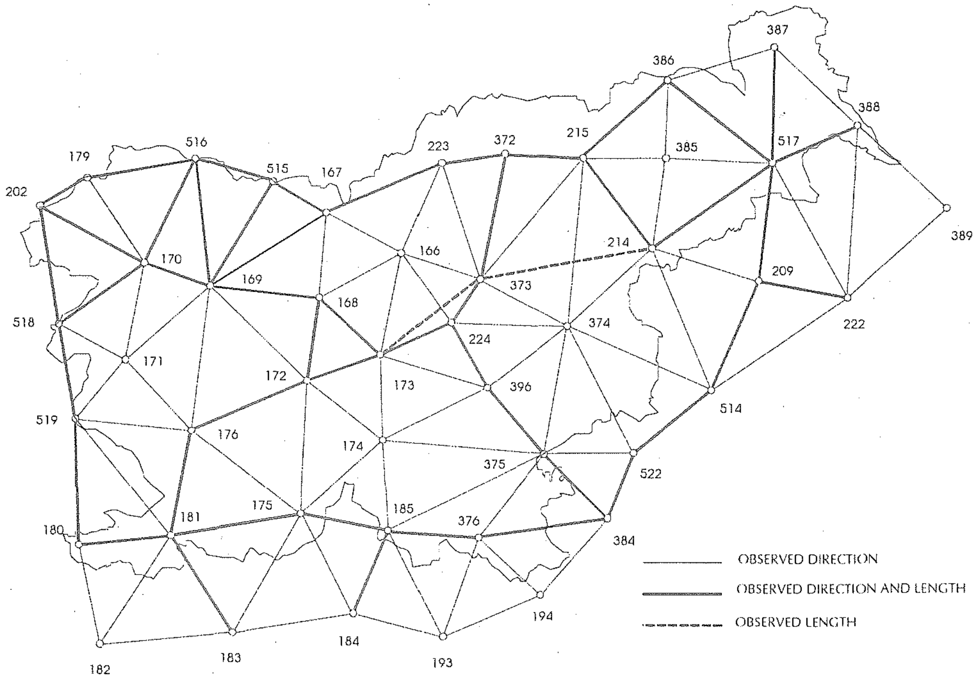

Figure 1

To complete the network renovation, a trial astronomic-geodetic orientation of the national geodetic network was also performed. Four celestial longitudes, six celestial latitudes, six astronomical azimuths measured at six Laplace points, and twelve geoid points with given astronomical coordinates were used as orientation. The geodetic coordinates of the network points were the result of orientation of the network. The values of displacements of individual points in the official national geodetic network were determined from the comparison of geodetic coordinates obtained after orientation of the geodetic network and officially valid coordinates. These ranged from –335,3 to –341,9 m in the direction of the y axis and from –84,8 m to –91,7 m in the direction of the x axis (Jenko, 1986). Due to a lack of knowledge about the geoid during network renovation, the renovated network obtained after trial adjustment suffered from two shortcomings:

- in the observed directions, the influence of deflections of the vertical was not corrected,
- lengths were reduced onto the reference ellipsoid on the basis of the heights above sea level and not heights of points in the ellipsoid.

3 DETERMINATION OF THE GEOID PLANE IN THE TERRITORY COVERED BY THE ASTROGEODETIC NETWORK OF SLOVENIA

After the completion of latest research projects involving the national geodetic network, the relative geoid was determined for the territory of Slovenia and a part of Croatia (Čolić et al., 1992). The relative astrogeodetic geoid available at that time was determined on the basis of astronomic coordinates of 42 network points and geodetic coordinates which were obtained as results of trial astronomic-geodetic orientation of the network. The calculation of the geoid yielded relative undulations of the geoid which represent the relative shape of the geoid plane (Figures 2 and 3). In addition to the relative shape of the geoid plane, the authors also wished to achieve an absolute orientation of the geoid. This was done using the OSU91A geopotential model (Ohio State University 91A). Absolute undulations of the geoid ranged within the interval from 44,3 to 48,5 m. Accuracy in determining geoid undulations was 1 dm (Čolić et al., 1992).

On the basis of conclusions from the renovation of the astrogeodetic network of Slovenia, it is assumed that the officially valid astrogeodetic network of Slovenia has been discussed in detail and it will not be further discussed in this paper. We will only be concerned with the final trial adjustment of the astrogeodetic network which was performed within the framework of the renovation programme for the astrogeodetic network of Slovenia.

4 CORRECTIONS OF TERRESTRIAL OBSERVATIONS

In reducing observations onto the surface of the reference ellipsoid, two groups of corrections need to be considered. The first one consists of corrections which take into account the influence of the Earth's gravitational field on the observations. The second one includes geometrical corrections which arise from the geometry of the rotational ellipsoid. Let us assume that the geometrical corrections of observations were performed correctly. Since the values of corrections of observations from both groups of corrections are added, the corrections of observations which have not yet been calculated can be calculated from the known parameters of the geoid. The relative geoid heights and relative deflections of the vertical are currently available and they can be used to reduce observations onto the surface of the reference ellipsoid.

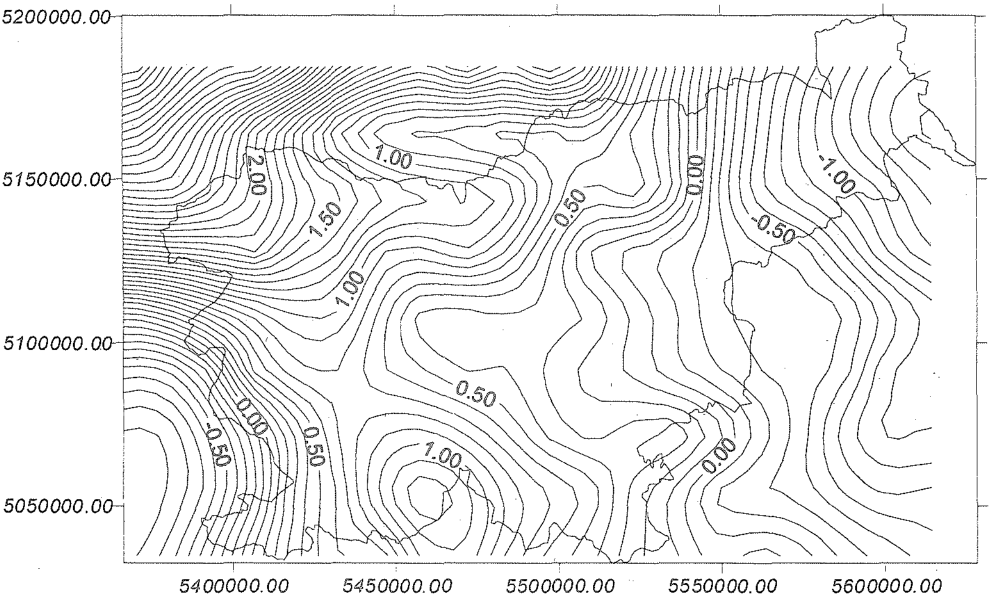

Figure 2

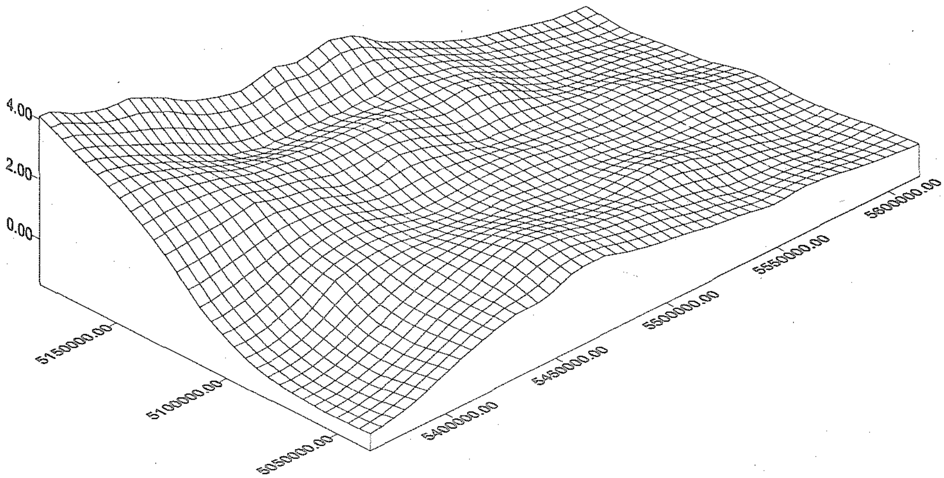

Figure 3

4.1 Reduction of observations in the astrogeodetic network of Slovenia

The observed direction s_{ij} between points P_i and P_j must be reduced from the vertical onto the normal in point P_i by the influence of components of the deflection of the vertical in the station point ξ_i and η_i, or corrected by the value of $C_{2_{ij}}$ (Sideris, 1990):

$$C_{2_{ij}} = -\left(\xi_i \sin\alpha_{ij} - \eta_i \cos\alpha_{ij}\right) \cot Z_{ij} \tag{4.1-1}$$

where α_{ij} is the geodetic azimuth and Z_{ij} is the zenith distance between points P_i and P_j. If the zenith distance is not observed, it can be determined from the following expression:

$$\cot Z_{ij} = \frac{h_j - h_i}{D^E_{ij}} - \frac{D^E_{ij}}{2R_m}, \tag{4.1-2}$$

where h_i and h_j are ellipsoid heights of points P_i in P_j, D^E_{ij} is the length of the geodetic line between the points, and R_m is the mean radius of curvature of the ellipsoid in azimuth α_{ij} between points P_i and P_j. The influence of the deflection of the vertical on the value of zenith distance can be neglected. It can be established on the basis of expressions for geometrical reduction of observations that the lack of knowledge about geoid undulations affects the value of azimuthal reduction. Since the difference of geoid undulations between neighbouring points in the network is always smaller than 2 m (i.e. lower than 0,0001"), this correction can be neglected.

As can be seen from the above equations, the corrections of observed directions depend on the magnitude of deflection of the vertical at the station point, the azimuth of the observed direction and the zenith distance from the observed point. In absolute terms, the greatest correction of observed direction in the astrogeodetic network of Slovenia is in point 518 Korada towards point 202 Kanin and it amounts to C_2 = 0,5231". However, data on the deflection of the vertical in points 179 Mangart, 515 Košuta, 202 Kanin and 194 Privis are not available. With regard to the fact that points 179, 515 and 202 are located at a large height above sea level, and that the observed directions towards the neighbouring points which also lie at a large height are located in these points, it can be expected that in absolute terms there are no greater corrections of observed directions in the entire network.

The components of deflection of the vertical in points P_i and P_j do not affect the value of reduced length, but they do affect the value of geoid undulations N_i and N_j. Since heights above sea level were used for length reduction onto the surface of the reference ellipsoid instead of ellipsoidal heights, the reduced values should be reduced (algebraically) by the following value (Sideris, 1990):

$$\Delta D^E_{ij} = \frac{N_i - N_j}{2R_m} D^E_{ij}, \tag{4.1-3}$$

where R_m is the mean radius of curvature of the ellipsoid in azimuth α_{ij} between points P_i and P_j. In this manner the actual lengths of geodetic lines on the reference ellipsoid are obtained. The absolutely greatest correction of observed length is the length correction between points 170 Rodica and 202 Kanin and it amounts to ΔD^E_{ii} = 0,0102 m. It is of interest that the longest measured length in the network, i.e. the length between points 373 Mrzlica and 214 Donačka Gora requires the smallest correction, $\Delta D_{ij}^{E} = -0{,}0001$ m.

Because the reductions of observations were made correctly, in addition to the values of observed quantities, the positions of points in the network also change. Reduced observations were therefore readjusted in the free network. The new positions of points differ from the old ones by up to 4 cm. Point 519 Kamenek changes its position most: by $\Delta y = +0{,}0295$ m and $\Delta x = +0{,}0275$ m, which means a difference in position of ΔP = 0,0403 m. Figure 4 presents changes in the positions of points in the network as a result of reduction of observations onto the reference ellipsoid.

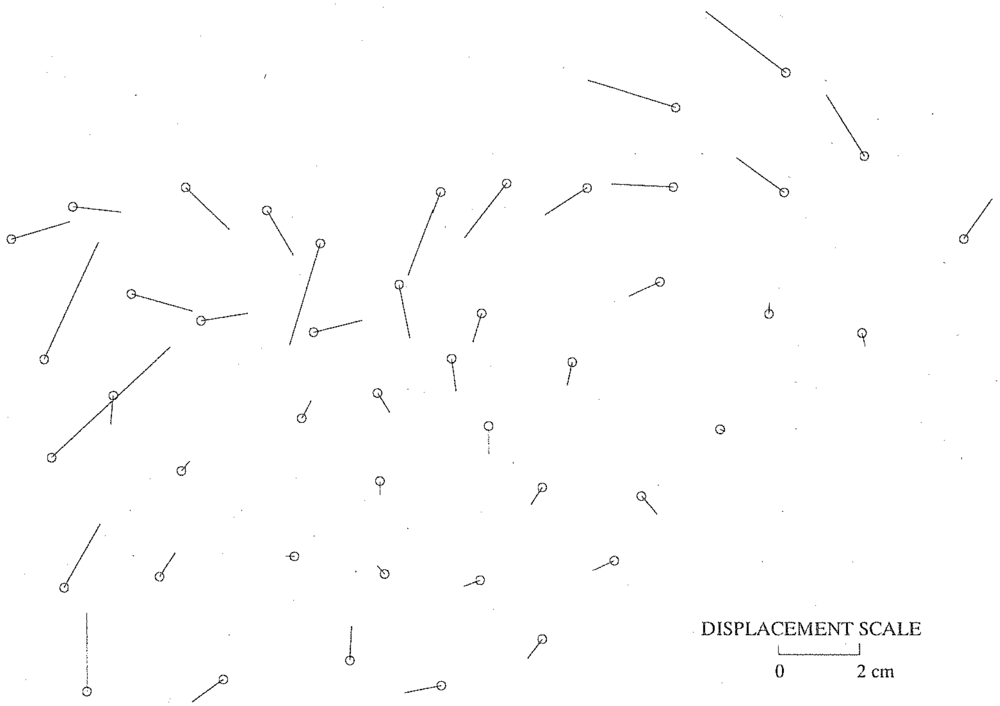

Figure 4

5 THE GEOID'S INFLUENCE ON THE ACCURACY OF THE ASTROGEODETIC NETWORK OF SLOVENIA

In addition to changes in the position of points in the network, it was also desired to establish whether the accuracy of the astrogeodetic network of Slovenia changes due to correctly reduced observations onto the reference computation plane. The following overall accuracy criteria were applied to establish changes in network accuracy:

- mean standard deviation $\bar{s}_{\hat{x}} = \sqrt{\dfrac{\text{sled}\ (\Sigma_{\hat{x}})}{u - d}}$

- generalised standard deviation $\overline{\overline{s}}_{\hat{x}} = \sqrt[u-d]{\det(\Sigma_{\hat{x}})}$
- maximum own value λ_{max} of the covariance matrix $\Sigma_{\hat{x}}$ and
- homogeneity of the astrogeodetic network, i.e. the value of $\frac{\lambda_{min}}{\lambda_{max}}$.

The position of the network in the coordinate system is determined by the position of point 173 Kucelj. The network's scale and orientation are determined in the same manner as for the free network. The covariance matrix of the network is singular in all cases, with a defect of d = 1, or with a rank equal to $(\Sigma_{\hat{x}}) = u - d = u - 1$, where u is the number of coordinate unknowns in the network. The determinant and the trace of the covariance matrix refer only to the coordinate part of the covariance matrix $\Sigma_{\hat{x}}$. All criteria for overall accuracy collected in Table 1 are therefore directly comparable. Since the covariance matrix $\Sigma_{\hat{x}}$ is singular, the value of the determinant of the covariance matrix was taken to be the product of multiplication of all own values of the covariance matrix $\Sigma_{\hat{x}}$ which differed from zero. The minimum own value given in Table 1 is the smallest own value of the covariance matrix $\Sigma_{\hat{x}}$ which differs from zero. The determinant of the covariance matrix $\Sigma_{\hat{x}}$ is proportional to the volume of a hyperellipsoid which is represented by the quadratic equation $(x - \hat{x})^T \Sigma_{\hat{x}} (x - \hat{x}) - \chi^2_{1-\alpha}(u)$. In the same manner as the hyperellipsoid equation depends on all these parts of the covariant matrix, its determinant is a scalar which depends on all parts of the covariance matrix. For this reason, generalised standard deviation $\overline{\overline{s}}_{\hat{x}}$ was taken as the most important criterion of network accuracy for the comparison of individual versions of adjustment.

As was mentioned above, the entire astrogeodetic network included in trial adjustment during network renovation (Jenko, 1986) is discussed separately, and the network located in the territory of the Republic of Slovenia separately. In the entire network, 49 lengths and 222 directions are observed on 46 points, while on 9 points of the network no lengths are observed. In the network which covers only the territory of the Republic of Slovenia, 40 lengths and 160 directions are observed. No lengths are observed on 5 points of the network. The ratio of the number of observed directions to lengths is more favourable for the network covering the territory of Slovenia; i.e. 4 : 1, than that of the entire network, which amounts to 4,5 : 1. The ratio of the number of points which are not connected with lengths to the number of all points of the network is also more favourable for the network in the territory of Slovenia. The ratio of the number of observations to the number of unknowns in the network is equal and sufficiently favourable for both networks.

Trial adjustment refers to:

1) adjustment of nonreduced observations in the entire network with the values of reference standard deviations of lengths of σ_{0d} = 0,038 m and for directions σ_{0s} = 0,45", and this represents adjustment with equal weights of observations as in the trial final adjustment during network renovation;

2) adjustment of observations reduced onto the reference ellipsoid in the entire network;

3) adjustment of nonreduced observations in the network in the territory of Slovenia;

4) adjustment of observations reduced onto the reference ellipsoid in the network in the territory of Slovenia.

As regards the value of the generalised reference variance $\bar{\bar{s}}_{\hat{x}}$, the network in the territory of Slovenia which was adjusted on the basis of nonreduced observations (case 3) is the one with the highest accuracy. In both cases and taking into account all accuracy criteria, the network adjusted on the basis of nonreduced observations is better than that in the case of adjustment of reduced observations. The requirements for homogeneous accuracy are not fulfilled to any greater extent neither by the entire network nor by the network in the territory of Slovenia.

Adjustment	$\bar{s}_{\hat{x}}$	$\bar{\bar{s}}_{\hat{x}}$	λ_{min}	λ_{max}	$\frac{\lambda_{min}}{\lambda_{max}}$	$\hat{\sigma}_0^2$
1)	0,0511	0,0282	0,000031	0,058199	0,000532	1,00523
2)	0,0530	0,0292	0,000033	0,062545	0,000532	1,04209
3)	0,0533	0,0272	0,000033	0,076472	0,000436	1,04755
4)	0,0552	0,0282	0,000036	0,082120	0,000436	1,08555

Table 1

If the results of the first and second adjustments are analysed together, and those of the third and fourth adjustments together, it can be established that the differences in network accuracy for networks adjusted on the basis of reduced and nonreduced observations are caused by different values of the reference variance $\hat{\sigma}_0^2$ estimated a posteriori, since the reference variance $\hat{\sigma}_0^2$ is used for the calculation of the covariance matrix $\Sigma_{\hat{x}} = \hat{\sigma}_0^2\ (A^T PA)^+$ of estimated coordinate points of the network. The covariance matrix depends on the matrix of coefficients in correction equations, A, which is practically equal for both cases, from the matrix of observation weights P and the value of the reference variance $\hat{\sigma}_0^2$ estimated a posteriori. If the value of the reference variance $\hat{\sigma}_0^2$ is taken to be equal for all cases, the overall and local accuracy criteria are obtained which are completely equal in the first and second adjustments and the third and fourth adjustments. This means that the differences in astrogeodetic network accuracy which would result from the disregard of reductions of observations onto the reference ellipsoid cannot be estimated. In our opinion it is impossible to state that network accuracy changes due to the reduction of observations onto the reference ellipsoid, or that the reduction of observations means an improvement in geodetic network accuracy.

If the influence of the reference variance $\bar{\bar{s}}_{\hat{x}}$ is removed from the value of the generalised standard deviation $\bar{s}_{\hat{x}}$ and mean standard deviation $\hat{\sigma}_0^2$, and the results of adjustment of the entire network are compared with the results of adjustment of the part of the network in the territory of Slovenia, it can be seen that the network in the territory of Slovenia is slightly more accurate than the entire network.

Differences in the accuracy of individual adjustments therefore occur due to different values of the reference variance $\hat{\sigma}_0^2$. The values of the reference variance $\hat{\sigma}_0^2$ exceed 1 in all adjustments. This means that at least for one of the values of the reference standard deviations σ_{0s} and σ_{0d}, the taken value is too small, which means that the weights of at least one type of observations are excessive.

The fact that in the first adjustment $\hat{\sigma}_0^2 \cong 1$ follows from the ratio of the reference standard deviations of directions and lengths which is determined on the basis of extensive analyses for the geoid's influence in nonreduced observations. In these analyses, only the accuracy of observed directions in individual triangles is analysed, and the total accuracy of the entire network, and the accuracy of observed lengths is observed separately (Jenko, 1986). On the basis of these analyses, the ratio of the reference standard deviations of direction and length σ_{0s} and σ_{0d} is determined.

If one wished to establish the ratio direction and length accuracy after reduction for the values of deflections of the vertical and the values of geoid heights, a detailed analysis such as was performed in (Jenko, 1986) would need to be performed again. However, it is clear that the estimated length accuracy would not change at all - only the accuracy of observed directions could change. The original angular observations were not available during the preparation of this paper, and the analysis of accuracy of observed directions was therefore not performed. The values determined by (Jenko, 1986) were taken for the calculation of the ratio of the reference standard deviations of directions and lengths σ_{0s} and σ_{0d}.

In order to obtain an objective estimate of the ratio of weights of observed directions and lengths, a posteriori estimate of weights of observed quantities could be made. Reference variances of reasonably composed groups of observations are the starting point for the procedure of a posteriori estimation of weights. In our case such groups are the groups of observed directions and lengths. For a correct a posteriori estimate of weights of observations, a sufficient number of correctly distributed observations are required, in order to be able to estimate the coordinates of network points only on the basis of one type of observations. This means that the number of each type of observations must be higher and that the observations of each type must be uniformly distributed over the entire network (Kogoj, 1992). In the astrogeodetic network of Slovenia, these requirements are not fulfilled. The number of observed lengths is too small to obtain a correct estimate of weights for groups of observations.

By comparing the results of adjustment of the entire network and the results of adjustment of the network in the territory of Slovenia, and disregarding the reference variance $\hat{\sigma}_0^2$ in the calculation of the covariance matrix of adjusted network point coordinates, it can be seen that the network in the territory of Slovenia is slightly more accurate than the entire network. But since the value of the reference matrix for the network in the territory of Slovenia is $\hat{\sigma}_0^2 > 1$, it can be concluded that too high values were taken for weights of observations. It is impossible to predict now what changed weights of observations would mean for network accuracy. It can only be claimed that by reducing the network, its overall accuracy is not reduced. The adjustments of observations were performed using the GEM 3 computer program (Ambrožič, 1988) which was slightly modified for the needs of preparing this paper.

6 CONCLUSIONS

The adjustment of reduced observations in the national geodetic network was performed with the intention of comparing this network with the renovated astrogeodetic network treated in the trial adjustment as part of the renovation of the astrogeodetic network of Slovenia (Jenko, 1986). The objective of the comparison of the results of adjustments of the entire network with the results of adjustment of its part in the territory of the Republic of Slovenia was to establish whether network reduction affects network accuracy. On the basis of the above results it can be claimed that network accuracy is not improved with a correctly performed reduction of observations for values of deflections of the vertical and geoid heights, at least not to a detectable extent, and that the reduction of the network only to the territory of Slovenia does not reduce the overall accuracy of the geodetic network.

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