RAZLIČNI PRISTOPI ZA IZRAČUN TIRNIC GLONASS-SATELITOV IZ ODDANIH EFEMERID

DIFFERENT APPROACHES IN GLONASS ORBIT COMPUTATION FROM BROADCAST EPHEMERIS

Kamil Maciuk

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IZVLEČEK

Enačbo gibanja umetnega satelita lahko rešujemo z različnimi metodami. Pri nekaterih diskretne položaje satelitov dobimo v enem, pri drugih v več korakih. Pri numerični integraciji na natančnost izračuna vpliva izbor velikosti koraka integracije med zaporednimi iteracijami. Nepravilen izbor integracijskega koraka med posameznimi iteracijami vodi do odstopanj, ki so lahko večja od dosegljive natančnosti izračuna s posamezno metodo. Zaradi tega moramo v izračunih integracijski korak toliko zmanjšati, da ne vpliva več na natančnost izračuna. V prispevku smo analizirali uporabnost različnih metod Runge-Kutta za numerično reševanje gibanja umetnega satelita, in sicer metodo Runge-Kutta 4. in 5. stopnje ter metodo Runge-Kutta-Fehlerberg 4. in 5. stopnje. Rezultate izračunov smo primerjali s klasično metodo Runge-Kutta in ugotovili, da z ustreznim izborom integracijskega koraka in metode nekoliko upočasnimo računski čas, vendar pridobimo bolj kakovostne rezultate izračunov.

ABSTRACT

Several types of methods can solve equations of satellite motion numerically. These methods are divided into single and multi-step methods. The accuracy of each method depends directly on adopted integration step size between successive iterations. To achieve result with required accuracy it is important to maintain appropriate size of integration step. Inappropriate step size could cause local errors between iterations greater than accuracy of the method. Therefore, integration step size needs to be reduced until it does not affect accuracy of the final solution. Group of Runge-Kutta (RK) methods for solving equations of satellite motion have been analysed in this article. Five different methods: Runge Kutta 4th order, Runge Kutta 5th order and Runge Kutta Fehlberg 4th and 5th order methods were discussed. Compared to the classical Runge-Kutta integration method other methods are slower, but give results that are slightly more accurate.

KLJUČNE BESEDE

numerična integracija, GNSS, GLONASS, satelitska geodezija

KEY WORDS

numerical integration, GNSS, GLONASS, satellite geodesy

1 INTRODUCTION

Equations of satellite motion could be solved both analytically (Góral and Skorupa, 2012) and numerally (Gaglione et al., 2011). Runge-Kutta (RK) methods are one of the well-known numerical methods for solving differential equations (Kosti et al., 2009; Ozawa, 1999; Sermutlu, 2004), while 4th order Runge-Kutta method is recommended to solve equations of satellite motion by GLONASS Interface Control Document (ICD-GLONASS, 2008).

Currently there are very few publications referring to comparison of numerical methods to solve GNSS equations of satellite motion. Numerical integration of low Earth orbiting satellites was performed by (Es-hagh, 2005). The author compared two variable step integration methods: Adams and RungeKuttaFehlberg (RKF) methods. Adams' method is recommended for long arc orbit integration or in low resolutions (large step size) orbit integration. In contrast, RKF method is better to be used for highresolution (small step size) solutions. (Sermutlu, 2004) presented the comparison of accuracy and speed tests of RungeKutta 4th and 5th order for solving Lorenz equation. He noticed that 4th order method gives more accurate results for shorter running times, but as step sizes decline, 5th order method gives more accurate results. (Khodabin and Rostami, 2015) obtained the same results. The authors analysed different orders of Runge-Kutta methods for applications in electric circuits. They confirmed superiority of higher order RK methods over other methods. (Montenbruck, 1992) compared multistep, interpolation and Runge-Kutta methods for the numerical integration of ordinary differential equations of orbital motion. The author showed that both single-step and multi-step methods are competitive. Equations of satellite motion were also solved by many different approaches, e.g. RungeKuttaFehlberg method (Atanassov, 2010), analytically (Kudryavtsev, 1995), by MATLAB ODE45 function (Bradley et al., 2014) or by new types of Runge Kutta methods (Gonzalez et al., 1999).

Runge-Kutta 4th order method to solve equations of satellite motion was presented by (ICD-GLONASS, 2008), but without any data concerns accuracy. It is clear that the error in orbit integration strongly depends on a step size. GLONASS satellite integration results have no explicit differences between solutions from 1 to 300 s integration step size. The author suggested that 60 s GLONASS integration step width is sufficient in any case, because for small angular distances the satellite orbit could be considered as nearly linear.

2 KEPLERIAN MOTION

Simplified satellite orbiting is called Keplerian motion (Zare, 1982). In Earth-artificial satellite, setting the mass of a satellite can be considered negligible and does not enter the motion equations system (Breiter and Elipe, 2006). This is due to its size and mass that are negligibly small relatively to the mass of the Earth. The satellites motion is governed by the Newton's second law hence, according to the formula:

$$\ddot{\vec{\mathbf{r}}} = -\frac{\mu}{r^2}\frac{\vec{\mathbf{r}}}{r} \tag{1}$$

where:

$\mu = GM$ - the product of Newton's gravitational constant and mass of the Earth,

r - distance between the Earth and satellite centres.

Equation 1 relates to a motion in an inertial system. Two vectors or 6 scalars are the solutions of this second order differential equation (Keplerian elements). They are the results of double integration of (1). In case of the Earth's artificial satellite, perturbing forces affecting its position should also be taken into account (Bobojć and Drożyner, 2011):

$$\ddot{\vec{\mathbf{r}}} = -\frac{\mu}{r^2}\frac{\vec{\mathbf{r}}}{r} + \mathrm{K} \qquad (2)$$

where:

K - perturbing forces.

Gravitational forces due to the Earth as well as the strength of perturbing forces determine satellites motion. Table 1 shows the magnitude of perturbing forces and their effect on a GNSS satellite.

Table 1: Perturbing accelerations acting on a GPS satellite (Dach et al., 2007).

Source	*Acceleration [m/s2]*	*Orbit error after 24 hours [m]*
Two-body term of the Earth's gravity field	0.59	∞
Oblateness of the Earth	$5 \cdot 10^{-5}$	10.000
Lunar gravitational attraction	$5 \cdot 10^{-6}$	3.000
Solar gravitational attraction	$2 \cdot 10^{-6}$	800
Other terms of the Earth's gravity field	$3 \cdot 10^{-7}$	200
Radiation pressure (direct)	$9 \cdot 10^{-8}$	200
Y-bias	$5 \cdot 10^{-10}$	2
Solid Earth tides	$1 \cdot 10^{-9}$	0.3

The main perturbing force affecting a satellite is the Earth's oblateness that characterizes polar flattening of the Earth. The effect of accelerations due to the luni-solar gravitational perturbations is an order of magnitude smaller than the second zonal harmonic. We can consider other forces as negligible. It may be assumed that perturbing forces acting on a GPS satellite affect will be different that on a GLONASS satellites due to two reasons. Firstly, GLONASS satellite orbit the Earth much lower, that is mean they are much sensitive to gravitational perturbations. Secondly, GLONASS satellites have larger area-to-mass ratios than GPS satellites, which implies that the impact of solar radiation pressure is larger for GLONASS.

3 RUNGE-KUTTA METHODS

Numerical integration methods can be divided into single and multi-step methods. In case of multi-step methods, to calculate the predicted value of the function, we must know values of the function at some previous time points (e.g. t_{n-1}, t_{n-2}). The best known multi-step methods used to solve equations of satellite motion are Cowell and Encke methods (Liu and Liao, 1994). Whereas single-step methods based on a single initial point of time, allow us to calculate predicted values of the function. The best-known single-step methods for solving satellite equations of motion are Runge-Kutta 4th and higher order methods.

The equation of satellite's motion is a second order differential equation. Therefore, it has to be converted to the system of first order differential equations to be solved by RK methods as following:

$$y'(x) = f(x, y(x))$$
$$y(t_0) = y_0 \tag{3}$$

Runge-Kutta method allows calculation of the approximate value of the function $y(x_n)$ for $a = x_0 < x_1 < \ldots < x_n = b$, as in the formula:

$$y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i$$
$$x_{n+1} = x_n + h \tag{4}$$

where:

$$k_i = f\left(x_n + c_i h,\ y_n + h\sum_{j=1}^{i-1} a_{ij} k_j\right) \tag{5}$$

and

$$c_i = \sum_{j=1}^{s} a_{ij} \tag{6}$$

where:

a, b	-	constants,
h	-	step size.
s	-	Runge-Kutta method's order,

i=1,2,… ,s.

Expanding (2) into first order differential equations still makes it impossible to solve them analytically in a fast and simple way. GLONASS Interface Control Document (ICD-GLONASS, 2008) recommends the use of Runge-Kutta 4th order method for this purpose, as it ensures adequate accuracy altogether with the simplicity of the solution. Equation (7) is an extension of (2) into a form of scalar functions. It takes into account perturbing forces due to the flattening of the Earth (second zonal harmonic) and influence of the Sun and Moon (Poutanen et al., 1996):

$$\frac{dx}{dt} = \dot{x}$$
$$\frac{dy}{dt} = \dot{y}$$
$$\frac{dz}{dt} = \dot{z}$$
$$\frac{d\dot{x}}{dt} = -\frac{\mu}{r^3}x + \frac{3}{2}C_{20}\frac{\mu a^2}{r^5}x\left(1 - \frac{5z^2}{r^2}\right) + \ddot{x}_{LS} + \omega^2 x + 2\omega\dot{y} \tag{7}$$
$$\frac{d\dot{y}}{dt} = -\frac{\mu}{r^3}y + \frac{3}{2}C_{20}\frac{\mu a^2}{r^5}y\left(1 - \frac{5z^2}{r^2}\right) + \ddot{y}_{LS} + \omega^2 y - 2\omega\dot{x}$$
$$\frac{d\dot{z}}{dt} = -\frac{\mu}{r^3}z + \frac{3}{2}C_{20}\frac{\mu a^2}{r^5}z\left(3 - \frac{5z^2}{r^2}\right) + \ddot{z}_{LS}$$

where:

x, y, z - satellite coordinates,

$\dot{x}, \dot{y}, \dot{z}$ - satellite velocities,

$\ddot{x}_{LS}, \ddot{y}_{LS}, \ddot{z}_{LS}$ - lunisolar accelerations,

a - semi-major axis of the ellipsoid,

ω - the Earth rotation rate,

C_{20} - second zonal harmonic coefficient of the geopotential,

$r = \sqrt{x^2 + y^2 + z^2}$,

$\mu = GM$.

Second zonal harmonic is known from parameters of current PZ-90 (Параметры Земли 1990 года, Parameters of the Earth 1990) realization. In calculations, it is adopted as the known parameter. Lunisolar accelerations are varying in time, thus they are transmitted in GLONASS navigational (broadcast) message in 15 min intervals, and they are assumed constant within ±15 min from the initial position.

4 GLONASS NAVIGATION MESSAGE

GLONASS navigation message contains information regarding satellites' position parameters for a single observation epoch. Those data are recorded in RINEX format *.yyG (Gurtner and Estey, 2007) with 30-minutes interval as vector components of satellite position, velocity and acceleration (Table 2).

Table 2: GLONASS data record description (Gurtner and Estey, 2007).

Observation record	*Description*	*Format*
SV / EPOCH / SV CLK	- Satellite system (R), satellite number (slot number in sat. constellation) - Epoch: Toc - Time of Clock (UTC) __- year (4 digits) __- month, day, hour, minute, second - SV clock bias (sec) (-TauN) - SV relative frequency bias (+GammaN) - Message frame time (tk+nd*86400) _ in seconds of the UTC week	A1, I2.2 1X, I4 5 (1X, I2, 2), 3D19.12
BROADCAST ORBIT – 1	- Satellite position X __ __ __ (km) - Satellite velocity X dot__ __ (km/sec) - Satellite X acceleration__ __ (km/sec2) - Satellite health (0=OK)	4X, 4D19.12
BROADCAST ORBIT – 2	- Satellite position Y________(km) - Satellite velocity Y dot__ __ (km/sec) - Satellite Y acceleration__ __ (km/sec2) - Satellite frequency number__(-7…+12)	4X, 4D19.12
BROADCAST ORBIT – 3	- Satellite position Z________ (km) - Satellite velocity Z dot__ __ (km/sec) - Satellite Z acceleration__ __ (km/sec2) - Age of oper. information__ _(days)	4X, 4D19.12

Figure 1 contains a single record of GLONASS navigational message in RINEX format. It relates to the satellite PRN 1 from 9th June 2013 at 0:00 GLONASS time.

```
PRN y m d h m s            SV clock bias (sec)        SV relative frequency bias Message frame time
1 13 6 9 0 0 0.0           -0.172111205757E-03        0.000000000000E+00         0.846000000000E+05
Satellite position X (km)  X velocity (km/sec)        X acceleration (km/sec2)   Health
0.144409179688E+05         -0.264622497559E+01        0.000000000000E+00         0.000000000000E+00
Satellite position Y (km)  Y velocity (km/sec)        Y acceleration (km/sec2)   Frequency number
0.522635791016E+04         0.877996444702E+00         0.000000000000E+00         0.100000000000E+01
Satellite position Z (km)  Z velocity (km/sec)        Z acceleration (km/sec2)   Age of oper. information
0.203675307617E+05         0.165256118774E+01         -0.279396772385E-08        0.000000000000E+00
```

Figure 1: Example of GLONASS navigation message.

Contrary to GPS, GLONASS message contains information about satellites' positions in ECEF coordinate system (Gaglione et al., 2011). Those data for a single satellite are stored in four 80-byte lines (Figure 1). GLONASS ephemeris message contains information about satellites' position in current PZ-90 realization (Boucher and Altamimi, 2001). PZ-90.02 realization was obligatory since 2007 (Montenbruck et al., 2015), currently PZ-90.11 is in use (IGSMAIL-6896).

5 GENERAL COMPARISON

In this paper, a group of Runge-Kutta methods were analysed in resolving equations of satellite motion for GLONASS satellite. Parameters of GLONASS space segment are presented in Table 3.

Table 3: GLONASS space segment parameters (Angrisano et al., 2013).

Parameter	*Value*
Number of SV	24
Orbital planes	3
Orbital altitude (km)	19 100
Orbital inclination	64.8°
Ground track period	8 sidereal days
Layout	Symmetric
Broadcast ephemerides	ECEF
Datum	PZ-90

This paper discusses four variants of Runge-Kutta method: best-known 4th order method (RK4), 5th order method (RK5) and Runge-Kutta-Fehlberg 4th (RKF4) and 5th (RKF5) order methods. Table 4 shows formulas of analysed RK methods.

The determination error of satellite position depends on Runge-Kutta method order and adopted for calculations integration step. In principle, position determination is more accurate for smaller integration steps. Smaller integration step (h) carries a serious increase of intermediate positions thus, it increases computation time. Each step h, depending on the adopted formula requires calculation of four, five or six intermediate values of the function analysed in this paper (Table 4). Therefore, the best solution appears to be a method, which provides required accuracy of a satellite position solution combined with the highest execution speed. It is especially important in case of real-time solutions.

Table 4: Parameter of analysed Runge-Kutta methods (Rentrop et al. 1989; Sermutlu, 2004).

Runge-Kutta 4th order (RK4)	*Runge-Kutta 5th order (RK5)*	*Runge-Kutta-Fehlberg (RKF45)*
$k_1 = hf(t_n, y_n)$	$k_1 = hf(t_n, y_n)$	$k_1 = hf(t_n, y_n)$
$k_2 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$	$k_2 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$	$k_2 = hf\left(t_n + \frac{1}{4}h, y_n + \frac{1}{4}k_1\right)$
$k_3 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$	$k_3 = hf\left(t_n + \frac{1}{4}h, y_n + \frac{3k_1 + k_2}{16}\right)$	$k_3 = hf\left(t_n + \frac{3}{8}h, y_n + \frac{3}{32}k_1 + \frac{9}{32}k_2\right)$
$k_4 = hf(t_n + h, y_n + k_3)$	$k_4 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_3\right)$	$k_4 = hf\left(t_n + \frac{12}{13}h, y_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right)$
$y_{n+1} = y_n + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$	$k_5 = hf\left(t_n + \frac{3}{4}h, y_n + \frac{-3k_2 + 6k_3 + 9k_4}{16}\right)$	$k_5 = hf\left(t_n + h, y_n + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right)$
	$k_6 = hf\left(t_n + h, y_n + \frac{k_1 + 4k_2 + 6k_3 - 12k_4 + 8k_5}{7}\right)$	$k_6 = hf\left(t_n + \frac{h}{2}, y_n - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 + \frac{11}{40}k_5\right)$
	$y_{n+1} = y_n + \frac{7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6}{90}$	$y_{n+1}^{[4]} = y_n + \left(\frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5\right)$
		$y_{n+1}^{[5]} = y_n + \left(\frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6\right)$

6 RESULTS

This paper shows research of GLONASS' satellite position determination by RK methods according to the integration step size and its effect on the accuracy and speed of solution. There has been analysed position of #10 GLONASS satellite (SV 717, orbit 2, launched 25/12/2006, active from 03/04/2007) due date 01/01/2012 at three different moments of time: 10^{15}, 10^{45} and 11^{15} UTC. The survey is based on broadcast orbit coordinates taken with maximum available accuracy of 12 decimal digits (Figure 1). Accuracy analysis was performed based on ORBGEN results, which is a part of Bernese GPS Software 5.0 (Dach et al., 2007). Comparison of numerical solutions of (2) was carried out based on the author's own scripts implemented in Matlab R2010b®. They were run on Lenovo L420 computer equipped with Windows 7 Professional, with the Intel Core i5-2410M 2.30 GHz, 4.00 GB RAM.

Based on known initial function values of position, velocity and acceleration it is possible to determine satellite's position for any moment within the range ±15 min (900 s). This time span comes from the fact that the GLONASS ephemeris is updated every 30 minutes. If ephemeris data are used in the range exceeding ±15min difference between calculated and actual position expected is grow rapidly every ±15 min (Figure 2).

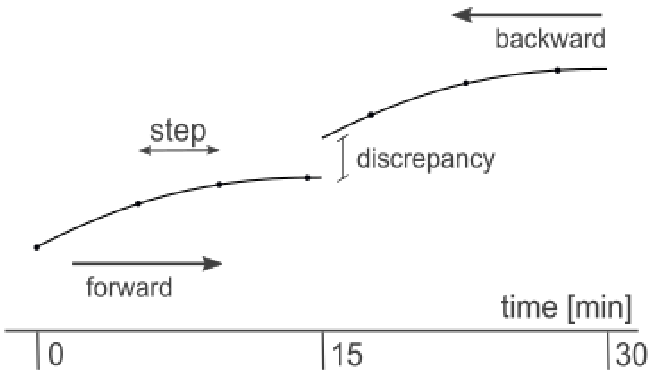

Figure 2: Discrepancy of forward and backward 15 - minute integration.

Figure 3 shows errors of XYZ components calculated based on initial satellite position by RK4 with integration step h = 30 s. After 30 minutes, the error of each component does not exceed 1 meter, after 60 minutes error is smaller than 5 meters, and after 4 hours exceeds value of several meters. Therefore, in an application of numerical methods for solving equations of satellite motion information on satellite position in the shortest possible time intervals is very important.

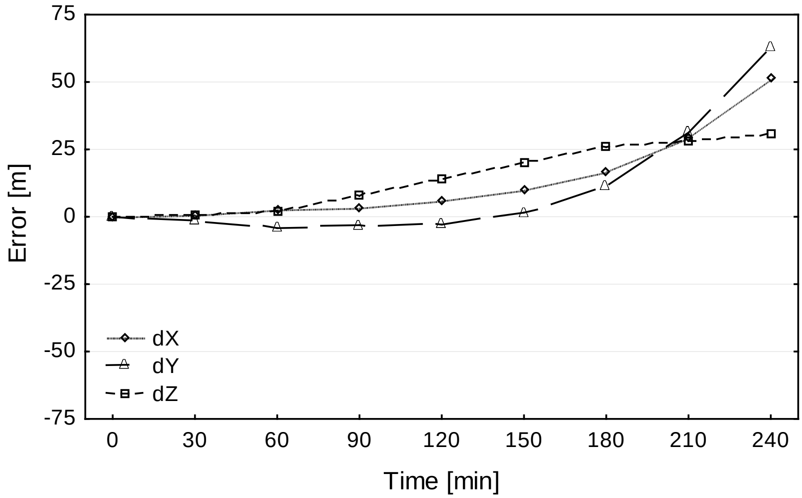

Figure 3: Increase of satellite position error (RK4, h = 30 s)

Figure 4 shows more detailed data presented on Figure 3. "Known" coordinate and speed components are at t = 0 s. At t = 900 s follows update of satellite ephemeris data and then should be used next "known" position coordinate (t = 1800 s for this figure) and solved backward. Therefore, the increase of XYZ components error magnitude due to the updated ephemeris parameters is clearly visible.

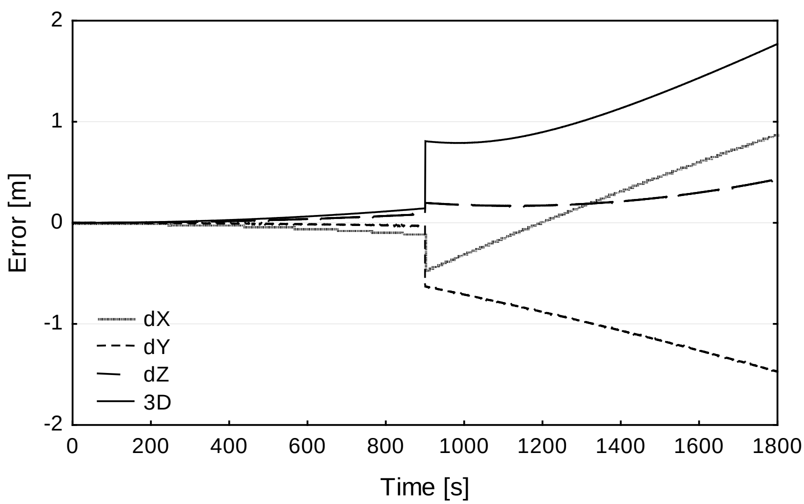

Figure 4: Increase of satellite position error (RK4, h = 1 s).

Figure 5 shows the difference between RK4 method with the step *h* = 1 s solution and the reference solution. The figure presents three consecutive "backward" and "forward" solutions within 900 s interval. At 900 s, 2700 s and 4500 s moment, satellite coordinates, velocity and acceleration values are known. Solutions of three analysed, successive time points have similar errors. The offset of each component is a result of its update. That is why the determination of a single satellite position should be done within ±900 s around the known position. Maximum error in X component is around -0.1 m, Y around 0.9 m, and Z component up to 0.1 m error. Consequently, maximum 3D position error is 0.15 m. Thus, this type of calculation can be considered as sufficient for GLONASS broadcast orbit determination, due to its accuracy of about several meters.

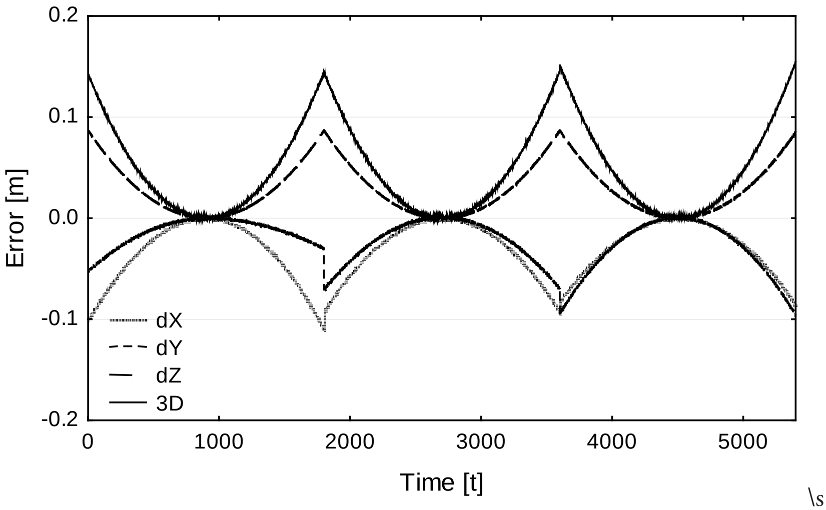

Figure 5: Example of three consecutive integration steps.

Table 5: Average duration of positions calculation and percentage changes in relation to RK4 [ms].

Step size h [s]	*1*	*3*	*5*	*10*	*20*	*30*	*90*	*180*	*300*	*900*
Number of steps	*900*	*300*	*180*	*90*	*45*	*30*	*10*	*5*	*3*	*1*
RK4	4.816	1.605	0.962	0.480	0.241	0.160	0.054	0.027	0.016	0.006
	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
RK5	7.374	2.881	1.758	0.992	0.468	0.353	0.147	0.084	0.060	0.038
	153%	180%	183%	207%	194%	221%	272%	311%	375%	633%
RKF4	7.712	2.539	1.557	0.776	0.401	0.275	0.107	0.066	0.049	0.031
	160%	158%	162%	162%	166%	172%	198%	244%	306%	517%
RKF5	8.047	2.719	1.608	0.774	0.403	0.275	0.113	0.079	0.054	0.033
	167%	169%	167%	161%	167%	172%	209%	293%	338%	550%

Table 5 presents a comparison of average speed of satellite position determination. These values are means of 100 000 consecutive solutions of Runge-Kutta methods. It depends on the adopted integration step size *h*. Increased integration step size decreases time of position determination. For each integration step the most efficient is Runge-Kutta 4th order method (RK4), due to the least complexity. The other three methods depending on the step length are between 2 to 6 times slower than RK4 method. Despite of the most complex equations RKF5 method is the second fastest after the RK4 method among analysed. RKM is the slowest method for each step size. Speed of calculation in this method is comparable to other only for 1 and 3 s integration step sizes.

Figure 6 presents calculated errors of "forward" satellite position. It reveals the difference between the author's and model solution based on integration step size. In case of small step length, less than 180 s, results are comparable for all tested methods and the maximum error does not exceed 0.15 m. This accuracy is sufficient for navigation purposes.

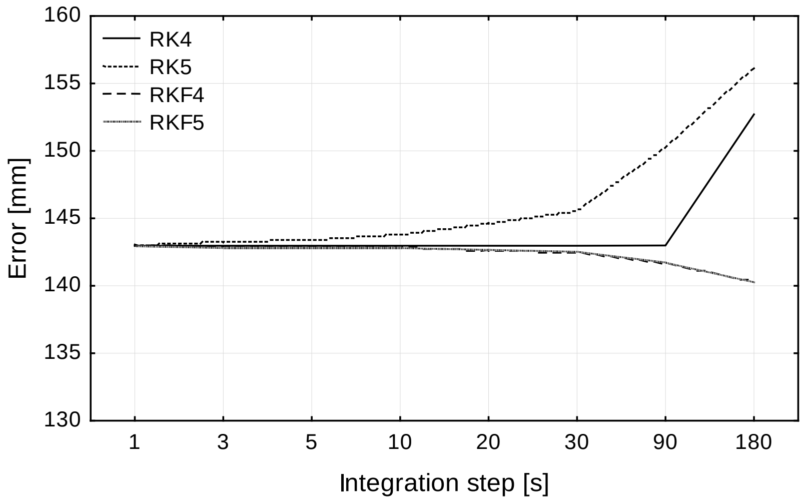

Figure 6: Runge-Kutta method determination error [mm].

With the increase of integration step length a distinct advantage of higher order RungeKutta methods may be observed. It is clearly visible for integration steps h = 300 s and h = 900 s. RKF method projects satellite's trajectory with 0.60 cm accuracy for a single, 900 s step. If you need to determine denser number between consecutive positions/coordinates (e.g. coordinates are available every 60 s, but you want to have coordinates every 1 s) all you have to do is decrease step-size to needed. Therefore, simplicity is the main advantage of using this method against GPS, where navigation message data contain Keplerian elements, which must have analytical solution. Moreover, GLONASS navigation message contains Cartesian coordinates and velocities in current PZ-90 realization every 30 min, so it is much affordable data than Keplerian elements in GPS navigation message available every 2 hours.

7 CONCLUSIONS

The accuracy of GLONASS satellite's position calculated numerically depends mostly on integration step size. The influence of applied RK method type and order is smaller. Short integration step allows a relatively high precision, but it involves extension of solution time. Error of calculated position from initial parameter (epoch) increases together with "distance" from known coordinates. This study confirmed that higher order RK methods are more accurate. This fact is more evident especially in large-size integration steps of RK computations. The previous studies showed that the 5th order method or modified RKF methods are more accurate than the RK4 recommended by the GLONASS-ICD. On the other hand, due to the simplicity of equations RK4 order method is the fastest of the all analysed methods. However, an argument of economical saving time was more important in the 90s, when PCs' computing power was less efficient smaller than today. Currently due the highest accuracy of analysed methods, the most suitable for calculation of GLONASS satellite position is RungeKuttaFehlberg 5th order method.

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Kamil Maciuk, Ph.D.
AGH University of Science and Technology
Faculty of Mining Surveying and Environmental Engineering
al. Mickiewicza 30
30-059 Kraków, Poland
E-mail: maciuk@agh.edu.pl