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Angular momentum content of the $\rho(1450)$ from chiral lattice fermions*

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Abstract. We identify the chiral and angular momentum content for the leading quark-antiquark Fock component for the $\rho(770)$ and $\rho(1450)$ mesons using a lattice simulation with chiral fermions. Our analysis shows that in the angular momentum basis the $\rho(770)$ is a $^3S_1$ state, in accordance with the quark model. The $\rho(1450)$ is a $^3D_1$ state, showing that the quark model wrongly assumes the $\rho(1450)$ to be a radial excitation of the $\rho(770)$.

1 Introduction

An interesting question in hadronic physics is the origin of spin and distribution of angular momentum. How the spin of a hadron is generated, and by which constituents it is carried, is a priori not clear. In the non-relativistic, constituent quark model [2], which has been quite successful in delivering a classification scheme for the low-lying hadron spectrum, the spin of a hadron is assigned solely to its valence quarks. Being an effective classification scheme, it does not care about foundations in terms of underlying QCD dynamics. Despite its successes the non-relativistic description clearly has limitations.

In this project we investigate the angular momentum content of the $\rho(770)$ and $\rho(1450)$ mesons. In the spectroscopic notation $n\ ^{2S+1}l_J$ the $\rho(770)$ is assigned to the $1\ ^3S_1$ state by the quark model. The $\rho(1450)$ is assigned to the $2\ ^3S_1$ state, hence being the first radial excitation of the $\rho(770)$. However, this assumption is by far not clear from the underlying QCD dynamics, and is an output of the non-relativistic potential description of a meson as a two-body system.

The angular momentum content of the leading quark-antiquark Fock components of mesons can in principle be identified by lattice simulations. Studies like [3], which rely on heavy quarks for the non-relativistic reduction of hadrons, find good agreement with the quark model classification. However, there is an alternative approach to project non-perturbative lattice results onto the quark model assuming ultra-relativistic quarks. Latter method, which is explained and has been applied in previous studies [4–8], makes use of the chiral-parity group and an unitary transformation to the $^{2S+1}l_J$ basis.

Main ingredients to such an investigation are the overlap factors of operators obtained in lattice calculations. In our study it is crucial that these operators

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form a complete set with respect to the chiral-parity group. From these overlap factors the chiral content of a state can be identified, and using the unitary transformation also the angular momentum content. Since the chiral properties are important for such a study, we need a proper lattice fermion discretization, which respects chiral symmetry. For this purpose we use overlap fermions, which distinguishes the present study from the previous ones.

2 Method and Simulation

The full details of this study, its methodology and simulation parameters, can be found in the main paper [1] and references therein. Here we present the idea and summarize the most important components.

To generate states with ρ quantum numbers $(1, 1^{--})$ two different local interpolators can be used, which belong to two distinct chiral representations

$$J^V_\rho(x) = \bar{\Psi}(x)(\tau^a \otimes \gamma^i)\Psi(x) \quad \in (0,1) \oplus (1,0) \tag{1}$$

$$J^T_\rho(x) = \bar{\Psi}(x)(\tau^a \otimes \gamma^0\gamma^i)\Psi(x) \quad \in (1/2, 1/2)_b. \tag{2}$$

We denote them according to their Dirac structure as *vector (V)* and *pseudotensor (T)* interpolators. In a next step we connect the chiral basis to the angular momentum basis with quantum numbers isospin I and ${}^{2S+1}l_J$. For spin-1 isovector mesons there are only two allowed states $|1; {}^3S_1\rangle$ and $|1; {}^3D_1\rangle$, which are connected to the chiral basis by a unitary transformation:

$$|\rho_{(0,1)\oplus(1,0)}\rangle = \sqrt{\frac{2}{3}}\,|1; {}^3S_1\rangle + \sqrt{\frac{1}{3}}\,|1; {}^3D_1\rangle \ , \tag{3}$$

$$|\rho_{(1/2,1/2)_b}\rangle = \sqrt{\frac{1}{3}}\,|1; {}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|1; {}^3D_1\rangle \ . \tag{4}$$

Note that the operators (1),(2) form a complete and orthogonal basis with respect to the chiral group. Through the unitary transformation (3),(4) they also form a complete and orthogonal basis with respect to the angular momentum content.

On the lattice we evaluate the correlators $\langle J(t)J^\dagger(0)\rangle$. We apply the variational technique, where different interpolators are used to construct the correlation matrix $\langle J_l(t)J^\dagger_m(0)\rangle = C(t)_{lm}$. By solving the generalized eigenvalue problem

$$C(t)_{lm}u^{(n)}_m = \lambda^{(n)}(t, t_0)C(t_0)_{lm}u^{(n)}_m \tag{5}$$

the masses of states can be extracted in a standard way. Denoting $a^{(n)}_l = \langle 0|\, J_l\, |n\rangle$ as the overlap of interpolator J_l with the physical state $|n\rangle$, the relative weight of the chiral representations is now given by

$$\frac{C(t)_{lj}u^{(n)}_j}{C(t)_{kj}u^{(n)}_j} = \frac{a^{(n)}_l}{a^{(n)}_k} \ . \tag{6}$$

We can extract the ratio a_V/a_T for each state n. Then via the unitary transformation (3),(4) we arrive at the angular momentum content of the ρ mesons.

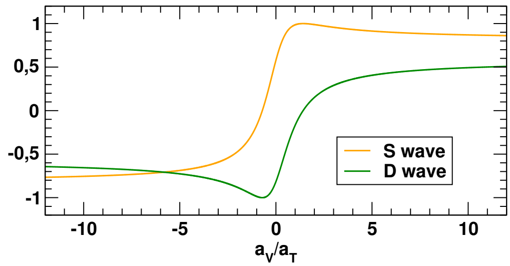

Fig. 1. Partial wave content of ρ mesons in dependence of the relative chiral contribution a_V/a_T, which are connected via transformation (3),(4).

For any lattice simulation an intrinsic resolution scale is set by the lattice spacing a. This means that probing the hadron structure with point-like sources gives results at a scale fixed by the ultraviolet regularization a.

In order to measure the structure close to the infrared region we introduce a different resolution scale by smearing the sources of the quark propagators. We use four different smearing widths in this study. The radius σ of a given source $S(x;x_0)$ is calculated by

$$\sigma^2 = \frac{\sum_x (\mathbf{x}-\mathbf{x}_0)^2 |S(x;x_0)|^2}{\sum_x |S(x;x_0)|^2} , \tag{7}$$

where we define the resolution scale as $R = 2\sigma a$. The smeared profiles of the sources used in this study are pictured in Figure 2. The *Ultra Wide* source does not resolve details smaller than ~ 0.9 fm and marks our infrared end, where we ultimatively extract the resolution scale dependent quantities.

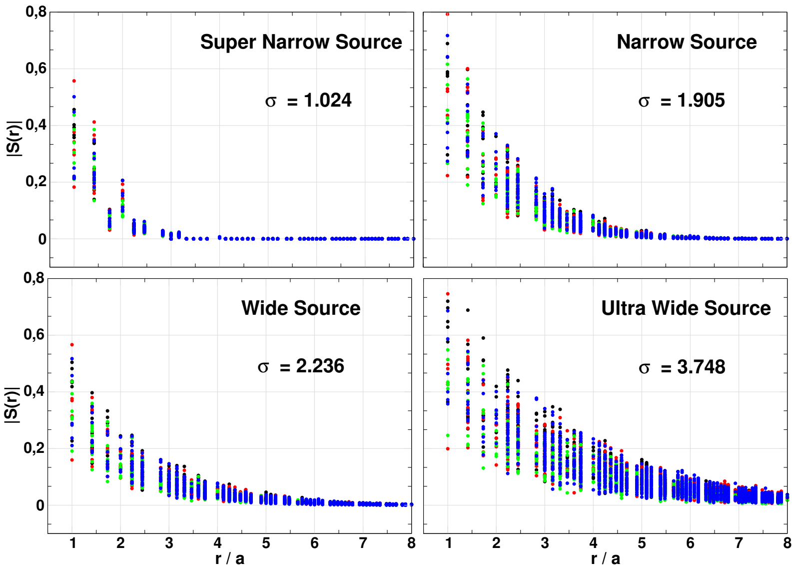

Fig. 2. Different source profiles. σ is their radius in lattice units.

3 Results

To study the ratio a_V/a_T at different resolution scales R we solve the eigenvalue problem (5) with operators (1) and (2) and four different smearings. Then using (6) we extract the ratio a_V/a_T as a function of R. In Fig. 4 we show the ratio a_V/a_T at different resolution scales R. We find a clear R-effect for the ratio a_V/a_T: both ρ and ρ' states are linear dependent on the resolution scale.

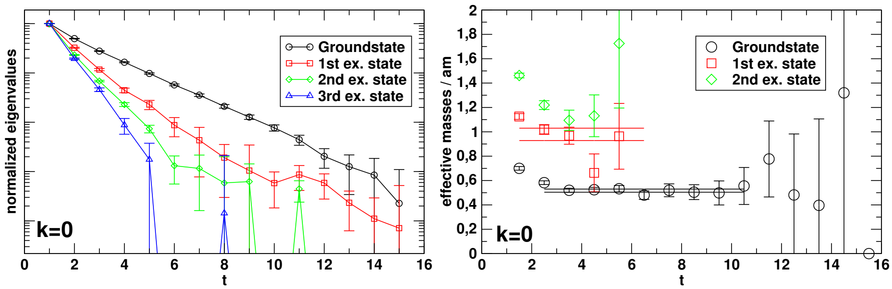

Fig. 3. Normalized eigenvalues and effective masses.

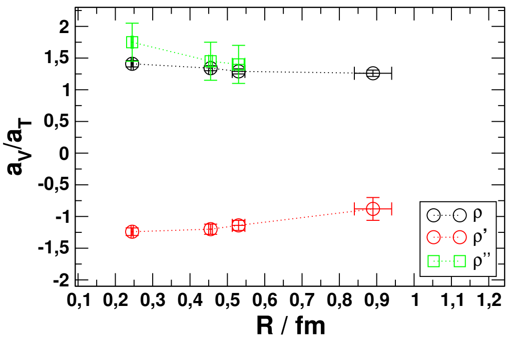

Fig. 4. a_V/a_T ratio for different resolutions.

Using now transformations (3),(4) we find:

$$|\rho(770)\rangle = +\,(0.998 \pm 0.002)\,|^3S_1\rangle - (0.05 \pm 0.025)\,|^3D_1\rangle\ , \tag{8}$$

$$|\rho(1450)\rangle = -\,(0.106 \pm 0.09)\,|^3S_1\rangle - (0.994 \pm 0.005)\,|^3D_1\rangle\ . \tag{9}$$

$$|\rho(1700)\rangle = +\,(0.99 \pm 0.01)\,|^3S_1\rangle - (0.01 \pm 0.12)\,|^3D_1\rangle\,. \tag{10}$$

The ground state ρ is therefore practically a pure $^3S_1$ state, in agreement with the potential quark model assumption.

The first excited ρ is, however, a $^3D_1$ state with a very small admixture of a $^3S_1$ wave. The second excited state is almost pure $^3S_1$ state. The latter results are in clear contradiction with the potential constituent quark model that attributes the first excited state of the ρ-meson to a radially excited $^3S_1$ state and the next excited state to a $^3D_1$ state.

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