Closed-Loop Handling Stability of 4WS Vehicle with Yaw Rate Control

Hongming Lv* – Shaona Liu

Yancheng Institute of Technology, School of Automotive Engineering, China

To improve the vehicle handling stability, a four-wheel steering (4WS) controller with yaw rate tracking is proposed in this paper. A driver/ vehicle closed-loop system is built based on the preview-follower theory to evaluate the system's manoeuvring performance. Next, the system driving on a double lane-change road and on a low adhesion road are simulated and compared with conventional front-wheel steering vehicle. The Fourier transform method is developed to analyse the dynamics of the closed-loop system in the frequency domain. The results show that the proposed 4WS vehicle has the preferred response characteristics of sideslip angle and yaw rate, and is robust regarding tire cornering stiffness, which guarantees the handling stability in emergency braking condition.
Keywords: four-wheel steering, vehicle, handling stability, closed-loop system, driver, yaw rate

0 INTRODUCTION

With the rapid development of global transportation, the amount of traffic accidents has been increasing year by year. Consequently, many countries have enhanced the development of automotive active safety systems. For the potential of improving vehicle handling stability, the 4WS (Four-Wheel Steering) system has attracted much attention from companies and researchers over the previous 40 years [1] and [2]. Although 4WS is one of the current focuses of vehicle dynamics control and many researchers have presented various opinions on the configuration and control strategy, automotive companies still tend to adopt the active rear steering configuration, which is simple, economic and reliable [3] to [5].

For a 4WS vehicle, the conventional zero sideslip control strategy [6] can help driver to have the correct feeling of the vehicle body movement when vehicle corners, but it decreases the steady-state gain of the vehicle yaw rate a lot at high speed, and thus leads the vehicle to excessive understeering in that it is impossible to decouple the two variables of sideslip angle and yaw rate completely with only one control variable of the rear wheel steering angle [7].

Due to the above mentioned reasons, some researchers began to incorporate 4WS into the chassis integrated control system with direct yaw-moment control (DYC) or an active suspension system (ASS) and so on [8] to [10]. Some others took the optimising the control objective in order to improve the vehicle handling performance by only using 4WS technology, which not only considered the coordination of the sideslip angle and yaw rate, but also the nonlinearity of tire and the variation of road condition [11] to [14].

In the 1970s, Sorgatz and Weir proposed a driver model to build a driver/vehicle closed-loop system to handle the problem of evaluating the handling characteristics of designed vehicle [15] and [16]. However, for most 4WS control methodologies, the emphases were put on the vehicle dynamics models and 4WS controller logic themselves, i.e. the analysis and synthesis only considered the independent system including the vehicle model and 4WS controller. Thus, there have only been a few studies taking into account the human effect of the driver in the controller design and synthesis of 4WS [17] and [18].

The objective of this paper is to investigate the design of a 4WS controller to reduce the dynamic errors of yaw rate and sideslip angle by tracking the proposed vehicle model, and to evaluate the closed-loop system performance including driver and the 4WS vehicle. The paper is organised as follows. In Section 1, a 2DOF (degree-of-freedom) lateral dynamics model of 4WS vehicle is built. A control algorithm with yaw rate tracking is proposed in Section 2. In Section 3, the closed-loop system of driver and 4WS vehicle is built. The handling characteristics of the driver/vehicle system on a double lane-change road and limited road conditions are simulated and compared to those of a front-wheel steering (FWS) vehicle. A frequency analysis method is proposed to evaluate the handling performance of the vehicle/driver closed system. Finally, some conclusions are drawn in Section 4.

1 LATERAL DYNAMICS OF 4WS VEHICLE

A vehicle is a very complex system, so many of parameters are usually required if a multi-body dynamics method is adopted to model it in detail. To

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minimise the complexity and difficulty of developing a vehicle dynamics control system, it is common to build a relatively simple vehicle model and thus validate the feasibility in the concept design stage.

In order to study the essential vehicle dynamics and simplify the analysis procedure, the influence of roll on lateral motion is assumed to be small and not taken into account in this paper. A 2DOF model containing sideslip and yaw rate, commonly used in vehicle dynamics control, is utilised to study the handling stability of 4WS vehicle, as shown in Fig. 1 [13] and [19].
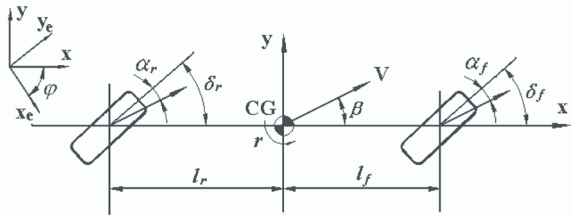

Fig. 1. *2DOF vehicle lateral dynamic model*

The inertial coordinate system, (x_e, y_e, z_e) is fixed on the ground, where the z_e axis represents the direction normal to the (x_e, y_e) plane. This serves as a reference frame for vehicle motions. The body (chassis) coordinate system, denoted by (x, y, z) with its origin at the CG, is introduced to describe vehicle motion. Next, the chassis system, (x, y, z) is rotated a yaw angle φ with respect to the inertial system (x_e, y_e, z_e).

In Fig. 1, β and r denote the sideslip angle and yaw rate of vehicle at the CG; m is the vehicle mass；J_z is the yaw moment of inertia about its mass centre z-axis; V denotes the velocity of vehicle; l_f and l_r are the distances from the CG (centre of gravity) to the front and rear axles; δ_f and δ_r denote the steering angles of front and rear wheels; α_f and α_r are the slip angles of front and rear tires; C_f and C_r denote the lateral stiffnesses of the front and rear tires, respectively.

Only considering lateral and yaw motions, the vehicle dynamic equations can be derived by applying Newton's second law.

Lateral motion:

$$mV(\dot{\beta} + r) = F_f \cos\delta_f + F_r \cos\delta_r. \qquad (1)$$

Yaw motion, or moments about the vertical z-axis through the CG:

$$J_z\dot{r} = l_f F_f \cos\delta_f - l_r F_r \cos\delta_r. \qquad (2)$$

In general, lateral tire force is a non-linear function of the slip angle. In this study, the cornering stiffness for the front (rear) wheel is denoted by $C_f(C_r)$ and its value depends on the tire-road interaction. As long as the tire slip angle is small, a linear relationship between tire force and slip angle can be justified. Then, the lateral forces generated by the front and rear tires vary linearly with their slip angles.

$$F_f = C_f\alpha_f \quad \text{and} \quad F_r = C_r\alpha_r. \qquad (3)$$

If the sideslip angle β is small and vehicle velocity V varies slowly, α_f and α_r will be given by:

$$\alpha_f = \delta_f - \beta - \frac{l_f}{V}r \quad \text{and} \quad \alpha_r = \delta_r - \beta + \frac{l_r}{V}r. \qquad (4)$$

In addition, δ_f and δ_r are generally small,

$$\cos\delta_f \approx 1 \quad \text{and} \quad \cos\delta_r \approx 1. \qquad (5)$$

Hence, the vehicle motion equations are expressed by

$$\begin{cases} mV(\dot{\beta} + r) = -2(C_f + C_r)\beta - \dfrac{2(l_f C_f - l_r C_r)}{V}r \\ +2C_f\delta_f + 2C_r\delta_r \\ J_z\dot{r} = -2(l_f C_f - l_r C_r)\beta - \dfrac{2(l_f^2 C_f + l_r^2 C_r)}{V}r \\ +2l_f C_f\delta_f - 2l_r C_r\delta_r \end{cases}. \qquad (6)$$

Define the state vector $x = \begin{bmatrix} \beta & r \end{bmatrix}^T$, input vector $u = \begin{bmatrix} \delta_f & \delta_r \end{bmatrix}^T$ and output vector $y = \begin{bmatrix} \beta & r \end{bmatrix}^T$, the vehicle model can be written in the state-space form as follows:

$$\dot{x} = Ax + B\delta_f + E\delta_r \qquad (7)$$
$$y = Cx$$

where, the coefficient matrices are:

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{mV} & -\dfrac{2(l_f C_f - l_r C_r)}{mV^2} - 1 \\ -\dfrac{2(l_f C_f - l_r C_r)}{J_z} & -\dfrac{2(l_f^2 C_f + l_r^2 C_r)}{J_z V} \end{bmatrix},$$

$$B = \begin{bmatrix} \dfrac{2C_f}{mV} \\ \dfrac{2C_f}{J_z} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} \dfrac{2C_r}{mV} \\ -\dfrac{2C_r}{J_z} \end{bmatrix}.$$

2 4WS CONTROLLER

One of the main advantages of a 4WS vehicle is that there are more choices to realise the desired handling performance. Proportional control based on steady zero sideslip can improve the vehicle manoeuvrability at low speeds [20]. However, the control strategy may significantly decrease the yaw rate gain in that only one control input is used in the 2DOF system, which is not suitable for the handling of vehicle at high speed. The proposed control objective in this paper is to reduce both the harmonic peak values of yaw rate and sideslip angle as much as possible [13], viz. to improve the vehicle handling stability at high speed while avoid excessive understeer, an optimal controller for 4WS vehicle based on yaw rate tracking is designed as shown in Fig. 2.
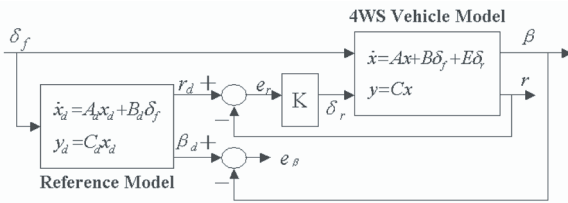

Fig. 2. *4WS control system*

The desired reference yaw rate and sideslip angle are first order time lag transfer functions of the steering angle of front wheel, i.e.

$$r_d = \frac{r_0}{1+\tau_r s}, \qquad (8)$$

$$\beta_d = \frac{\beta_0}{1+\tau_\beta s} \qquad (9)$$

where, τ_r and τ_β are the time constants of yaw rate and sideslip angle respectively; r_0 is the steady-state gains of yaw rate of FWS vehicle, and β_0 is the corresponding steady-state gains of sideslip angle. They are derived from Eq. (6).

$$r_0 = \frac{r}{\delta_f}\bigg)_{steady} = \frac{V/(l_f+l_r)}{1+\frac{m}{l_f+l_r}\left(\frac{l_f}{2C_r}-\frac{l_r}{2C_f}\right)V^2}, \qquad (10)$$

$$\beta_0 = \frac{\beta}{\delta_f}\bigg)_{steady} = \frac{1-\frac{ml_f}{2(l_f+l_r)l_rC_r}V^2}{1+\frac{m}{l_f+l_r}\left(\frac{l_f}{2C_r}-\frac{l_r}{2C_f}\right)V^2}\frac{l_r}{(l_f+l_r)}. \qquad (11)$$

The state-space representation of the desired reference vehicle model is given by:

$$\begin{cases} \dot{x}_d = A_d x_d + B_d \delta_f, \\ y_d = C_d x_d \end{cases} \qquad (12)$$

where the state vector x_d and output vector y_d are written as follows, $x_d = \begin{bmatrix} \beta_d & r_d \end{bmatrix}^T$ and $y_d = \begin{bmatrix} \beta_d & r_d \end{bmatrix}^T$.

And the coefficient matrices are:

$$A_d = \begin{bmatrix} A_{d1} & A_{d2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_\beta} & 0 \\ 0 & -\frac{1}{\tau_r} \end{bmatrix},$$

$$B_d = \begin{bmatrix} B_{d1} \\ B_{d2} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ r_0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The physical interpretation of this control algorithm is that when the front wheel is steered, the yaw rate is measured and compared with an ideal yaw rate response r_d. Next, the rear wheel steering angle is controlled by controller K.

$$\delta_r = K \cdot (r_d - r). \qquad (13)$$

In order to make the outputs of an actual vehicle to follow the outputs of desired model, the error variables are defined as $e_\beta = \beta_d - \beta$, $e_r = r_d - r$ while the state vector $\hat{x} = \begin{bmatrix} \beta & r & e_\beta & e_r \end{bmatrix}^T$ and output vector $\hat{y} = \begin{bmatrix} e_\beta & e_r \end{bmatrix}^T$. Note that only e_r is used to control the system directly, and e_β is used to optimise the controller in the design stage. The equivalent state equations of 4WS vehicle including the controller K are obtained from Eqs. (7), (12) and (13).

$$\begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\delta_f, \\ \hat{y} = \hat{C}\hat{x} \end{cases} \qquad (14)$$

where

$$\hat{A} = \begin{bmatrix} A & 0 & K \cdot E \\ A_d - A & A_{d1} & A_{d2} - K \cdot E \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} B \\ B_d - B \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

For the above standard output feedback problem, the controller K can be designed according to optimal control theory or the H∞ control theory, etc.

To minimise all the harmonic peak values of vehicle yaw rate and sideslip angle while not decreasing the steady-state response gain of yaw rate, a H∞ optimal controller is designed. The mathematic expression is described as,

$$\min J =: \|WS\|_\infty, \qquad (15)$$

where the weight $W = [w_\beta \quad w_r]$; S is the sensitivity function matrix of β and r to δ_f.

$$S = \begin{bmatrix} e_\beta & e_r \end{bmatrix}^T = \hat{C}(sI - \hat{A})^{-1}\hat{B}. \qquad (16)$$

It is noted that the weight W is mainly dependent on the experience and experiment. However, it is also found that the values of w_β and w_r are related closely to e_β and e_r in that the sensitivities of them to controller K are quite different.

3 CLOSED-LOOP HANDLING STABILITY OF 4WS VEHICLE

3.1 Modelling of Closed-Loop System including Driver and Vehicle

Modelling the driver/vehicle closed-loop system has been a subject of research since the 1970s [15] and [16]. The driver's use of visual information in controlling the direction of a vehicle is obviously very significant and has been investigated extensively. There is a general consensus that control occurs at two levels [21] and [22]: preview control (open-loop feedforward), in which the driver anticipates the path ahead and makes an appropriate steering action based on knowledge of the vehicle dynamics; and compensatory control (closed-loop feedback), in which the driver compensates for errors in the preview control and for disturbances. The compensatory task involves the human operator controlling a system to minimise an error.

Guo presented an algorithm to perform the modelling of the closed-loop driver/vehicle directional control system from a mathematical point of view, i.e. preview follower theory, which can be used to determine the main frame and all parameters of the system. The driver/vehicle closed-loop system mode is built as shown in Fig. 3. T_p denotes the driver's lead time constant; C_0 represents the driver's compensatory gain, and equals to the reciprocal of the steady-state lateral acceleration gain; T_c is the driver's correction time constant; t_d is the driver's cognitive time delay; T_h is a first-order lag time constant which represents the driver's neuromuscular time delay [23].

Ashkens and McRuer stated that the driver's delay time constants of t_d and T_h have nothing to do with the characteristics of vehicles and usually take the values about $t_d = 0.2$ s and $T_h = 0.1$ s while C_0, T_p, and T_c depend on the vehicle characteristics, and driver experience [24].

On the basis of the previous investigation [23], Guo promoted the further work of driver model parameter identification by utilising a car driver simulator and presented the parameter's reference range [25]: $T_p \in [0.58, 2.072]$, $T_c \in [0.02, 2.76]$, $t_d \in [0.17, 0.53]$, $T_h \in [0.08, 0.36]$.

The vehicle model in Fig. 3 is rebuilt by defining new state variables $[\beta \ r \ \varphi \ y]^T$ and output variables $[\beta \ r \ y]^T$ while δ_f and δ_r are control variables. The yaw rate r is the differential of yaw angle φ.

$$r = \dot{\varphi}. \qquad (17)$$

According to the vehicle dynamic characteristic shown in Fig. 1, the vehicle lateral velocity $\dot{y}$ can be deduced as,

$$\dot{y} = V \sin(\beta + \varphi) \approx V(\beta + \varphi). \qquad (18)$$

The new matrixes A, B, C and E of state equations are rewritten as following,

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{mV} & -\dfrac{2(l_f C_f - l_r C_r)}{mV^2} - 1 & 0 & 0 \\ -\dfrac{2(l_f C_f - l_r C_r)}{J_z} & -\dfrac{2(l_f^2 C_f + l_r^2 C_r)}{J_z V} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ V & 0 & V & 0 \end{bmatrix},$$
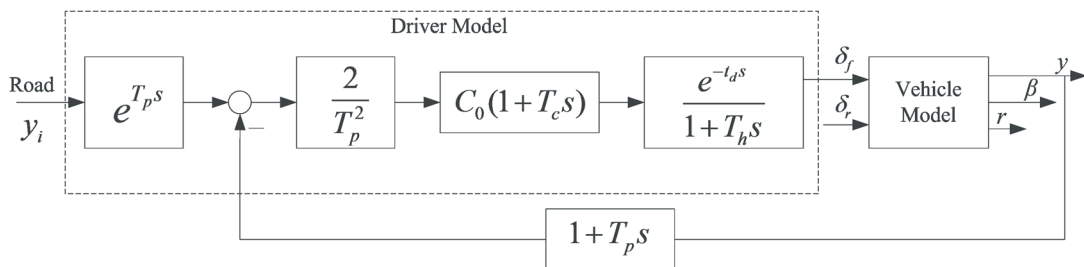
Fig. 3. *Closed-loop system including driver and vehicle*

$$B = \begin{bmatrix} \dfrac{2C_f}{mV} \\ \dfrac{2C_f}{J_Z} \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} \dfrac{2C_r}{mV} \\ -\dfrac{2C_r}{J_Z} \\ 0 \\ 0 \end{bmatrix}.$$

The test parameters of a Buick LeSabre, a full-size sedan, are used to build the vehicle model [26] while the driver handling parameter ranges presented by Guo [23] and [25] are referred and synthesised to build the driver model. The key parameters of driver/vehicle system are summarised in Table 1.

Table 1. *Parameters of the driver/vehicle system*

Parameter	Value
Vehicle mass (m)	1740 kg
Yaw inertia (J_Z)	3214 kg·m2
Front axle to CG (l_f)	1.058 m
Rear axle to CG (l_r)	1.756 m
Front tire cornering stiffness (C_f)	29000 N/rad
Rear tire cornering stiffness (C_r)	60000 N/rad
Reference yaw rate time lag (τ_r)	0.1 s
Reference sideslip angle time lag (τ_β)	0.1 s
Driver's lead time (T_p)	1 s
Driver's compensatory gain (C_0)	1.7 s2/m
Driver's correction time (T_c)	0.2 s
Driver's cognitive time delay (t_d)	0.28 s
Driver's neuromuscular time delay (T_h)	0.1 s

3.2 Simulation of Double Lane-Change Manoeuvre

A double lane-change manoeuvre is a typical driving condition and commonly used to estimate vehicle handling performance. In this paper, it is adopted to simulate the lateral dynamics for both the 4WS and FWS vehicles at a speed of 30 m/s, see Fig. 4. The behaviour of front wheel steering angles of them during the manoeuvring process is given as shown in Fig. 5.

The trajectories of both vehicles are similar to the actual steering process, which proves the validity of the simulation. The steering angles of them are also very similar, which shows the simulation results of FWS and 4WS vehicles have comparability because they are at the same input condition.

Their sideslip angle and yaw rate responses on a double lane-change road are compared in Fig. 6. When a 4WS vehicle is cornering, the maximum
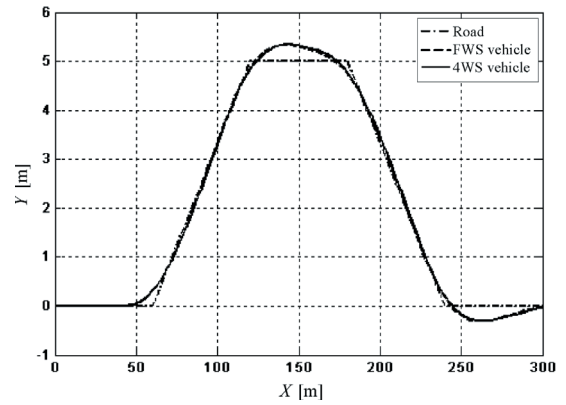

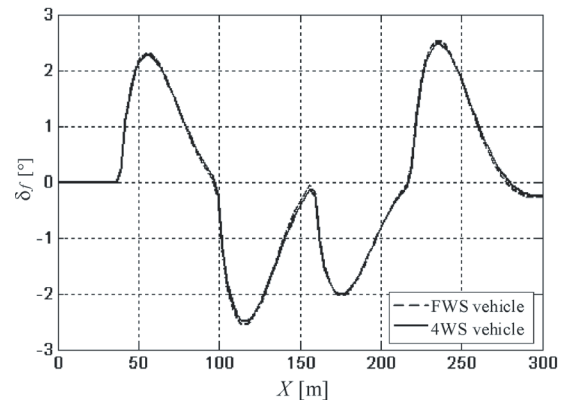**Fig. 4.** *Trajectories of road, 4WS and FWS vehicle motion*

Fig. 5. *Steering angles of front wheel*
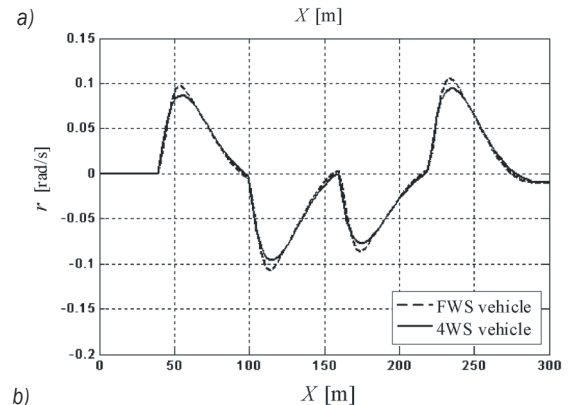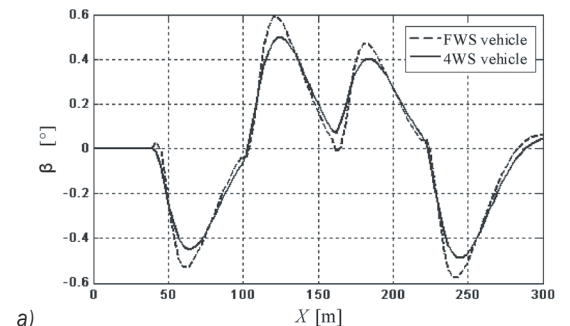

a)

b)

Fig. 6. *Responses of vehicle state variables on double lane-change road; a) Sideslip angle, b) Yaw rate*

sideslip angle, compared to FWS vehicle, is obviously reduced while the maximum yaw rate is decreased to a certain extent.

The relationships of β and r to δ_f of both 4WS and FWS vehicles at a speed of 30 m/s are compared in Fig.7.

The response curves of 4WS and FWS are similar to each other, but the maximum values of sideslip angle and yaw rate of 4WS vehicle are smaller than those of FWS vehicle, as the same in Fig. 6. Therefore, the time responses indicate that 4WS can improve the vehicle's dynamic handling behaviour to make the vehicle easier to drive at high speeds.
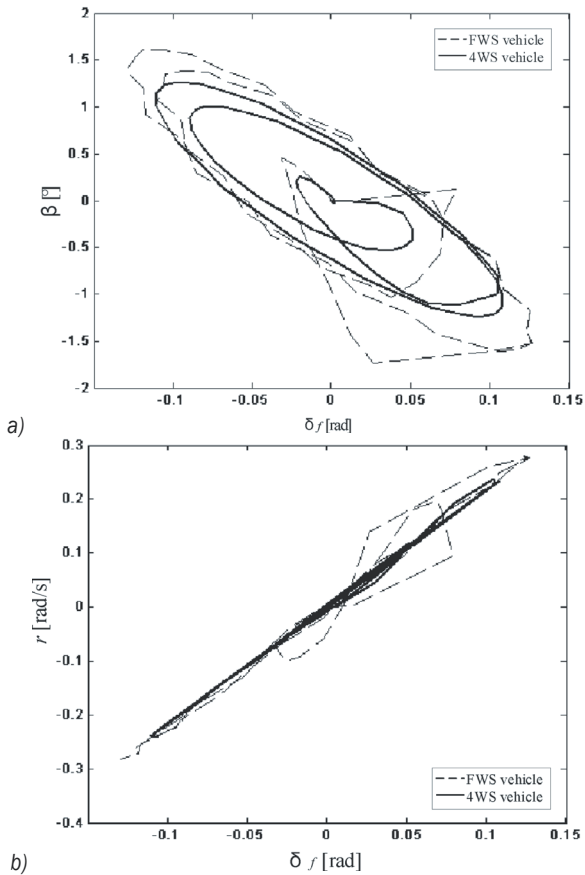

a)

b)

Fig. 7. *Relationship of β and r to δ_f on double lane-change road; a) β - δ_f, b) r - δ_f*

3.3 Analysis of Closed-Loop System in the Frequency Domain

Though a double lane-change road is usually described its length in space, it can also be converted by time if the road length is divided by vehicle velocity. Thus, for a closed-loop system including driver and vehicle, the road is the time input signal while β and r are the outputs. It is known that the system transfer

function is so complex that it cannot be written in a simple format. There is a direct way to obtain the system transfer function: the Fourier Transform method is used to the input and output variables in the simulation.

$$H(j\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{F[y(t)]}{F[x(t)]} = \frac{\int_0^\infty y(t)e^{-j\omega t}dt}{\int_0^\infty x(t)e^{-j\omega t}dt}. \quad (19)$$

Here, $H(j\omega)$ is the system transform function; $X(\omega)$ and $Y(\omega)$ are the input and output expressed by frequency; $x(t)$ and $y(t)$ are the input and output by time, respectively. Based on the above theory, the closed-loop dynamic characteristics in the frequency domain can be obtained by FFT (Fast Fourier Transform).

The dynamic characteristics of closed-loop system in frequency domain are depicted in Figs. 8 and 9.

The figures show that the resonances of driver/ vehicle closed-loop system mainly lie in the middle frequency range between 1~20 rad/s. The important natural frequencies, caused by vehicle and driver characteristics, are 1.5, 3, 4.5, 6.3, 8 and 9.4 rad/s, individually. They also indicate that the proposed 4WS controller is equivalent to a mid-pass filter for the driver/ vehicle closed-loop system. Compared to the FWS vehicle, the sideslip angle resonance peak values of the 4WS vehicle are decreased distinctly. The second resonance peak value is decreased is from 0.88 to 0.7 deg, the third is from 0.85 to 0.55 deg, and the fourth decreases especially, from 1.1 to 0.57 deg, respectively.

The yaw rate resonance peak values of 4WS vehicle are also decreased. The third peak value is decreased from 0.17 to 0.15 rad/s while the fourth is from 0.25 to 0.18 rad/s.

In fact, the closed-loop handling stability evaluation is still developing and not effectively applied for the variation of driver's subjective properties. It is meaningful to evaluate the handling stability of a developing vehicle by utilising the resonance frequency analysis method to the driver/ vehicle closed-loop system.

3.4 Simulation of Emergency Braking Condition

Many traffic accidents occur in condition of emergency braking, which lead to the vertical loads of front and rear tires changing greatly. According to the lateral properties of tires, the equivalent
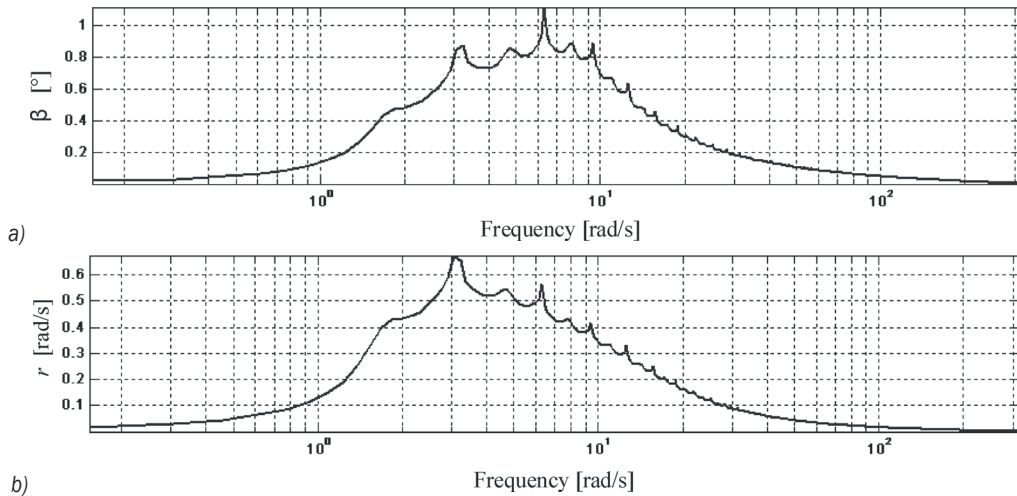
a)

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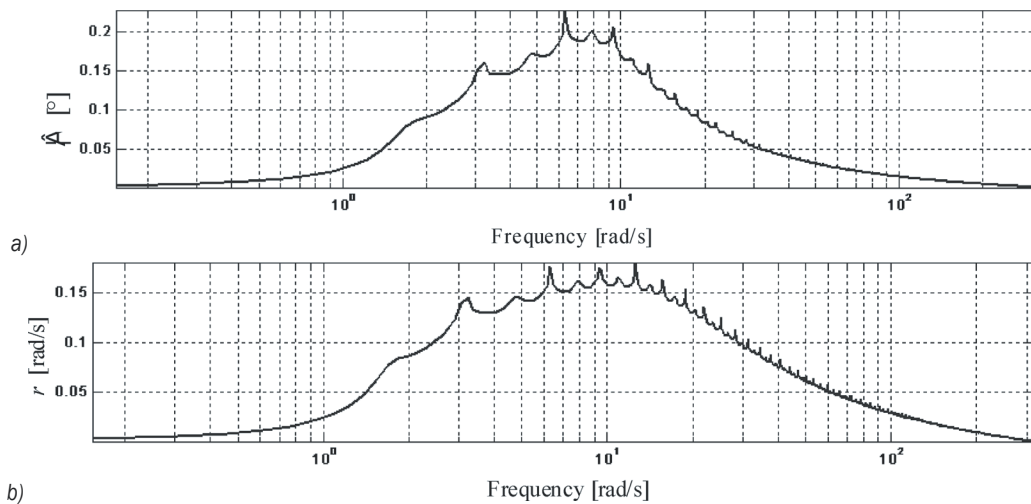**Fig. 8.** *Closed-loop dynamic characteristics of sideslip angle; a) FWS vehicle, b) 4WS vehicle*

a)

b)

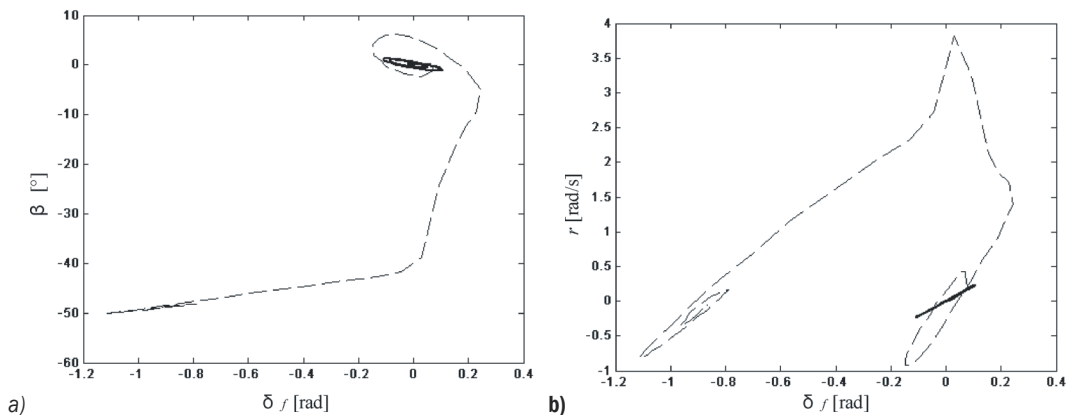**Fig. 9.** *Closed-loop dynamic characteristics of yaw rate; a) FWS vehicle, b) 4WS vehicle*

a)

b)

Fig. 10. *Relationship of β and r to δ_f in emergency braking condition; a) β - δ_f, b) r - δ_f*

cornering stiffness of the front tire increases while the rear decreases, which reduces the vehicle's natural frequency and increases the response peak value of each state variable. Thus the handling stability of the vehicle is deteriorated, and the vehicle may be out of control.

To simplify the study, it is assumed that the cornering stiffness of rear tire decreases by 50% [8]. The closed-loop systems of 4WS and FWS vehicles on a double lane-change road are simulated and the relationships of state variables are shown in Fig. 10.

When both types of vehicle are steered in such emergency braking condition, all the responses of FWS vehicle exceed the stable region while the 4WS vehicle still has the original characteristics, which shows that the 4WS vehicle is robust to the variation of tire cornering stiffness, and thus guarantees preferable handling stability.

4 CONCLUSIONS

According to the lateral dynamics simulation of a driver/vehicle closed-loop system on a double lane-change road, the 4WS vehicle with yaw rate tracking control has the following advantages.

It can track the referenced sideslip angle and yaw rate, and thus improves vehicle handling stability.

It has a preferable robustness to tire cornering stiffness. The vehicle can remain stable even in emergency braking condition.

Though the resonance frequency analysis of driver/vehicle closed-loop system is still in development, it may be an effective method to study the handling stability of vehicle.

5 ACKNOWLEDGEMENT

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