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Upper embeddability of graphs and products of transpositions associated with edges*

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Abstract

Given a graph, we associate each edge with the transposition which exchanges the endvertices. Fixing a linear order on the edge set, we obtain a permutation of the vertices. Dénes proved that the permutation is a full cyclic permutation for any linear order if and only if the graph is a tree.

In this article, we characterize graphs having a linear order such that the associated permutation is a full cyclic permutation in terms of graph embeddings. Moreover, we give a counterexample for Eden's question about an edge ordering whose associated permutation is the identity.

Keywords: Full cyclic permutation ordering, upper-embeddable graph, 2*-cell embedding, rotation system.*

Math. Subj. Class. (2020): 05C25, 05C10, 57M15

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Zgornja vložljivost grafov in produkti transpozicij, pridruženih povezavam*

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Povzetek

V danem grafu vsaki povezavi pridružimo transpozicijo, ki zamenja krajišči. Če določimo linearno ureditev na množici povezav, dobimo permutacijo točk grafa. Dénes je dokazal, da je ta permutacija polna ciklična permutacija za vsako linearno ureditev, če in samo če je graf drevo.

V tem članku karakterizramo grafe, ki imajo linearno ureditev, za katero je pridružena permutacija polna ciklična permutacija v smislu grafovskih vložitev. Poleg tega podamo protiprimer za Edenino vprašanje o ureditvi povezav, katere pridružena permutacija je identiteta.

Ključne besede: Popolna ciklična permutacijska ureditev, zgornje vložljiv graf, 2*-celična vložitev, rotacijski sistem.*

Math. Subj. Class. (2020): 05C25, 05C10, 57M15

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