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Imprecise data envelopment analysis model for robust design with multiple fuzzy quality responses

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A B S T R A C T

In this research, Imprecise Data Envelopment Analysis (IDEA) model was utilized to improve fuzzy multiple responses in robust design. The combination of process factor levels at each experiment was considered as a Decision Making Unit (DMU) with responses treated as inputs and outputs for all DMUs. The Fuzzy C-Means Clustering (FCMC) technique is used to fit the response fuzziness by clustering the average values, relative to each response, into a suitable number of clusters with triangular / trapezoidal membership functions. IDEA models were used to estimate the fuzzy triangular / trapezoidal efficiency values for each DMU. Finally, the preference degree-based ranking approach was used to discriminate between the fuzzy efficiency values and identifying the best combination of factors levels that would improve fuzzy multiple responses. Two case studies are utilized to illustrate the proposed approach, including optimizing wire electrical discharge machining and sputtering process parameters. The results showed that the proposed approach provides better anticipated improvements than the fuzzy multiple regression based approach. This approach would provide great assistant to process engineers in improving process performance with fuzzy multiple responses over a wide range of business applications.

A R T I C L E I N F O

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1. Introduction

To survive in today's competitive markets, manufacturers produce their products considering multiple quality responses of main customer interest. Therefore, engineers aim to determine the best combination of process settings that reduces the variability of the quality responses and simultaneously shift the mean to the desired target [1]. For this reason, several approaches are proposed to optimize product/process performance with multiple responses [2-8].

In reality, dealing with response fuzziness becomes a challenging task for process engineers. The fuzzy responses are captured as an imprecise value rather than crisp one. The imprecise value could be interval, triangular, trapezoidal, or even linguistic. Conceptually, response fuzziness can be justified by four reasons [9, 10]. The first reason is the vague and complex process behaviour which may be explained by the nondiscretionary factors. The second is the inability to fix the process settings at precise values or in words the fuzziness inherent in the settings physical values. The third is the qualitative nature of the response itself. Finally, the fuzziness occurs due to customer preference. Several approaches are proposed to deal with response fuzziness problem in robust design [9-16].

The Taguchi method utilizes an orthogonal array to provide experimental format. Let DMU_j denotes the combination of factor settings at each experiment in Taguchi's orthogonal array. For DMU_j, the fuzzy inputs and outputs are denoted by $\breve{\tilde{y}}_{ij}$ and $\breve{\tilde{y}}_{rj}$, respectively. In fuzzy goal programming (FGP), the fuzzy efficiency value, $\breve{E}_j$, of each DMU_j is calculated as follows. For a fuzzy triangular inputs, $\breve{\tilde{y}}_{ij}$, and outputs, $\breve{\tilde{y}}_{rj}$, values, the relative efficiency value of each DMU_j is also considered as a fuzzy triangular value with three parameters, E_j^L , E_j^M, and E_j^U, which represent the lower, nominal, and the upper efficiency values, respectively. That is, $\tilde{E}_j = (E_j^L, E_j^M, E_j^U)$ and is formulated as shown in Eq. 1:

$$\tilde{E}_j = \begin{cases} \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^L}{\sum_{i=1}^{m} vi\, y_{ij}^U} \\ \\ \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^M}{\sum_{i=1}^{m} v_i\, y_{ij}^M} \\ \\ \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^U}{\sum_{i=1}^{m} v_i\, y_{rj}^L} \end{cases} \tag{1}$$

where s and m denote the number of outputs and inputs, respectively. Let the lower and upper inputs values are denoted by y_{ij}^L and y_{ij}^U, respectively. Also, the lower and upper outputs values are expressed as y_{rj}^L and y_{rj}^U, respectively. For a DMU, $DMU_{k\in j}$, under consideration, the upper desired efficiency value, E_k^U, is calculated by using Model 1.

Model 1 is:

$$E_k^U = max \sum_{r=1}^{s} u_r\, y_{rk}^U \tag{2}$$

subject to

$$\sum_{i=1}^{m} v_i\, y_{ik}^L = 1 \tag{3}$$

$$\sum_{r=1}^{s} u_r y_{rk}^U - \sum_{i=1}^{m} v_i\, y_{ik}^L \leq 0 \tag{4}$$

$$\sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i\, y_{ij}^U \leq 0, \quad \forall\, j \neq k \tag{5}$$

$$u_r, v_i \;\geq \varepsilon\,, \forall\, r, i \in q \tag{6}$$

where u_r, and v_i, are the weights assigned to the outputs and inputs, and ε is the non-Archimedean value. In model 1, the objective function seeks to maximize the upper relative efficiency for each DMU_k under the most favorable situation. The first constraint keeps the inputs weighted sum of the DMU_k equals one. The second and the third constraints represent the most favorable condition for DMU_k, where the highest score of the upper efficiency value is attained by settings the relative interval outputs at their upper bounds and the interval inputs at their lower bounds. Meanwhile, the outputs of all other $DMU_{j\neq k}$ reach their corresponding lower bounds and the interval inputs reach their corresponding upper bounds. The last constraint keeps the inputs and outputs weights larger than a small positive value. Similarly, the lower efficiency value E_k^L, is calculated by using Model 2.

Model 2 is:

$$E_k^L = max \sum_{r=1}^{s} u_r\, y_{rk}^L \tag{7}$$

subject to

$$\sum_{i=1}^{m} v_i\, y_{ik}^U = 1 \tag{8}$$

$$\sum_{r=1}^{s} u_r y_{rk}^L - \sum_{i=1}^{m} v_i\, y_{ik}^U \leq 0 \tag{9}$$

$$\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i\, y_{ij}^L \leq 0, \quad \forall\, j \neq k \tag{10}$$

$$u_r, v_i \ \geq \varepsilon\, , \forall\, r, i \tag{11}$$

In Model 2, the objective function seeks to maximize the lower relative efficiency E_k^L, for each DMU_k under the least favorable situation. The first constraint keeps the upper weighted sum of the DMU_k inputs equals one. The second and the third constraints represent the least favorable condition for DMU_k, where the highest score of the relative efficiency value is attained by setting the relative interval outputs at their lower bounds and the interval inputs at the upper bounds, while the interval outputs of all other $DMU_{j \neq k}$ reach their relative upper bounds and the interval inputs reach their corresponding lower bounds. The last constraint keeps the inputs and outputs weights larger than a small positive value.

Further, let the middle inputs and outputs values are denoted by y_{ij}^M and y_{rj}^M , respectively. Then, for DMU_k the nominal efficiency value, E_k^M, is calculated by using Model 3 as follows:

Model 3 is:

$$E_k^M = max \sum_{r=1}^{s} u_r\, y_{rk}^M \tag{12}$$

subject to

$$\sum_{i=1}^{m} v_i\, y_{ik}^M = 1 \tag{13}$$

$$\sum_{r=1}^{s} u_r y_{rk}^M - \sum_{i=1}^{m} v_i\, y_{ik}^M \leq 0, \qquad j = k \tag{14}$$

$$\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i\, y_{ij}^L \leq 0, \quad \forall\, j \neq k \tag{15}$$

$$u_r, v_i \ \geq \varepsilon\, , \forall\, r, i \in q \tag{16}$$

The objective function in Model 3 seeks the optimal setting of outputs and inputs weights, u_r and v_i, that maximize the nominal efficiency value, E_k^M, for each DMU_k. The second and the third constraints keep the input weighted sum for each DMU_k constant and at the same time the relative efficiency value less than one. The fourth constraint represents the nominal desired condition for each DMU_k such that the nominal efficiency value is achieved when its relative outputs and inputs values reach their middle level, while the outputs reach their corresponding higher levels and the inputs reach their corresponding lower levels for $DMU_{j \neq k}$. The last constraint keeps the values of the inputs and outputs weights more than a small non Archimedean variable.

On the other hand, for a fuzzy trapezoidal inputs, $\breve{\tilde{y}}_{ij}$ and outputs $\breve{\tilde{y}}_{rj}$, values the relative efficiency value of each DMU_j have four parameters, E_j^L, E_j^{LM}, E_j^{UM}, and E_j^U which represent the lower, lower mid, upper mid, and the upper efficiency values, respectively. The fuzzy trapezoidal efficiency value of each DMU_j can be written as $\tilde{E}_j = (E_j^L, E_j^{LM}, E_j^{UM}, E_j^U)$ which is shown in Eq. 17.

$$\tilde{E}_j = \begin{cases} \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^L}{\sum_{i=1}^{m} v_i\, y_{ij}^U} \\[2ex] \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^{LM}}{\sum_{i=1}^{m} v_i\, y_{ij}^{LM}} \\[2ex] \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^{UM}}{\sum_{i=1}^{m} v_i\, y_{ij}^{UM}} \\[2ex] \dfrac{\sum_{r=1}^{s} u_r\, y_{rj}^U}{\sum_{i=1}^{m} v_r\, y_{rj}^L} \end{cases} \tag{17}$$

Then, Model 1 and Model 2 are used to calculate the upper and lower relative efficiencies for each DMU_j, respectively. The lower mid efficiency value, E_j^{LM}, is calculated as follows:

Model 4 is:

$$E_k^{LM} = max \sum_{r=1}^{s} u_r\, y_{rk}^{LM} \tag{18}$$

subject to

$$\sum_{i=1}^{m} v_i\, y_{ik}^{LM} = 1 \tag{19}$$

$$\sum_{r=1}^{s} u_r y_{rk}^{LM} - \sum_{i=1}^{m} v_i\, y_{ik}^{LM} \leq 0, \tag{20}$$

$$\sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i\, y_{ij}^{L} \leq 0, \quad j \neq k \tag{21}$$

$$u_r, v_i \ \geq \varepsilon, \forall\, r, i \in q \tag{22}$$

In Model 4, the lower mid values of the inputs are set as $\bar{y}_{ij}^{LM}$, while the lower mid values of the outputs are set as $\bar{y}_{rj}^{LM}$. The objective function seeks the optimal setting of outputs and inputs weights u_r, v_i that maximize the relative efficiency value, E_k^{LM}, for each DMU_k. The second and the third constraints keep the lower mid efficiency value for each DMU_k less than one and the input weighted sum equal to one. The fourth constraint represents the lower mid desired condition for each DMU_k such that the lower mid efficiency value is achieved when its relative outputs and inputs values are at their lower mid-level, while the outputs and inputs for $DMU_{j \neq k}$ are at their corresponding higher and lower relative levels respectively. The last constraint keeps the values of the inputs and outputs weights larger than a small non Archimedean variable [17].

By the same way, the upper mid efficiency value, E_k^{UM}, is calculated by using Model 5 as follows.

Model 5 is:

$$E_k^{UM} = max \sum_{r=1}^{s} u_r \, y_{rk}^{UM} \tag{23}$$

subject to

$$\sum_{i=1}^{m} v_i \, y_{ik}^{UM} = 1 \tag{24}$$

$$\sum_{r=1}^{s} u_r y_{rk}^{UM} - \sum_{i=1}^{m} v_i \, y_{ik}^{UM} \leq 0, \tag{25}$$

$$\sum_{r=1}^{s} u_r y_{rj}^{U} - \sum_{i=1}^{m} v_i \, y_{ij}^{L} \leq 0, \quad j \neq k \tag{26}$$

$$u_r, v_i \ \geq \varepsilon \, , \forall \, r, i \in q \tag{27}$$

Utilizing the Models 1 to 5, the optimal factor settings can then be determined. Therefore, this research proposes an extension to ongoing research by proposing a procedure for solving the fuzzy multiple responses problem in robust design using DEA approaches. The remaining of this research including introduction is organized as follows. Section two presents the proposed approach. Section three illustrates the proposed approach using two cases. Section four compares the results. Finally, section four highlights the research conclusions.

2. The proposed approach

In robust design method, several combinations of process factor levels are conducted to determine the best combination that improves multiple responses of main concern. The proposed approach for solving the multiple fuzzy quality characteristics is outlined in the following steps:

Step 1: Let y_{qj} denotes the value of the q-th response at the j experiment. Then, the combination of factor levels at each experiment is treated as a DMU_j. Let $\bar{y}_{qj}$ be the response average of the-smaller-the-better (STB), the-larger-the-better (LTB) response or the quality loss value for the-nominal-the-best (NTB) response to a number of triangular or trapezoidal membership functions.

Step 2: The FCMC technique is used to cluster the average values of $\bar{y}_{qj}$ into a number of classes, d, each is treated either as a triangular or trapezoidal membership function. The parameter of each membership function is determined such that, the class center, c_{qd}, for the triangular membership function is considered as the most likely value, $\bar{y}_{qd}^{M}$. The upper, $\bar{y}_{qd}^{U}$, and lower, $\bar{y}_{qd}^{L}$, parameters are determined by the centers of the neighbor classes or the DMU with least considerable membership value. For the trapezoidal membership function, the lower mid, $\bar{y}_{qd}^{LM}$, and the upper mid, $\bar{y}_{qd}^{UM}$, of each class are considered as the DMUs of the largest membership value relative to the same class. The upper and lower parameters $\bar{y}_{qd}^{U}$ and $\bar{y}_{qd}^{L}$, are determined by the centers of the neighbor classes or the DMU with least considerable membership value.

Step 3: The fuzzy efficiency values are computed by using models 1 to 5. The upper mid values inputs are set as y_{ij}^{UM} and the upper mid outputs are set y_{rj}^{UM} in Model 5, which has similar to that of Model 4 except the fourth constraint represents the upper mid desired condition for each DMU_k where the upper mid efficiency value is achieved when its relative outputs and inputs values are at their upper mid-level, while the outputs and inputs reach their corresponding higher and lower levels respectively for $DMU_{j\neq k}$.

Step 4: The preference degree based ranking approach [18] is used for clear-cut discrimination among the DMUs. In this regard, the complete ranking order for *n* fuzzy efficiency values can be obtained as (1) the triangular efficiency values and (2) the trapezoidal efficiency values.

(1) For the triangular efficiency values:

In the preference degree based ranking approach, let $\tilde{E}_k = (E_k^L, E_k^M, E_k^U)$ and $\tilde{E}_{j\neq k} = (E_j^L, E_j^M, E_j^U)$, are two fuzzy triangular efficiency values. According to fuzzy arithmetic, there are four possible relationships to compare $\tilde{E}_k$ with $\tilde{E}_{j\neq k}$ as shown in Eq. 28.

$$P(\tilde{E}_k > \tilde{E}_{j\neq k}) = \\ = \begin{cases} 1, & \text{IF } (E_k^L \geq E_{j\neq k}^U) \\ 0, & \text{IF } (E_k^U \leq E_{j\neq k}^L), \\ \dfrac{\left(E_k^U - E_{j\neq k}^L\right)^2}{\left(E_k^U - E_{j\neq k}^L + E_{j\neq k}^M - E_k^M\right)\left(E_k^U - E_k^L + E_{j\neq k}^U - E_{j\neq k}^L\right)}, & \text{IF } (E_k^U > E_{j\neq k}^L) \cap (E_k^M \leq E_{j\neq k}^M), \\ \dfrac{\left(E_{j\neq k}^U - E_k^L\right)^2}{\left(E_{j\neq k}^U - E_k^L + E_k^M - E_{j\neq k}^M\right)\left(E_k^U - E_k^L + E_{j\neq k}^U - E_{j\neq k}^L\right)}, & \text{IF } (E_k^L < E_{j\neq k}^U) \cap (E_k^M > E_{j\neq k}^M). \end{cases} \tag{28}$$

The preference matrix $\boldsymbol{P_{k,j}}$ is calculated as follows:

$$\boldsymbol{P_{k,j}} = \begin{bmatrix} p_{k,j} & \breve{E}_{j=1} & \tilde{E}_{j=k} & \tilde{E}_{j=n} \\ \tilde{E}_{j=1} & 0.5 & p_{1,k} & p_{1,n} \\ \tilde{E}_{j=k} & \dots & 0.5 & p_{k,n} \\ \tilde{E}_{J=n} & \dots & \dots & 0.5 \end{bmatrix} \tag{29}$$

where $\boldsymbol{P_{k,j}}$ is the preference matrix for all DMUs. Find a row from the matrix, $\boldsymbol{P_{k,j}}$, whose elements except the diagonal are larger than or equal to 0.5. If this row corresponds to $\tilde{E}_k$, then DMU_k is considered as the most efficient DMU and its relative settings are the best. The k^{th} row is eliminated from the matrix. In the reduced matrix, if $\tilde{E}_{h\neq k}$ stands out with the largest preference values compared to the remaining efficiency values, then $\tilde{E}_{h\neq k}$ is ranked in the second place. Repeat this step until all of the fuzzy efficiency values are properly ranked.

(2) For the trapezoidal efficiency values:

Let $\tilde{E}_{\mathrm{k}}(E_k^L, E_k^{LM}, E_k^{UM}, E_k^U)$ and $\tilde{E}_{j\neq k}(E_{j\neq k}^l, E_{j\neq k}^{LM}, E_{j\neq k}^{UM}, E_{j\neq k}^U)$ be two fuzzy trapezoidal efficiency values. According to fuzzy arithmetic, there are five possible relationships to compare $\tilde{E}_k$ with $\tilde{E}_{j\neq k}$, which are stated in Eq. (30).

$$P(\tilde{E}_k > \tilde{E}_{j\neq k}) =$$

$$= \begin{cases} 1, & IF\ (E_k^L \geq E_{j\neq k}^U) \\ 0, & IF\ (E_k^U \leq E_{j\neq k}^L) \\ \dfrac{\left(E_k^u - E_{j\neq k}^l\right)^2}{\left(E_k^U - E_{j\neq k}^L + E_{j\neq k}^{ML} - E_k^{MU}\right)\left(E_k^U - E_k^L + E_k^{MU} - E_k^{ML} + E_{j\neq k}^{MU} - E_{j\neq k}^{ML} + E_{j\neq k}^U - E_{j\neq k}^L\right)}, & (E_k^U > E_{j\neq k}^{LM}) \cap (E_k^{UM} \leq E_{j\neq k}^{LM}) \\ \dfrac{\left(E_k^u - E_{j\neq k}^l + E_k^{UM} - E_{j\neq k}^{LM}\right)}{\left(E_k^U - E_k^L + E_k^{MU} - E_k^{ML} + E_{j\neq k}^{MU} - E_{j\neq k}^{ML} + E_{j\neq k}^U - E_{j\neq k}^L\right)}, & (E_k^{UM} > E_{j\neq k}^{LM}) \cap (E_k^{LM} \leq E_{j\neq k}^{UM}) \\ \dfrac{\left(E_{j\neq k}^u - E_j^l\right)^2}{\left(E_{j\neq k}^u - E_k^l + E_k^{ml} - E_{j\neq k}^{mu}\right)\left(E_k^U - E_k^L + E_k^{MU} - E_k^{ML} + E_{j\neq k}^{MU} - E_{j\neq k}^{ML} + E_{j\neq k}^U - E_{j\neq k}^L\right)}, & (E_k^{LM} > E_{j\neq k}^{UM}) \cap (E_k^L < E_{j\neq k}^U) \end{cases} \tag{30}$$

The preference matrix, $\boldsymbol{P_{k,j}}$, is calculated using Eq. (30):

$$\boldsymbol{P_{k,j}} = \begin{bmatrix} p_{k,j} & \check{E}_{j=1} & \tilde{E}_{j=k} & \tilde{E}_{j=n} \\ \tilde{E}_{j=1} & 0.5 & p_{1,k} & p_{1,\mathrm{n}} \\ \tilde{E}_{j=k} & \dots & 0.5 & p_{k,n} \\ \tilde{E}_{J=n} & \dots & \dots & 0.5 \end{bmatrix} \tag{31}$$

where $\boldsymbol{P_{k,j}}$ is the preference matrix for all DMUs. Repeat until all of the fuzzy efficiency values are properly ranked.

Step 5: The anticipated improvements are calculated by using the proposed approach, then the improvements gained by the proposed approach are compared to fuzzy multiple regression approach (FMRA).

3. Two cases for illustration

Two cases adopted in the literature are applied to illustrate the proposed approach. The first case deals with response fuzziness that is best fit by fuzzy triangular membership function, whereas the second case considers trapezoidal membership function as the best fit to response fuzziness.

3.1 Case I: Optimization of Inconel on machining of CNC WEDM process

Al-Refaie et al. [8] conducted nine experiments utilizing Taguchi's L9 array to optimize the multi quality responses of Inconel 718 on machining of CNC WEDM process using fuzzy multiple regression approach (FMRA). The two quality responses are surface roughness (SR), y_1, which is a STB type response and material removal rate (MRR), y_2, which is a LTB type response. Table 1 shows the four process factors considered which are: pulse in time (A), delay time (B), wire feed speed (C), ignition current (D) as well as the corresponding levels. The combination of factor settings at each experiment is treated as DMU_j, where the average values of SR, $\bar{y}_{1j}$, are considered as inputs, while the average values of MRR, $\bar{y}_{2j}$, are the outputs for DMUs. Table 1 displays the experimental results.

Table 1 Experimental data for WEDM process optimization

DMU_j	Process factors				Inputs	Outputs
	A	B	C	D	$\bar{y}_{1j}$	$\bar{y}_{2j}$
DMU_1	1	1	1	1	3.15	46.00
DMU_2	1	2	2	2	3.25	47.50
DMU_3	1	3	3	3	3.30	41.50
DMU_4	2	1	2	3	3.75	55.50
DMU_5	2	2	3	1	3.45	49.50
DMU_6	2	3	1	2	3.25	52.50
DMU_7	3	1	3	2	4.10	70.50
DMU_8	3	2	1	3	3.65	73.50
DMU_9	3	3	2	1	3.35	64.00

Table 2 The center values for the three triangular membership functions

Class	Membership function center	
	c_{1d}	c_{2d}
d_q^{low}	3.15	47
d_q^{medium}	3.65	57
d_q^{High}	4.00	67

Then, the Fuzzy C-Means Clustering (FCMC) technique is used to determine the center values for the three triangular membership functions which are listed in Table 2. Each defined class is considered as a triangular membership function, whose parameters are tuned such that the center of the relative class is considered as the most likely parameter, $\bar{y}_{qd}^{M}$, while the centers of the neighbor classes are considered as the upper, $\bar{y}_{qd}^{U}$, and lower, $\bar{y}_{qd}^{L}$, parameters. Consequently, the experiments results shown in Table 1 are transformed into the fuzzy triangular numbers shown in Table 3.

Model 1 is used to calculate the upper efficiency values, E_j^U for all DMUs. Similarly, Model 2 is used to calculate the lower efficiency values, E_j^L for all DMUs. Model 3 is used to calculate the nominal efficiency values, E_j^U for all DMUs. Models 1, 2, and 3 are solved and the fuzzy triangular relative efficiency values are shown in Table 3.

Table 3 Fuzzy efficiency values for WEDM process optimization

DMU_j	$\breve{y}_{1j}$	$\breve{y}_{2j}$	$\tilde{E}_k$
DMU_1	3.15 ≈ $(3.15,3.15,3.65)^{low}$	46.0 ≈ $(40,47,57)^{low}$	(0.460, 0.625, 1.000)
DMU_2	3.25 ≈ $(3.15,3.15,3.65)^{low}$	47.5 ≈ $(40,47,57)^{low}$	(0.460, 0.625, 1.000)
DMU_3	3.30 ≈ $(3.15,3.15,3.65)^{low}$	41.5 ≈ $(40,47,57)^{low}$	(0.460, 0.625, 1.000)
DMU_4	3.75 ≈ $(3.4,3.65,4.0)^{medium}$	55.5 ≈ $(47,57,67)^{medium}$	(0.550, 0.640, 1.000)
DMU_5	3.45 ≈ $(3.15,3.15,3.65)^{low}$	49.5 ≈ $(40,47,57)^{low}$	(0.460, 0.625, 1.000)
DMU_6	3.25 ≈ $(3.15,3.15,3.65)^{low}$	52.5 ≈ $(47,57,67)^{medium}$	(0.587, 0.731, 1.000)
DMU_7	4.1 ≈ $(3.65\ ,4.2,4.2)^{high}$	70.5 ≈ $(57,67,74)^{high}$	(0.555, 0.565, 1.000)
DMU_8	3.65 ≈ $(3.4,3.65,3.8)^{medium}$	73.5 ≈ $(57,67,74)^{high}$	(0.606, 0.752, 1.000)
DMU_9	3.35 ≈ $(3.15,3.15,3.65)^{low}$	64.0 ≈ $(57,67,74)^{high}$	(0.636, 0.864, 1.000)

Then, the preference degree based ranking approach is used for more clear-cut discrimination among the DMUs. Eq.(28) is used to calculate the preference matrix as shown in Table 4, where it is found that $DMU_{j=9}$ has the largest value in each column. The minimum of these nine largest values is 0.609. Hence, it is the most preferred DMU.

Table 4 Preference matrix for WEDM process optimization

DMU_j	Preference value $P(\tilde{E}_k > \tilde{E}_{j \neq k})$									Rank
	1	2	3	4	5	6	7	8	9	
DMU_1	0.500	0.500	0.500	0.461	0.500	0.357	0.458	0.306	0.227	5
DMU_2	0.500	0.500	0.500	0.461	0.500	0.357	0.458	0.306	0.227	5
DMU_3	0.500	0.500	0.500	0.461	0.500	0.357	0.458	0.306	0.227	5
DMU_4	0.539	0.539	0.539	0.500	0.539	0.383	0.491	0.327	0.240	4
DMU_5	0.500	0.500	0.500	0.461	0.500	0.357	0.458	0.306	0.227	5
DMU_6	0.643	0.643	0.643	0.617	0.643	0.500	0.589	0.435	0.328	3
DMU_7	0.542	0.542	0.542	0.509	0.542	0.411	0.500	0.359	0.359	4
DMU_8	0.694	0.694	0.694	0.673	0.694	0.565	0.640	0.500	0.390	2
DMU_9	0.773	0.773	0.773	0.759	0.773	0.672	0.719	0.609	0.500	1

3.2 Case II: Optimizing of the sputtering process parameters

Al-Refaie et al. [8] conducted eighteen experiments utilizing L_{18} array to optimize sputtering process parameters using fuzzy multiple regression based method. Five process factors considered, including: the R.F. power (P), the sputtering pressure (Q), the deposition time (R), the substrate temperature (S), and the post-annealing temperature (T). Further, three quality responses were considered including the electrical resistivity (ER), y_1, which is a STB type response, the deposition rate (DR), y_2, which is also a STB type response and the optical transmittance (OT), y_3, which is a LTB type response. The experimental results are shown in Table 5 in term of the responses average values, $\bar{y}_{qj}$. In Table 5, the average values of ER quality response $\bar{y}_1$, are considered as inputs, while the outputs are considered as the average values of DR, $\bar{y}_2$, and OT, $\bar{y}_3$. The FCMC technique is employed to categorize each response average values into three clusters. The class center value, c_{qd} with respect to each class are calculated and listed in Table 6. The trapezoidal membership functions for all responses are shown in Table 7. Further, the fuzzy trapezoidal efficiency values and preference degree matrix for sputtering process are listed in Tables 8 and 9, respectively.

Table 5 Experimental data for sputtering process optimization

Exp. No.	Process factors					Responses		
	P	Q	R	S	T	$\bar{y}_{1j}$	$\bar{y}_{2j}$	$\bar{y}_{3j}$
1	50.00	0.13	30.00	25.00	0.00	15.10	4.60	88.40
2	50.00	0.67	60.00	50.00	100.00	9.75	5.60	87.70
3	50.00	1.33	90.00	100.00	200.00	7.85	4.95	88.10
4	100.00	0.13	30.00	50.00	100.00	5.50	9.45	89.25
5	100.00	0.67	60.00	100.00	200.00	4.45	11.20	87.05
6	100.00	1.33	90.00	25.00	0.00	6.55	10.00	84.70
7	200.00	0.13	60.00	25.00	200.00	1.65	20.00	86.60
8	200.00	0.67	90.00	50.00	0.00	1.95	21.60	82.35
9	200.00	1.33	30.00	100.00	100.00	1.70	20.90	85.45
10	50.00	0.13	90.00	100.00	100.00	7.15	4.70	87.60
11	50.00	0.67	30.00	25.00	200.00	7.00	4.95	89.10
12	50.00	1.33	60.00	50.00	0.00	7.75	4.85	87.40
13	100.0	0.13	60.00	100.00	0.00	6.00	9.70	87.00
14	100.00	0.67	90.00	25.00	100.00	5.90	11.35	83.70
15	100.00	1.33	30.00	25.00	200.00	5.60	10.75	88.35
16	200.00	0.13	90.00	50.00	200.00	1.05	19.45	83.10
17	200.00	0.67	30.00	100.00	0.00	1.25	22.05	85.70
18	200.00	1.33	60.00	25.00	100.00	1.35	20.50	83.80

Table 6 The center values for the three triangular membership functions

Class	Membership function center		
	c_{1d}	c_{2d}	c_{3d}
Low (d^{low})	1.57	4.95	83.50
Medium (d^{medium})	6.70	10.50	86.50
High (d^{high})	14.80	21.00	88.40

Table 7 Fuzzy trapezoidal experimental data for sputtering process

DMU_j	$\bar{y}_{1j}$	$\bar{y}_{2j}$	$\bar{y}_{3j}$
DMU_1	15.10 ≈ $(6.7,14.8,15.1,15.1)^H$	4.60 ≈ $(4.6,4.6,4.95,10.5)^L$	88.40 ≈ $(86.5,88.4,90,90)^H$
DMU_2	9.75 ≈ $(6.7,14.8,15.1,15.1)^H$	5.60 ≈ $(4.95,9.7,11.35,21)^M$	87.70 ≈ $(86.5,88.4,90,90)^H$
DMU_3	7.85 ≈ $(1.57,6,7.3,14.8)^M$	4.95 ≈ $(4.6,4.6,4.95,10.5)^L$	88.10 ≈ $(86.5,88.4,90,90)^H$
DMU_4	5.50 ≈ $(1.57,6,7.3,14.8)^M$	9.45 ≈ $(4.95,9.7,11.35,21)^M$	89.25 ≈ $(86.5,88.4,90,90)^H$
DMU_5	4.45 ≈ $(1.57,6,7.3,14.8)^M$	11.20 ≈ $(4.95,9.7,11.35,21)^M$	87.05 ≈ $(86.5,88.4,90,90)^H$
DMU_6	6.55 ≈ $(1.57,6,7.3,14.8)^M$	10.00 ≈ $(4.95,9.7,11.35,21)^M$	84.70 ≈ $(83.5,86,87,88.4)^M$
DMU_7	1.65 ≈ $(1,1,1.95,5.5)^L$	20.00 ≈ $(10.5,19,22,22)^H$	86.60 ≈ $(83.5,86,87,88.4)^M$
DMU_8	1.95 ≈ $(1,1.57,1.95,6.7)^L$	21.60 ≈ $(12,19,22,22)^H$	82.35 ≈ $(82,82,83.5,86.5)^L$
DMU_9	1.70 ≈ $(1,1.57,1.95,6.7)^L$	20.90 ≈ $(10.5,19,22,22)^H$	85.45 ≈ $(83.5,86,87,88.4)^M$
DMU_{10}	7.15 ≈ $(1.57,6,7.3,14.8)^M$	4.70 ≈ $(4.6,4.6,4.95,10.5)^L$	87.6 ≈ $(83.5,86,87,88.4)^M$
DMU_{11}	7.00 ≈ $(1.57,6,7.3,14.8)^M$	4.95 ≈ $(4.6,4.6,4.95,10.5)^L$	89.1 ≈ $(86.5,88.5,90,90)^H$
DMU_{12}	7.75 ≈ $(1.57,6,7.3,14.8)^M$	4.85 ≈ $(4.6,4.6,4.95,10.5)^L$	87.4 ≈ $(86.5,88.4,90,90)^H$
DMU_{13}	6.00 ≈ $(1.57,6,7.3,14.8)^M$	9.70 ≈ $(4.95,9.7,11.35,21)^M$	87.00 ≈ $(83.5,86,87,88.4)^M$
DMU_{14}	5.90 ≈ $(1.57,6,7.3,14.8)^M$	11.35 ≈ $(4.95,9.7,11.35,21)^M$	83.70 ≈ $(82,82,83.5,86.5)^L$
DMU_{15}	5.60 ≈ $(1.57,6,7.3,14.8)^M$	10.75 ≈ $(4.95,9.7,11.35,21)^M$	88.35 ≈ $(86.5,88.5,90,90)^H$
DMU_{16}	1.05 ≈ $(1,1.57,1.95,6.7)^L$	19.45 ≈ $(10.5,19,22,22)^H$	83.10 ≈ $(82,82,83.5,86.5)^L$
DMU_{17}	1.25 ≈ $(1,1.57,1.95,6.7)^L$	22.05 ≈ $(10.5,19,22,22)^H$	85.70 ≈ $(83.5,86,87,88.4)^M$
DMU_{18}	1.35 ≈ $(1,1.57,1.95,6.7)^L$	20.50 ≈ $(10.5,19,22,22)^H$	83.80 ≈ $(82,82,83.5,86.5)^L$

Table 8 Fuzzy trapezoidal efficiency values for sputtering process

DMU_j	$\tilde{E}_k$
DMU1	(0.070,0.073,0.073,0.619)
DMU2	(0.117,0.121,0.123,1.000)
DMU3	0.116,0.121,0.123,1.000)
DMU4	(0.117,0.121,0.123,1.000)
DMU5	(0.117,0.121,0.123,1.000)
DMU6	(0.114,0.116,0.118,1.000)
DMU7	(0.229,0.235,0.244,1.000)
DMU8	(0.224,0.226,0.244,1.000)
DMU9	(0.229,0.235,0.244,1.000)
DMU10	(0.114,0.116,0.117,1.000)
DMU11	0.116,0.121,0.123,1.000)
DMU12	(0.116,0.121,0.123,1.000)
DMU13	(0.114,0.116,0.118,1.000)
DMU14	(0.112,0.112,0.114,1.000)
DMU15	(0.117,0.121,0.123,1.000)
DMU16	(0.224,0.226,0.244,1.000)
DMU17	(0.229,0.235,0.244,1.000)
DMU18	(0.224,0.226,0.244,1.000)

Table 9 Preference degree matrix for the fuzzy trapezoidal efficiency values (columns 1 to 9)

	Preference value $P(\tilde{E}_k > \tilde{E}_{j\neq k})$									Rank
	DMU_j									
DMU_k	1	2	3	4	5	6	7	8	9	
DMU1	0.500	0.320	0.320	0.320	0.320	0.324	0.207	0.212	0.207	7
DMU2	0.680	0.500	0.500	0.500	0.500	0.504	0.403	0.407	0.403	2
DMU3	0.679	0.499	0.50	0.499	0.499	0.503	0.402	0.406	0.402	3
DMU4	0.680	0.500	0.500	0.500	0.500	0.504	0.403	0.407	0.403	2
DMU5	0.680	0.500	0.500	0.500	0.500	0.504	0.403	0.407	0.403	2
DMU6	0.676	0.495	0.495	0.495	0.495	0.500	0.400	0.404	0.400	4
DMU7	0.792	0.595	0.596	0.595	0.595	0.598	0.500	0.504	0.500	1
DMU8	0.641	0.393	0.394	0.393	0.393	0.398	0.495	0.500	0.495	6
DMU9	0.792	0.595	0.596	0.595	0.595	0.598	0.500	0.504	0.500	1
DMU10	0.676	0.495	0.495	0.495	0.495	0.500	0.400	0.404	0.400	4
DMU11	0.679	0.499	0.50	0.499	0.499	0.503	0.402	0.406	0.402	3
DMU12	0.679	0.499	0.50	0.499	0.499	0.503	0.402	0.406	0.402	3
DMU13	0.676	0.495	0.495	0.495	0.495	0.500	0.400	0.404	0.400	4
DMU14	0.672	0.492	0.492	0.492	0.492	0.495	0.398	0.401	0.398	5
DMU15	0.680	0.500	0.500	0.500	0.500	0.504	0.403	0.407	0.403	2
DMU16	0.641	0.393	0.394	0.393	0.393	0.398	0.495	0.500	0.495	6
DMU17	0.792	0.595	0.596	0.595	0.595	0.598	0.500	0.504	0.500	1
DMU18	0.641	0.393	0.394	0.393	0.393	0.398	0.495	0.500	0.495	6

Table 9 Preference degree matrix for the fuzzy trapezoidal efficiency values (continuation, columns 10 to 18)

	Preference value $P(\tilde{E}_k > \tilde{E}_{j\neq k})$									Rank
	DMU_j									
DMU_k	10	11	12	13	14	15	16	17	18	
DMU1	0.324	0.320	0.320	0.324	0.327	0.320	0.212	0.207	0.212	7
DMU2	0.504	0.500	0.500	0.504	0.506	0.500	0.407	0.403	0.407	2
DMU3	0.503	0.500	0.500	0.503	0.503	0.499	0.406	0.402	0.406	3
DMU4	0.504	0.500	0.500	0.504	0.506	0.500	0.407	0.403	0.407	2
DMU5	0.504	0.500	0.500	0.504	0.506	0.500	0.407	0.403	0.407	2
DMU6	0.500	0.495	0.495	0.500	0.502	0.495	0.404	0.400	0.404	4
DMU7	0.598	0.596	0.596	0.598	0.601	0.595	0.792	0.595	0.596	1
DMU8	0.398	0.394	0.394	0.398	0.398	0.393	0.500	0.495	0.500	6
DMU9	0.598	0.596	0.596	0.598	0.601	0.595	0.792	0.595	0.596	1
DMU10	0.500	0.495	0.495	0.500	0.502	0.495	0.404	0.400	0.404	4
DMU11	0.503	0.500	0.500	0.503	0.503	0.499	0.406	0.402	0.406	3
DMU12	0.503	0.500	0.500	0.503	0.503	0.499	0.406	0.402	0.406	3
DMU13	0.500	0.495	0.495	0.500	0.502	0.495	0.404	0.400	0.404	4
DMU14	0.495	0.492	0.492	0.495	0.500	0.492	0.401	0.398	0.401	5
DMU15	0.504	0.500	0.500	0.504	0.506	0.500	0.407	0.403	0.407	2
DMU16	0.398	0.394	0.394	0.398	0.398	0.393	0.500	0.495	0.500	6
DMU17	0.598	0.596	0.596	0.598	0.601	0.595	0.792	0.595	0.596	1
DMU18	0.398	0.394	0.394	0.398	0.398	0.393	0.500	0.495	0.500	6

4. Research results and discussion

4.1 Results of case I

For this case, Table 4 reveals that DMU_9 is the most preferred DMU. Table 10 shows the results of the proposed approach against that of the fuzzy multiple regression approach (FMRA). Using the proposed approach, the MRR fuzzy response value, $\tilde{\bar{y}}_1$, which is a LTB type response improved from (56.4, 59.10, 62.46) to (57, 67, 74), where the $\bar{y}_2^U$ and $\bar{y}_2^M$ values are significantly increased. Also, the SR fuzzy response value, $\tilde{\bar{y}}_2$, which is a STB type response improves from (2.94, 3.32, 3.75) to (3.15, 3.15, 3.65). Note that the proposed approach provides smaller mean and upper bound value than FMRA. Therefore, to improve the performance of WEDM process, the best combination of factor settings is pulse in time A_3, delay time B_3, wire feed speed C_2, ignition current D_1.

Table 10 Improvement comparison for case I

Response	$\tilde{\bar{y}}_1$ (LTB)	$\tilde{\bar{y}}_2$ (STB)
Initial condition	≈ (49.4,51.9,54.58)	≈ (2.99,3.42,3.92)
Fuzzy multiple regression approach (FMRA)	≈ (56.4,59.10,62.46)	≈ (2.94,3.32,3.75)
Proposed approach results (IDEA)	≈ (57,67,74)	≈ (3.15,3.15,3.65)

4.2 Results for case II

For case II, it is found that DMU_9 is the best DMU, which corresponds as shown in Table 5. Table 11 displays the anticipated improvements using the proposed approach and FMRA.

Using the proposed approach the fuzzy trapezoidal value of the ER, $\bar{y}_1$, which is a STB type response decreased from (1.39, 2.91, 3.19, 4.32) to (1.0, 1.57, 1.95, 6.7). Although the proposed approach increased the upper response value, $\bar{y}_1^U$ it significantly decreased the lower mid and upper mid response values. For DR, $\bar{y}_2$, which is a LTB type response, the proposed approach enhances the response fuzzy trapezoidal value from (11.84, 12.26, 12.75 21.94) to (10.5, 19, 21, 22). Finally for OT, $\bar{y}_3$, which is a LTB type response, the proposed approach improves the upper response value from 87.7 to 88.5. Consequently, the best combination of factor settings for the sputtering process is the R.F. power P = 200, the sputtering pressure Q = 1.33, the deposition time R = 30, the substrate temperature S = 100, and the post-annealing temperature T = 100.

Table 11 Improvement comparison for case II

Methods	Quality responses											
	y_1 (STB, input)				y_2 (LTB, output)				y_3 (LTB, output)			
	y_1^L	y_1^{ML}	y_1^{UL}	y_1^U	y_2^L	y_2^{ML}	y_2^{UL}	y_2^U	y_3^L	y_3^{ML}	y_3^{UL}	y_3^U
FMRA results	1.39	2.91	3.19	4.32	11.84	12.26	12.75	21.94	86.4	86.5	87.6	87.7
IDEA results	1.0	1.57	1.95	6.7	10.5	19.0	21.0	22.0	83.5	86.0	87.0	88.5

5. Conclusions

In this research, a fuzzy DEA based procedure is proposed to solve the fuzzy multiple responses problem in robust design. DEA models are utilized to calculate the fuzzy efficiencies. Then, the preference matrix is adopted to identify the best decision making unit. Two real case studies from previous literature are employed to illustrate the proposed approach including improving performance of the WEDM and sputtering processes, where the response fuzziness is fitted by a triangular and trapezoidal membership functions in the first and second case study, respectively. In both studies, the proposed approach efficiently identified the best combination of factor settings and provides better anticipated improvements than the fuzzy multiple regression based approach. In conclusion, the proposed approach may provide great assistant to process engineers in determining the best combination of factor settings that improves fuzzy multiple responses in a wide range of business applications. Nevertheless, this approach ignores process factor settings and preferences on quality responses. Another limitation is its complexity when many fuzzy responses are considered simultaneously. Future research will consider these issues.

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