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FINITE-ELEMENT THERMAL ANALYSIS OF A NEW COOLER DESIGN

TERMIČNA ANALIZA NOVE OBLIKE HLADILNE MIZE PO METODI KONČNIH ELEMENTOV

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A mathematical model of the cooling process of an octagonal cross-section steel semi-product was constructed on the basis of the finite-element analysis technique. The temperature variation throughout the structure causes thermal contraction stresses and deformations. Transient heat-conduction analyses and thermal stress analyses were performed to determine the allowable rate of cooling with regard to the temperature distribution within the structure and the stresses and deformations resulting from the variation of the heat-transfer coefficient of forced convection.

Keywords: finite-element analysis, thermal stress analysis, continuous casting steel semi-products, cooling rate

Na osnovi metode končnih elementov je bil razvit matematični model procesa ohlajanja jeklenega polproizvoda z osmerokotnim prerezom. Sprememba temperature po prerezu povzroči termične krčilne napetosti in deformacije. Izvršene so bile analize toplotne prevodnosti in toplotnih napetosti z namenom, da se določi dopustna hitrost ohlajanja z upoštevanjem porazdelitve temperature po prerezu ter napetosti in deformacije zaradi spremembe koeficienta prenosa toplote pri prisiljeni konvekciji.

Ključne besede: metoda končnih elementov, toplotne napetosti, polproizvodi, kontinuirno lito jeklo, hitrost ohlajanja

1 INTRODUCTION

Finite-element thermal analysis has been used to estimate the temperature changes during material testing under hot-working conditions [1,2] and as a general procedure for thermal optimization to determine an optimum set of design variables [3,4]. In ref. [5] a mathematical model based on the finite-element analysis technique was constructed to simulate the cooling process of an octagonal cross-section steel semi-product in conditions of natural convection. Transient heat-conduction analysis is performed to determine the temperature distribution within the structure and the cooling rate. For the verification of the model, the obtained results are compared with those obtained from measurements of cooling parameters in real conditions.

In order to increase the cooling capacity for continuous-casting steel semi-products, it was decided to examine a new cooler design with natural- and forced-air cooling. The finite-element analysis technique was used to model the cooling process. The thermal analysis was performed in order to establish the stress distributions within the octagonal cross-section steel semi-product as a result of the forced convection.

2 FINITE-ELEMENT COOLING MODEL

Since the steel semi-product under consideration (J-55, a product of Željezara Sisak) is very long (l = 3.6 m) in relation to its cross section, the length of a side of the octagonal is 101 mm, any cross-section will have the same temperature distribution, i.e., the temperature distribution is constant in the axial direction. Because of the "infinite" length in the Z-direction it is sufficient to consider only the cross-section in the XY plane. For this reason, the modeling is based on the assumption of a 2-D heat flow. The discretization of the physical problem over a continuous domain into numerous elements is the first step in any analysis. The cross-section under consideration was divided into two-dimensional four-node quadrilateral and three-node triangular elements, with temperature as the only degree-of-freedom at each node. Since the geometry and the boundary conditions of the cross-section are symmetrical with respect to the Y-axis, only one half of the cross-section was modeled. The discretization is presented in **Figure 1**.

Boundary conditions

The problem under consideration requires convection and radiation boundary conditions for all the elements around the closed domain. In the thermal analysis the evaluation of the heat-exchange coefficients by radiation, natural and forced convection is critical to the reliability of the results. In this paper the mean heat-transfer coefficients, h, over the entire length of the individual boundary surfaces, under the natural convection conditions, were calculated with the following expressions based on experimental data [6]:

1. The empirical correlation for the natural convection on the vertical wall, which applies to laminar flow only and holds for all values of the Prandtl number,

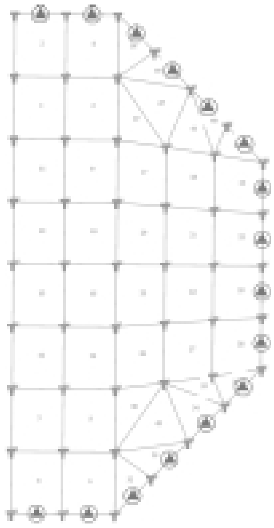

Figure 1: Discretization of the half octagonal cross-section
Slika 1: Diskretizacija polovice osmerokotnega prereza

is given for a uniform wall temperature (i.e., an isothermal surface) by

$$Nu_m = 0.68 + \frac{0.67(Gr_L Pr)^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}$$

for $10^{-1} < Gr_L Pr < 10^9$ (1)

where the characteristic length L is the height of the vertical boundary surfaces.

The physical properties are evaluated at the air film temperature

$$T_f/(\mathrm{K}) = 1/2\ (T_w - T_\infty) \quad (2)$$

2. The average Nusselt number for natural convection on a horizontal plate depends on whether the surface is facing up or down and whether the octagonal surface is warmer or cooler than the surrounding air. In this paper the mean Nusselt number for free convection on horizontal surfaces at uniform temperature was calculated with the following expressions:

- for the horizontal hot surface facing upward:

$$Nu_m = 0.54\ (Gr_L Pr)^{1/4} \quad (3)$$

for $10^5 < Gr_L Pr < 2{\cdot}10^7$,

$$Nu_m = 0.14\ (Gr_L Pr)^{1/3} \quad (4)$$

for $2 \cdot 10^7 < Gr_L Pr < 3 \cdot 10^{10}$

- for the horizontal hot surface facing downward:

$$Nu_m = 0.27\ (Gr_L Pr)^{1/4} \quad (5)$$

for $3 \cdot 10^5 < Gr_L Pr < 3 \cdot 10^{10}$

The characteristic length L of the horizontal surface can be taken as the arithmetic mean of the two dimensions of a rectangular surface. However, recent correlations suggest that an improved accuracy may be obtained if the characteristic length L for the horizontal surface is defined as

$$L/(\mathrm{m}) \approx \frac{A}{P} \quad (6)$$

where A is the surface area and P is the perimeter that encloses the area.

3. The heat-transfer coefficient for natural convection on an inclined surface can be predicted by the vertical plate formulas if the gravitational term in the Grashof number is adjusted to accommodate the effect of the inclination. The orientation of the inclined surface, whether the surface is facing upward or downward, is also a factor that affects the Nusselt number. The following expressions were used:

- for the inclined hot surface facing downward

$$Nu_m = 0.56\ (Gr_L Pr \cos\theta)^{1/4} \quad (7)$$

for $+\theta < 88°$, $10^5 < Gr_L Pr < 10^{11}$

- for the inclined hot surface facing upward

$$Nu_m = 0.145\ [(Gr_L Pr)^{1/3} - (Gr_c Pr)^{1/3}] + 0.56\ (Gr_L Pr \cos\theta)^{1/4} \quad (8)$$

for $Gr_L Pr < 10^{11}$, $Gr_L > Gr_c$, and $-15° < \theta < -75°$.

Here, the value of the transition Grashof number Gr_c depends on the angle of inclination θ. In Eqs. (5-6) all physical properties are evaluated for the mean temperature

$$T_m/(\mathrm{K}) = T_\infty - 0.25\ (T_w - T_\infty) \quad (9)$$

The Grashof number is defined as

$$Gr_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \quad (10)$$

and the mean Nusselt number over the length L as

$$Nu_m = \frac{h_m L}{k} \quad (11)$$

The average heat-transfer coefficient for the flow of air across a single long body of octagonal cross-section, i.e., h_m over the entire length of the single boundary surfaces under the forced convection conditions, was calculated using the following simple Jacob's relationship [6]

$$Nu_m = \frac{h_m D_e}{k_f} = c\left(\frac{u_\infty D_e}{\nu_f}\right)^n \cdot Pr^{1/3} \quad (12)$$

where the constant c and the exponent n are

Nu_m	C	n
$5{\cdot}10^3$–$1.95{\cdot}10^4$	0.160	0.638
$1.95{\cdot}10^4$–10^5	0.0385	0.782

The characteristic dimension D_e is the octagonal outer diameter (= 0.264 m). The properties for use with Eq. (12) are evaluated at the film temperature according to Eq. (2) as indicated by the subscript *f*. Actually, the local value of the heat-transfer coefficient $h(\theta)$ varies with the angle θ around the octagonal section[7], but for low Reynolds numbers (below 70 800 and 101 300), as in this case, the variation is minimal.

The above convective coefficients are temperature-dependent. In specifying the convective coefficients for a given element surface the temperature-scaling factor curves were referenced to give the dependence of the coefficient on temperature.

Radiant boundary conditions are imposed on elemental surfaces to take into account incident/emitted radiation energy. The general equation governing the heat exchange by radiation between two surfaces was used in the form

$$q/(\mathrm{W/m}) = \sigma\, \varepsilon\, F_{1-2}\,(T_s^4 - T_\infty^4) \qquad (13)$$

The program needs the value of the Stefan-Boltzman constant ($\sigma = 5.669\cdot10^{-8}$ W/(m^2 K^4)), the "view factor" ($F_{1-2} = 1$) and the emissivity factor ($\varepsilon = 0.3$). The emissivity was assumed to be temperature independent.

Initial conditions and material properties

The initial temperature value, T = 1473 K, was specified for the overall structure (nodes) of the octagonal cross-section at the beginning of the cooling. The ambient temperature is assumed to have the measured time-independent value (T_∞ = 286 K).

The steel semi-product is cooled by radiation and:

- Natural convection in atmospheric air for 10 min;
- Forced convection up to the environment temperature in a free-stream velocity of atmospheric air u_∞ = 10 m/s at T_∞ = 286 K.

In this model all the thermal and heat-transfer properties were assumed to be directionally independent, as in the "isotropic" material model, i.e., they do not change with any orthogonal transformation of the axes.

The material properties, such as density (ρ), specific heat (c_p) and thermal conductivity, are assumed to be temperature dependent [8,9] (**Table 1**).

Table 1: Material properties of the steel J-55
Tabela 1: Lastnosti jekla J-55

T/K	293	373	473	573	673	773	873	973	1073	1173
ρ/(kg/m^3)	7850	7827	7794	7759	7724	7687	7648	7611	7599	7584
c_p/(J/(kg K))	469	490	519	553	599	657	741	879	699	703
k/(W/(mK))	54	51	48	46	42	38	34	30	25	26

3 THERMAL STRESSES ANALYSIS

A thermal stress arises because of the existence of a temperature gradient in a body. During the heating of a body, the external surfaces are hot and tend to expand but are restrained by the cooler center. This causes compression at the surface and tension in the center. During cooling, the temperature of the surface layers of the metal is lower than that of the deeper layers. After cooling of the surface layers to a temperature at which the metal loses its plasticity the maximum surface-tension stress is achieved.

If the metal is plastic enough, it deforms plastically under the action of thermal stresses. With less plastic metals, the stresses can exceed the elastic limit, and if this limit is close to the ultimate strength, then fracture can occur. The effect of temperature on the static properties of steel is such that the tensile strength changes little up to a certain temperature. Above this limit it falls rapidly. The yield strength, however, decreases continuously as the temperature is increased. As might be expected, there is a substantial increase in ductility at higher temperatures. Steels are sufficiently elastic only at temperatures up to about 773 K, except for some special grades, and become plastic above that temperature. With carbon steels this transformation takes place at about 673 K. Therefore, thermal stresses can be dangerous only below that temperature in the case of the maximum temperature variation throughout the body.

When the body is loaded by a uniaxial stress state, the stress or the strength can be compared directly with the *yield strength*, the *ultimate strength* or the *shear strength*. The problem becomes more complicated in the case of a biaxial or a tri-axial stress state. In such cases there are a multitude of stresses, but only one significant strength. A number of *failure theories* have been proposed to answer the question of whether the body is safe during heating or cooling. In this paper the *maximum-normal-stress theory* and the *von Mises strain-energy theory* were used.

The *maximum-normal-stress theory* states that failure occurs whenever one of the three principal stresses reaches or exceeds the ultimate strength, i.e.,

$$\sigma_1 > \sigma_2 > \sigma_3 \qquad \sigma_1 \geq R_m \qquad (14)$$

where R_m *is the ultimate strength* (Pa).

The *maximum-strain-energy theory* [10] predicts that failure by yielding occurs when the total strain energy in a unit volume reaches or exceeds the strain energy in the same volume corresponding to the ultimate strength in tension or in compression. The von *Mises stress* σ' (after Dr. R. von Mises, who contributed to the theory), which originated from the *distortion-energy theory,* is

$$\sigma = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}\right]^{\frac{1}{2}} \qquad (15)$$

Thermal stress analysis was performed in order to determine the thermal stresses and deformations of the structure as a consequence of the temperature distribution within the structure.

Table 2 lists approximate values [8,9] for the *coefficients of thermal expansion* α (linear mean coefficients

Table 2: Coefficients of thermal expansion and mechanical properties of the steel J-55
Tabela 2: Koeficienti toplotnega raztezka in mehanske lastnosti za jeklo J-55

T/K	293	373	473	573	673	773	873
α/(10^6 K^{-1})	12	12.9	13.6	14.2	14.6	15.0	15.2
E/GPa	206	197	187	156	168	–	160
R_m/MPa	–	520	635	670	630	450	310

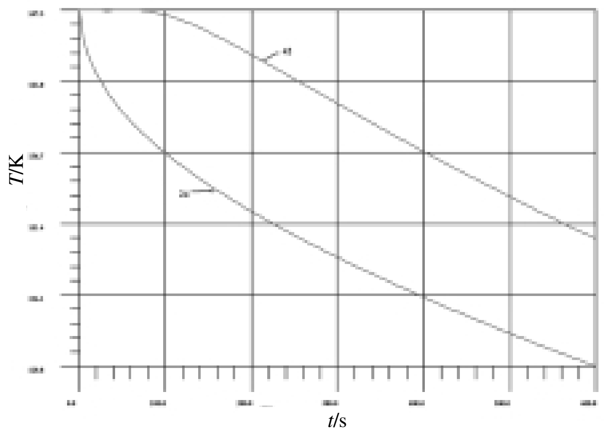

Figure 2: Temperature history obtained by model under the natural convection conditions
Slika 2: Temperaturna zgodovina, izračunana z uporabo modela pri naravni konvekciji

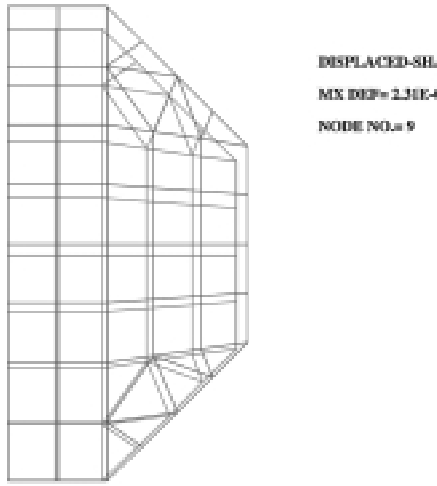

Figure 4: Resultant deformations
Slika 4: Toplotne napetosti

for the temperature range from 293 K to *T*), *ultimate strength* R_m and *modulus of elasticity E*, which were used as material data in this analysis. The value of *Poisson's ratio* ν is taken as 0.3.

4 RESULTS OF THE SIMULATION

Nonlinear transient heat-transfer analysis is performed when the material properties and the boundary conditions are time dependent and the process involves radiation. These types of problem are solved using an incremental iterative solution technique. In this case of non-linear transient heat transfer analysis of the cooling of the octagonal steel semi-product, the modified Newton-Raphson methods, the option of recalculating the conductivity matrix at the beginning of each increment, was used in conjunction with the trapezoidal Crank-Nicholson integration techniques.

The heat flux tolerance (0.001) was specified to control the accuracy of the converged solution.

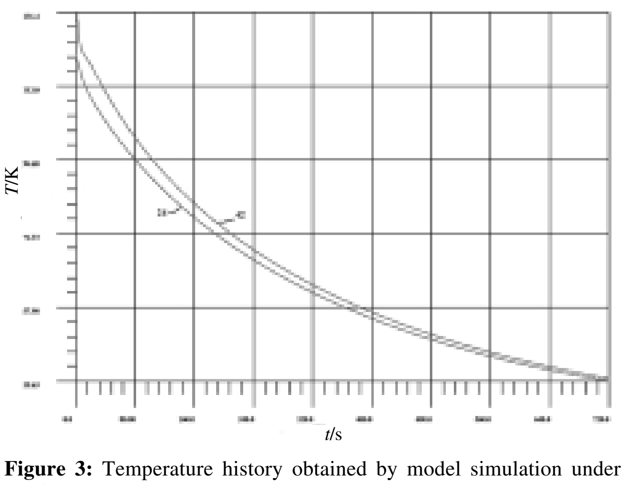

Figure 3: Temperature history obtained by model simulation under the forced convection conditions
Slika 3: Temperaturna zgodovina, dosežena pri modelni simulaciji s prisiljeno konvekcijo

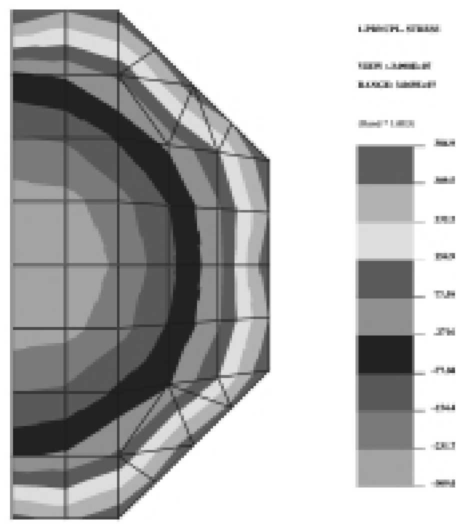

Figure 5: Principal stresses σ_1
Slika 5: Glavne napetosti σ_1

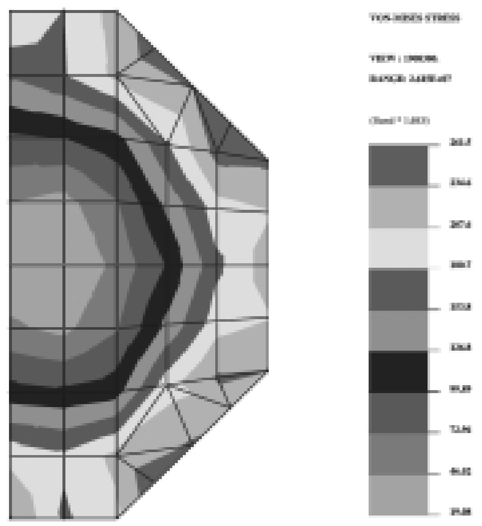

Figure 6: Von Mises stresses σ'
Slika 6: Von Misesove napetosti σ'

The transient heat-conduction analysis is performed for:

- 10 min (600 s) under natural convection conditions. The change of temperature of the nodes 42 (the lower corner of the vertical side) and 24 (the center of the cross-section) is shown in **Figure 2**. Between these nodes the maximum temperature difference was reached;
- the period from 10 min to 130 min (7800 s) under the forced convection conditions. The change of temperature of the nodes 42 and 24 is shown in **Figure 3**.

According to the performed analysis, the maximum thermal stresses in the octagonal semi-product were reached after 2500 s from the beginning of the cooling. Up to this moment, the material had a permanent plastic deformation, and as a result of that it will be elastically deformed. Right at the beginning of the elastic deformation, the maximum temperature variation throughout the structure is achieved.

The calculated resultant deformations (displacement) are shown in **Figure 4**.

The following stress intensities are computed from the stress components:

- principal stresses σ_1 (**Figure 5**);
- von Mises stresses σ' (**Figure 6**).

The maximum deformation is $2.31 \cdot 10^{-3}$ m (Node 9), the maximum values of the principal stress σ_1 and of the von Mises stress σ' were reached.

5 CONCLUSIONS

The results shown in **Figures 4, 5 and 6** provide useful information for the interpretation of the behavior of the material with respect to the various theories of failure. In this case, the maximum thermal stresses are much lower than the ultimate strength because the carbon steel was cooled in atmospheric air. The presumed value of 10 m/s is close to the maximum value to be applied in real cooling conditions. Accordingly, further calculations on the influence of air free-stream velocity change on the cooling rate were not performed. In spite of that, this verified the mathematical model based on the finite-element analysis technique, which was developed to simulate the cooling process of an octagonal cross-section steel semi-product, might be useful, especially for analyses of the cooling and heating of special grades of alloy steels.

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