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OPTIMALNA KONFORMNA PROJEKCIJA ZA VSEEVROPSKO KARTIRANJE

THE OPTIMAL CONFORMAL PROJECTION FOR PAN-EUROPEAN MAPPING

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IZVLEČEK

V Evropi je predvsem v povezavi s podporo različnim politikam opazna vse večja potreba po vseevropskih prostorskih podatkovnih nizih. Za vizualizacijo teh podatkov bi bilo treba sprejeti ustrezne kartografske projekcije. Na delavnici na temo kartografskih projekcij združenja EuroGeographic *leta 2001 so bila oblikovana priporočila Evropski komisiji, in sicer da se v namene kartiranja Evrope v merilih, manjših ali enakih 1 : 500.000, uporabi Lambertova konformna stožčna projekcija. V članku razpravljamo, ali je predlagana kartografska projekcija optimalna rešitev z vidika linearnih deformacij na območju kartiranja. Kot alternativna rešitev je predlagana optimizirana prilagojena konformna projekcija rotacijskega elipsoida CAMPREL (angl.* Conformal Adaptive Mapping Projection of Rotation Ellipsoid*). Izračunali smo vrednosti cenilk za oceno kakovosti predlagane projekcije in jih primerjali z vrednostmi za Lambertovo konformno stožčno projekcijo. Dodatno je izračunana največja mogoča absolutna linearna popačenost pri konformnem kartiranju Evrope. Izkazalo se je, da predlagana projekcija CAMPREL bolje ustreza izbranim kriterijem glede primernosti za vseevropsko kartiranje.*

ABSTRACT

There is an increasing need for pan-European spatial datasets, mainly to support the common European Union policies. This has inevitably raised demands for adopting pan-European cartographic projections to visualize the spatial data. The Map Projection Workshop organized by EuroGeographics in 2001 provided a recommendation to the European Commission to adopt the Lambert conformal conic projection for conformal pan-European mapping at the scales smaller or equal to 1:500,000. This paper discusses if the projection is an optimal solution in terms of linear deformations over the mapping region. An optimized CAMPREL (conformal adaptive mapping projection of rotation ellipsoid) projection for the area of interest is proposed as an alternative solution. The projection quality criteria were calculated and compared with those of the Lambert conformal conic projection. The maximal possible absolute linear distortion for conformal mapping of the pan-European area is also given. It has been shown that the CAMPREL projection designed for pan-European mapping better meets the projection selection criteria.

KLJUČNE BESEDE

kartografska projekcija, CAMPREL, Chebyshev-Grave teorem, Meshcheryakov teorem, Laplaceove parcialne diferencialne enačbe, vseevropsko kartiranje, linearno popačenje

KEY WORDS

cartographic projection, CAMPREL, Chebyshev-Grave's theorem, Meshcheryakov's theorem, Laplace's partial differential equations, pan-European mapping, maximum absolute linear distortion

1 INTRODUCTION

There are increasing demands for seamless pan-European spatial datasets (e.g. GMES land monitoring data, LUCAS, LPIS.), mainly aimed to support definition and monitoring of the European Union policies, e.g. Common Agriculture Policy, Environment (Annoni et al., 2001; Annoni and Smits, 2003). To cartographically represent and store such spatial data, there is a need for a common spatial reference system for Europe. According to ISO19111:2019 (ISO, 2019), a spatial reference system consists of a datum and a coordinate system that includes the definition of a map projection.

The European Commission Interservices Committee for Geographic Information-COGI adopted ETRS89 with the underlying GRS80 ellipsoid, like the geodetic datum (Annoni et al., 2000). The Map Projection Workshop, organized in 2001 by EuroGeographics following a request from the JRC (Joint Research Centre) of the European Commission, resulted in the recommendation to the European Commission to adopt the Lambert conformal conic projection with two standard parallels (namely, 35°N and 65°N) for conformal pan-European mapping at the scales smaller or equal to 1:500.000 (Annoni et al., 2003). It is not, however, clear the exact reason for such a choice. There is, however, an implicit indication within the final document of the workshop stating that *an area with a large West-East extent is better represented in a normal conical projection (i.e. Lambert conformal conic - ETRS-LCC).*

The general rule of selecting a projection for conformal mapping is to achieve the least possible exaggeration of the area within the region to be mapped. This paper aims to show that this requirement was not fulfilled by selecting the Lambert conformal conic projection for the pan-European area. To support this claim, the CAMPREL (conformal adaptive mapping projection of rotation ellipsoid) projection (Nestorov, 1996) has been derived for the area of interest, and values of the projection quality criteria were calculated and compared with those of the Lambert conformal conic projection.

The CAMPREL projection (Nestorov, 1996 and 1997), tries to define the "best projection"[1] for a subject area following Meshcheryakov (1968) criteria. It satisfies both Chebyshev's minimax criteria (Chebyshev, 1856) and variational criteria based on numerical distortion measures. Such projection is not just a great candidate for the conformal projection of the area. Still, it can also answer what the smallest possible maximal linear distortion with which the area of interest can be conformally mapped is.

The next section presents the general idea of the CAMPREL projection, with theoretical background, derivation of equations for direct mapping, and numerical distortion criteria measures used to select the most optimal projection. In the following sections, a computer program and methodology finding the CAMPREL projection of the pan-European area is presented, and a comparison of the CAMPREL and

[1] *There are several ideas of "best map projection", among all possible projections, or among a certain class of projections. Some of them are:*

1. ***Kavraysky's idea of best map projection:*** *Map projection founded under unique conditions that maximal linear distortions on the whole area of mapping are minimal. (Kavrayskiy, 1958)*
2. ***Milnor's idea of best map projection:*** *Map projection where ratio between logarithms of maximal and minimal linear scales is minimal. Milnor's theorem proofs the existence of such projection for the mapping sphere on to the plane. (Milnor, 1969).*
3. ***Chebyshev's idea of best map projection:*** *Map projections having a minimal ratio between maximal and minimal linear scale on the whole area of mapping (Chebyshev, 1856). The last definition provides so-called minimax criteria and projection satisfying this criterion is called Chebyshev's projection or "the best projection in minimax sense" (Nestorov, 1996). The Chebyshev's - Grave's theorem, also known as the theorem of conformal mapping, proofs the existence of such projection.*
4. ***Meshcheryakov's idea of best map projection:*** *Map projection satisfying both minimax and variational criteria. (Meshcheryakov, 1968). Map projection satisfying variational criteria is the one having minimal some of the numerical criteria measures like total distortion, range of the absolute distortion, relative linear scale and the like.*

ETRS-LCC projections is made based on numerical distortion measures. The concluding remarks and observation are given the final section.

2 THE IDEA OF CAMPREL PROJECTION

2.1 Theoretical background

The idea of the CAMPREL projection has been presented by Nestorov (1996) in the Serbian language. Nestorov (1997) summarized the idea in English. However, the full explanation of the projection formulas has never been completely presented in the English language to the community. Therefore, this section presents the basics of the CAMPREL projection idea, following both references mentions above.

The concept of the CAMPREL projection is defined using the "reverse cartographic assignment method" (Urmayev, 1947; Meshcheryakov, 1968), which assumes that a new map projection can be developed if the conditions of the projection characteristics are known. Meshcheryakov's theorem (Meshcheryakov, 1968) says that if one knows exactly four conditions of projection characteristics[2], then the unique equations for direct and inverse mapping with positive Jacobian at the whole area of mapping exist. The projection equations and conditions can be defined analytically or by their numerical values (non-analytically) at any locations of the mapping region using numerical methods with the desired accuracy.

A conformal map projection can be obtained by integrating Laplace's differential equation that describes a condition for conformal mapping of a rotational ellipsoid onto a plane:

$$\frac{\partial^2(\ln v)}{\partial q^2}+\frac{\partial^2(\ln v)}{\partial l^2}=0 \tag{1}$$

where:

$$v=m\cdot\frac{\cos(\varphi)}{\sqrt{1-e_p^2\cdot\sin^2(\varphi)}}; \tag{2}$$

and:

$$q=\ln\left[tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\cdot\left(\frac{1-e_p\cdot\sin(\varphi)}{1+e_p\cdot\sin(\varphi)}\right)^{\frac{1}{2}\cdot e}\right] \text{ and } l=\lambda-\lambda_0. \tag{3}$$

q and l - isometric latitude and isometric longitude,

φ and λ - geographic latitude and longitude,

λ_0 - geographic longitude of the central meridian of the area,

m - linear scale distortion along the meridian,

e_p - eccentricity of meridian ellipsis.

[2] *Characteristics of a projection are characterizing distortions created during the mapping. These functions can be linear scale in an arbitrary direction, extreme linear scales, principal directions, linear scales in the direction of the meridians and parallels, azimuths of the images of meridians and parallels, angle between the images of the meridians and parallels, the area scale, the distortion of azimuth, maximal angular distortion, etc.*

The CAMPREL projection also utilizes Chebyshev-Grave's theorem[3] (Chebyshev, 1856; Grave, 1911; Urmayev, 1947) stating that the best conformal map projection for a region is the one with the constant logarithm of linear scale along the region's boundary. Therefore, the following condition applies:

$$\ln(v)\big|_{\Gamma} = \ln\left[K \cdot \frac{\cos(\varphi)}{\sqrt{1-e_p^2 \cdot \sin^2(\varphi)}}\right] \quad (4)$$

where:

Γ - the boundary of the mapping area;

K - the value of the constant linear scale assigned along the mapping boundary.

Since for conformal mapping is valid, that the linear scale c in any direction is constant and therefore $m = c$, the function of the linear scale in an arbitrary direction can be obtained by solving the Laplace's differential equation (1) under the boundary condition (4). The solutions of Laplace's differential equation, by definition, are harmonic functions, like a harmonic polynomial u_k of degree k in equation (5), where P_k and Q_k are the real and imaginary parts of harmonic polynomial u_k.

$$u_k = (q + i \cdot l)^k = P_k + i \cdot Q_k; \qquad i^2 = -1 \quad (5)$$

One can obtain an expression for any degree of harmonic polynomials P_k and Q_k using Morozov's recurrent formulas (6) with given null terms $P_0 = 1$ and $Q_0 = 0$. Their equality can be proved by mathematical induction. (Mitrinović, 1989)

$$P_{j+1} = q \cdot P_j - l \cdot Q_j \quad P_0 = 1; P_{j+1} = q \cdot P_j - l \cdot Q_j \quad (6)$$

$$Q_{j+1} = q \cdot Q_j + l \cdot P_j \quad Q_0 = 0; Q_{j+1} = q \cdot Q_j - l \cdot P_j$$

The real P_k and imaginary Q_k parts are the solution of the of Laplace's differential equation (1) as well as their arbitrary linear combination presented in equation (7), where a_j and b_j are unknown Chebyshev's coefficients (Urmayev, 1947).

$$u_k = \sum_{j=0}^{k} a_j \cdot P_j + b_j \cdot Q_j \quad (7)$$

Equation (7) can further be simplified if the boundary of the mapping area (Γ) and isocoles are symmetrical in relation to the central meridian. These symmetries exclude imaginary part Q_j of the polynomial and the solution takes a form of a symmetrical harmonic polynomial presented in equation (8).

$$u_k = \sum_{j=0}^{k} a_j \cdot P_j \quad (8)$$

Using the boundary condition (4) and the symmetrical harmonic polynomial above, one can express the more general solution of the Laplace's differential equation for the complete mapping area as:

[3] *Chebyshev-Grave's theorem reads precisely:* In the class of all conformal map projections minimal ratio between maximal and minimal linear scale on the whole area of mapping, will be on the map projection having constant logarithm of linear scale along the contour of the mapping area.

$$\ln(c)=\sum_{j=0}^{k} a_j\cdot P_j-\ln\left[\frac{\cos(\phi)}{\sqrt{1-e_p^2\cdot\sin^2(\phi)}}\right] \tag{9}$$

As explained by Nestorov (1997), equation (9) can be formulated for $n \geq k + 1$ discrete points on the boundary of the mapping area as a system of equations presented with (10).

$$\ln(K)=\sum_{j=0}^{k} a_j\cdot P_j-\ln\left[\frac{\cos(\varphi_i)}{\sqrt{1-e_p^2\cdot\sin^2(\varphi_i)}}\right],\quad i=1,2,3,\ldots,n \tag{10}$$

With the constant linear scale, K assigned to the boundary of the mapping area, and with enough discrete points on the boundary, one can determine values of polynomial coefficients a_j using the least square method.

Knowing the coefficients of the symmetrical harmonic polynomial makes it possible to calculate the values of the linear scale c for all the points of the mapping region using equation (11), and meridian convergence γ with equation (12) according to Nestorov (1996). In equation (11), e is the base of the natural logarithm.

$$c=e^{\sum_{j=0}^{k} a_j\cdot P_j-\ln\left[\frac{\cos(\phi)}{\sqrt{1-e_p^2\cdot\sin^2(\phi)}}\right]} \tag{11}$$

$$\gamma=\sum_{j=0}^{k}-a_jQ_j \tag{12}$$

As already mentioned above, the four independent mapping characteristics in every point are needed for the complete map projection definition (Meshcheryakov, 1968). For the conformal mapping, it is enough to know two independent mapping characteristics, since the linear scale along the meridian m and the linear scale along the parallel n are both equal to the linear scale c, i.e. $m = n = c$ and also the angle between a meridian and a parallel always remains right ($\Theta = \pi/2$). Therefore, two mapping characteristics c and γ, defined with equations (11) and (12), satisfy the Meshcheryakov's theorem, and one can determine unique equations of the direct and inverse mapping and also functions of all other mapping characteristics.

2.2 Equations for direct mapping

When deriving equations of direct mapping based on isometric latitude (q) and longitude (l), $x = x(q, l)$, $y = y(q, l)$ the following two facts were taken into account:

1. The rectangular coordinates in the plane, x and y, are conjugated harmonic functions of isometric coordinates, and one can use an analytic function that determines the conformal mapping in the form of a harmonic polynomial:

$$\begin{aligned} y&=\sum_{j=1}^{n} A_j\cdot P_j+B_j\cdot Q_j \\ x&=\sum_{j=1}^{n} A_j\cdot P_j-B_j\cdot Q_j \end{aligned} \tag{13}$$

where A_i and B_i are real coefficients, and P_i and Q_i are real and imaginary parts of this harmonic polynomial.

2. For conformal mapping, it is known (Bugayevskiy, 1974; Pedzich, 2005; Tutić, 2009) to be valid:

$$\begin{aligned} x_q &= \nu \cdot \cos(\gamma), \quad x_l = -\nu \cdot \sin(\gamma) \\ y_q &= \nu \cdot \sin(\gamma), \quad y_l = \nu \cdot \cos(\gamma) \end{aligned} \tag{14}$$

where x_q, x_l, y_q, y_l, are partial derivatives of the mapping function, γ is the meridian convergence and the function ν is previously given by expression (2).

The polynomial real coefficients, A_i and B_i, can be determine using partial derivatives of the expression (13) by isometric latitude q.

$$\begin{aligned} y_q &= \sum_{j=1}^{n} A_j \cdot (P_j)_q + B_j \cdot (Q_j)_q \\ x_q &= \sum_{j=1}^{n} A_j \cdot (P_j)_q - B_j \cdot (Q_j)_q \end{aligned} \tag{15}$$

The partial derivative of a harmonic polynomial u_k. by isometric latitude q is:

$$(P_n + i \cdot Q_n)_q = n \cdot (q + i \cdot l)^{n-1} = n \cdot (P_{n-1} + i \cdot Q_{n-1}). \tag{16}$$

Therefore, the real and imaginary parts can be written as:

$$\begin{aligned} (P_n)_q &= n \cdot P_{n-1} \\ (Q_n)_q &= n \cdot Q_{n-1} \end{aligned} \tag{17}$$

Hence, the expressions (15) becomes:

$$\begin{aligned} y_q &= \sum_{j=1}^{n} j \cdot (A_j \cdot P_{j-1} + B_j \cdot Q_{j-1}) \\ x_q &= \sum_{j=1}^{n} j \cdot (A_j \cdot P_{j-1} - B_j \cdot Q_{j-1}) \end{aligned} \tag{18}$$

On the other hand, the relations in equations (14) are valid at each point of the mapping area. The meridian convergence (γ) and the function ν in these equations can be replaced using expressions (2), (11), and (12), and the numeric values of x_q and y_q can be calculated as follows:

$$\begin{aligned} y_q &= e^{\sum_{i=0}^{k} a_i \cdot P_i} \cdot \sin\left(\sum_{i=0}^{k} -a_i \cdot Q_i \right) = T_y \\ x_q &= e^{\sum_{i=0}^{k} a_i \cdot P_i} \cdot \cos\left(\sum_{i=0}^{k} -a_i \cdot Q_i \right) = T_x \end{aligned} \tag{19}$$

With known numeric values of partial derivatives y_q and x_q as T_y, and T_x, one can write for each point of the mapping area a system of two linear equations, where the coefficients A_j and B_j ($i = 1,2,3,\ldots,n$) are $2n$ unknown variables:

$$\sum_{j=1}^{n} j\cdot\left(A_j\cdot P_{j-1}+B_j\cdot Q_{j-1}\right)=T_y$$
$$\sum_{j=1}^{n} j\cdot\left(A_j\cdot P_{j-1}-B_j\cdot Q_{j-1}\right)=T_x \qquad (20)$$

To define the unknown coefficients A_j and B_j (j = 1,2,3,...,n), it is necessary to use at least n different points over the mapping area and solve the system of $2n$ linear equations with $2n$ unknowns. One can take a larger number of points than necessary and use the least-squares method to optimize the accuracy of the results obtained. Once the polynomial coefficients A_j and B_j are determined, the rectangular coordinates x and y can be calculated from the expression (13) for the whole mapping area.

For the calculation of geographic coordinates φ and λ based on known y and x coordinates, also known as inverse mapping, authors suggesting established numerical methods since the equations for the direct mapping are derived numerically.

2.3 Numerical distortion criteria for projection optimization

Since the idea of the CAMPREL is to find a projection satisfying both Chebyshev's (presented in the first subsection above) and variational criteria, this subsection presents numerical measures (distortion parameters) for projection optimization used to satisfy the lateral criterion. The "best projection" in a variational sense would, therefore, have minimal values of all these parameters.

All presented measures are based on linear distortion[4]. Besides maximal and minimal linear scales, *c_max* and *c_min*, the following measures were also utilized:

1. Jordan's total distortion criteria (Kavrayskiy, 1958):

$$E_J^2=\frac{1}{2\cdot\pi\cdot P}\int_P\int_0^{2\pi}(c-1)^2\cdot d\alpha\cdot dP \qquad (21)$$

For the case of the finite number of points, the simplified form can be used (Nestorov, 1996):

$$E_J=\sqrt{\sum_{i=1}^{n}(c_i-1)^2} \qquad (22)$$

2. Jordan-Kavraysky's total distortion criteria (Kavrayskiy, 1958):

$$E_{JK}^2=\frac{1}{2\cdot\pi\cdot P}\int_P\int_0^{2\pi}\left(\ln c\right)^2\cdot d\alpha\cdot dP \qquad (23)$$

And in the simplified form of the above formula for the finite number of points (Nestorov, 1996):

$$E_{JK}=\sqrt{\sum_{i=1}^{n}(\ln c_i)^2} \qquad (24)$$

[4] *When defining the projection quality criteria, it should be borne in mind that map projections generally cause inevitable distortions of the lengths, angles, and areas of the original surface, but also that mapping is possible in which eliminated either angular distortions (conformal projections) or distortions of area (equivalent projections). However, linear distortions are always present. Therefore, the overall measure of projection quality must contain linear distortions, namely linear distortions throughout the mapping area. In other words, the basic factor in defining the selection criteria and the quality of the projection should be the distribution of linear distortion over the entire mapping area.*

3. Range of linear distortions:

$$RLD = c\_max - c\_min \tag{25}$$

4. Relative linear scale:

$$RLS = \frac{c\_max\text{-}\ c\_min}{c\_max} \tag{26}$$

5. The ratio of maximal and minimal linear scales:

$$RMMS = \frac{c_{max}}{c_{min}} \tag{27}$$

6. Ratio of logarithms of maximal and minimal linear scales:

$$RLMMS = \frac{\ln(c\_max)}{\ln(c\_min)} \tag{28}$$

7. Range of absolute linear distortions:

$$RALD = d\_absmax - d\_absmin \tag{29}$$

8. Average absolute linear distortion:

$$AALD = \frac{\sum_{i=1}^{n} abs(d_i)}{n} \tag{30}$$

9. Root mean square of linear distortions:

$$RMSALD = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}} \tag{31}$$

where *c*, α, *P*, *dP*, *d_max*, *d_min*, *d_absmax*, and *d_absmin* are respectively: linear scale, azimuth, the area of mapping, a differential element of the area, maximal linear distortion, minimal linear distortion, the absolute value of maximal linear distortion and the absolute value of minimal linear distortion.

3 CAMPREL PROJECTION FOR THE AREA OF PAN-EUROPEAN MAPPING

3.1 Methodology for finding the CAMPREL projection

To find the CAMPREL projection for pan-European mapping area - the area bounded by parallels of 27°N and 71°N and meridians of 30°W and 45°E, (Annoni et al., 2001) - the task is to determine the degree (*k*) of the harmonic polynomial (u_k) and the constant value of the linear distortion along the area border (*K*) for which all numerical criteria presented in section 2.3 would be minimized. A computer program that generates and tests Chebyshev's projection variants were developed for this task.

Based on the boundary parallels and meridians, the program varies the degree of the harmonic polynomial (*k*), the constant value of the linear distortion along the area border (*K*), and the number of points on the contour of the region (*n*), all from expression (10), to create projection variants. It increases the degree of the harmonic polynomial (*k*) until the difference of the calculated total distortion (based on formulas (22) and (24)) for the two successive degrees of the harmonic polynomials is less than $3x10^{-4}$. For the initial value of the constant linear scale along the boundary meridians and parallels (*K*), a value of 1.00700 was taken and varied to 1.00710 in steps of 0.00002. This range is based on the estimation that the maximum positive linear distortion is expected to be approximately equal to the maximum negative linear distortion. The number of boundary points (*n*) varied from 36 to 60 until their number ceased to affect the accuracy of calculation of linear scales along the boundary meridians and parallels. The points along the border were evenly distributed. In total, the program generates 210 Chebyshev's projection variants.

For every variant, the program then calculates numerical criteria measures to test the projection quality (distortion parameters): Jordan's total distortion (E_J), Jordan-Kavraysky's total distortion (E_{JK}), maximal linear scale (*c_max*), minimal linear scale (*c_min*), range of linear distortions (*RLD*), relative linear scale (*RLS*), the ratio of maximal and minimal linear scales (*RMMS*), the ratio of logarithms of maximal and minimal linear scales (*RLMMS*), maximal linear distortion (*d_max*), minimal linear distortion (*d_min*), maximal absolute linear distortion (*d_absmax*), minimal absolute linear distortion (*d_absmin*), range of absolute linear distortions (*RALD*), average absolute linear distortion (*AALD*), and root mean square of absolute linear distortion (*RMSALD*). Each numerical measure is based on 3420 evenly distributed grid points (cross-sections of meridians and parallels for every degree of latitude and longitude) over the area of interest.

All variant cases are tested in less than one minute of CPU time.

3.2 The optimal CAMPREL projection and its comparison to ETRS-LCC projection

From the 210 projection variants, Table 1 gives Chebyshev's coefficients, $a_0 - a_{10}$, for the optimal CAMPREL projection designed for the pan-European mapping area. The total distortions of the projection have minimum values, and the absolute maximum positive linear distortion is approximately equal to the absolute maximum negative linear distortion value. The optimal variant has the degree of the harmonic polynomial 10, the constant linear scale along the border is 1.00706, and 60 evenly distributed points on the boundary of the mapping area are used to define the coefficients. The precision of values presented in Table 1 is limited to the accuracy of the numerical methods used to generate the variant. Authors believe that the precision here is more than sufficient for the purpose of mapping in the scales smaller or equal to 1:500,000.

Table 1: Chebyshev's coefficients of the optimal CAMPREL projection for Pan-European area.

j	a_j
0	-0.567667536
1	-0.783326923
2	-0.083929018
3	0.011493356
4	0.055463877
5	-0.019925516
6	0.003009703
7	0.003545522
8	-0.006029762
9	0.000535393
10	-0.003798708

The numerical quality criteria measures (distortion parameters) for ETRS-LCC projection and the optimal CAMPREL projection were calculated from a grid of points for each degree of latitude and longitude within the pan-European mapping area. In total, 3420 points were used. The results are presented in Table 2.

All distortion parameters are convincingly better for the CAMPREL projection compared to the ETRS-LCC. The Jordan's and Jordan-Kavraysky's distortion ratios of the two projections show that the CAMPREL projection is almost twice as good (the ratio is over 180%). Similarly, both mean values of linear distortion, average (AALD) and mean square (RMSALD), show the same outperformance. It should also be noted that the linear distortion range is about 39.17% better with the CAMPREL projection compared to the ETRS-LCC, as well as the absolute linear distortion range, which is improved for 55.70%. The linear distortion of the ETRS-LCC ranges from -0.034374 to 0.043704, while at CAMPREL projections symmetrically ranges from -0.028069 to 0.028069. With the CAMPREL projection, the maximum absolute linear distortion of conformally mapped pan-European area, therefore, drops to 0.028069.

Table 2: Numerical quality criteria measures (distortion parameters) for the ETRS-LCC and CAMPREL projections.

CRITERIA/PROJECTION	ETRS-LCC	CAMPREL	ETRS-LCC/CAMPREL
Number of points:	3420	3420	-
Jordan's total distortion - EJ	1.481778	0.821865	180.29%
Jordan-Kavrayskiy's total distortion - EJK	1.488330	0.826696	180.03%
Maximal linear scale - c_max	1.043704	1.028034	101.52%
Minimal linear scale - c_min	0.965626	0.971931	99.35%
Range of linear distortions - RLD	0.078078	0.056103	139.17%
Relative linear scale - RLS	7.48%	5.46%	137.08%
Ratio of maximal and minimal linear scales - RMMS	1.080857	1.057723	102.19%
Ratio of logarithms of maximal and minimal linear scales - RLMMS	-1.222914	-0.971120	125.93%
Maximal linear distortion - d_max	0.043704	0.028034	155.90%
Minimal linear distortion - d_min	-0.034374	-0.028069	122.46%
Maximal absolute linear distortion - d_absmax	0.043704	0.028069	155.70%
Minimal absolute linear distortion - d_absmin	0.000000	0.000000	-
Range of absolute linear distortions - RALD	0.043704	0.028069	155.70%
Average absolute linear distortion - AALD	0.022567	0.011804	191.19%
Root mean square of absolute linear distortion - RMSALD	0.025338	0.014054	180.29%

Figure 1 and 2 shows linear scale isocole maps and absolute linear distortion isocole maps for the ETRS-LCC and CAMPREL projections. These maps were generated from the values of linear scales, and absolute linear distortions in the grid of 3420 points used to calculate numerical criteria measures in Table 2.

When comparing projections in each figure, it is evident that the values of distortions are overall smaller in the CAMPREL projection compare to the ETRS-LCC, as it was indicated in Table 2. From the maps, one can also notice different distortion pattern of the CAMREL projection compares to the established ETRS-LCC. In the ETRS-LCC, the linear distortion changes only in one direction, south-to-north, and with fast decrease when approaching the standard parallels. In the CAMREL projection, the linear distortion changes radially and relatively evenly, in all directions, following the form of the border of the mapping area. This pattern is part of a very important feature of the CAMPREL projection that should be emphasized: the CAMPREL is an adaptive projection (Nestorov 1996 and 1997). What this means is that the distortion and its isocoles adapts and follow the boundary of the mapping region, which results in linear distortions optimally distributed and stretched within the subject area.

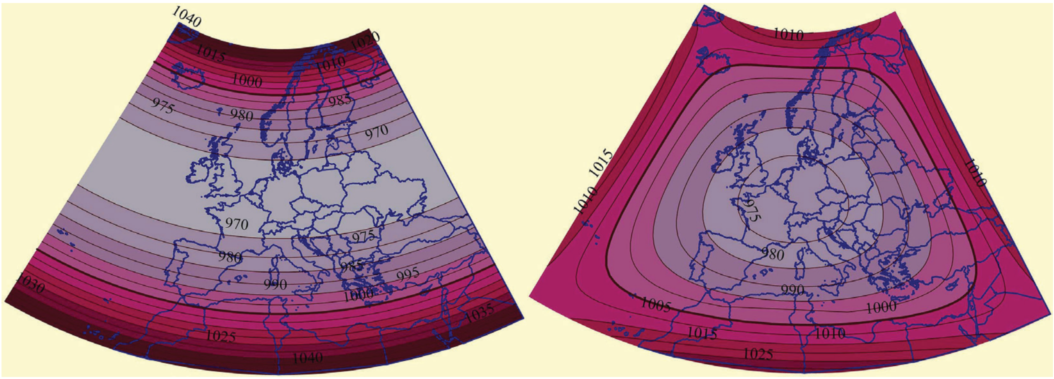

Figure 1: Isocole map of linear scale distribution (multiplied by 1000) for the ETRS-LCC projection (left), and for the optimal CAMPREL projection (right).

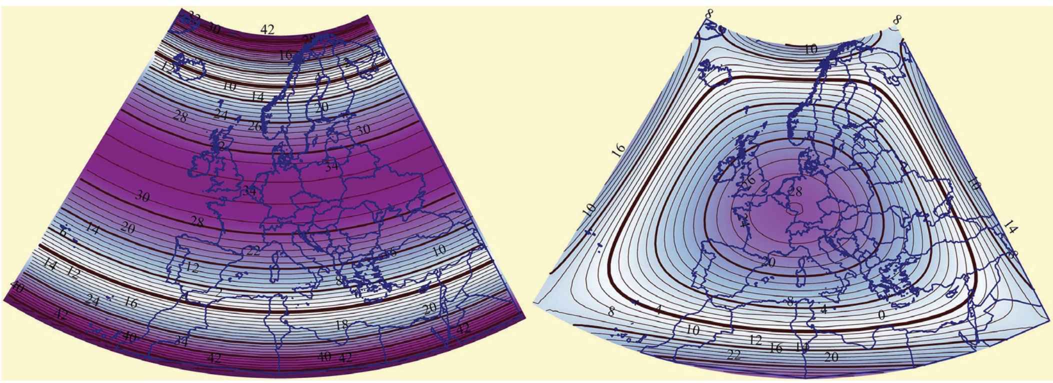

Figure 2: Isocole map of absolute linear distortion (multiplied by 1000) for the ETRS-LCC projection (left), and for the optimal CAMPREL projection (right).

4 CONCLUSION

The need for common European spatial datasets induces a strong demand for adopting standard map projections for pan-European mapping. The Lambert conformal conic projection (ETRS-LCC) was chosen for the scales equal or smaller to 1:500.000 that require the conformal mapping of an ellipsoid to a plane. However, if one adopts the general criteria when selecting a projection, which is "to select a projection in which the extreme distortions are smaller than would occur in any other projection used to map the same area" (Maling, 1989), then the general rule for selecting a projection for conformal mapping would be to achieve the least possible linear distortions. This paper proves that this rule has not been met with ETRS-LCC projection. It is shown that the CAMPREL projection defined for the area of interest has significantly better characteristics. The maximum absolute linear distortion is 0.028069 compare to the maximum absolute linear distortion of 0.043704 for the ETRS-LCC. Also, linear distortion distribution for the CAMPREL projection changes radially and relatively evenly, in all directions, following the form of the area boundary, while in ETRS-LCC, the linear distortion changes just in the direction of south-to-north and with faster decrease approaching the standard parallels.

Clearly, the CAMPREL projection designed for pan-European mapping better meets the projection selection criteria. Authors believe that the subject area cannot be better conformally mapped, that is, any other conformal projection will cause a maximum absolute linear distortion greater than 0.028069. They leave to other map projection experts to refute or confirm this claim.

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