Also available at http://amc-journal.eu
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
Ars Mathematica Contemporanea Volume 4, Issue 1, Year 2011, Pages 177-204

Dominating sets in triangulations on surfaces

Hong Liu, Michael J. Pelsmajer

Abstract

A *dominating set* $D \subseteq V(G)$ of a graph G is a set such that each vertex $v \in V(G)$ is either in the set or adjacent to a vertex in the set. Matheson and Tarjan (1996) proved that any n-vertex plane triangulation has a dominating set of size at most $n/3$, and conjectured a bound of $n/4$ for n sufficiently large. King and Pelsmajer recently proved this for graphs with maximum degree at most 6. Plummer and Zha (2009) and Honjo, Kawarabayashi, and Nakamoto (2009) extended the $n/3$ bound to triangulations on surfaces.

We prove two related results: (i) There is a constant c_1 such that any n-vertex plane triangulation with maximum degree at most 6 has a dominating set of size at most $n/6 + c_1$. (ii) For any surface S, $t \geq 0$, and $\varepsilon > 0$, there exists c_2 such that for any n-vertex triangulation on S with at most t vertices of degree other than 6, there is a dominating set of size at most $n(1/6 + \varepsilon) + c_2$.

As part of the proof, we also show that any n-vertex triangulation of a non-orientable surface has a non-contractible cycle of length at most $2\sqrt{n}$. Albertson and Hutchinson (1986) proved that for n-vertex triangulation of an orientable surface

other than a sphere has a non-contractible cycle of length √(2n), but no similar result was known for non-orientable surfaces.

Keywords: Dominating set, triangulation, graphs on surfaces, non-contractible cycle, non-orientable surface.

Math. Subj. Class.: 05C69

Math Sci Net: 05C69 (05C10)

Dominantne množice v triangulacijah ploskev

Povzetek

Dominantna množica $D \subseteq V(G)$ grafa G je množica, za katero velja, da je vsako vozlišče $v \in V(G)$ bodisi element te množice bodisi je sosednje kakemu vozlišču, ki pripada tej množici. Matheson in Tarjan (1996) sta dokazala, da ima vsaka triangulacija ravnine z n vozlišči dominantno množico velikosti največ $n/3$, in postavila domnevo, da je zgornja meja $n/4$ za vse dovolj velike n. King in Pelsmajer sta pred nedavnim to domnevo dokazala za grafe z maksimalno stopnjo največ 6. Plummer in Zha (2009) ter Honjo, Kawarabayashi in Nakamoto (2009) so posplošili mejo $n/3$ na triangulacije na ploskvah.

V članku dokažemo dva sorodna rezultata: (i) Obstaja taka konstanta c_1, da ima vsaka triangulacija ravnine z n vozlišči z maksimalno stopnjo največ 6 dominantno množico velikosti največ $n/6 + c1$. (ii) Za vsako ploskev S, $t \geq 0$ in $\varepsilon > 0$, obstaja tak c_2, da za vsako triangulacijo z n vozlišči na ploskvi S, kjer

je največ t vozlišč stopnje, različne od 6, obstaja dominantna množica velikosti največ $n(1/6 + \varepsilon) + c_2$.

Eden od pomožnih rezultatov dokaza je tudi ta, da ima vsaka n-vozlična triangulacija neorientabilne ploskve ne-kontraktibilen cikel dolžine največ $2\sqrt{n}$. Albertson in Hutchinson (1986) sta dokazala, da ima n-vozliščna triangulacija orientabilne poskve, različne od sfere, nekontraktibilen cikel dolžine $\sqrt{(2n)}$, za neorientabilne ploskve pa doslej ni bil znan noben podoben rezultat.

Ključne besede: Dominantna množica, triangulacija, grafi na ploskvah, nekontraktibilen cikel, neorientabilna ploskev.