BLED WORKSHOPS
IN PHYSICS
VOL. 20, NO. 1
p. 93

Proceedings of the Mini-Workshop
Electroweak Processes of Hadrons
Bled, Slovenia, July 15–19, 2019

Measurement of G_A and the GDH sum rule at high energies at Jefferson Lab: two proposals

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Abstract. We present two developing experimental proposals for measurements to be performed by using electron scattering, with TJNAF (Jefferson Lab) as the most likely facility: a clean measurement of the nucleon axial form-factor, G_A, and a measurement of the high-energy contribution to the Gerasimov-Drell-Hearn (GDH) sum rule. The work on G_A is done in collaboration with A. Deur (Jefferson Lab) and C. M. Camacho (IPN-Orsay), and the GDH effort is pursued in collaboration with A. Deur, M. Dalton (Jefferson Lab) and J. Stevens (College of William & Mary).

1 Clean measurement of G_A

The nucleon electro-magnetic form-factors, $G_E(Q^2)$ and $G_M(Q^2)$, parameterize the (nucleon) electro-magnetic current operator, and they are well known over a range of Q^2 from e–p and e–"n" scattering. With certain approximations, they can be considered as Fourier transforms of spatial distributions of nucleon charge and magnetization. On the other hand, the *axial* and *pseudoscalar* form-factors, $G_A(Q^2)$ and $G_P(Q^2)$, entering the axial current,

$$\left\langle N(p') \left| \overline{q}\gamma_\mu\gamma_5 \frac{\tau^\alpha}{2}\, q \right| N(p) \right\rangle = \overline{u}(p') \left[\gamma_\mu G_A(Q^2) + \frac{(p'-p)_\mu}{2M}\, G_P(Q^2) \right] \gamma_5 \frac{\tau^\alpha}{2} u(p),$$

are less well known. The axial form-factor, in particular, can be thought to probe the spatial distribution of the nucleon spin, as can be seen from the terms containing $\boldsymbol{\sigma}$ appearing upon a non-relativistic reduction of the axial current:

$$\boldsymbol{\sigma}\,, \quad \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\,, \quad \frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{E'+M}\,, \quad \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\,\boldsymbol{\sigma}\,\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{E'+M}\,.$$

The axial form-factor is conventionally parameterized in the so-called "dipole" form, i. e. by using the same traditional functional form as used in the electromagnetic form-factors:

$$G_A(Q^2) = \frac{G_A(0)}{\left(1 + \dfrac{Q^2}{{M_A}^2}\right)^2}\,, \qquad G_A(0) = g_A \approx 1.27\,, \tag{1}$$

where g_A is the axial coupling constant and $M_A \approx 1$ GeV is an adjustable "axial mass" (cut-off parameter). Different (and better justified) parameterizations exist, e. g. based on axial-vector dominance, large N_c and high-energy-QCD constraints:

$$G_A(Q^2) = g_A \sum_n c_n \frac{1}{1 + Q^2/m_n^2} \, .$$

Such a representation uses a sum of "monopole" forms, with the index n running over isovector/axial-vector mesons (a_1, a_1', ...), and $c_n = f_n \, g_{nNN}/g_A$, where f_n is the vacuum amplitude of meson n and g_{nNN} its coupling to the nucleon [1].

1.1 Existing determinations of G_A

Thus far $G_A(Q^2)$ has been extracted by using two methods: elastic or quasi-elastic neutrino scattering, and electron scattering. In the first case one measures the cross-sections for the processes $\nu n \to l^- p$ and $\overline{\nu} p \to l^+ n$ (in nuclei),

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{8\pi} \frac{M^2 \cos^2\theta_c}{E_\nu^2} \left[A(Q^2) \mp B(Q^2) \frac{s - M^2}{M^2} + C(Q^2) \frac{(s - M^2)^2}{M^4} \right] \, ,$$

where $A(Q^2)$, $B(Q^2)$ and $C(Q^2)$ are known functions of $G_E(Q^2)$, $G_M(Q^2)$ and $G_A(Q^2)$. The axial form-factor is then determined by fitting the Q^2-dependence of the cross-section; the cut-off parameter M_A is then typically extracted by assuming the dipole form of $G_A(Q^2)$ (see [2] and references therein).

In the second case, one exploits the $p(e, e'\pi^+)n$ process near threshold [3], for which the cross-section can be written as

$$\frac{d\sigma}{dE_e' \, d\Omega_e' \, d\Omega_\pi^\star} = \Gamma_v \frac{d\sigma_v}{d\Omega_\pi^\star} \, , = \Gamma_v \left[\frac{d\sigma_T}{d\Omega_\pi^\star} + \varepsilon_L^\star \frac{d\sigma_L}{d\Omega_\pi^\star} \right] \, ,$$

where Γ_v is the virtual photon flux. The longitudinal part of the cross-section probes $F_\pi(Q^2)$, while its transverse part is sensitive to $G_A(Q^2)$ and, in turn, to the axial RMS radius,

$$\langle {r_A}^2 \rangle = -\frac{6}{G_A(0)} \left. \frac{dG_A(Q^2)}{dQ^2} \right|_{Q^2=0} = \frac{12}{{M_A}^2} \, .$$

It is well known from χPT [4] that the axial radius picks up a correction due to pion loops, such that the "true" axial radius (measured in neutrino scattering) becomes modified in electro-production experiments:

$$\langle {r_A}^2 \rangle \to \langle {r_A}^2 \rangle + \frac{3}{64 f_\pi^2} \left(1 - \frac{12}{\pi^2} \right) \, . \tag{2}$$

As suggested by the extractions shown in Fig. 1, this indeed seems to be the case: the neutrino experiments yield a world average of $\langle M_A \rangle = (1.026 \pm 0.009)$ GeV, while the pion electro-production experiments give $\langle M_A \rangle = (1.062 \pm 0.015)$ GeV. There is an $\approx 2.5\ \sigma$ difference in $\langle M_A \rangle$ between the two extraction methods, but one should not overlook the large statistical and systematic uncertainties and possible data inconsistencies. In particular, the MiniBooNE collaboration, performing a state-of-the-art neutrino scattering experiment, has reported values as high as $M_A \approx 1.35$ GeV!

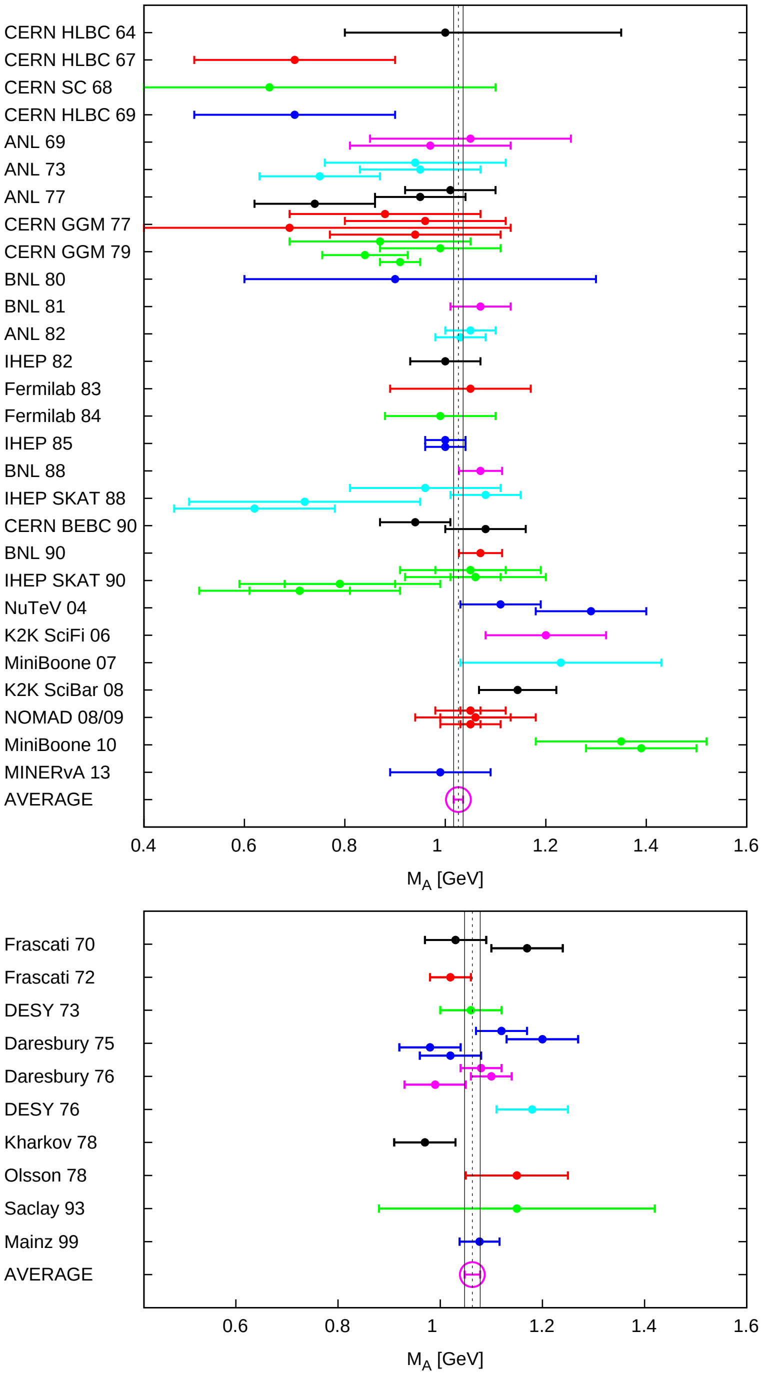


Fig. 1. Extractions of the axial mass parameter from neutrino experiments (top panel) and electron scattering experiments (bottom panel). The difference in their averages, $\langle M_A \rangle = (1.026 \pm 0.009)$ GeV and $\langle M_A \rangle = (1.062 \pm 0.015)$ GeV, respectively, may have their origin in the chiral correction (2) — but may also hint at a limitation of the dipole parameterization.

1.2 Proposed measurement of G_A by using inverse β decay

Clearly our knowledge of the axial form-factor would benefit from a third, independent and ideally cleaner, way to access G_A. The theoretically cleanest way to access G_A (but, as it turns out, experimentally very challenging) is through the weak interaction, as in neutrino experiments — but it can also be probed in weak electron scattering, i. e. in inverse β decay shown in Fig. 2:

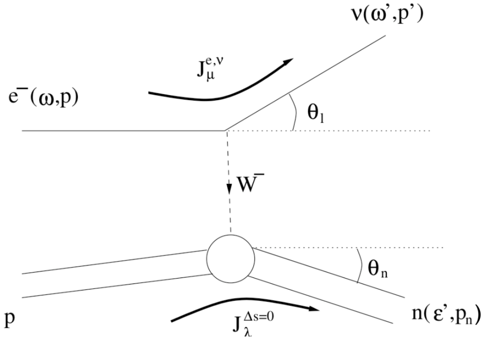


Fig. 2. Kinematics of inverse β decay used to access G_A in an electron scattering experiment, with detection of neutrons the final state.

The weak charged-current cross-section is given by

$$\frac{d\sigma}{d\omega'} = M\frac{G_F^2\cos^2\theta_c}{\pi}\frac{\omega'}{\omega}\left[\cos^2\left(\frac{\theta_l}{2}\right) f_2 + \left(2f_1 + \frac{\omega+\omega'}{M} f_3\right)\sin\left(\frac{\theta_l}{2}\right)\right] ,$$

where the structure functions f_1, f_2 and f_3 are known functions of the electro-magnetic form-factors and G_A. In contrast to pion electro-production experiments, the extraction of G_A from this purely weak process is model-independent, and with recent advances in polarized beams high precision is possible.

The main experimental challenges are: tiny cross sections (on the order of $\approx 10^{-40}\,\mathrm{cm}^2/\mathrm{sr}$), neutron detection with accurate kinematics; and (very) large electro-magnetic backgrounds. The strategies to deal with these challenges are presently being developed, but we will certainly wish to exploit the available high-intensity polarized electron beams at either JLab or MAMI, in conjunction with a long LH2 target; we would wish to remain at low beam energy (less than ≈ 120 MeV) in order to stay below the pion production threshold; and design a suitable backward kinematics to enhance the weak cross-section (forward neutrons). The beam must be polarized and pulsed so that the electro-magnetic background can be cleanly removed: the weak process has a 100 % asymmetry while the electro-magnetic process has a vanishing asymmetry, and they can be separated on a pulse-by-pulse basis.

So far several facilities have been considered where this experiment could take place: MESA at Mainz, FEL at JLab, Hall D tagger at JLab, at the JLab injector, or at Cornell. Each has its particular instrumental constraints, its own pros and cons regarding beam conditions and available infrastructure. Regardless of the peculiarities of the setup, we will need to remove the scattered electrons (Møller, nuclear scattering) by means of a sweeper magnet; reduce the prompt electro-magnetic radiation (γ-flash, electrons) by timing cuts; reduce the background from the target cell window by minimizing window thickness and using a backwards veto detector. We do possess preliminary background estimates, and a detailed Monte Carlo simulation is underway. Assuming 100 % efficiency and no further backgrounds, the precision of the extracted G_A that we could achieve in about 2 months of running (order of magnitude estimate) is indicated in Fig. 3.

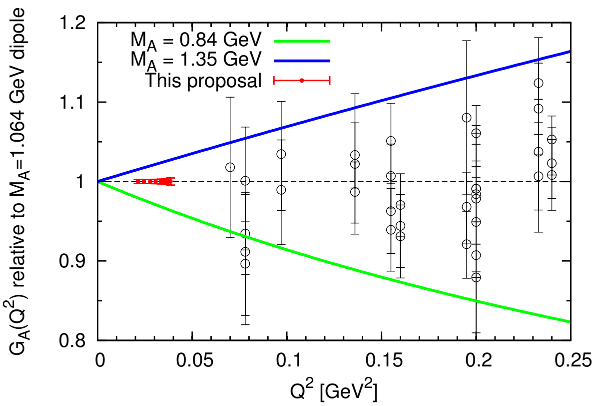


Fig. 3. The expected precision of the extracted G_A with $\approx$ 2 months runtime at a typical high-luminosity facility. The $M_A = 0.84$ GeV and $M_A = 1.35$ GeV curves correspond to the dipole parameterization (1) with the two rather extreme axial masses (one far below and one far above the world average). If nothing else, with the shown precision we should be able to reject or confirm the dipole form itself.

2 Ascertaining the high-energy behavior of the GDH integrand

The Gerasimov-Drell-Hearn (GDH) sum rule is a sum rule that relates the energy-weighted difference of the spin-dependent cross-section for photo-production off a given target to the spin (S) and anomalous magnetic moment (κ) of that target:

$$\int_{\nu_{\text{thr}}}^{\infty} \left(\sigma_{3/2} - \sigma_{1/2}\right) \frac{d\nu}{\nu} = \frac{4\alpha S \pi^2 \kappa^2}{M^2} ,$$

where α is the fine-structure constant. This is a generic QFT prediction valid for any type of target. In its derivation, one relies on causality, unitarity, Lorentz and gauge invariances. In addition, one assumes that in the forward Compton scattering amplitude,

$$\frac{1}{8\pi M} T(\nu, \theta = 0) = f(\nu) \varepsilon'^* \cdot \varepsilon + i\, g(\nu) \boldsymbol{\sigma} \cdot (\varepsilon'^* \times \varepsilon) ,$$

the spin-dependent amplitude $g(\nu)$ vanishes at large ν to derive the dispersion relation, and that $\operatorname{Im} g(\nu)$ decreases fast enough with ν (faster than $\sim 1/\log \nu$) for the integral to converge. Note that the integral of the unpolarized cross-section, $\int(\sigma_{3/2}+\sigma_{1/2})\,d\nu$, *without* the $1/\nu$ weight, *does not converge*.

Looking at Fig. 4 which shows a prediction of the running GDH integral to very high energies one could claim that the sum rule is saturated already at $\nu \approx 3\,\mathrm{GeV}$ but in fact no measurements exist above that energy: all existing experiments (at LEGS, MAMI and ELSA) were performed below it. The polarized cross-section at large ν is *unknown*, yet it is usually expected to be described by Regge theory: it considers isoscalar $(p+n)$ and isovector $(p-n)$ contributions to $\sigma_{3/2}-\sigma_{1/2}$ as coming from different meson families: $f_1(1285)$ and $a_1(1260)$, respectively, resulting in the parameterization

$$\Delta\sigma^{(p\pm n)} = \sigma_{3/2}-\sigma_{1/2} = c_2\, s^{\alpha_{f_1}-1} \pm c_1 s^{\alpha_{a_1}-1} \;,$$

where $s = 2M\nu + M^2$, α_{f_1} and α_{a_1} are the Regge intercepts of $f_1(1285)$ and $a_1(1260)$ trajectories, respectively, and c_1 and c_2 are parameters.

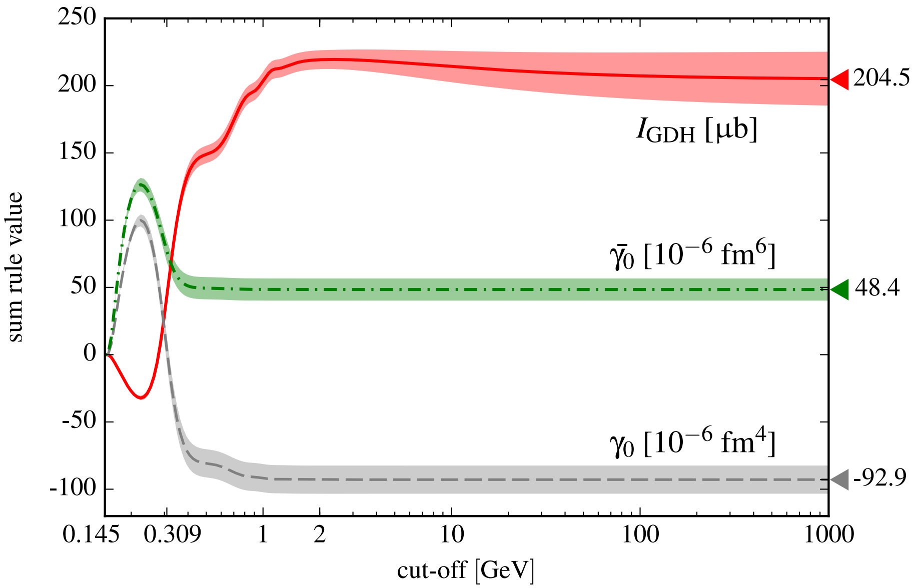


Fig. 4. The value of the GDH integral on the proton as a function of the upper integration bound. (Figure taken from [5].)

If the sum rule fails, its derivation implies that this would occur at high energies, and there are several conceivable violation mechanisms [6]. For instance, the appearance of a fixed $J = 1$ pole of the Compton amplitude in the complex angular-momentum plane or the existence of an anomalous charge-density commutator, i. e. $[J^0(x), \mathbf{J}(y)]_{\mathrm{e.t.}} \neq 0$ would both cause the sum rule to fail; other, more exotic possibilities have been proposed.

2.1 Experimental strategy

The main task of the experiment currently being devised is to measure the energy dependence of the GDH integrand at high energies for both proton and neutron (deuteron) to allow for isospin separation. Assuming

$$\sigma_{3/2} - \sigma_{1/2} = a\nu^b$$

(for a given target), the primary goal is to get b, without a need to extract an accurate a. Initially, we would measure only the *yield difference*, $N_{3/2} - N_{1/2}$, and consider proper normalization (absolute cross-sections) later on. The ideal facility to run the proposed experiment would be Hall D at JLab with a circularly polarized tagged photon beam, longitudinally polarized target and large solid-angle ($\approx 4\pi$) detector: this setup would allow us to measure $\Delta\sigma(\nu)$ at high ν where no data exist, and to cover four times the existing ν range (3 GeV $\to$ 12 GeV).

2.2 Impact of the proposed experiment

Intercept of the a_1 Regge trajectory

The high-energy behaviors of the isovector (non-singlet) and isoscalar (singlet) cross-section differences are driven by the $a_1(1260)$ and $f_1(1285)$ Regge trajectories,

$$\Delta\sigma^{(p-n)} \sim s^{\alpha_{a_1}-1} , \qquad \Delta\sigma^{(p+n)} \sim s^{\alpha_{f_1}-1} .$$

From DIS data one typically extracts $\alpha_{a_1} \approx 0.4$, $\alpha_{f_1} \approx -0.5$, while a recent fit [7] yields $\alpha_{a_1} \approx 0.45$, $\alpha_{f_1} \approx -0.36$. A naive Regge expectation gives $\alpha_{a_1} \approx -0.27$, $\alpha_{f_1} \approx -0.32$, so there appears to be a discrepancy in the a_1 intercept between the two extractions. Measuring $\Delta\sigma$ at high ν for both proton and neutron targets would help to remove this uncertainty.

Spin-dependent Compton amplitude

Figure 5 shows the real and imaginary parts of the spin-dependent Compton amplitude g. The imaginary part is measured directly in a GDH experiment as

$$\mathrm{Im}\, g(\nu) = -\frac{\nu}{8\pi}\, \Delta\sigma(\nu) .$$

The real part, however, is given by a dispersion relation,

$$\mathrm{Re}\, g(\nu) = -\frac{\nu}{4\pi^2} \mathcal{P} \int_0^\infty \frac{\nu' \Delta\sigma(\nu')}{\nu'^2 - \nu^2}\, d\nu' ,$$

and is therefore very sensitive to the quality of the integrand.

If both $\mathrm{Re}\, g(\nu)$ and $\mathrm{Im}\, g(\nu)$ were known precisely enough (and given $f(\nu)$ which is well measured), the two complex amplitudes could be used to determine the forward-scattering ($\theta = 0$) quantities

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=0} = \left|f\right|^2 + \left|g\right|^2 , \qquad \Sigma_{2z}|_{\theta=0} = -\frac{fg^* - f^*g}{|f|^2 + |g|^2} .$$

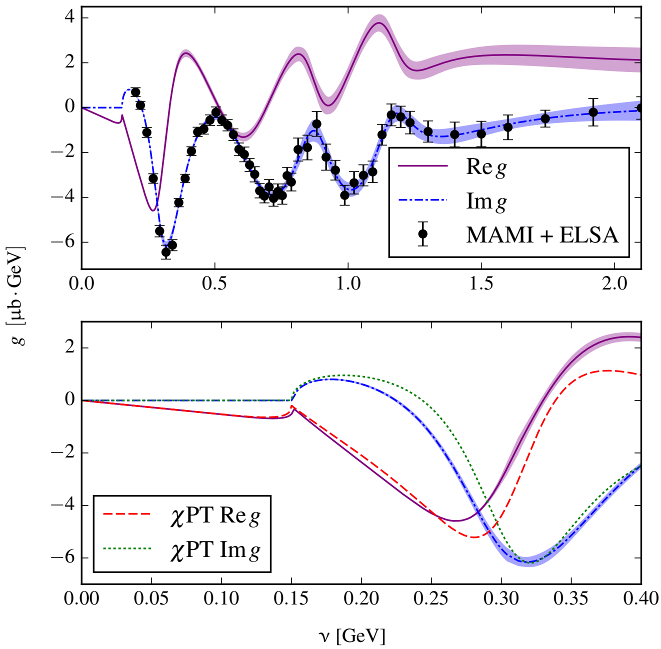


Fig. 5. The spin-dependent Compton amplitude $g(\nu)$. Top: real and imaginary parts, the latter fitted to GDH data, the former calculated via dispersion relations. Bottom: χPT calculation. Figure from [5].

The asymmetry for circularly polarized photons and nucleons polarized along the z axis,

$$\Sigma_{2z} = \frac{d\sigma_{3/2} - d\sigma_{1/2}}{d\sigma_{3/2} + d\sigma_{1/2}} ,$$

as well as its counterpart Σ_{2x} (with transverse polarization of the nucleons), can provide information on all four spin polarizabilities appearing in Compton scattering. In addition, Σ_{2z} (in particular its behavior near $\theta = 0$) is very sensitive to chiral loops. Moreover, the uncertainty of the product of the unpolarized XS and Σ_{2z} for $\theta = 0$ increases rapidly for $\nu > 2\,\mathrm{GeV}$, hence a precise measurement of $\Delta\sigma(\nu)$ in the ν range up to about 10 or 12 GeV could significantly reduce the uncertainty on Σ_{2z}.

Polarizability correction to hyperfine splitting in muonic hydrogen

The third impact of the proposed measurement is related to the "proton radius puzzle", specifically to the effect of proton structure on the hyperfine splitting of the nS levels in muonic hydrogen,

$$E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_{\mathrm{Fermi}}(nS) .$$

The proton-structure correction can be split into three terms: the Zemach radius, the recoil contribution, and the polarizability contribution,

$$\Delta_{\mathrm{structure}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}} .$$

At present, the relative uncertainties of the three terms are 140 ppm, 0.8 ppm and 86 ppm, respectively, which need to be put into the perspective of the forthcoming PSI measurement of the hyperfine splitting whose precision is expected to be as low as 1 ppm. Our proposed measurement can contribute to the uncertainty reduction in the third correction term. It can be written as

$$\Delta_{\text{pol}} = \frac{Z\alpha m}{2\pi(1+\kappa)M}[\delta_1 + \delta_2] \, ,$$

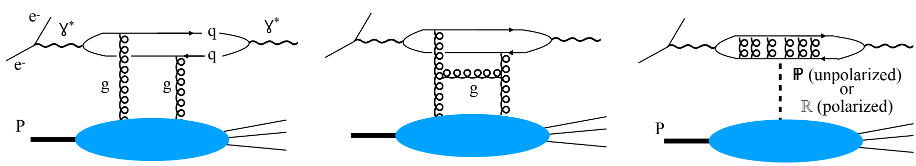
where m is the electron mass. Here δ_1 involves an integral of the polarized distribution function $g_1(x, Q^2)$ over both x and Q, while δ_2 involves a similar integration of $g_2(x, Q^2)$: see [8] for explicit formulas. Since g_1 at low Q is essentially the GDH integrand,

$$\sigma_{1/2} - \sigma_{3/2} = \frac{4\pi\alpha^2}{m\mathcal{F}} \left(g_1 - \frac{Q^2}{\nu^2} g_2 \right) \, ,$$

a precise measurement of $\Delta\sigma$ would constrain δ_1. To calculate δ_1, one indeed needs the Q^2 dependence of g_1, but *the integrand is weighted by* $1/Q^3$, thus knowing the value at $Q^2 = 0$ would stabilize the integration. This is badly needed, as the above mentioned 86 ppm uncertainty needs to be brought down to the 1 ppm level, and this implies that our knowledge of g_1 needs to be improved by two orders of magnitude!

Transition from polarized DIS to diffractive regime

The proposed experiment would also have the capability to explore the transition between DIS and low-x regime of diffractive scattering. This regime has been investigated e. g. at HERA, but only in the unpolarized case. Such processes are traditionally described in terms of a diquark picture: a hard γ^* hadronizes into a $\bar{q}q$ pair of coherence length $1/(xM)$, with high Q^2 dominated by gluon exchange and low Q^2 dominated by Pomeron/Reggeon exchange as shown in the Figure:



The spin-0 Pomeron couples to the proton components irrespectively of their helicity, i. e. controls unpolarized diffractive scattering, while double-polarized $\vec{e}\vec{p}$ scattering filters out Pomeron exchange to reveal the non-singlet Reggeon exchange. This is relevant for the physics of the envisioned Electron-Ion-Collider (EIC), and a measurement of $\Delta\sigma$ would provide a $Q^2 = 0$ baseline for the study of the transition from hard partonic picture to soft Reggeon exchange picture.

2.3 The physics goals in brief

The primary physics goal is to determine the α_{f_1} and α_{a_1} intercepts (in case $N_{3/2} - N_{1/2}$ follows Regge) and thus validate the convergence of the GDH integral. We intend for instance, acquire the data precise enough to result in uncertainties $\Delta\alpha_{a_1} = \pm 0.008$, $\Delta\alpha_{f_1} = \pm 0.016$ as compared to $\Delta\alpha_{a_1} = \pm 0.23$,

$\Delta\alpha_{f_1} = \pm 0.22$ from ELSA: see Fig. 6. The secondary physics goal is to improve the current experimental accuracy of the GDH integral by $\approx 25\,\%$ (with reasonable assumptions on ΔP_e, ΔP_t, and absolute normalization). Finally, regardless of the convergence and sum rule validity, we would be able to explore the region of diffractive QCD relevant to EIC physics.

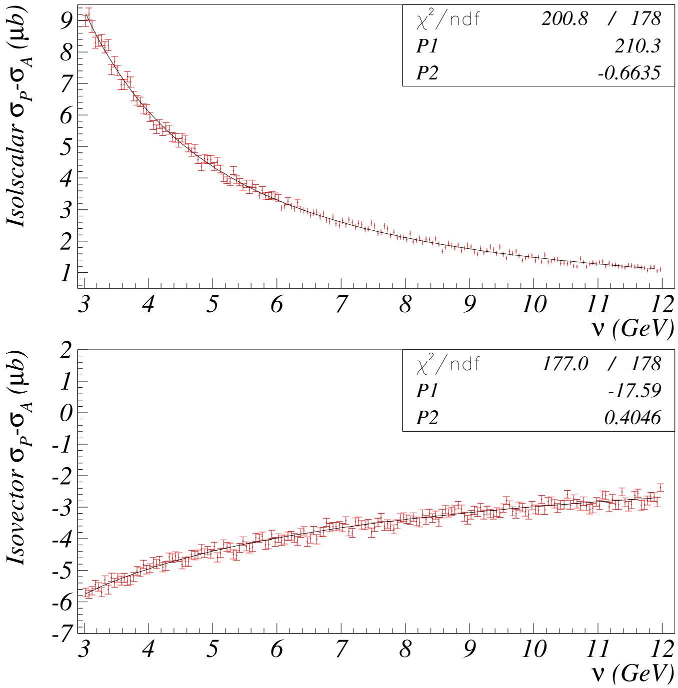


Fig. 6. The expected precision of the isoscalar (top panel) and isovector (bottom panel) polarized cross-section differences (plotted on a Regge curve).

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