Strojniški vestnik - Journal of Mechanical Engineering 55(2009)2, 141-149
UDC 621.8

Paper received: 00.00.200x
Paper accepted: 00.00.200x

Effects of Couple Stresses on the Unsteady Performance of Finite Lubricated Bearings

Adolfo Senatore[1,*] - Alessandro Ruggiero[1] - Vladeta Jevremović[2] - Valentin Nedeff[3]
[1] Department of Mechanical Engineering, University of Salerno, Italy
[2] High Technical Mechanical School, Trstenik, Serbia
[3] Faculty of Engineering, University of Bacau, Romania

Based upon the Stokes micro-continuum theory, the influence of the behaviour of journal bearings with couple stress fluids on the dynamics of rotor-systems have been studied by several Authors during the last decade.

This paper is a part of a general approach aimed to the performances of the couple stress lubricants used to minimize the friction losses in steady operating conditions. Its purpose is to illustrate a method to formulate with closed-form solutions the steady/unsteady fluid film forces for the "infinitely long" and "finite" lubricated couple stress journal bearings, assuming the micro-continuum Stokes model. The model allows the advantage of minimising the computational time required for the analysis dynamic states of couple stress journal bearings without any significant loss of accuracy, while the analytical form of the solution involves a better readability of the parameter effects on the system unsteady behaviour.

Keywords: journal bearings, friction, lubricants, stability of lubricants

0 INTRODUCTION

Hydrodynamic journal bearings are commonly used for supporting rotating shafts subjected to high radial loads. Applications can be seen in a wide variety of machines where satisfactory performances are necessary for proper functioning, such as pumps, turbines, compressors, etc. The design of the journal bearings focuses the first analysis on the static characteristics such as hydrodynamic film force, load-carrying capacity and friction coefficient. Under certain external unexpected disturbances the bearing system involves self excited oscillating behaviors. The problem of oil film instability is of primary importance in high speed rotating machines. It is well known that the dynamic performance of a rotor on lubricated bearings, in fact, is strongly affected by the fluid film characteristics. The instability occurs when the speed exceed a certain value and appears as self excited orbital motions induced by action of fluid dynamic forces. The fluid film forces rise up directly by the gap oil film pressure field which is essentially related to the lubricant viscosity of the used lubricant.

It is well known that the additives are typically added to petroleum oils to modify the physical properties such as pour point, foaming or viscosity–temperature behaviour, chemical actions such as detergency, oxidation, or corrosion and to improve wear and extreme pressure resistance.

With reference to long-chain organic compounds additives, e.g., the length of the polymer chain may be a million times the diameter of a water molecule, the experimental studies showed good load enhancement and friction reduction effects due to their presence [1] to [3].

The increasing use of complex fluids as lubricants has received widespread interests owing to the development of modern machine elements. Common complex fluids are polymer-thickened oils, lubricants with various additives, synthetic fluids, liquid crystals and bio-fluids. Experimental investigations have also shown that the use of complex fluids can decrease the sensitivity to shear rate change, improving the stabilization of lubricating properties. According to the observation in strip squeeze film flow, polymer thickened oil gives significant load enhancement as compared to a Newtonian one under similar conditions [3].

In the first work about the short journal bearing by Oliver [4], the presence of dissolved

*Corr. Author's Address: University of Salerno, Department of Mechanical Engineering, via Ponte don Melillo, 84084 Fisciano (SA), Italy, a.senatore@unisa.it

polymer in the lubricant produces load enhancement and friction reduction.

Since the classical continuum theory neglects the fluid particles size, this approach is not suitable for describing the rheological behaviour of these kinds of non-Newtonian complex fluids. However, the micro-continuum theory takes into account the intrinsic motion of material constituents; it is developed by polar theory of complex fluids characterized by classical Cauchy stresses as well as by couple stresses resulting from the spin of microelements in fluids (Ariman and Sylvester [5], and Stokes [6]).

In particular, the Stokes micro-continuum theory [6] is a generalization of conventional theory which allows the study of the polar effects such as the presence of couple stresses, body couples and non-symmetric tensors, involving the rotational velocity field with the dimensional effect of the particles.

Such an approach can be found for other applications, as in the following list: peristalsis mechanisms by Srivastava [7] and Shehawey and Mekheimer [8]; line contacts by Das [9]; rolling elements by Sinha and Singh [10] as well as by Bujurke and Naduvinami [11]; externally pressurized bearings by Lin [12]; squeezing films by Bujurke and Jayaraman [13] and Lin [14]; slider bearings by Ramaniah [15]; finite bearings by Lin [16] and Chiang et al. [17]; short journal bearings by Naduvinamani et al. [18], Chiang et al. [19], Ruggiero and Senatore [20].

Focusing on the literature about finite bearings with couple stress fluid, both the papers [16] and [17] study the performance of these tribological components considered in steady state conditions through implementation of finite difference schemes; the latter also takes into account the surface roughness effect.

However, an approximate closed form analysis for the finite journal bearings considering the gap cavitation zone due to the unsteady operating conditions was not known. Therefore, it has inspired further interests toward the journal bearings with couple stress fluids.

In this paper, the inference of couple stress fluid property on the film forces in unsteady operating conditions of infinitely long and finite journal bearings is investigated.

The unsteady Reynolds equation governing the film pressure is achieved through the Stokes equations of motion for accounting the couple stress effects resulting from the flow behaviour of non-Newtonian complex fluids. Based on the product function approach, the pressure solution for the infinitely long bearing in an approximate closed-form description has been extended to Finite Journal Bearing (FJB) configuration. The outcomes allow the availability of analytical expressions for very fast assessment on large motion unsteady behaviour of shafts rotating on oil bearings with couple stress fluids.

1 OIL FILM PRESSURE MODEL

The system here analysed consists of a rigid, symmetric and balanced rotor supported by equal cylindrical bearings. Symmetry about the rotor middle plane allows limiting the analysis to one of the two halves into which the system is subdivided by the above mentioned plane.

Based upon the classical conception of hydrodynamics, the Stokes model allows for the inspection of polar effects such as the presence of couple stresses, body couples and non-symmetric tensor. This couple stress fluid is a peculiar case of a non-Newtonian lubricant and takes account of particle-size effects of the blending additives with a large molecule [6]. Isothermal conditions will be assumed to prevail throughout the present investigation. Couple stresses might be expected to appear in noticeable magnitudes in liquids containing additives with a large molecule.

These couple stresses may be significant particularly under lubrication conditions where thin films usually exist. Couple stresses introduce non-linear terms in the relationship between shear stresses and velocity gradients. As a result the lubricant should be considered as non-Newtonian and it's characterized by two constants, the shear viscosity and the couple stress property.

The continuity and momentum equations governing the motion of an incompressible coupled stress fluid under the Stokes' assumptions are [6]:

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \rho \mathbf{F} + \frac{1}{2}\rho \nabla \times \mathbf{C} + \mu \nabla^2 \mathbf{V} - \eta \nabla^4 \mathbf{V} \quad (1)$$

$$\nabla \cdot \mathbf{V} = 0 \quad (2)$$

Where the vectors V, F and C represent the velocity, the body force per unit mass, and body couple per unit mass, respectively; ρ is the density, p is the pressure, μ is the shear viscosity and η is a material constant responsible for the couple stress fluid property; the following assumptions have been made: thin fluid film, body forces and body moments are absent and fluid inertia is small as compared to the viscous shear.

Then the field equations governing the motion of the lubricant given in cartesian coordinates reduce to:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \quad (3)$$

$$\frac{\partial p}{\partial y} = 0 \quad (4)$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^4 w}{\partial y^4} \quad (5)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (6)$$

The boundary conditions at the bearing surface:

$$u(x,0,z) = v(x,0,z) = w(x,0,z) = 0 \quad (7.1)$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=0} = 0 \quad (7.2)$$

While the boundary conditions at journal surface are described by:

$$u(x,h,z) = U \quad (8.1)$$

$$v(x,h,z) = V \quad (8.2)$$

$$w(x,h,z) = 0 \quad (8.3)$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=h} = 0 \quad (8.4)$$

Integrating the (3) and (5) by applying the above boundary conditions, the velocity components can be derived as:

$$u = U\frac{y}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[y(y-h) + 2\ell^2 \left(1 - \frac{\cosh\left(\frac{2y-h}{2\ell}\right)}{\cosh\frac{h}{2\ell}} \right) \right] \quad (9)$$

$$w = \frac{1}{2\mu}\frac{\partial p}{\partial z}\left[y(y-h) + 2\ell^2 \left(1 - \frac{\cosh\left(\frac{2y-h}{2\ell}\right)}{\cosh\frac{h}{2\ell}} \right) \right] \quad (10)$$

Where ℓ is the characteristic length of additives:

$$\ell = \sqrt{\frac{\eta}{\mu}} \quad (11)$$

The measurement methods and procedures for ℓ have been proposed in [6]. However, the available published data just give its theoretical value. In (11), η has the dimensions of momentum. Integrating the continuity equation (6) with respect to y using the velocity components u and w with boundary conditions (7.1), (8.2) and (8.3), with reference to the bearing in fig. 1, the modified form of the Reynolds equation can be derived [16]:

$$\frac{1}{R^2}\frac{\partial}{\partial \theta}\left(\bar{g}(\bar{h};\ell)\frac{\partial \bar{p}}{\partial \theta} \right) + \frac{\partial}{\partial \bar{z}}\left(\bar{g}(\bar{h};\ell)\frac{\partial \bar{p}}{\partial \bar{z}} \right) = = 6\mu\left(\frac{U}{R}\frac{\partial \bar{h}}{\partial \theta} + 2V \right) \quad (12)$$

with:

$$\bar{g}(\bar{h};\ell) = \bar{h}^3 - 12\ell^2\left(\bar{h} - 2\ell \tanh\frac{\bar{h}}{2\ell} \right) \quad (13)$$

The journal speeds are given as:

$$U = \omega R \qquad V = \frac{\partial \bar{h}}{\partial \bar{t}} \quad (14)$$

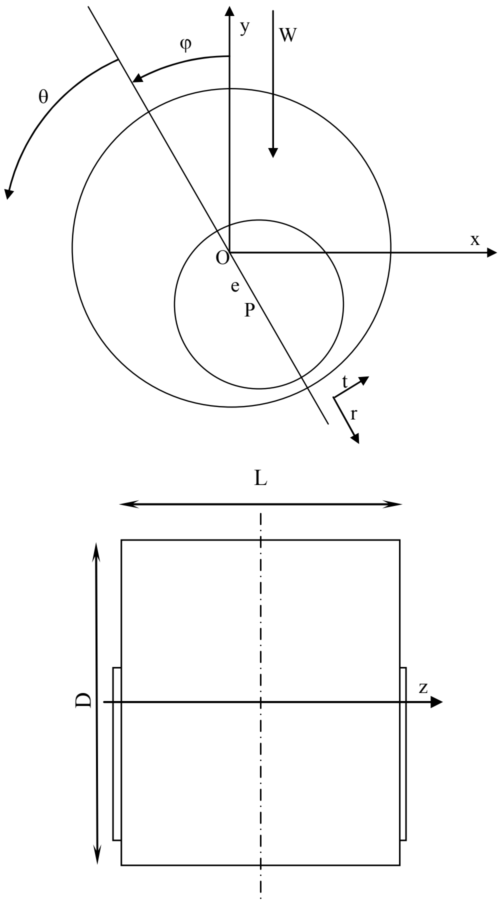

Fig. 1. Finite journal bearing scheme

The governing model for the hydrodynamic lubrication in the shaft-bearing wedge is the dimensionless form of (12):

$$\frac{\partial}{\partial\theta}\left(g(h;\tau)\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(g(h;\tau)\frac{\partial p}{\partial z}\right)=\frac{\partial h}{\partial\theta}+2\frac{\partial h}{\partial t} \quad (15)$$

where:

$$g(h;\tau)=h^3-12\tau^2\left(h-2\tau\tanh\frac{h}{2\tau}\right) \quad (16)$$

In (15) and (16) these dimensionless variables have been introduced:

$$p_{ref}=6\mu\omega\left(\frac{R}{C}\right)^2 \qquad p=\frac{\overline{p}-p_0}{p_{ref}}$$

$$h(\theta)=\frac{\overline{h}(\theta)}{C}=1+\varepsilon\cos\theta$$

$$\tau=\frac{\ell}{C} \qquad z=\frac{\overline{z}}{L} \quad (17)$$

For the journal bearings with couple stress fluids, analytical solutions of the Reynolds equation are not generally achievable and numerical methods must be involved; this is the case of the ‘infinitely long’ bearing, with the partial differential equation (15) reduced as follows, for which an exact solution can’t be obtained:

$$\frac{\partial}{\partial\theta}\left[g(h;\tau)\frac{\partial p}{\partial\theta}\right]=\frac{\partial h}{\partial\theta}(1-2\dot{\varphi})+2\dot{\varepsilon}\cos\theta \quad (18)$$

In this equation, the function for accounting the couple stress effect:

$$g(h;\tau)=h^3-12\tau^2\left(h-2\tau\tanh\frac{h}{2\tau}\right) \quad (19)$$

replaced by:

$$\tilde{g}(h;\tau)=(h(\theta)-\tau\varepsilon)^3=(1+\varepsilon\cos\theta-\tau\varepsilon)^3 \quad (20)$$

allows a closed-form integration of an approximation of (18), while numerical calculations show that (20) gives good results within the normal operating conditions of this tribological pair [20].

In this way, the differential equation for the infinitely long bearing can be integrated to give the following expressions:

$$p_L\left(\theta,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau\right)=(1-2\dot{\phi})\,I_1-2\,\dot{\varepsilon}\,I_2+c_1\,I_3 \quad (21)$$

where:

$$I_1=\int\frac{h(\theta)}{\tilde{g}(h;\tau)}d\theta$$

$$I_2=\int\frac{\sin\theta}{\tilde{g}(h;\tau)}d\theta \quad (22)$$

$$I_3=\int\frac{1}{\tilde{g}(h;\tau)}d\theta$$

The constant c_1 is calculated by analysing the pressure discontinuity at $\theta = \pi$ and the right/left limits.

Then, the unsteady infinitely long bearing approximate solution can now be written as:

$$p_L(\theta,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau)=\frac{\dfrac{(1-2\dot{\varphi})\varepsilon^2(2-2\varepsilon\tau+\varepsilon\cos\theta)\sin\theta}{2+\varepsilon(\varepsilon-4\tau+2\varepsilon\tau^2)}-\dot{\varepsilon}}{\varepsilon(1-\varepsilon\tau+\varepsilon\cos\theta)^2} \tag{23}$$

2 PRODUCT FINITE JOURNAL BEARING SOLUTION

The flow correction factor for modifying the fluid film pressure for the infinitely long bearing with couple stress fluid (23) is analysed through the characteristic scalar value λ and the end leakage 'shape' function s, given for the bearing in unsteady conditions, respectively, by [21]:

$$\lambda(\varepsilon,\dot{\varepsilon},\dot{\varphi};\tau)=\frac{\int_{\alpha}^{\alpha+\pi}\tilde{g}(h;\tau)\left[\dfrac{dp_L}{d\theta}\right]^2 d\theta}{\int_{\alpha}^{\alpha+\pi}\tilde{g}(h;\tau)p_L^2\, d\theta} \tag{24}$$

$$s(z,\varepsilon,\dot{\varepsilon},\dot{\varphi};\tau,L/D)=1-\frac{\mathrm{Cosh}\left[2\lambda(\varepsilon,\dot{\varepsilon},\dot{\varphi};\tau)\, z\,\dfrac{L}{D}\right]}{\mathrm{Cosh}\left[\lambda(\varepsilon,\dot{\varepsilon},\dot{\varphi};\tau)\dfrac{L}{D}\right]} \tag{25}$$

The following product solution produces the complete unsteady oil film pressure for finite length configuration in approximate closed form (not presented for the sake of briefness); the following integrations on the thrust domain provide the forces acting on the journal, which are also analytical expressions:

$$p_F(\theta,z,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau)=p_L(\theta,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau)\;\; s(z,\varepsilon,\dot{\varepsilon},\dot{\varphi};\tau,L/D) \tag{26}$$

$$\begin{Bmatrix} f_r \\ f_t \end{Bmatrix}(\varepsilon,\dot{\varepsilon},\dot{\phi};\tau,L/D)= \\ =\int_{-1/2}^{1/2}\int_{\alpha}^{\alpha+\pi} p_F(\theta,z,\varepsilon,\dot{\varepsilon},\dot{\phi};\tau,L/D)\begin{Bmatrix} -\cos\theta \\ \sin\theta \end{Bmatrix} d\theta\, dz \tag{27}$$

The angle α which defines the fluid film boundaries in (24) and (27) is evaluated by using the following relationships, according to [22]:

$$\begin{cases} (2+\varepsilon^2)\dot{\varepsilon}\cos\alpha-\varepsilon(1-2\dot{\varphi})\sin\alpha=0 \\ (2+\varepsilon^2)\dot{\varepsilon}\sin\alpha+\varepsilon(1-2\dot{\varphi})\cos\alpha\geq 0 \end{cases} \tag{28}$$

3 RESULTS

The following graphs show the dimensionless oil film forces in the rotating system frame for three aspect ratios (L/D=0.5, 1, 2) where the 'short' and 'infinitely long' bearing solutions lack in accuracy and a finite bearing solution is required.

Two typical couple stress parameter values (τ) as well as the newtonian case (τ = 0) are plotted.

The figures 2 and 3 depict the effects of the couple stress parameter on the oil film forces f_r and f_t for different static equilibrium eccentricity ratio ε. The two Figs. 4 and 5 show the maps achieved through the analytical knowledge of oil film forces in unsteady oil film response (journal 'squeeze').

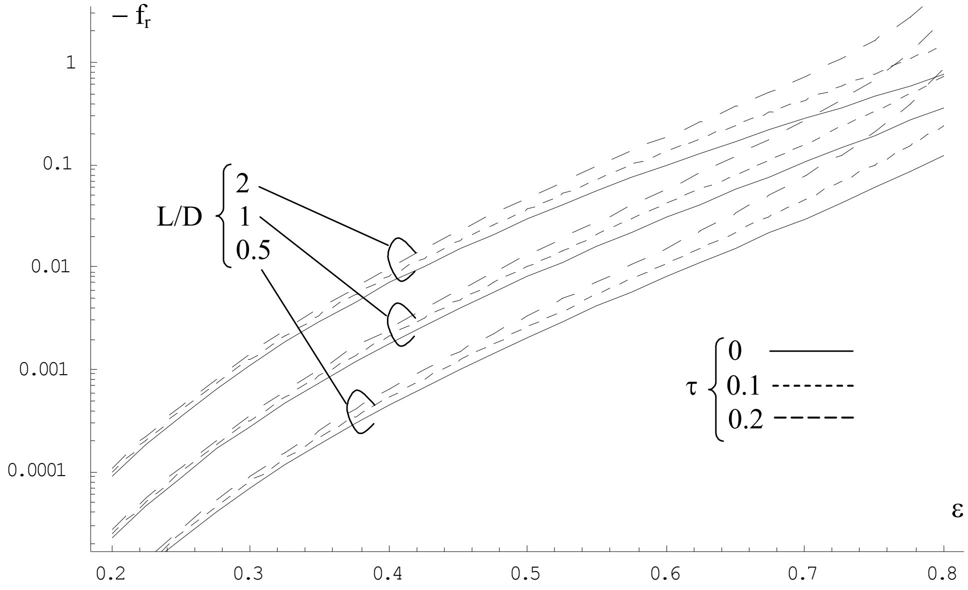

Fig. 2. Oil film force f_r in the rotating system frame: steady operating conditions

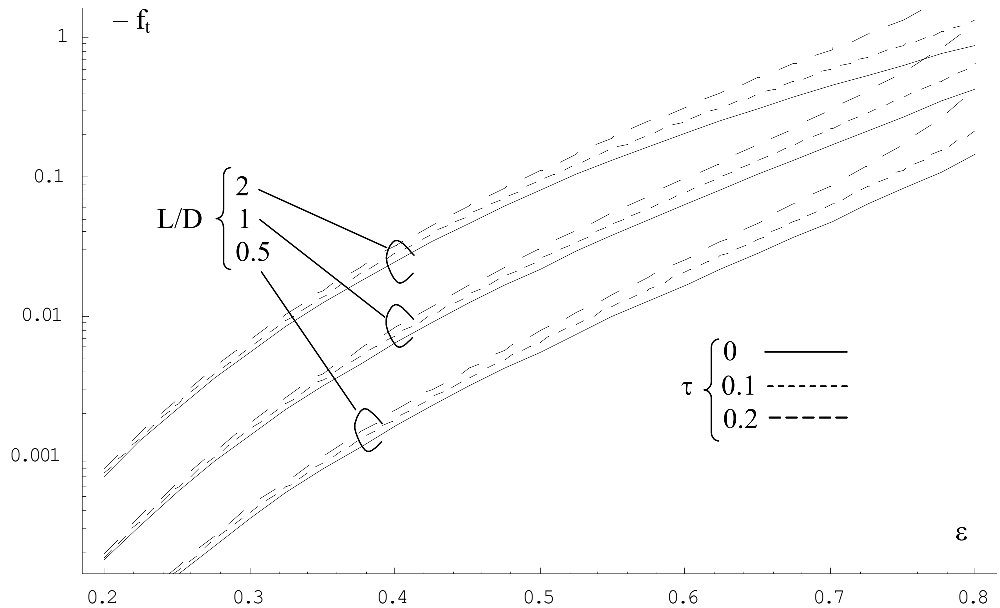

Fig. 3. Oil film force f_t in the rotating system frame: steady operating conditions

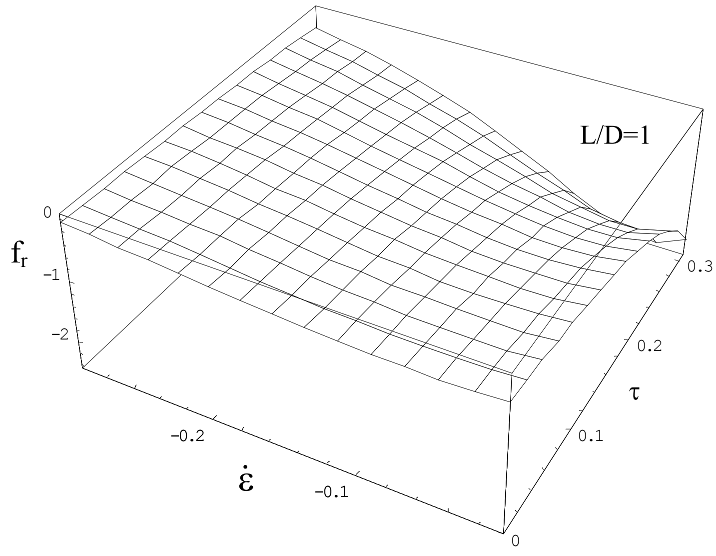

Fig. 4. Oil film force f_r in the rotating system frame: unsteady operating conditions, aspect ratio L/D=1

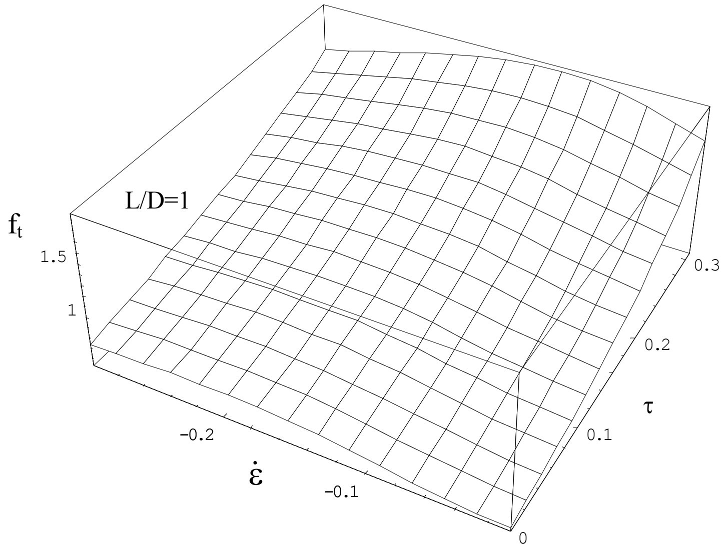

Fig. 5. Oil film force f_t in the rotating system frame: unsteady operating conditions, aspect ratio L/D=1

4 CONCLUSIONS

The present solution scheme for the evaluation of the oil film forces provides satisfactory results in the unsteady oil film forces computation for infinitely long and finite journal bearings with couple stress fluids, mainly for typical values of couple stress parameter for which the approximation introduced in (20) allows an enough accurated evaluation of the integrals (22).

The proposed approach allows generating approximated closed-form oil film expressions as function of the eccentricity ratio, eccentricity and attitude angle variation rates. The knowledge of the analytical form of the fluid film response is very useful to reduce drastically the calculation time in the computer simulations of flexible rotors supported by several bearings with non-newtonian fluids.

The effects of couple stresses result in a significant increase of the unsteady action of fluid film forces for a given journal centre radial speed.

The linearized stability analysis can be seen as an effortless application of the present work outcomes: in fact, the derivatives of the unsteady expressions of the oil film forces lead to the stable/unstable onset values and stability map for each aspect ratio.

5 REFERENCES

[1] Spikes, H. A., *The behaviour of lubricants in contacts: current understanding and future possibilities*. Proceedings of IMechE, 1994; Vol. 28; pp. 3-15.

[2] Scott ,W., Suntiwattana, P., *Effect of oil additives on the performance of a wet friction clutch material*. Wear 1995, Volumes 181-183, Part 2, pp. 850-855.

[3] Oliver, D. R., Shahidullah, M., *Load enhancement effects by polymer-thickened oils in short squeeze film flow,* Journal of Non-Newtonian Fluid Mechanics 1983, Vol. 3, pp. 93-102.

[4] Oliver, D. R., *Load enhancement effects due to polymer thickening in a short model journal bearings*. Journal of Non-Newtonian Fluid Mech 1988, Vol. 30, pp. 185–196.

[5] Ariman, T., Sylvester, N. D., *Microcontinuum fluid mechanics, a review*, Int. J. Eng. Sci. 1973, Vol. 11, pp. 905-930.

[6] Stokes, V. K., *Couple stresses in fluids*. Phys. Fluids 1966; Vol. 9, pp. 1709–1715.

[7] Srivastava, L.M., *Peristaltic transport of couple stress fluid*, Rheol. Acta 1986, Vol. 25, pp. 638-641.

[8] Shehawey, E. F. E., Mekheimer, K. S., *Couple stresses in peristaltic transport of fluids*, J. Phys. D: Appl. Phys 1994, Vol. 27, pp. 1163-1170.

[9] Das, N. D., *Elastohydrodynamic lubrication theory of line contact: couple stresses fluid model*, STLE Tribology Transactions 1997, Vol. 40, pp. 353-359.

[10] Sinha, P., Singh, C., *Couple stresses in the lubrication of rolling contact bearings considering cavitation*, Wear 1981, Vol. 67, pp. 85-91.

[11] Bujurke, N. M., Naduvinami, N.G., *The lubrication of lightly cylinders in combined rolling, sliding and normal motion with couple stress fluid*, Int. J. Mech. Sci. 1990, Vol. 32, pp. 969-979.

[12] Lin, J. R., *Static and dynamic characteristics of externally pressurized circular step thrust bearings lubricated with couple stress fluids*, Tribology International 1999, Vol. 32, pp. 207-216.

[13] Bujurke, N.M., Jayaraman, G., *The influence of couple stresses in squeeze films*, Int. J. Mech. Sci. 1982, Vol. 24, pp. 369-376.

[14] Lin, J. R., *Couple-stress effects on the squeeze film characteristics of hemispherical bearings with reference to synovial joints*, Appl. Mech. and Eng. 1996, Vol. 1, pp. 317-322.

[15] Ramaniah, G., *Slider bearings lubricated by fluids with couple stress*, Wear 1979, Vol. 52, pp. 27-36.

[16] Lin, J. R., *Effects of couple stresses on the lubrication of finite journal bearings*, Wear 1997, Vol. 206, pp.171-178.

[17] Chiang, H. L., Lin, J. R., Hsu, C. H., *Lubrication performance of finite journal bearings considering effects of couple stresses and surface roughness*, Tribology International 2004, Vol. 37, pp. 297–307.

[18] Naduvinamani, N. B., Hiremath, P. S., Gurubasavaraj, G., *Squeeze film lubrication of a short porous journal bearing with couple stress fluids*, Tribology International 2001, Vol. 34, pp. 739–747.

[19] Chiang, H. L., Lin, J. R., Hsu, C. H., Chang, Y. P., *Linear stability analysis of a rough short journal bearing lubricated with non-Newtonian fluids*, Tribology Letters 2004, Vol. 17, No. 4, pp. 867-877.

[20] Ruggiero, A., Senatore, A., *Approximate closed-form solution for the dynamical analysis of short bearings with couple stress fluid*, Lubrication Science 2007, Vol. 19, No. 4, pp. 247-267.

[21] D'Agostino, V., Ruggiero, A., Senatore, A.: *Approximate model for unsteady finite porous journal bearings fluid film force*

calculation, Proceedings of IMechE Vol. 220, Part J: Journal of Engineering Tribology 2006, pp. 227-234.

[22] Constantinescu, V. N., *Sliding Bearings* 1985, Allerton Press.

NOMENCLATURE

C	*Radial clearance*	u, v, w	*Velocity components (circum., radial, axial directions)*
$\boldsymbol{C}$	*Body couple per unit mass*	U	*Journal rotational speed*
$D=2R$	*Bearing diameter*	V	*Journal radial speed*
$\boldsymbol{F}$	*Body force per unit mass*	$\boldsymbol{V}$	*Fluid Velocity*
f_r, f_t	*Dimensionless oil film force (rot. system frame)*	W	*Journal load*
$f_{ref} = p_{ref} RL$	*Reference oil film force*	$\overline{z}$	*Axial coordinate*
g, $\widetilde{g}$	*Dimensionless functions*	α	*Pressure field boundary angle*
$h = \overline{h} / C$	*Dimensionless oil film thickness*	$\varepsilon = e / C$	*Eccentricity ratio*
ℓ	*Characteristic additives length*	φ	*Attitude angle*
L	*Bearing length*	η	*Couple stress oil property*
$p = (\overline{p} - p_0)/ p_{ref}$	*Dimensionless pressure*	λ	*Characteristic scalar value*
$p_{ref} = 6\mu\omega(R / C)^2$	*Reference pressure*	μ	*Oil shear viscosity*
p_L	*Infinitely long bearing film pressure*	θ	*Circumferential coordinate*
p_F	*Finite bearing film pressure*	τ	*Couple stress parameter*
s	*End leakage axial 'shape' function*	ω	*Journal angular speed*
t	*Time*		