

## DVODIMENZIONALNO MODELIRANJE TOKA S PROSTO GLADINO TWO-DIMENSIONAL MODELLING OF FREE SURFACE FLOW

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*Najprej so opisane teoretične osnove dvodimenzionalnih (2D) globinsko povprečnih modelov PCFLOW2D in PCFLOW2D-CURVE. Temeljne enačbe kontinuitete, ohranitve gibalne količine ter transporta turbulentne kinetične energije na enoto mase in stopnje njene disipacije so rešene s pomočjo numerične metode končnih volumnov. Prikazanih je več primerov izračuna tokov: stalni tok v ožini reke Idrijce, dvodimenzionalni tok reke Savinje v Laškem, nestalni tok po porušitvi pregrade v hipni razširitvi in tok zaradi plime in oseke v sistemu treh mehiških pacifiških lugun Jitzamuri, Agiabampo in Bacorehuis. Primerjava rezultatov z meritvami v naravi in na fizičnih modelih je pokazala zadovoljivo ujemanje.*

**Ključne besede:** *dvodimenzijsko matematično modeliranje, globinsko povprečni model, krivočrtne koordinate, ožina Idrijce, Savinja, Laško, porušitveni val, mehiške lagune*

*Some theoretical background of the two-dimensional (2D) depth-averaged mathematical models PCFLOW2D and PCFLOW2D-CURVE is described first. Basic equations of continuity, momentum conservation and the transport of turbulent kinetic energy per unit mass and its rate of dissipation are solved by using the finite volume numerical method. Several practical flow computations are shown: a steady flow in the Idrijca River narrow, a 2D flow of the Savinja River in the city of Laško and an unsteady dam-break flow in the sudden expansion and tidal flow in the system of three Mexican Pacific lagoons - Jitzamuri, Agiabampo and Bacorehuis. The comparison between the computed results and the field measurements or the physical models showed satisfactory agreement.*

**Key words:** *two-dimensional modelling, depth-averaged model, curvilinear coordinates, Idrijca River narrow, Savinja, Laško, dam-break wave, Mexican lagoons*

### 1. UVOD

Na področju računske hidravlike so simulacije toka s prosto gladino zelo pomembne. V preteklosti so se največ uporabljali enodimenzionalni (1D) hidrodinamični modeli, npr. za račun valov zaradi porušitev pregrad v naravnih dolinah (Rajar, 1978) ali za propagacijo poplavnih valov v rekah (Rajar & Četina, 1985). V zadnjem času pa te modele vse bolj nadomeščajo sodobnejši dvodimenzionalni (2D) modeli za simulacijo nestalnega toka (Četina et al., 1996; Četina & Rajar, 1994), lokalnih tokov v rekah (Četina & Rajar, 1993) ter toka in transporta sedimentov v plitvih jezerih (Krzyk & Četina, 1996). Za račun cirkulacije v stratificiranih globokih jezerih in morjih pa so potrebni še kompleksnejši in zahtevnejši tridimenzionalni (3D) modeli (Rajar & Četina, 1997).

### 1. INTRODUCTION

In the field of computational hydraulics, free surface flow simulations are very important. One-dimensional (1D) hydrodynamic models have been widely used in the past, e.g. for dam-break wave simulations in natural valleys (Rajar, 1978) or flood propagation in rivers (Rajar & Četina, 1985). These models have been recently replaced by up-to-date two-dimensional (2D) models to simulate an unsteady flow (Četina et al., 1996; Četina & Rajar, 1994), local flow phenomena in rivers (Četina & Rajar, 1993) and flow and sediment transport dynamics in shallow lakes (Krzyk & Četina, 1996). For circulation patterns in deep stratified lakes and seas, even more sophisticated three-dimensional (3D) models are needed (Rajar & Četina, 1997).

V rekah z relativno velikim razmerjem med širino in globino vode, v plitvih jezerih ali priobalnih morjih je mogoče uporabiti 2D globinsko povprečne modele. V prispevku so opisane nekatere teoretične osnove in praktične izkušnje z našima hidrodinamičnima modeloma PCFLOW2D in PCFLOW2D-CURVE. Temeljne enačbe kontinuitete, ohranitve gibalne količine ter transporta turbulentne kinetične energije na enoto mase  $k$  in stopnje njene disipacije  $\varepsilon$  so rešene s pomočjo numerične metode končnih volumnov. Prikazanih je več praktičnih primerov izračuna tokov: stalni tok v ožini reke Idrijce, tok reke Savinje v Laškem, nestalni tok po poružitvi pregrade v hipni razširitvi pravokotnega laboratorijskega kanala in tok zaradi plime in oseke v sistemu treh mehiških pacifiških lugun.

## 2. TEORETIČNE OSNOVE 2D MATEMATIČNIH MODELOV

### 2.1 TEMELJNE ENAČBE

V modelu PCFLOW2D je uporabljen Kartezijev koordinatni sistem. Kontinuiteta je opisana z enačbo (1), dinamični enačbi (2) in (3), ki opisujeta dvodimenzionalni nestalni globinsko povprečni tok, pa sta podani v konservativni obliki. Zadnja dva člena na desnih straneh enačb izražata vpliv turbulentne viskoznosti, ki jo določimo s pomočjo znanega  $k - \varepsilon$  modela turbulence. Zato sta potrebni dve dodatni transportni enačbi za turbulentno kinetično energijo na enoto mase  $k$  ter stopnjo njene disipacije  $\varepsilon$  (enačbi 4 in 5).

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial(hu)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -gh \frac{\partial h}{\partial x} - gh \frac{\partial z_b}{\partial x} - ghn^2 \frac{u\sqrt{u^2+v^2}}{h^{4/3}} + \frac{\partial}{\partial x} (hv_{ef} \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} (hv_{ef} \frac{\partial u}{\partial y}) \quad (2)$$

$$\frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -gh \frac{\partial h}{\partial y} - gh \frac{\partial z_b}{\partial y} - ghn^2 \frac{v\sqrt{u^2+v^2}}{h^{4/3}} + \frac{\partial}{\partial x} (hv_{ef} \frac{\partial v}{\partial x}) + \frac{\partial}{\partial y} (hv_{ef} \frac{\partial v}{\partial y}) \quad (3)$$

$$\frac{\partial(hk)}{\partial t} + \frac{\partial(huk)}{\partial x} + \frac{\partial(hvk)}{\partial y} = \frac{\partial}{\partial x} (h \frac{v_{ef}}{\sigma_k} \frac{\partial k}{\partial x}) + \frac{\partial}{\partial y} (h \frac{v_{ef}}{\sigma_k} \frac{\partial k}{\partial y}) + hG - c_D h\varepsilon + hP_{kv} \quad (4)$$

$$\frac{\partial(hu\varepsilon)}{\partial x} + \frac{\partial(hv\varepsilon)}{\partial y} = \frac{\partial}{\partial x} (h \frac{v_{ef}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x}) + \frac{\partial}{\partial y} (h \frac{v_{ef}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y}) + c_1 \frac{\varepsilon}{k} hG - c_2 \frac{\varepsilon^2}{k} h + hP_{\varepsilon v} \quad (5)$$

Two-dimensional (2D) depth-averaged mathematical models can be applied in rivers where the ratio between width and depth is relatively large, or in shallow lakes or coastal seas. Some theoretical features and practical experiences with our hydrodynamic models PCFLOW2D and PCFLOW2D-CURVE are described in the paper. Basic equations of continuity, momentum conservation and the transport of turbulent kinetic energy  $k$  and its rate of dissipation  $\varepsilon$  are solved numerically by using the finite volume approach. Several practical flow computations are shown: the steady flow in the Idrijca River narrow, the flow of the Savinja River in the city of Laško and an unsteady flow in a sudden enlargement of the rectangular laboratory flume and the tide flow in the system of three Mexican lagoons.

## 2. THEORETICAL BACKGROUND OF 2D MATHEMATICAL MODELS

### 2.1 BASIC EQUATIONS

The Cartesian co-ordinate system is used in the model PCFLOW2D. The continuity (1) and momentum equations ((2) and (3)) describing a two-dimensional unsteady depth-averaged flow are written in the conservation form. The last two terms on the right-hand side express the influence of turbulent viscosity, which is determined by the well known  $k - \varepsilon$  turbulence model. Thus, two additional transport equations for the turbulent kinetic energy per unit of mass  $k$  and its rate of dissipation  $\varepsilon$  are needed (equations 4 and 5).

$t$  je čas,  $h$  je globina vode,  $u$  in  $v$  sta komponenti hitrosti v  $x$  in  $y$  smereh,  $z_b$  je kota dna,  $n$  je koeficient hrapavosti po Manningu,  $g$  je zemeljski pospešek in  $\nu_{ef}$  efektivni koeficient viskoznosti. Izraze za  $G$  (produkcija  $k$  zaradi horizontalnih gradientov hitrosti),  $P_{kv}$  in  $P_{\epsilon v}$  (izvorna člena zaradi trenja ob dno) kakor tudi vrednosti standardnih turbulentnih konstant ( $c_D$ ,  $c_\mu$ ,  $c_1$ ,  $c_2$ ,  $\sigma_k$  in  $\sigma_\epsilon$ ) je mogoče najti v literaturi (Rodi, 1980).

## 2.2 NUMERIČNA METODA

Povezan sistem parcialnih diferencialnih enačb (1) do (5) se rešuje z numerično metodo končnih volumnov, ki jo je predlagal Patankar (1980). Glavne značilnosti metode so premaknjeni kontrolni volumni, t. im. HIBRIDNA shema in iterativni postopek popravkov globin (SIMPLE). Za integracijo po času je uporabljena polna implicitna shema, ki zagotavlja stabilno in točno rešitev tudi pri relativno visokih Courantovih številih (nekje do 10). Mogoče je simulirati tako mirni kot deroči tok (Četina & Rajar, 1993).

Hibridna shema je kombinacija gorvodne in centralnodiferenčne sheme (uporaba določene sheme je odvisna od celičnega Pecletovega števila). Gorvodna shema, ki je prvega reda točnosti, zagotavlja preprostost in zanesljivost (Patankar, 1980) in ostaja stabilna tudi pri zelo kompleksni geometriji, relativno grobi numerični mreži in zahtevnih robnih pogojih. Vendar je splošno znano, da lahko včasih vsebuje določeno mero t. im. "numerične difuzije". Problem je resnejši pri reševanju transportnih enačb za skalarnne parametre, vendar so lahko včasih tudi hidrodinamični rezultati vprašljivi (npr. pri simulaciji rečnega toka pri večjih hitrostih, s stranskim dotokom in recirkulacijskimi območji; Rajar & Četina, 1997). Ker je preprosteje spremeniti obstoječo shemo kot pa uvesti popolnoma novo, lahko v literaturi najdemo številne poskuse zmanjšanja numerične difuzije pri gorvodni shemi. Preizkusili smo dve izboljšani shemi: CONDIF (Runchal, 1987) in NONDIF (Hedberg, 1989). Njihovo uporabnost smo

$t$  is the time,  $h$  is the water depth,  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions,  $z_b$  is the bottom level,  $n$  is Manning's friction coefficient,  $g$  is the acceleration due to gravity and  $\nu_{ef}$  is an effective coefficient of viscosity. The expressions of  $G$  (the production of  $k$  due to horizontal velocity gradients),  $P_{kv}$  and  $P_{\epsilon v}$  (source terms due to the bottom friction), as well as the values of the standard turbulent constants ( $c_D$ ,  $c_\nu$ ,  $c_1$ ,  $c_2$ ,  $\sigma_k$  and  $\sigma_\epsilon$ ), can be found in Rodi (1980).

## 2.2 NUMERICAL METHOD

The set of coupled partial differential equations (1) – (5) is solved by using the finite volume numerical scheme proposed by Patankar (1980). The main characteristics of the method are staggered control volumes, the so-called HYBRID scheme and an iterative procedure of depth corrections (SIMPLE). A fully implicit scheme is used for time integration, providing stable and accurate solution even at relatively high Courant numbers (up to about 10). It is possible to simulate both subcritical or supercritical flow (Četina & Rajar, 1993).

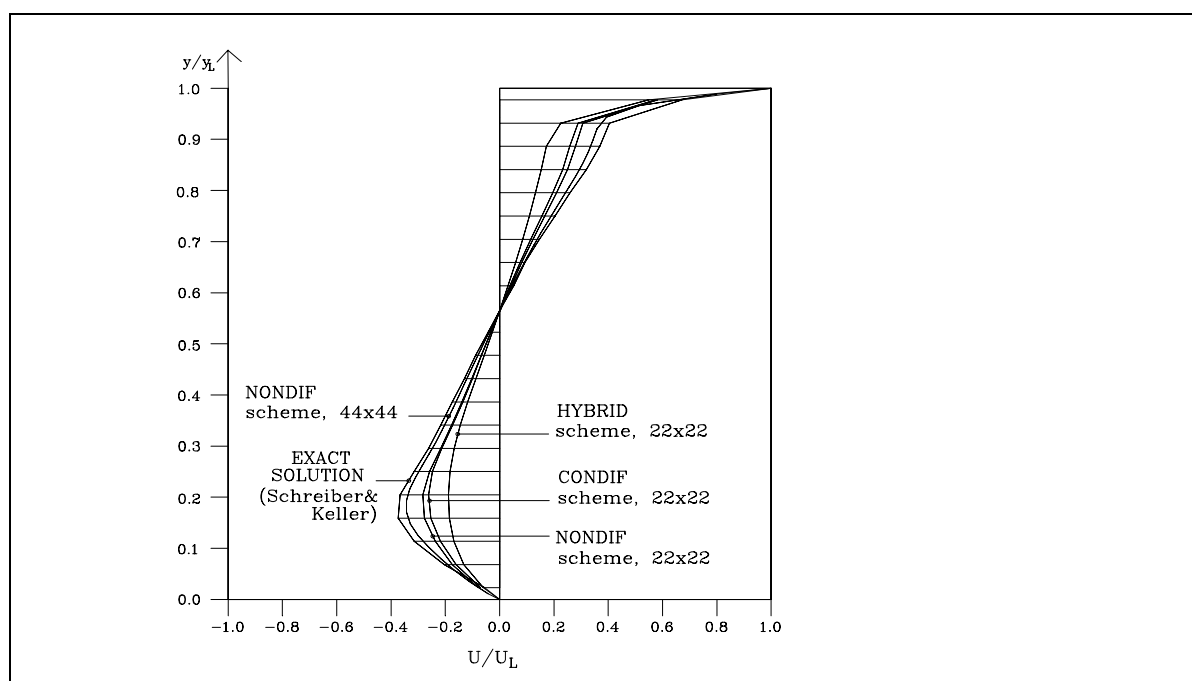
The HYBRID scheme is a combination of the upwind and central difference scheme (application of the scheme depending on the value of the cell Peclet number). The first order upwind scheme assures simplicity and robustness (Patankar, 1980), and it remains stable even with very complex geometry, a relatively coarse numerical grid and complicated boundary conditions. But it is well known that it can sometimes involve a certain amount of the so-called "numerical diffusion". The problem is more serious when solving transport equations of scalar quantities, but can sometimes render the hydrodynamic results questionable, as well (e.g. when simulating high velocity river flow with lateral inflow and recirculation zones; Rajar & Četina, 1997). Since it is simpler to modify an existing scheme than to introduce a completely new one, several attempts can be found in the literature for reducing the numerical diffusion of the upwind scheme. We tried two improved schemes: CONDIF (Runchal, 1987) and NONDIF (Hedberg, 1989). Their behaviour was tested by various 2D flows. In the case of laminar flow in a square cavity, the NONDIF scheme produced the most accurate

testirali pri različnih 2D tokovih. V primeru toka v kvadratni kotanji je dala NONDIF shema najbolj točne rezultate (slika 1). Toda pri praktičnih izračunih toka v pravokotnem laboratorijskem kanalu s stranskim dotokom ter v naravni rečni strugi je NONDIF shema odpovedala zaradi numerične nestabilnosti. V teh primerih je CONDIF shema dala stabilne in točne rezultate z občutno manjšo (30 do 50 odstotkov) numerično difuzijo. Treba pa je omeniti, da čas računanja naraste za 200 do 300 odstotkov. Več podrobnosti je opisanih v Rajar & Četina (1997).

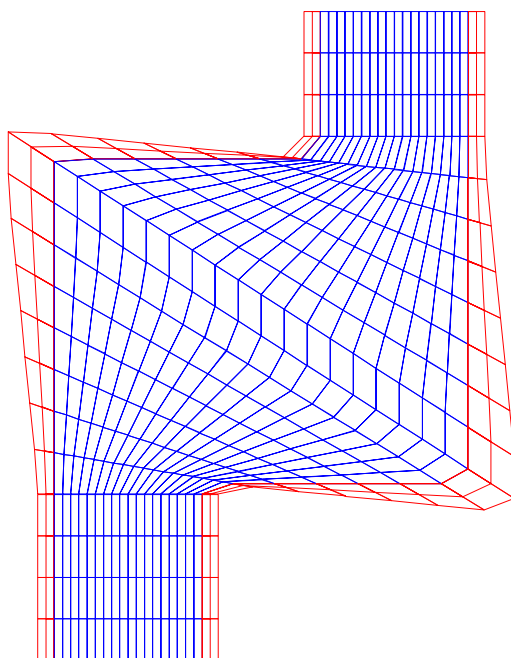
Druga možnost, da se izognemo problemu numerične difuzije, je uporaba gostejših numeričnih mrež. To lahko dosežemo z uvedbo krivočrtnih koordinatnih sistemov, ki se lahko prilagodijo nepravilnim robovom računskega področja. Uporabimo lahko pravokotne (Mikoš et al., 2000) ali nepravokotne krivočrtne mreže (Gerčer, 2000). Kot primer je na sliki 2 prikazana nepravokotna mreža za turbulentni tok v kanalu v obliki črke Z. Primerjava hitrostnih polj, ki so bila dobljena z različnimi modeli, je prikazana na sliki 3.

results (Figure 1). But for practical computations of turbulent flows in a rectangular laboratory channel with a side discharge, and in the natural river, the NONDIF scheme failed due to numerical instability. In these cases, the CONDIF scheme produced stable accurate solutions with a significantly lower (30 - 50%) amount of numerical diffusion. It should be mentioned, however, that the computational time is increased by about 200 to 300 %. More details can be found in Rajar & Četina (1997).

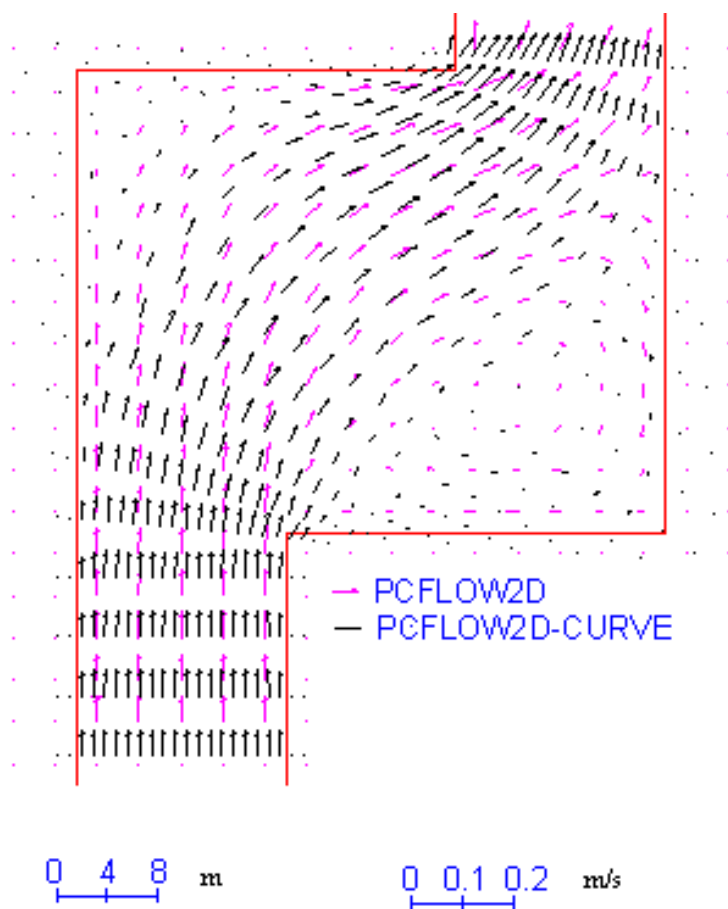
The other method for avoiding the problem of numerical diffusion is to use denser numerical grids. This can be achieved by introducing curvilinear co-ordinate systems, which are able to fit the irregular boundaries of the computational domain. It is possible to apply orthogonal (Mikoš et al., 2000) or non-orthogonal curvilinear meshes (Gerčer, 2000). Details about the equations in curvilinear co-ordinate systems, a description of the discretisation method and the basic features of the PCFLOW2D-CURVE computer code can be found in Gerčer (2000). As an example, a curvilinear non-orthogonal grid for the turbulent flow in a Z - shaped channel is shown in Figure 2. A comparison of the computed velocity fields obtained by various models is given in Figure 3.



Slika 1. Laminarni tok v kvadratni kotanji pri Reynoldsovem številu  $Re = 1000$ .  
Figure 1. Laminar flow in a square cavity at Reynolds number  $Re = 1000$ .



Slika 2. Krivočrtna nepravokotna numerična mreža v kanalu v obliki črke Z.  
Figure 2. Curvilinear non-orthogonal numerical grid in a Z-shaped channel.



Slika 3. Primerjava hitrostnih polj v kanalu v obliki črke Z.  
Figure 3. Comparison of velocity fields in a Z-shaped channel.

## 2.3 RAČUNALNIŠKI PROGRAMI

Izvorni kodi računalniških programov PCFLOW2D za Kartezične in PCFLOW2D-CURVE za nepravokotne mreže sta napisani v jeziku FORTRAN77 in tečeta na osebnih računalnikih, delovnih postajah in večjih centralnih računalnikih. Za pripravo vhodnih topografskih podatkov (predprocesiranje) in grafično predstavitev končnih rezultatov modela (postprocesiranje) sta uporabljena grafična paketa AutoCAD in QuickSurf (Četina&Krzyk, 1999). Za pripravo nepravilne mreže je bil razvit poseben računalniški program GEO-CURVE (Gerčer, 2000).

V matematičnem modelu PCFLOW2D se lahko v poljubni celici upoštevajo naslednji robni pogoji: a) trdne stene (kjer so normalne hitrosti enake 0); b) vtok rek (predpisani so lahko pretoki ali hitrosti); c) globine ali kote vode; d) kritični tok; e) enačba preлива. vzdolž odprtih robov so lahko podane znane funkcije poteka plime ali druge časovno odvisne funkcije gladine, lahko pa se upošteva tudi t.im. "radiacijski" ali "kontinuitetni" (Stravisi, 1977) robni pogoj. Na gladini so lahko upoštevane strižne napetosti zaradi vetra, pri čemer je treba podati njegovo hitrost in strižni koeficient.

## 3. PRAKTIČNI PRIMERI

### 3.1 STALNI TOK V OŽINI REKE IDRIJCE

Na reki Soči so zgrajene tri hidroelektrarne (HE). Prva v verigi je HE Dobljar, ki je bila dokončana leta 1938. Za pregrado v Podselu je nastalo akumulacijsko jezero, ki vsebuje  $5.5 \times 10^6 \text{ m}^3$  vode. Vpliv zaježitve sega po Soči gorvodno do Tolmina in po njenem pritoku Idrijci gorvodno do naselja Bača. Nekaj nižjeležečih hiš v omenjenem naselju je bilo v mokrem obdobju med leti 1961 in 1965 ter kasneje v letih 1979, 1990 in 1992 občasno poplavljenih.

Vzrokov za poplave je več, dva med njimi

## 2.3 THE COMPUTER CODES

The source computer codes PCFLOW2D for Cartesian and PCFLOW2D-CURVE for non-orthogonal numerical meshes are written in FORTRAN77 language and run on personal computers, workstations or large mainframe systems. For the preparation of the input topographic data (pre-processing) and graphical presentation of the final results of the model (post-processing), AutoCAD and Quick Surf graphic packages are used (Četina&Krzyk, 1999). For the irregular mesh generation procedure, a special GEO-CURVE computer program was developed (Gerčer, 2000).

In the mathematical model PCFLOW2D, the following boundary conditions can be taken into account at arbitrarily chosen cells of the computational domain: a) solid boundaries (with zero normal velocities); b) inflow of rivers (discharges or velocities can be prescribed); c) water depths or water surface elevations; d) critical flow; e) an equation of a weir. Along the open boundaries, known tidal or other time-dependent functions of the water surface can be given, or the "radiation" and "continuity" (Stravisi, 1977) boundary conditions prescribed. Wind stress can be taken into account at the water surface by giving the wind speed and the wind friction coefficient.

## 3. PRACTICAL APPLICATIONS

### 3.1 STEADY FLOW IN THE IDRIJCA RIVER NARROW

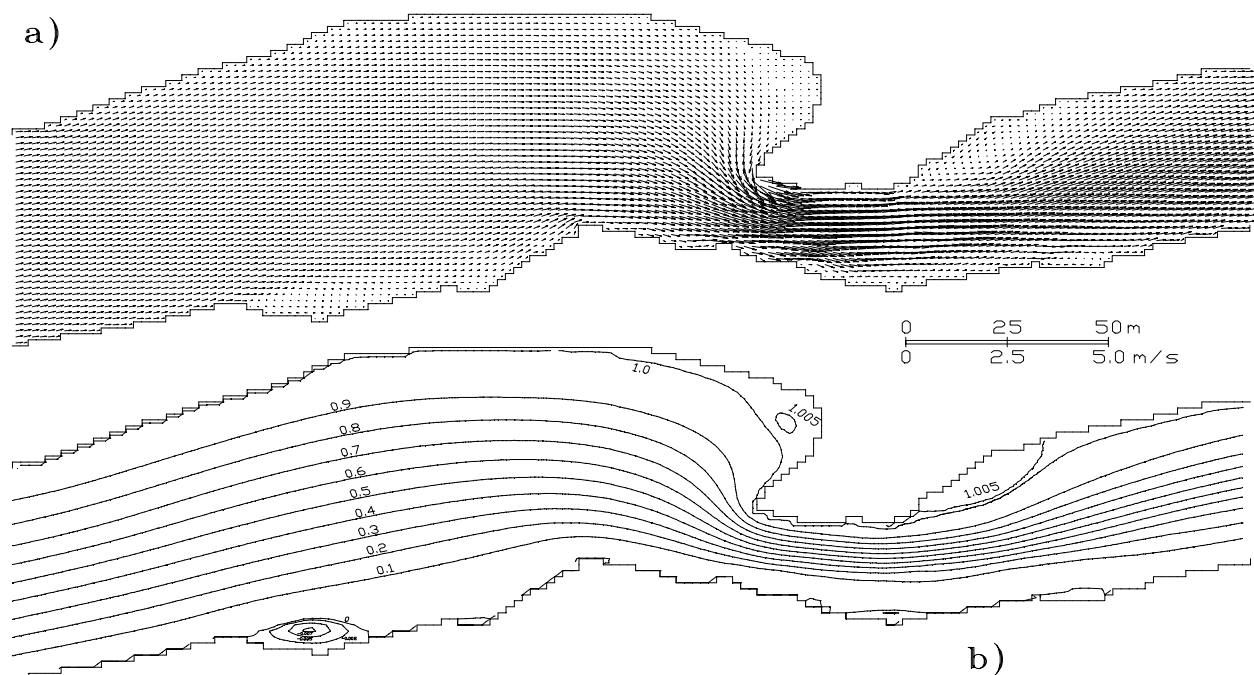
Three hydroelectric power plants (HEPP) have been built on the Soča River. The first one in the chain, Dobljar HEPP, was constructed in the year 1938. A reservoir containing approx.  $5.5 \cdot 10^6 \text{ m}^3$  of water was formed on the Soča River behind the dam at Podselo village. A backwater effect influenced the river reach from the dam to the upstream-situated town of Tolmin, and Soča's tributary, the Idrijca River reach, upstream to the village of Bača. This village was occasionally flooded due to high discharges in the wet period during the years 1961 to 1965, and later in the years 1979, 1990 and 1992.

pa sta najpomembnejša: dvigovanje dna Idrijce zaradi zmanjšane transportne sposobnosti zaradi učinka zaježitve in lokalna zožitev toka tik dolvodno od naselja Bača.

Možni ukrepi za znižanje gladine gorvodno od ožine so bili raziskani s pomočjo kombinacije 1D in 2D modela Idrijce (Četina&Krzyk, 1994). Zaradi hidravlične zapletenosti toka v ožini je bil lokalno uporabljen 2D model PCFLOW2D. Za pripravo vhodnih topografskih podatkov je bilo izmerjenih 17 prečnih profilov na odseku dolžine 308 m in širine 136 m. Interpolirana numerična mreža je imela 155 točk v vzdolžni y smeri ( $\Delta y = 2$  m) in 69 točk v prečni x ( $\Delta x = 2$  m) smeri.

There are several possible causes for these floods. Among others, two of them are most important: the rise of the Idrijca riverbed due to the lower sediment transport capacity caused by the backwater effect, and a local narrowing of the flow just downstream of Bača.

To suggest possible measures for lowering the upstream water levels, a combination of 1D and 2D mathematical models of the Idrijca River was used (Četina&Krzyk, 1994). Due to the hydraulic complexity of the flow in the narrow, a 2D model PCFLOW2D was applied locally. For the preparation of the input topographic data, 17 cross section profiles were measured on a 308 m long and 136 m wide reach. The interpolated numerical mesh consisted of 155 points in the longitudinal y direction ( $\Delta y = 2$  m) and 69 points in the lateral x direction ( $\Delta x = 2$  m).



Slika 4. Izračunani tok v ožini Idrijce pri pretoku  $Q = 1000 \text{ m}^3/\text{s}$ : a) hitrosti, b) tokovnice.  
Figure 4. Computed flow in the Idrijca River narrow at the discharge  $Q = 1000 \text{ m}^3/\text{s}$ .

Model je bil najprej umerjen pri pretoku  $Q = 1000 \text{ m}^3/\text{s}$  s približno 10-odstotno verjetnostjo nastopa (sliki 4a in 4b). Nadaljnje simulacije pri različnih pretokih so pokazale najbolj primerne ukrepe za izboljšanje hidravličnih pogojev v ožini. Z miniranjem

The model was calibrated first at a discharge of  $Q = 1000 \text{ m}^3/\text{s}$  of approx. 10% probability (Figures 4a and 4b). After several further runs at various discharges, the results of the simulations showed the most appropriate measures for improving hydraulic conditions in the narrow. By mining the rocks

skalnega masiva na levem delu najožjega dela in z odstranitvijo približno 2 m debelega sloja plavin bi dosegli znatno znižanje gladine tik gorvodno od ožine: približno 1.2 m pri pretoku  $Q = 1000 \text{ m}^3/\text{s}$  in 1.1 m pri pretoku  $Q = 1600 \text{ m}^3/\text{s}$  z 1-odstotno verjetnostjo nastopa.

### 3.2 TOK SAVINJE V LAŠKEM

V Laškem je načrtovana izgradnja novega zdraviliškega centra. Ležal bo na levem poplavnem območju Savinje tik dolvodno od novega cestnega mostu v Jagoče. Ker je Laško poplavno ogroženo območje, je bila sprejeta odločitev, da se z modelom PCFLOW2D naredi podroben hidravlični izračun za približno 780 m dolg odsek Savinje. Topografija dna za referenčno stanje pred spremembami zaradi izgradnje zdraviliškega centra je bila določena na podlagi 27 prečnih profilov, izmerjenih na medsebojni razdalji približno 25 do 30 m. Koordinate  $x$ ,  $y$ ,  $z$  za približno 1200 izmerjenih točk so bile vnešene v grafični paket AutoCAD. S pomočjo orodja QuickSurf in interpolacije na podlagi trikotnih elementov je bil zgrajen digitalni model terena (DMT). Ta je bil potem uporabljen za generacijo numerične mreže s 130 točkami v prečni  $x$  smeri ( $\Delta x = 2$  do 10 m) in 175 točkami v  $y$  smeri vzdolž toka ( $\Delta y = 5$  to 10 m). Enaka mreža je bila uporabljena tudi pri končnih izračunih, pri čemer so bile upoštevane spremembe topografije zaradi predvidenih gradenj na obravnavanem območju. Na gorvodnem koncu modela je bil podan pretok, na dolvodnem koncu modela pa gladina vode.

Karakteristični pretoki so bili znani z najbližje hidrološke postaje Laško:  $Q_{100} = 1412 \text{ m}^3/\text{s}$ ,  $Q_{25} = 1192 \text{ m}^3/\text{s}$ ,  $Q_5 = 834 \text{ m}^3/\text{s}$  in  $Q_2 = 619 \text{ m}^3/\text{s}$ . Pri referenčnem stanju, ki je bilo upoštevano tudi za umerjanje modela, korito ni bilo regulirano. Izračunano hitrostno polje in globine vode pri  $Q_{100} = 1412 \text{ m}^3/\text{s}$  kažejo, da sta poplavljeni levo in desno poplavno območje (sliki 5 in 6). Hitrosti v reki dosežejo do 4 m/s, velikosti na levem poplavnem območju pa so med 2 in 3 m/s.

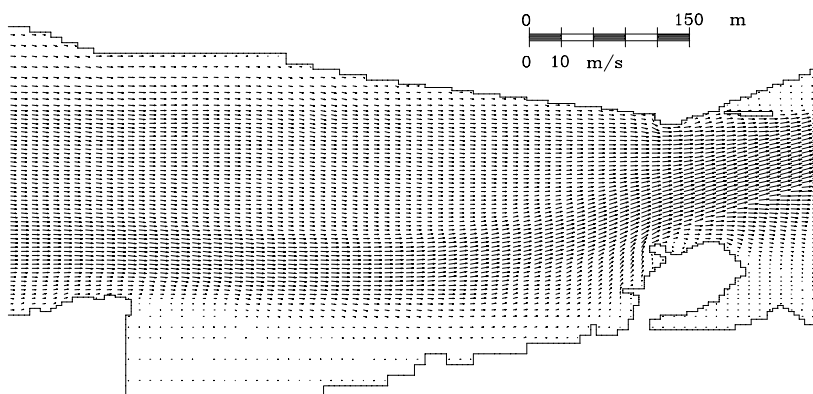
on the left-hand side of the narrowest part, and by excavating about 2 m of the bed material, a relatively significant lowering of the water levels just upstream of the narrow could be reached: about 1.2 m at the discharge of  $1000 \text{ m}^3/\text{s}$  and 1.1 m at the discharge of 1% probability,  $Q = 1600 \text{ m}^3/\text{s}$ .

### 3.2 FLOW OF THE SAVINJA RIVER IN THE CITY OF LAŠKO

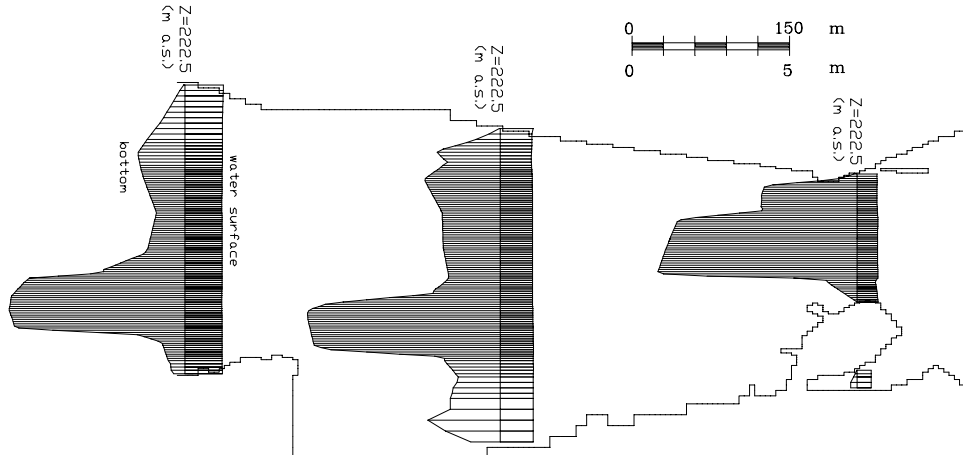
A new health centre is planned for the town of Laško. It will be situated on the left inundation area of the Savinja River, just downstream from the new motorway bridge to the village of Jagoče. Since the town of Laško is often endangered by floods, a decision was made to carry out a detailed hydraulic computation using the model PCFLOW2D. It was applied to a 780 m-long stretch of the river. The bottom topography for the reference state before changes due to construction work on the health centre was determined on the basis of 27 cross-sections measured at a longitudinal distance of approx. 25-30 m. The co-ordinates  $x$ ,  $y$ ,  $z$  of approx. 1200 measured points were added into the AutoCAD graphic package. Using the QuickSurf tool, an interpolation was made with triangles, and a Digital Terrain Model (DTM) was developed. The DTM was then used to generate a non-uniform numerical mesh with 130 points in the lateral  $x$  direction ( $\Delta x = 2$  to 10 m) and 175 points in the  $y$  direction along the flow ( $\Delta y = 5$  to 10 m). The same mesh was also used for the final computations, taking into account changes in the bottom topography due to the construction work in the area. At the upstream end of the model, the discharge was given, while at the downstream end of the model, the water-surface elevation was prescribed.

Characteristic discharges were known from the nearby Laško Hydrological Measuring Station:  $Q_{100} = 1412 \text{ m}^3/\text{s}$ ,  $Q_{25} = 1192 \text{ m}^3/\text{s}$ ,  $Q_5 = 834 \text{ m}^3/\text{s}$  and  $Q_2 = 619 \text{ m}^3/\text{s}$ . In the reference state, which was also taken into account during the calibration process at  $Q_{100}$ , the riverbed was not trained. The computed velocity field and water depths at  $Q_{100} = 1412 \text{ m}^3/\text{s}$  show that both left and right inundation areas were flooded (Figures 5 and 6). The velocities in the river reached up to 4 m/s, and in the left inundation, the magnitudes were from 2 to 3 m/s.





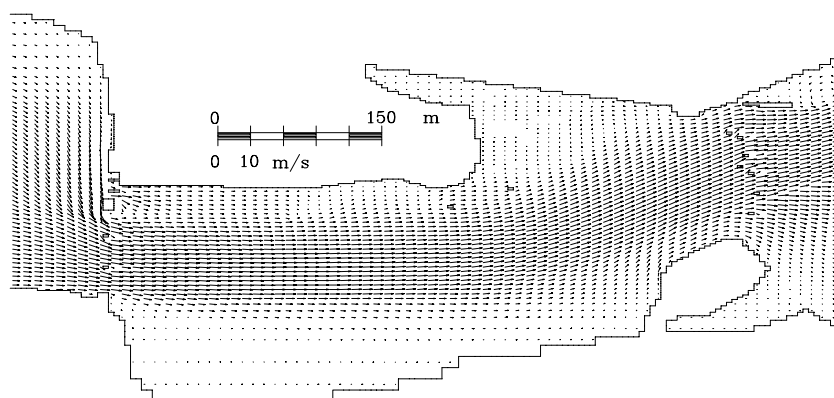
Slika 5. Hitrosti za referenčno stanje pri  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .  
Figure 5. Velocities for the reference state at  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .



Slika 6. Prečni prerezi dna in gladine pri referenčnem stanju in pretoku  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .  
Figure 6. Cross sections of bottom and surface elevations for the reference state at  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .

Pri končnem stanju so bile upoštevane vse izvedene ali načrtovane spremembe po letu 1990, tako v strugi kot na poplavnih območjih: regulacija struge, mehki jez s stopnjo v dnu, brv za pešce, stavba novega zdraviliškega centra in že zgrajen cestni most v Jagoče. Spremembo tokovne slike je mogoče razbrati iz slike 7, ki prikazuje izračunano hitrostno polje za pretok  $Q_{100}$ . Zaradi vpliva novega cestnega mostu in gradbenih del na levem poplavnem območju so gladine vode gorvodno od mostu višje za približno 18 cm v strugi in do 35 cm na levem poplavnem območju (Četina & Krzyk, 1999).

In the final state all constructed or planned measures in the inundation and riverbed area after 1990 were taken into account: river training, a rubber weir with a step in the river bottom, a small bridge for pedestrians, the new health centre building, and the already-constructed motorway bridge to the village of Jagoče. How the flow situation changed can be clearly seen from the computed velocity field at  $Q_{100}$  in Figure 7. Due to the influence of the new motorway bridge and construction work on the left inundation area, the water-surface elevations upstream of the bridge were higher by about 18 cm on the river and up to 35 cm in the left inundation area (Četina & Krzyk, 1999).



Slika 7. Hitrostno polje pri končnem stanju in pretoku  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .

Figure 7. Velocity field for the final state at  $Q_{100} = 1412 \text{ m}^3/\text{s}$ .

### 3.3 PORUŠITVENI VAL V HIPNI RAZŠIRITVI

Ena od težav, ki ima pri standardnih računih porušitvenih valov v praksi zelo velik pomen, je obnašanje vala v hipnih in izrazitih razširitvah. V takšnih situacijah je tok dvodimenzionalen s hitrimi spremembami hitrosti in z nenadnim znižanjem globine vode v razširitvi. Model PCFLOW2D je bil preverjen z opazovanji na fizičnem modelu. V nadaljevanju je prikazana kvalitativna in kvantitativna primerjava rezultatov in podana ocena možnosti uporabe modela v naravnih pogojih. Podrobnosti lahko najdemo v Četina & Rajar (1994).

Fizični model je bil sestavljen iz pravokotnega betonskega kanala dolžine 20 m (Popovska, 1989). Iz glavnega rezervoarja, ki je bil širok 1.2 m in globok 0.6 m, je voda ob odprti zapornici tekla po 4 m dolgem in 0.4 m širokem kanalu, s padcem dna 0.2 odstotka. Ta se je nato v trenutku razširil na 2.8 m, padec dna pa je ostal enak. Pred dvigom zapornice je bilo dno kanala dolvodno suho. Umerjena vrednost Manningovega koeficienta hrapavosti je bila  $n = 0.0137 \text{ sm}^{-1/3}$ . Za nestalni tok po trenutnem dvigu zapornice so bile značilne hitre spremembe hidravličnih parametrov, zato so bili potrebni merski instrumenti z visoko

### 3.3 DAM-BREAK WAVE IN A SUDDEN ENLARGEMENT

One of the problems that is of great practical importance for standard dam-break wave calculations is the behaviour of the wave at the transition through a sudden, huge enlargement. In such situations, the flow is two-dimensional, with rapid changes in flow velocities and a sudden drop of flow depths in the enlargement. The model PCFLOW2D was verified by the flow observed on the physical model. Qualitative and quantitative comparisons of the results and the applicability of the model to predict the flow in natural conditions is briefly discussed below. More details can be found in Četina & Rajar (1994).

The physical model consisted of a concrete rectangular, 20 m long channel (Popovska, 1989). From the main reservoir, which was 1.2 m wide and 0.6 m deep, the water discharged through the gate into the 4 m long and 0.4 m wide channel, which had a bottom slope of 0.2 %. Then the channel suddenly became enlarged to 2.8 m, preserving the same bottom slope. Initially the channel bottom downstream of the gate was dry. The calibrated value of Manning's roughness coefficient was  $n = 0.0137 \text{ sm}^{-1/3}$ . The unsteady flow after the instantaneous lifting of the gate was characterised by a rapid change of hydraulic parameters. These imposed a need for measuring instruments of high dynamic

dinamično občutljivostjo. Časovni potek gladine vode kot osnovne merske količine je bil merjen s kapacitivnimi sondami v 31 točkah na prvih 5.15 m razširjenega dela kanala.

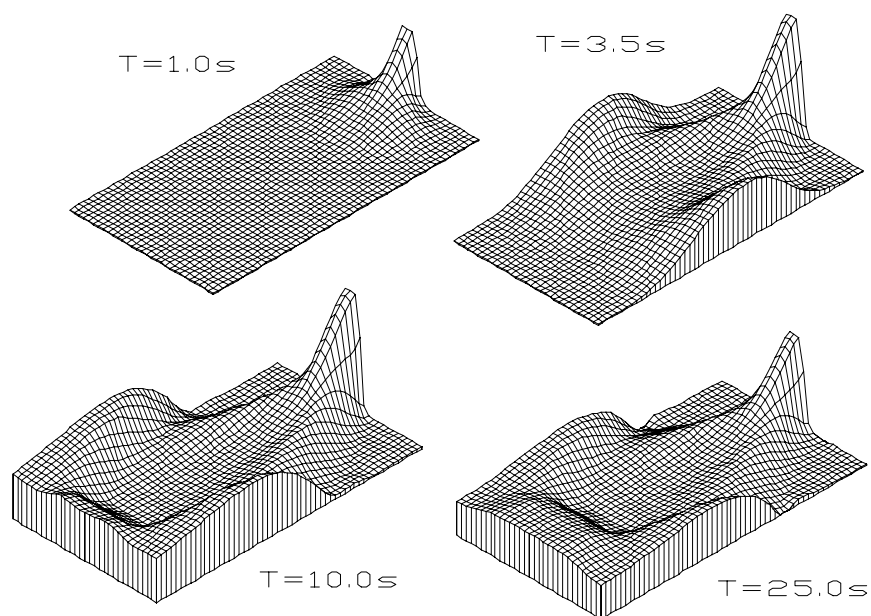
Zaradi simetrije je bila z neenakomerno Kartezijevo numerično mrežo pokrita samo polovica razširjenega dela kanala. Imela je 25 točk v  $y$  smeri vzdolž toka in 15 točk v prečni smeri  $x$ . Prostorski korak v  $y$  smeri se je spreminjal od  $\Delta y = 0.1$  m do 0.3 m, v  $x$  smeri pa od  $\Delta x = 0.05$  do 0.2 m. Časovni korak je bil  $\Delta t = 0.5$  s, povprečne vrednosti Courantovih števil pa med 2 in 3 (v nekaterih točkah pa tudi do 13). Enačbi (2) in (3) sta bili poenostavljeni z zanemaritvijo členov, ki izražajo turbulentne napetosti v vertikalnih ravninah, torej  $v_{ef} = 0$  (Četina & Rajar, 1994; Popovska, 1989). Za vzdolžne hitrosti je bil predpostavljen poenostavljen pogoj, da je normalni gradient ob steni enak 0. Na vtočnem robu je bil predpisan merjen nivogram  $h = h(t)$ , na iztočnem odprtem robu pa je bilo upoštevano, da so vzdolžni gradienti  $u$ ,  $v$  and  $h$  enaki 0. Začetna globina vode v rezervoarju je bila 0.45 m, dno kanala dolvodno od zapornice pa je bilo suho. Zaradi računskih razlogov pa je bila upoštevana začetna najmanjša globina vode 0.001 m.

Kvalitativno je model lahko napovedal odboj vala prečno od sten, deroči tok tik za razširitvijo s skoraj trenutnim zmanjšanjem globine in nato prehod v mirni tok ter manjši recirkulacijski območji v kotih razširitve (slika 8). Kvantitativno je bila izračunana hitrost čela vala v osi kanala za 28 odstotkov premajhna. V splošnem so bile izračunane gladine nekoliko višje od opazovanih. Razlike med največjo izračunano in opazovano globino so bile v osi kanala v rangi  $\pm 30$  odstotkov, pri sondah v bližini sten pa +25 odstotkov. Vendar pa je bila izračunana največja globina na obravnavanem območju samo 3 odstotke večja od izmerjene.

sensitivity. Water levels, which were basic measured values, were permanently recorded with capacity gauges at 31 points along the first 5.15 m of the enlarged channel.

Due to the symmetry, only one half of the enlarged part of the channel was covered by the non-uniform Cartesian numerical grid. It had 25 points in the  $y$  direction along the flow and 15 points in the cross direction  $x$ . The space step in the  $y$  direction varied from  $\Delta y = 0.1$  m to 0.3 m, and in the  $x$  direction from  $\Delta x = 0.05$  to 0.2 m. The time step  $\Delta t = 0.5$  s was used. The average values of Courant numbers were among 2 and 3 (but up to 13 at some points). Equations (2) and (3) were simplified by suppressing the terms for turbulent stresses in the vertical planes,  $v_{ef} = 0$  (Četina & Rajar, 1994; Popovska, 1989). For the longitudinal velocities, simplified conditions of zero normal gradients were assumed near the walls. At the inflow boundary, the measured hydrographs  $h = h(t)$  were prescribed. At the outflow open boundary, zero longitudinal gradients of  $u$ ,  $v$  and  $h$  were taken into account. At the symmetry plane, the normal velocities and the normal gradients of all the other variables were zero. Initial depth in the reservoir was 0.45 m, while the bottom of the channel downstream of the gate was dry. For computational reasons, a minimal depth of 0.001m instead of a dry bottom was assumed.

Qualitatively, the mathematical model was capable of predicting the reflection of the wave from the lateral walls; the supercritical flow just after the enlargement, with an almost sudden rise of depth at the transition to a subcritical flow; and the small recirculating zones in the corner of the enlargement (Figure 8). Quantitatively, the computed wave front velocity in the channel axis was underpredicted by about 28%. Generally, the computed depths were, to a certain degree, greater than the observed ones. Differences in the maximal computed and observed depths at the gauges on the central axis were within  $\pm 30\%$ , and at the gauges near the wall, +25%. However, the maximal depth in the area which results from the mathematical model, was only 3% greater than the experimental value.



Slika 8. Izračunana gladina v razširitvi.  
*Figure 8. Computed free surface in the enlargement.*

### 3.4 CIRKULACIJA ZARADI PLIME V LAGUNAH DOLINE CARIZO

Dolina Carizo v severnem delu pokrajine Sinaloa v SZ Mehiki, ki zajema 43 000 ha kmetijskih zemljišč, leži v bližini lagun Bacorehuis-Agiabampo-Jitzamuri. Gre za tipični sistem, kjer lahko kmetijsko onesnaženje ogrozi gojitveno marikulturo v lagunah (ribe, rakci, ostrige). Sistem lagun pokriva območje v obsegu 35 x 20 km. Polja se namakajo z vodo iz dveh akumulacij, ki ležita vzhodno od območja, odvodnjavajo pa se preko površinskih kanalov neposredno v lagune. Preko kanalov se v lagune izpirajo tudi hraniva in pesticidi. Posebno zadnji so zelo nevarni za onesnaževanje proizvodov marikulture.

S pomočjo dvodimenzionalnega modela PCFLOW2D za modeliranje hidrodinamike in širjenja onesnaženja je bilo narejenih nekaj prvih simulacij. Namen tega dela študije je bil določiti cirkulacijo v lagunah, izmenjavo vode z oceanom ter simulirati transport in disperzijo hipotetičnega konservativnega polutanta. V članku so prikazani samo hidrodinamični rezultati. Najpomembnejši je vpliv plime, ki ima amplitudo  $\pm 0.90$  m in periodo približno 12 ur. Upoštevan je bil tudi zahodni veter 3

### 3.4 TIDAL CIRCULATION IN THE LAGOONS OF THE CARIZO VALLEY

The Carizo Valley, in the northern part of Sinaloa in the NW part of Mexico, with 43 000 ha of agricultural area, is situated in the vicinity of the Bacorehuis-Agiabampo-Jitzamuri Lagoons. This is a typical system where agricultural pollution may endanger important mariculture production in the lagoons (fish, shrimps, oysters). The lagoon system covers a region of about 35 x 20 km. The fields are irrigated with water from two reservoirs lying east of the region, and the water, nutrients and pesticides are drained from the fields into the lagoons by surface channels. Pesticides, especially, are very dangerous for the pollution of the mariculture products.

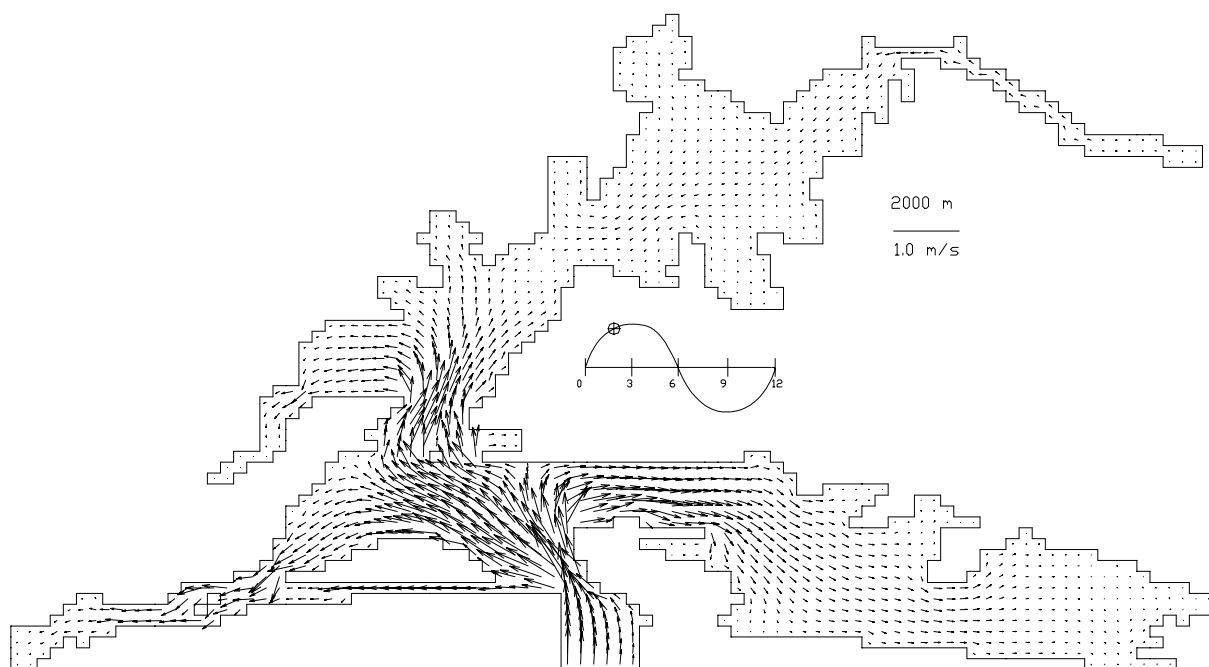
Some preliminary simulations with the two-dimensional hydrodynamic and pollutant transport model PCFLOW2D were carried out. The goal of this part of the study was to determine the circulation in the lagoons and the exchange of water with the ocean, and to simulate the transport-dispersion of a hypothetical conservative contaminant. Only the hydrodynamic results are shown in this paper. The main forcing factor is the tidal influence. Its amplitude is  $\pm 0.90$  m and the tidal period lasts approximately 12 hours. A

m/s, ki je bil določen za poletne razmere (avgust), vendar so simulacije pokazale, da je njegov vpliv na cirkulacijo zelo majhen. V računih je bilo upoštevanih 94 in 62 kontrolnih volumnov v  $x$  in  $y$  smereh (kot kaže slika 9, je os  $x$  usmerjena vzdolž pacifiške obale približno v smeri od severa proti jugu). Prostorska in časovni korak so bili  $\Delta x = 400$  m,  $\Delta y = 333$  m in  $\Delta t = 30$  s.

Izračunana hidrodinamična cirkulacija 2 uri po začetku plimnega ciklusa je prikazana na sliki 9. Največje hitrosti toka pri vtoku v lagune so približno 1 m/s. Zanimivo je, da je tok v osrednji laguni Bacorehuis usmerjen nasprotno, v smeri navzven iz lagune. Pojav gre pripisati dejstvu, da je prejšnji plimni val še vedno usmerjen navzven in se sreča s prihajajočim vtočnim plimnim valom. Iz izračunanega časovnega poteka gladin na različnih lokacijah v lagunah Bacorehuis in Jitzamuri je mogoče ugotoviti, da prihaja do močnega zmanjšanja amplitude vala in do časovnega zamika največjih kot v primerjavi z vtokom v lagune.

west wind of 3 m/s, which was defined for summer conditions (August), was also taken into account, although the simulations showed that its influence on the circulation is almost negligible. In computations, 94 and 62 control volumes in the  $x$  and  $y$  direction were used (the  $x$  direction is oriented along the Pacific coastline, approx. North to South, see Fig. 9). Space and time steps were  $\Delta x = 400$  m,  $\Delta y = 333$  m and  $\Delta t = 30$  sec.

The computed hydrodynamic circulation 2 hours after the beginning of the tidal cycle is presented in Figure 9. The maximum flow velocities at the lagoon entrance were near 1 m/s. It is interesting to notice that in the central Bacorehuis Lagoon, the direction of the flow was reversed - travelling out of the lagoon - which is due to the fact that the previous tidal wave was still propagating outwards and was meeting the new incoming wave. From the computed time variation of water levels at different locations, it can be seen that in both the Bacorehuis and Jitzamuri Lagoons, there was a strong attenuation of the wave amplitude and a time lag of the peaks, in comparison to the lagoon entrances.



Slika 9. Plimna cirkulacija v lagunah Bacorehuis, Agiabampo in Jitzamuri (Mehika).  
Figure 9. Tidal Circulation in the Bacorehuis, Agiabampo and Jitzamuri Lagoons (Mexico).

## 4. ZAKLJUČKI

Matematična modela PCFLOW2D in PCFLOW2D-CURVE sta lahko pomembno orodje za simulacije toka v rekah s kompleksno topografijo dna, za izračune tokov v plitvih jezerih in plimno cirkulacijo v priobalnih morjih. To se je pokazalo pri številnih prejšnjih uporabah (Četina & Rajar, 1993, Krzyk & Četina, 1996), kakor tudi pri primerih stalnega toka, prikazanih v tem članku. Porušitveni val v hipni razširitvi kot primer nestalnega toka je bil kvalitativno dobro simuliran. Kvantitativno pa daje matematični model sprejemljive rezultate za praktično uporabo, vendar bodo potrebni nadaljnji naporji za povečanje točnosti napovedane hitrosti širjenja vala. Prvi izračuni plimne cirkulacije v treh mehiških lagunah so tudi zadovoljivi in se kvalitativno ujemajo z opazovanji v naravi.

Treba je omeniti, da se lahko hidrodinamični model PCFLOW2D uporablja tudi kot osnovno orodje za nadaljnje simulacije transporta plavin in kakovosti vode. Za te namene je bil že dopolnjen s transportno-disperzijskim modulom TCONC, modulom za lebdeče plavine TSM (Krzyk & Četina, 1996) in modulom STATRIM za opis širjenja živega srebra v morskem okolju (Rajar et al., 1997).

## 4. CONCLUSIONS

The PCFLOW2D and PCFLOW2D-CURVE mathematical models can be useful tools for flow simulations in rivers with a complex bottom topography, for flow simulations in shallow lakes and tidal circulation in coastal seas. This has been proved in many previous applications (Četina & Rajar, 1993, Krzyk & Četina, 1996), as well as in the steady state cases presented in the recent paper. The unsteady dam-break wave in a sudden enlargement was qualitatively well simulated. Quantitatively, the mathematical model gave acceptable results for practical use, although further efforts should be undertaken to increase the accuracy of the predicted wave propagation speed. First computations of the tidal circulation in three Mexican lagoons also gave satisfactory results and are in qualitative agreement with the field observations.

It is worth mentioning that the hydrodynamic model PCFLOW2D can serve as a basic tool for further sediment transport and water quality simulations. For these purposes, it has been already accomplished by the transport-dispersion module TCONC, the suspended sediment transport module TSM (Krzyk & Četina, 1996) and the module STATRIM, describing a mercury cycling in the aquatic environment (Rajar et al., 1997).

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