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Entropijska analiza sotočnih prenosnikov toplote

Entropy Analysis of Parallel-Flow Heat Exchangers

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V prispevku obravnavamo povečanje entropije v ločilnih sotočnih prenosnikih toplote. Med delovanjem menjalnika se pojavljata dva vira entropije, vsled padca tlaka zaradi trenja in zaradi prenosa toplote. Obravnavajmo samo prenos toplote. Analiza je opravljena z matematičnim modelom, ki uporablja ista brezrazsežna števila, kakor jih je pred leti uvedel Bošnjaković pri energijski analizi prenosnikov toplote. Ovrednoten je vpliv posameznih brezrazsežnih spremenljivk (delovni pogoji sotočnega prenosnika toplote) na povečanje entropije. Izsledki so prikazani brezrazsežno v diagramih, kar jim daje bolj splošen pomen. Posebej so opisane delovne razmere sotočnega rekuperatorja, ki so povsem enake tudi pri protitočnih in križnih menjalnikih.

(Ključne besede: prenosniki toplote, povečevanje entropije, učinkovitost, rešitve analitične)

The entropy generation in a parall-flow recuperative heat exchanger is analysed in this paper. During the operation of a heat exchanger two sources of entropy generation normally exist: the pressure drop (friction) source and the heat-exchange source. Here, only the heat-exchange source is considered. The analysis is performed using an analytical mathematical model and the same non-dimensional numbers that were introduced into the energy analysis of heat exchangers by Bošnjaković, many years ago. The influence of the individual non-dimensional variables (the operating points of the parallel-flow heat exchanger) on the entropy generation is quantified. The results are presented non-dimensionally in diagrams, which gives them a more universal meaning. Special conditions, i.e. the boundary operating conditions, of the parallel-flow recuperator, which are identical to those for counter-flow and cross-flow exchangers, are also discussed.

(Keywords: heat exchangers, entropy generation, effectiveness, analytical solutions)

0 UVOD

V splošnem sta dva vzroka za povečevanje entropije v vsaki vrsti prenosnikov toplote: razlika temperatur med obema pretokoma in padec tlaka zaradi trenja v tokovih skozi prenosnik. Podrobno in sistematično analizo vpliva znižanja tlaka in temperaturne razlike dobimo v [1] in [2].

V tem prispevku ne bomo obravnavali vpliva padca tlaka na povečevanje entropije. Podobne analize najdemo tudi v novejših virih [3] in [4], ki med drugim ocenjujejo povezanost povečanja entropije in izkoristka z namenom pridobitve najugodnejših delovnih pogojev prenosnika toplote.

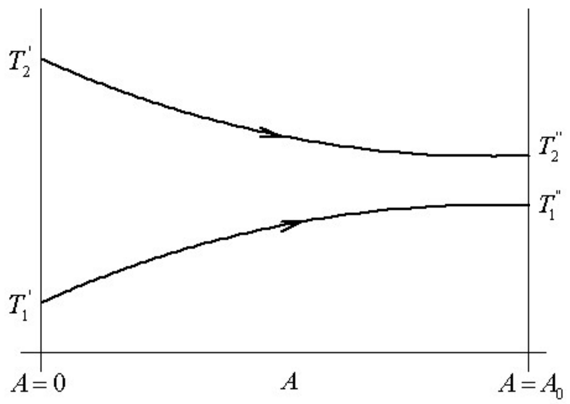
Namen tega prispevka je prikazati povezavo med povečanjem entropije in izkoristkom sotočnih prenosnikov toplote kot odvisnost znanih brezrazsežnih števil π_1, π_2 in π_3, ki jih je v energijsko

0 INTRODUCTION

There are generally two causes of entropy generation in any type of heat exchanger: the temperature difference between the flows and the pressure drop induced by the friction in the flows streaming through the exchanger. A detailed and systematic analysis of the influence of pressure drop and temperature difference of the flows may be found in [1] and [2].

In this paper the pressure drop influence on the entropy generation is neglected. Similar analyses may also be found in recent works [3] and [4] that analyse, amongst other factors, the relationship between entropy generation and the effectiveness in order to determine the optimum operating parameters of the heat exchanger.

The objective of this paper is to show the relationship between entropy generation and the effectiveness of parallel-flow heat exchangers as a function of the known non-dimensional numbers π_1, π_2 and π_3 introduced into



Sl. 1. *Potek temperature vzdolž površin sotočnega prenosnika toplote*
Fig. 1. *Temperature change along the surface area of a parallel-flow heat exchanger*

analizo uvedel Bošnjaković [4], kakor tudi pred kratkim uvedeno brezrazsežno število π_T za potrebe predstavljene entropijske analize. Dobljeni model tako lahko neposredno uporabimo v optimizacijski postopek sotočnih prenosnikov toplote.

the energy analysis by Bošnjaković [4], and the newly introduced non-dimensional number π_T, for the purposes of the present entropy analysis. The model obtained may thus be directly included into the parallel-flow heat-exchanger optimisation process.

1 MATEMATIČNI ZAPIS PROBLEMA

1 THE MATHEMATICAL FORMULATION OF THE PROBLEM

Enačbo povečanja entropije v odvisnosti od toplotne zmogljivosti ter vstopnih in izstopnih temperatur pretokov zlahka dobimo z integracijo vzdolž tokovnic obeh tekočin:

The equation for entropy generation as a function of thermal capacities and the inlet and outlet temperatures of the flows is easily obtained by an integration along the streamlines of both flows:

$$\Delta\dot{S}_{\text{gen}} = C_1 \ln\frac{T_1^{''}}{T_1^{'}} + C_2 \ln\frac{T_2^{''}}{T_2^{i}} \qquad (1),$$

kjer indeksa 1 in 2 pomenita šibkejši in močnejši pretok. Pretok z večjo toplotno zmogljivostjo $C = q_m \cdot c_p$ imejmo za močnejšega. Z delitvijo s toplotno zmogljivostjo močnejšega pretoka C_2 zgornja enačba postane brezrazsežna:

where the indices 1 and 2 designate the weaker and the stronger flows respectively. The flow with the larger heat capacity $C = q_m \cdot c_p$ is considered to be stronger. Dividing by the heat capacity of the stronger flow C_2 the above equation becomes non-dimensional:

$$\frac{\Delta\dot{S}_{\text{gen}}}{C_2} = \pi_3 \quad \ln\frac{T_1^{''}}{T_1^{'}} + \ln\frac{T_2^{''}}{T_2^{i}} \qquad (2).$$

Člen π_3 v enačbi (2) je po Bošnjakovićevi zasnovi razmerje toplotnih zmogljivosti šibkejšega in močnejšega pretoka:

The member π_3 in Equation (2), according to Bošnjaković's concept, is the ratio of the heat capacities of the weaker and stronger flows:

$$\pi_3 = \frac{C_1}{C_2} = \frac{q_{\text{m}1} c_{p1}}{q_{\text{m}2} c_{p2}} \qquad (3).$$

Z uporabo obrazca za brezrazsežno število π_1 prenosnikov toplote (kar je tudi enako izkoristku menjalnika ε [4])

Using the formula for the non-dimensional number π_1 of heat exchangers (which is also identical to the effectiveness of the exchanger ε [4]):

$$\pi_1 = \frac{T_1^{'} - T_1^{''}}{T_1^{'} - T_2^{'}} \qquad (4),$$

kakor zaradi enačbe ohranitve energije:

as well as the energy-conservation equation:

$$C_1\left(T_1^{'} - T_1^{''}\right) = C_2\left(T_2^{''} - T_2^{'}\right) \qquad (5),$$

enačbo (2) preoblikujemo v:

Equation (2) is transformed into:

$$\frac{\Delta\dot{S}_{\text{gen}}}{C_2} = \pi_3 \ln\left(1 - \pi_1\left(1 - \frac{T_2^{'}}{T_1^{'}}\right)\right) + \ln\left(1 + \pi_1\pi_3\left(\frac{T_1^{'}}{T_2^{'}} - 1\right)\right) \qquad (6),$$

Vrednost π_1 lahko izrazimo kot odvisnost med brezrazsežnima številoma π_2 in π_3, kakor navajajo Bošnjakovićevi učbeniki (izpeljavo funkcijske odvisnosti najdemo v [6]):

The value π_1 may be expressed as a function of the non-dimensional numbers π_2 and π_3 as it is stated in Bošnjaković's textbooks (the derivation of the functional relation may be found in [6]):

$$\pi_1 = \frac{1-\exp\left(-\left(1+\pi_3\right)\pi_2\right)}{1+\pi_3} \qquad (7),$$

kjer je

where:

$$\pi_2 = \frac{kA_0}{C_1} \qquad (8).$$

Z vstavitvijo (7) v (6) in označitvijo razmerja med vstopnimi temperaturami:

Inserting (7) into (6), and denoting the ratio of the inlet temperatures with:

$$\pi_{\mathrm{T}} = \frac{T_2^{'}}{T_1^{'}} \qquad (9),$$

dobimo enačbo (6) v njeni končni obliki

Equation (6) takes its final form:

$$\frac{\Delta\dot{S}_{gen}}{\dot{C}_2} = \pi_3 \ln\left(1-\frac{1-\exp\left(-\left(1+\pi_3\right)\pi_2\right)}{1+\pi_3}\left(1-\pi_{\mathrm{T}}\right)\right) + \ln\left(1+\pi_3\frac{1-\exp\left(-\left(1+\pi_3\right)\pi_2\right)}{1+\pi_3}\left(\frac{1}{\pi_{\mathrm{T}}}-1\right)\right) \qquad (10).$$

Enačba (10) pove, da povečanje entropije v sotočnem prenosniku toplote lahko izrazimo z uporabo istih brezrazsežnih števil π_2 in π_3, kakor jih je uporabil Bošnjaković na ravni energetske analize. Uporabiti moramo dodatno brezrazsežno število π_{T}.

Preden podamo grafično ponazoritev enačbe (10), zapišimo nekaj pripomb o posebnih primerih.

a) Ko je $\pi_{\mathrm{T}} = 1$, enačba (10) daje $\Delta\dot{S}_{gen}/C_2 = 0$, kar je pravilno, saj pomeni primer enakih vstopnih temperatur, ko ni ne spremembe toplote in ne povečanja entropije.

b) V primeru izmišljenega prenosnika toplote z neskončno veliko menjalno površino, ko $\pi_2 \to \infty$, enačba (10) postane:

Equation (10) indicates that the entropy generation of a parallel-flow exchanger may be expressed by means of the same non-dimensional numbers π_2 and π_3, as introduced by Bošnjaković in the energy-level analysis. However, the additional non-dimensional number π_{T} must be used.

Before the graphical presentation of Equation (10) is given, here are some comments on its special cases:

a) If $\pi_{\mathrm{T}} = 1$, eq. (10) yields $\Delta\dot{S}_{gen}/C_2 = 0$. This is physically correct, because it represents the case of equal inlet temperatures, where there is no exchange of heat and thus no entropy generation.

b) For the case of a hypothetical exchanger with an infinite heat-exchange surface area, where $\pi_2 \to \infty$, Eq. (10) becomes:

$$\left(\frac{\Delta\dot{S}_{\mathrm{gen}}}{\dot{C}_2}\right)_{\pi_2\to\infty} = \pi_3 \ln\frac{\pi_3+\pi_{\mathrm{T}}}{1+\pi_{\mathrm{T}}} + \ln\frac{\pi_{\mathrm{T}}+\pi_3}{\pi_{\mathrm{T}}\left(1+\pi_3\right)} \qquad (11),$$

kar pomeni pri določenih vrednostih π_3 in π_{T} povečanje entropije do končne vrednosti (vodoravna asimptota).

b1) Dodatno, v primeru pretokov enakih toplotnih zmogljivosti, ko je $\pi_3 = 1$, postane enačba (11):

indicating that the entropy generation for defined values of π_3 and π_{T} tends to a finite value (horizontal asymptote).

b1) Additionally, in the case of flows of equal heat capacities, where $\pi_3 = 1$, eq. (11) becomes:

$$\left(\frac{\Delta\dot{S}_{\mathrm{gen}}}{C_2}\right)_{\pi_2\to\infty,\pi_3=1} = \ln\frac{\left(1+\pi_{\mathrm{T}}\right)^2}{4\pi_{\mathrm{T}}} \qquad (12).$$

c) Kadar eden od pretokov kondenzira ali se uparja (fazna premena, npr. $C_2 = \infty$; $\pi_3 = 0$; $T_2^{'} = T_2^{''} =$ konst), postane enačba (10):

c) When one of the flows condenses or evaporates (phase change: e.g. $C_2 = \infty$; $\pi_3 = 0$; $T_2^{'} = T_2^{''} =$ const), where eq. (10) becomes:

$$\frac{\Delta\dot{S}_{\mathrm{gen}}}{C_2} = 0 \qquad (13),$$

dobimo nedoločeno vrednost:

the undetermined value occurs:

$$\Delta\dot{S}_{\mathrm{gen}} = 0\cdot C_2 = 0\cdot\infty \qquad (14).$$

V tem primeru je nedoločen tudi drugi člen v enačbi (1) in ga moramo spremeniti v skladu z drugim

In this case the second member in Eq. (1) is also undetermined and has to be modified according zakonom termodinamike, glede na nespremenljivo temperaturo močnejšega pretoka:

$$\Delta\dot{S}_{gen} = C_1 \ln\frac{T_1^{''}}{T_1^{'}} + \frac{C_1\left(T_1^{'} - T_1^{''}\right)}{T_2^{'}} \qquad (15),$$

ki se, po izločitvi $T_1^{''}$ in uporabi enačb (4), (7) in (9), spremeni v želeno obliko:

$$\frac{\Delta\dot{S}_{gen}}{C_1} = \ln\left(\exp\left(-\pi_2\right) + \pi_T\left(1-\exp\left(-\pi_2\right)\right)\right) + \left(\frac{1}{\pi_T} - 1\right)\left(1-\exp\left(-\pi_2\right)\right) \qquad (16).$$

Če enačbi (10) in (16) delimo z izkoristkom $\varepsilon = \pi_1$, dobimo razmerja $\Delta\dot{S}_{gen}/C_2\,\varepsilon$ in $\Delta\dot{S}_{gen}/C_1\,\varepsilon$, ki podajajo obseg pozitivnega povečanja entropije glede na hitrost prenosa toplote:

$$\frac{\Delta\dot{S}_{gen}}{C_2\varepsilon} = \frac{1}{\pi_1}\left[\pi_3 \ln\left(1 - \frac{1-\exp\left(-\left(1+\pi_3\right)\pi_2\right)}{1+\pi_3}\left(1-\pi_T\right)\right)\right] + \frac{1}{\pi_1}\left[\ln\left(1 + \pi_3\frac{1-\exp\left(-\left(1+\pi_3\right)\pi_2\right)}{1+\pi_3}\left(\frac{1}{\pi_T} - 1\right)\right)\right] \qquad (17),$$

$$\frac{\Delta\dot{S}_{gen}}{C_1\varepsilon} = \frac{1}{\pi_1}\left[\ln\left(\exp\left(-\pi_2\right) + \pi_T\left(1-\exp\left(-\pi_2\right)\right)\right) + \left(\frac{1}{\pi_T} - 1\right)\left(1-\exp\left(-\pi_2\right)\right)\right] \qquad (18),$$

kjer π_1 v (17) izračunamo po (7) oziroma v (18) z:

$$\pi_1 = 1 - \exp\left(-\pi_2\right) \qquad (19).$$

2 GRAFIČNA PONAZORITEV IZRAČUNA POVEČANJA ENTROPIJE

Podani matematični postopek smo prelili v računalniški zapis z uporabo Fortrana, rezultate pa prikažimo grafično. Seveda mora biti graf enačbe (10) prostorski, saj je odvisnost brezrazsežne vrednosti $\Delta\dot{S}_{gen}/C_2$ funkcija π_2, π_3 in π_T. Ustrezno diagram na sliki 2 kaže povečanje entropije v odvisnosti od π_2 in π_3 za ravni ploskev π_T = 2, 4, 6, 8 in 10, medtem ko diagram na sliki 3 predstavlja isto odvisnost za π_T = 0,2 , 0,4 , 0,6 , 0,8 in 1,0.

Iz diagramov lahko razberemo, da vse parametrične krivulje π_T kažejo ničelno povečanje entropije pri π_3 = 0, kar smo že poprej opisali pod c) med pripombami o posebnih primerih. Povečanje entropije v tem primeru je prikazan v diagramih na slikah 4 in 5. Diagrama na sliki 4a in 4b sta narisana po enačbi (16) za vse vrednosti parametra π_T= 0,2 do 1,0 in 2,0 do 10,0. Vrednosti odvisnosti iz (18) so prav tako podane v teh diagramih. Diagram na sliki 5, narisan za π_2 = 1,0 in π_3 = 0, jasno kaže vpliv π_T na brezrazsežno povečanje entropije. Diagrami na slikah 6 in 7 podajajo vrednosti, dobljene z uporabo enačb (10) in (17) za π_3 = 0,5 in π_T 0,2 do 1,0 oziroma za π_3 = 0,5 in π_T = 2,0 do 10,0. Diagrami na slikah 8 in 9 ovrednotijo iste spremenljivke za π_3 = 1,0.

to the Second law of thermodynamics, regarding the constant temperature of the stronger flow:

(15),

which, after eliminating $T_1^{''}$ and using Eq. (4), (7) and (9), is transformed into the required form:

(16).

If the Eq. (10) and (16) are divided by the effectiveness $\varepsilon = \pi_1$, the ratia $\Delta\dot{S}_{gen}/C_2\,\varepsilon$ and $\Delta\dot{S}_{gen}/C_1\,\varepsilon$ are obtained, as relevant indicators of the range where the entropy generation is acting positively on the exchanged-heat flow rate:

(17), (18),

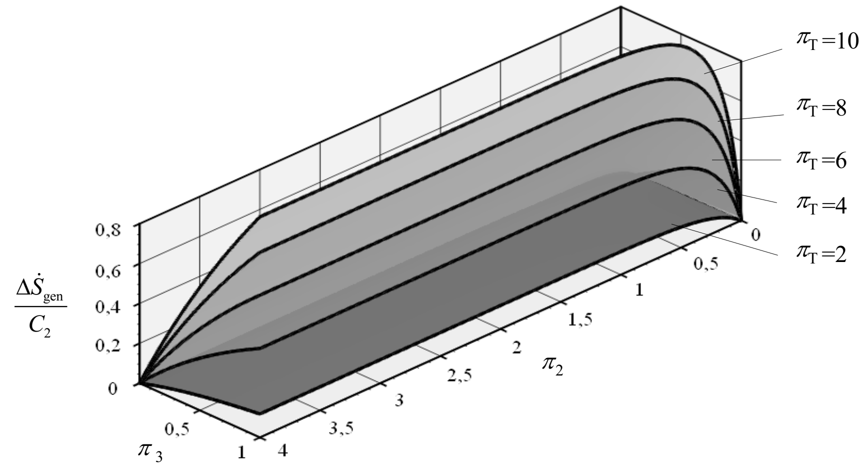
where π_1 in (17) is calculated according to (7), and in (18) according to:

(19).

2 GRAPHICAL PRESENTATION OF THE ENTROPY-GENERATION CALCULATION

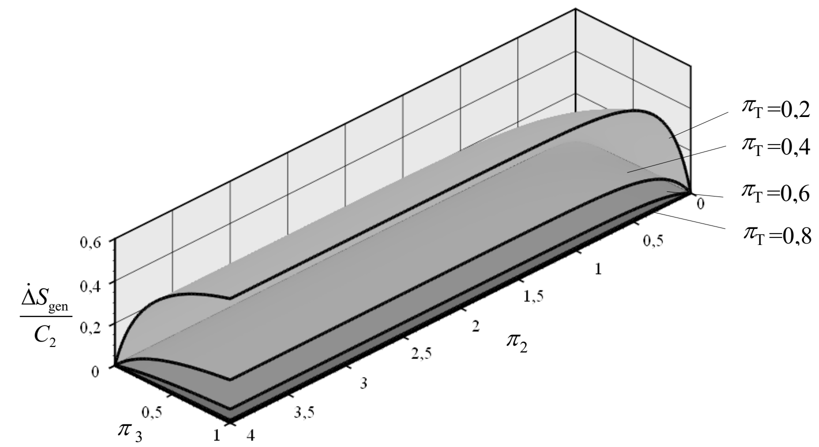
The presented mathematical procedure was put into a calculation algorithm using Fortran, and the results are presented graphically. It is obvious that the graph of Eq. (10) must be three dimensional, because the non-dimensional value $\Delta\dot{S}_{gen}/C_2$ is the function of π_2, π_3 and π_T. Accordingly, the diagram in Fig. 2 displays the entropy generation as a function of π_2 and π_3 through the niveau surfaces π_T = 2, 4, 6, 8 and 10, while the diagram in Fig. 3 represents the same relationship for π_T = 0.2, 0.4, 0.6, 0.8 and 1.0.

It can be seen from the diagrams that all the parametric curves π_T give a zero-entropy generation for π_3 = 0, which is explained above in part c) of the comments on special cases. The entropy generation for this case is presented in the diagrams in Figures 4 and 5. The diagrams in Figure 4a and 4b are plotted according to Eq. (16) for all the parametric curves π_T = 0.2 to 1.0 and 2.0 to 10.0 respectively. The values of the function defined by (18) are shown in the same diagrams. The diagram in Fig. 5, drawn for π_2 = 1.0 and π_3 = 0, clearly indicates the influence of π_T on the non-dimensional entropy generation. The diagrams in Figure 6 and 7 represent the values obtained by using equations (10) and (17) for π_3 = 0.5 and π_T = 0.2 to 1.0, and for π_3 = 0. 5 and π_T = 2.0 to 10.0, respectively. The diagrams in Figure 8 and 9 quantify the same variables for π_3 = 1.0.



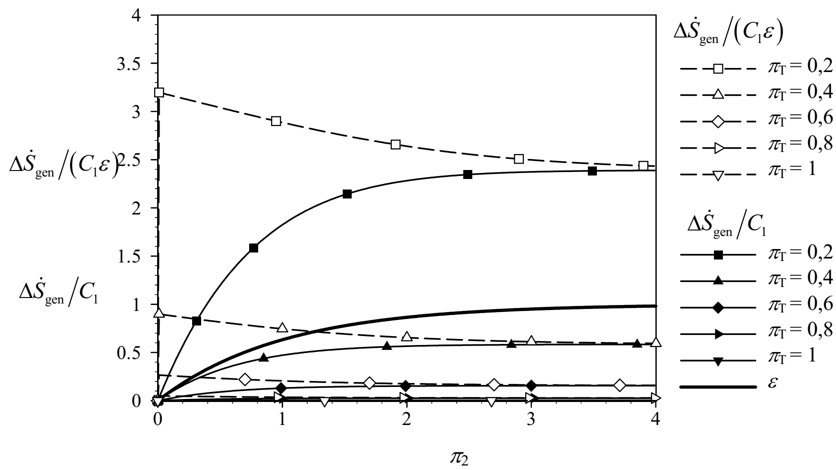
Sl. 2. *Brezrazsežno povečanje entropije v sotočnem prenosniku toplote v odvisnosti od π_2 in π_3 pri π_T = 2, 4, 6, 8 in 10 kot parametri*

Fig. 2. *Non-dimensional entropy generation of a parallel-flow heat exchanger as a function of π_2 and π_3 with π_T = 2, 4, 6, 8 and 10 as parameters*



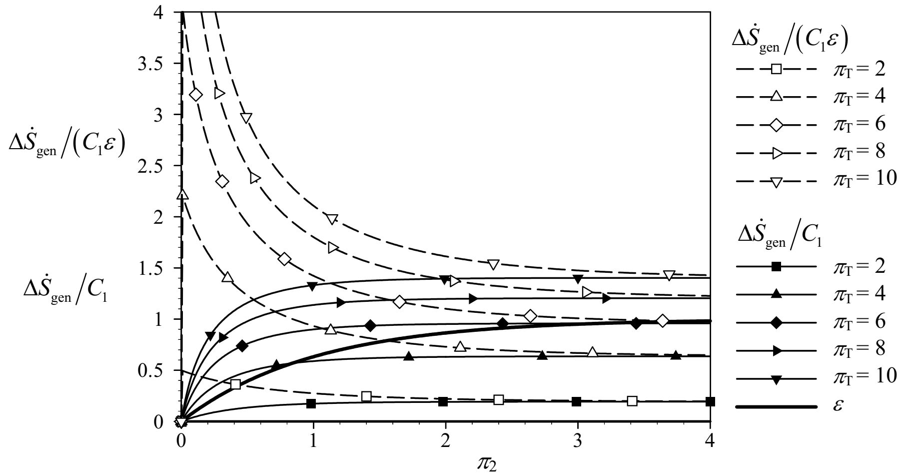
Sl. 3. *Brezrazsežno povečanje entropije v sotočnem prenosniku toplote v odvisnosti od π_2 in π_3 pri π_T = 0,2 , 0,4 , 0,6 , 0,8 in 10,0 kot parametri*

Fig. 3. *Non-dimensional entropy generation of a parallel-flow heat exchanger as a function of π_2 and π_3 with π_T = 0.2, 0.4, 0.6, 0.8 and 1.0 as parameters*



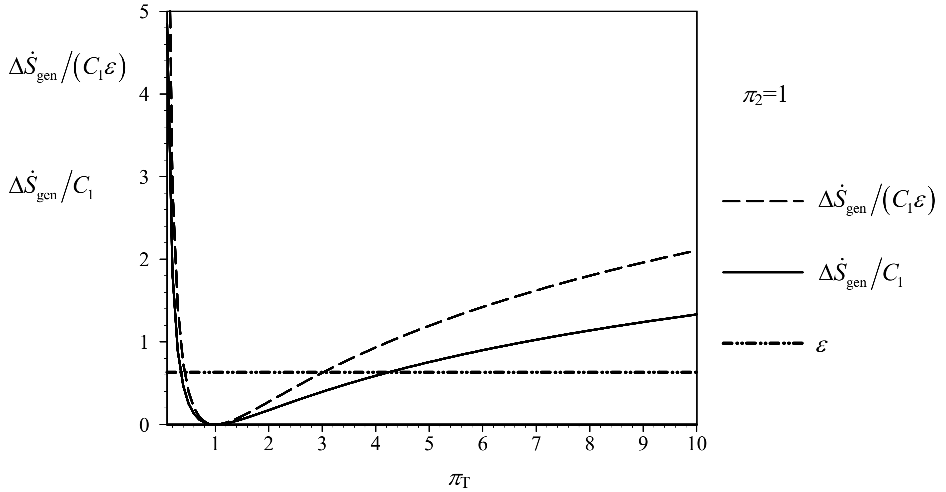
Sl. 4a. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 0,2 do 1,0 za π_3 = 0*

Fig. 4a. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_2 and the parametric curves π_T = 0.2 to 1.0 for π_3 = 0*



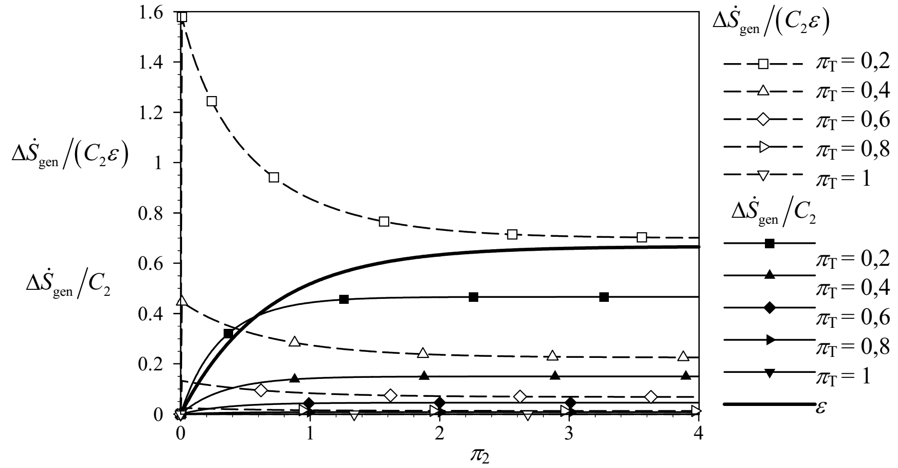
Sl. 4b. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 2,0 do 10,0 za π_3 = 0*

Fig. 4b. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_2 and the parametric curves π_T = 2.0 to 10.0 for π_3 = 0*



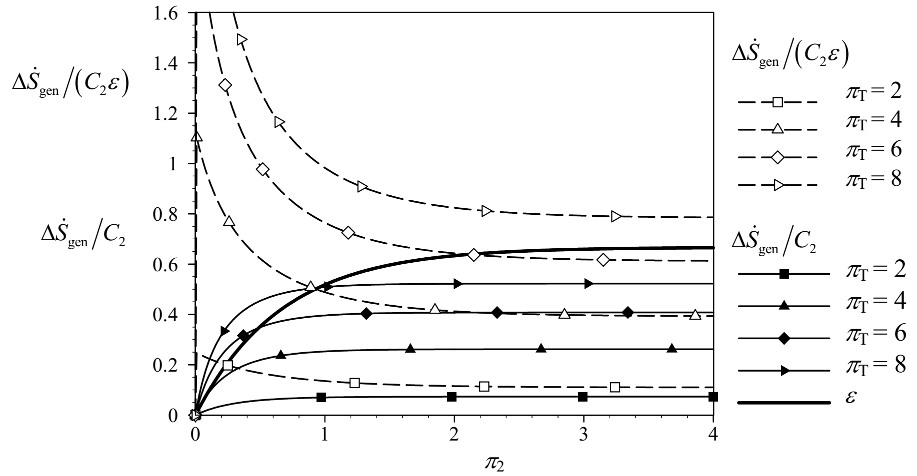
Sl. 5. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_T za π_2 = 1,0 in π_3 = 0*

Fig. 5. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_T for π_2 = 1.0 and π_3 = 0*



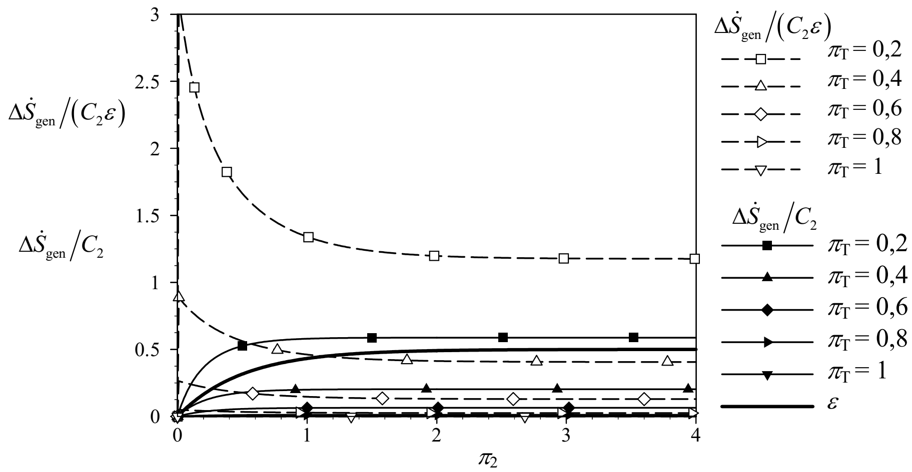
Sl. 6. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 0,2 do 1,0 za π_3 = 0,5*

Fig. 6. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_2 and the parametric curves π_T = 0.2 to 1.0 for π_3 = 0.5*



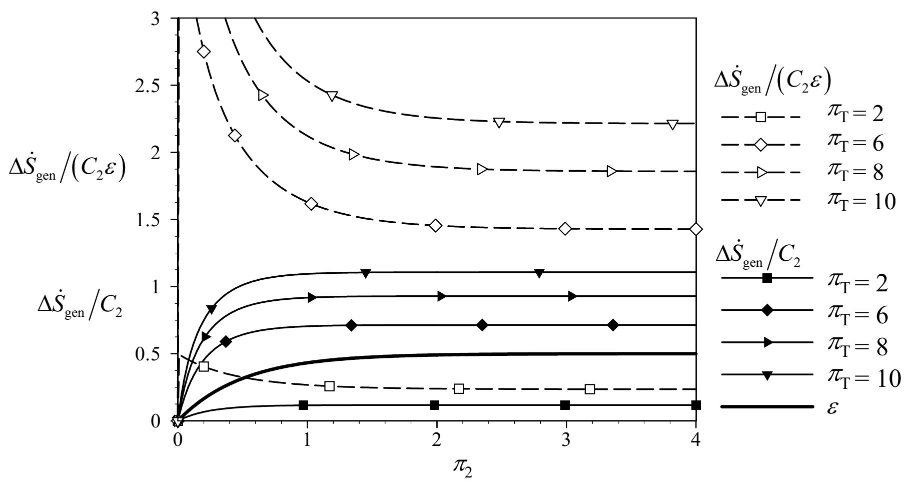
Sl. 7. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 2,0 do 10,0 za π_3 = 0,5*

Fig. 7. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_2 and the parametric curves π_T = 2.0 to 10.0 for π_3 = 0.5*



Sl. 8. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 0,2 do 1,0 za π_3 = 1,0*

Fig. 8. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat transfer effectiveness as a function of π_2 and the parametric curves π_T = 0.2 to 1.0 for π_3 = 1.0*



Sl. 9. *Brezrazsežno povečanje entropije in razmerje brezrazsežnega povečanja entropije s toplotnim izkoristkom v odvisnosti od π_2 in parametrične krivulje π_T = 2,0 do 10,0 za π_3 = 1,0*

Fig. 9. *Non-dimensional entropy generation and the ratio of non-dimensional entropy generation to heat-transfer effectiveness as a function of π_2 and the parametric curves π_T = 2.0 to 10.0 for π_3 = 1.0*

2.1 Razlaga diagramov

Prostorski diagrami na slikah 2 in 3 povedo, da povečanje entropije pri določeni vrednosti π_T praktično doseže svoje asimptotične vrednosti, kadar π_2 postane večji od ena. Te asimptotske vrednosti so podane z enačbami (11) in (12). Lahko tudi vidimo, da se brezrazsežno povečanje entropije zmanjša s povečanjem π_T med 0 in 1,0. Za π_T = 1,0 ni povečanja entropije. (To je razumljivo, saj sta pri π_T = 1,0 temperaturi vstopnih pretokov enaki, kar pomeni nespremenljivost entropije). Seveda novo povečanje π_T prek vrednosti 1 pomeni povečanje entropije.

Nespremenljivost entropije velja za vsak π_T, če je π_3 = 0. Zato definirano brezrazsežno povečanje entropije $\Delta\dot{S}_{gen}/C_2$ ni primerno za reševanje tega posebnega primera, saj daje nedoločeno vrednost $0\cdot\infty$ za $\Delta\dot{S}_{gen}$. Rešitev takega primera kažejo diagrami na slikah 4a, 4b in 5, ki so dobljeni z uporabo enačb (16), (18) in (19). Prekinjane črte v diagramih podajajo vrednosti $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ in polne črte ustrezno $\Delta\dot{S}_{gen}/C_1$.

Zanimivo je pripomniti o spremembi vrednosti $\Delta\dot{S}_{gen}/(\varepsilon C_1)$. Vidimo, da se vsaka krivulja za dano vrednost π_T monotono zniža od vrednosti za π_2 = 0 do asimptotične vrednosti (vodoravna asimptota), če $\pi_2 \to \infty$. Izračunane so iz enačbe (18). Če π_2 = 0, tj. $C_2 \to \infty$ vstavimo v (18), dobimo nedoločeno vrednost 0/0. Z uporabo L'Hospitalovega pravila zlahka pokažemo, da je:

2.1 Interpretation of the diagrams

The three-dimensional diagrams in Figure 2 and 3 indicate that the entropy generations for given values of π_T effectively reach their asymptotic values when π_2 becomes larger than one. These asymptotic values are quantified by Equations (11) and (12). It can be seen that the non-dimensional entropy generation becomes smaller with the rise of π_T from 0 to 1.0. For π_T = 1.0 the entropy generation is zero. (This is physically justified because for π_T = 1.0 the inlet temperatures of the flows are equal, thus yielding no entropy generation). However, further increasing π_T above the value of 1 increases the entropy generation again.

The entropy generation is zero for each π_T when π_3 = 0. Thus, the defined non-dimensional entropy generation $\Delta\dot{S}_{gen}/C_2$ is not suitable for the solution of this special case because it gives the undefined value $0\cdot\infty$ for $\Delta\dot{S}_{gen}$. The solution of this problem is presented in the diagrams in Figure 4a, 4b and 5, obtained by using Equations (16), (18) and (19). The dotted lines in the diagrams give the values of $\Delta\dot{S}_{gen}/(\varepsilon C_1)$, and the full lines the ones of $\Delta\dot{S}_{gen}/C_1$.

It is interesting to note the change of the value $\Delta\dot{S}_{gen}/(\varepsilon C_1)$. It is clear that each curve for a given value of π_T drops monotonously from the value for π_2 = 0 to the asymptotic value (horizontal asymptote), when $\pi_2 \to \infty$. They are calculated from Equation (18). If π_2 = 0, i.e. $C_2 \to \infty$ is inserted into (18), the undefined form 0/0 is obtained. Using the L'Hospital rule it is easily shown that:

$$\frac{\Delta\dot{S}_{gen}}{\varepsilon C_1}\left(\pi_2 = 0\right) = \frac{\left(\pi_T - 1\right)^2}{\pi_T} \qquad (20).$$

Kaj pravzaprav predstavlja desna stran enačbe (20)? Odgovor zlahka dobimo, če zapišemo enačbo brezrazsežne spremembe entropije zaradi menjave toplote pri različnih temperaturah, na različnih prenosnih površinah:

What is actually represented on the right-hand side of Equation (20)? The answer is easily obtained by establishing the non-dimensional equation of entropy generation due to heat exchange at different temperatures on a differential exchange surface area:

$$d\dot{S}_{gen} = \frac{\left(T_1 - T_2\right)^2}{T_1\ T_2}\ k\ \ dA \qquad (21).$$

Z vstavitvijo $T_1 = T_1'$ = konst. in $T_2 = T_2'$ = konst. v (21), ob uporabi (9), postane prvi člen na desni strani (21) povsem enak desni strani (20). To pomeni, da enačba (20) dejansko predstavlja največje povečanje entropije, ki je neposredno povezano z vstopnima temperaturama obeh pretokov. Praktično uporabo tega najdemo v prenosnikih toplote, kjer oba pretoka prestaneta fazni premeni. Tam imata tokova neskončno toplotno zmogljivost, saj eden kondenzira in se drugi uparja.

Asimptotično vrednost $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ na slikah 4a in 4b dobimo z vstavitvijo $\pi_2 \to \infty$ v enačbo (18):

By inserting $T_1 = T_1'$ = const. and $T_2 = T_2'$ = const. into (21), and by using (9), the first term on the right-hand side of (21) becomes identical to the right-hand side of (20). This means that equation (20) in fact represents the maximum entropy generation, which is directly related to the inlet temperatures of both flows. The practical application of this may be found in heat exchangers, where both flows undergo a phase change. There the flows have an infinite heat capacity because one condenses and the other evaporates.

The asymptotic value of $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ in Figure 4a and 4b are obtained by inserting $\pi_2 \to \infty$ into Equation (18):

$$\frac{\Delta\dot{S}_{gen}}{\varepsilon C_1}\left(\pi_2 \to \infty\right) = \ln\left(\pi_T\right) + \frac{1-\pi_T}{\pi_T} \qquad (22).$$

Diagram na sliki 5 predstavlja brezrazsežno spremembo entropije $\Delta\dot{S}_{gen}/C_1$ in $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ v odvisnosti od π_T za $\pi_2 = 1,0$ in $\pi_3 = 0$. Jasno se vidi iz diagrama, ob rasti π_T v območju $0 < \pi_T < 1$ se obe izračunani vrednosti strmo znižata in se izničita pri $\pi_T = 1$. Za $\pi_T > 1$ je povečanje izračunane vrednosti počasnejše, saj se sprememba entropije $\Delta\dot{S}_{gen}/C_1$ nagiba v neskončnost, ko se π_T nagiba k nič ali v neskončnost. To zlahka dokažemo z analizo enačbe (16). Kadar prenosnik toplote deluje v območju $0 < \pi_T < 1$, rahel upad π_T bistveno poveča entropijo. Čeprav so bili diagrami na slikah 4a, 4b in 5 izdelani z enačbami, ki veljajo za sotočne prenosnike toplote, so iste rešitve veljavne tudi za protitočne in križne prenosnike toplote, saj je $\pi_3 = 0$.

Diagrami na slikah 6 in 7 dajejo brezrazsežne vrednosti $\Delta\dot{S}_{gen}/C_1$ in $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ v odvisnosti od π_2 za $\pi_3 = 0,5$. Na sliki 6 so narisane krivulje za $\pi_T = 0,2$ do 1,0, na sliki 7 pa za $\pi_T = 2,0$ do 10,0. Na obeh diagramih so krivulje izkoristka

The diagram in Figure 5 presents the non-dimensional entropy generations $\Delta\dot{S}_{gen}/C_1$ and $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ as a function of π_T for $\pi_2 = 1.0$ and $\pi_3 = 0$. It is clear from the diagram that for π_T rising in the interval $0 < \pi_T < 1$, both the ordinate values drop steeply and become zero for $\pi_T = 1$. For $\pi_T > 1$ the rise of the ordinate values is slower because the entropy generation $\Delta\dot{S}_{gen}/C_1$ tends to infinity when π_T tends to zero or to infinity. This is easily proven by analysing Equation (16). If the heat exchanger is operated in the range $0 < \pi_T < 1$, a slight decrease of π_T significantly increases the entropy generation. Although the diagrams in Figure 4a, 4b and 5 were obtained according to the equations for parallel-flow heat exchangers, the same solutions are valid for counter-flow and cross-flow heat exchangers because of $\pi_3 = 0$.

The diagrams in Figure 6 and 7 represent the non-dimensional values $\Delta\dot{S}_{gen}/C_1$ and $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ as functions of π_2 for $\pi_3 = 0.5$. In Figure 6 the curves for $\pi_T = 0.2$ to 1 are plotted, and in Figure 7 for $\pi_T = 2.0$ to 10.0. In both diagrams the curve for the effectiveness:

$$\varepsilon = \pi_1 = \frac{2}{3}\left(1 - \exp\left(-1,5\pi_2\right)\right) \tag{23}$$

prav tako narisane.

V obeh primerih kažejo diagrami, da se pri takšnih prenosnikih toplote pojavi največja vrednost izkoristka, praktično pri $\pi_2 = 3,0$ ($\varepsilon = 0,659$). Nadaljnje večanje π_2 pomeni povečanje entropije, ob zanemarljivem izboljšanju izkoristka.

Nazadnje, diagrami na slikah 8 in 9 kažejo brezrazsežne vrednosti $\Delta\dot{S}_{gen}/C_1$ in $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ v odvisnosti od π_2 za $\pi_3 = 1,0$ oziroma vrednosti parametrov $\pi_T = 0,2$ do 1,0 in $\pi_T = 2,0$ do 10,0. Krivulja izkoristka

is also drawn.

For both cases the diagrams indicate that for such heat exchangers the maximum effectiveness value is practically achieved for $\pi_2 = 3.0$ ($\varepsilon = 0.659$). A further increase of π_2 only results in entropy generation, with an insignificant improvement of the effectiveness.

Finally, the diagrams in Figure 8 and 9 represent the non-dimensional values $\Delta\dot{S}_{gen}/C_1$ and $\Delta\dot{S}_{gen}/(\varepsilon C_1)$ as functions of π_2, for $\pi_3 = 1.0$ and parametric values of $\pi_T = 0.2$ to 1.0 and $\pi_T = 2.0$ to 10.0 respectively. The effectiveness curve:

$$\varepsilon = \pi_1 = \frac{1}{2}\left(1 - \exp\left(-2\pi_2\right)\right) \tag{24}$$

je prav tako vnesena v diagramih.

Slike jasno kažejo, da je povečanje entropije zelo blizu svoje asimptotične vrednosti pri $\pi_2 = 2,5$. Nadaljnje večanje π_2 daje enakomerno rast entropije ob zanemarljivem vplivu na izboljšanje izkoristka. Glede na sliko 8 smemo poudariti, da povečanje π_T v območju 0,2 do 0,8 enakomerne rasti entropije pomeni bistven vpliv na povečanje ε v širokem območju spremenljivke π_2. Po sliki 9 velja podoben izrek za zmanjšanje π_T od 10,0 do 2,0.

is also plotted in the diagrams.

The figures clearly indicate that the entropy generations are very close to their asymptotic values for $\pi_2 = 2.5$. A further increase of π_2 yields constant entropy generation with negligible influence on the improvement of effectiveness. Referring to Figure 8, it should be emphasized that increasing π_T in the range 0.2 to 0.8 the constant entropy generation has a significant influence on the increase of ε in a wide range of the variable π_2. According to Figure 9, the same can be said for decreasing π_T from 10.0 to 2.0.

3 SKLEP

Izvedena analiza entropije sotočnih prenosnikov toplote je dokazala, da povečanje entropije lahko podamo z istimi brezrazsežnimi veličinami π_1, π_2 in π_3, kakor pri energijski analizi, z dodatnim brezrazsežnim številom π_T. Entropijska analiza kaže neposredno povezanost med izkoristkom ε in povečanjem entropije. Prikazali smo,

3 CONCLUSION

The performed entropy analysis of parallel-flow heat exchangers has proven that entropy generation can be represented using the same non-dimensional characteristics π_1, π_2 and π_3, used in the energy analysis, and the additional non-dimensional number π_T. The entropy analysis reveals a direct connection between the effectiveness ε and the da je povečanje entropije veliko, kadar je razlika med toplotnima zmogljivostima obeh pretokov velika. Največja je, kadar en pretok spremeni fazo ($\pi_3 = 0$), ter najmanjša kadar sta toplotni zmogljivost enaki ($\pi_3 = 1$). Razmere so prav nasprotne glede izkoristka ε prenosa toplote. Prav tako je prikazano, da za $0 < \pi_3 < 1$ povečanje brezrazsežne značilnice menjalne površine $\pi_2 = kA_0/C_1$ učinkuje samo do določene vrednosti, nato pa se entropija le povečuje ob praktično nespremenljivem izkoristku.

Brezrazsežne značilnice π_1, π_2 in π_3, uporabljene v tem prispevku, je uvedel že Bošnjaković za svoje energijske izračune prenosnikov toplote. Sedanji način izračuna entropije z uporabo dodatne značilnice π_T dodaja več splošnosti pri problemih sotočnih prenosnikov toplote.

entropy generation. It was shown that the entropy generation is large when the difference between the heat capacities of the heat exchanging flows is big. It is largest when one of the flows changes its phase ($\pi_3 = 0$), and smallest when the flow heat capacities are equal ($\pi_3 = 1$). The situation is just the opposite for the heat-transfer effectiveness ε. Further, it was shown that for $0 < \pi_3 < 1$, increasing the non-dimensional exchanger area characteristic $\pi_2 = kA_0/C_1$ is useful only to a limited value, after which only entropy is produced with a practically constant effectiveness.

The non-dimensional characteristics π_1, π_2 and π_3 used in this paper were introduced by Bošnjaković for his energy calculation of heat exchangers. This approach to the entropy calculation, using the additional characteristic π_T, adds more generality to the problem of parallel heat-exchanger analysis.

4 OZNAKE
4 NOMENCLATURE

površina toplotne menjave	A	m^2	heat-exchange surface area
specifična toplota pri *p*=konst. šibkejšega oz. močnejšega pretoka	c_{P1}, c_{P2}	J/(kg K)	specific heat capacity at *p*=const. of the weaker and stronger flow respectively
toplotna zmogljivost šibkejšega oz. močnejšega pretoka	C_1,C_2	W/K	heat capacity of the weaker and stronger flow respectively
izkoristek prenosnika toplote	ε	-	heat-exchanger effectiveness
celotni količnik prenosa toplote	*k*	W/(m^{2}K)	overall heat-transfer coefficient
brezrazsežno razmerje šibkejšega oz. močnejšega pretoka vstopne temperature	π_T	-	non-dimensional relation of the weaker and stronger flow input temperatures
brezrazsežna temperaturna značilnica prenosnika toplote	π_1	-	non-dimensional temperature characteristic of the heat exchanger
brezrazsežna površinska značilnica prenosnika toplote	π_2	-	non-dimensional heat-exchanger area characteristic
brezrazsežna značilnica razmerja zmogljivosti šibkejšega oz. močnejšega pretoka	π_3	-	non-dimensional characteristic of the weaker and stronger flow heat-capacity ratio
pretočna količina šibkejšega oz. močnejšega pretoka	q_{m1}, q_{m2}	kg/s	mass flow rate of the weaker and stronger flow respectively
celotno povečanje entropije v prenosniku toplote	$\Delta \dot{S}_{gen}$	W/K	overall entropy generation of the heat exchanger
absolutna vstopna in izstopna temperatura šibkejšega pretoka	T_1', T_1''	K	absolute input and output temperatures of the weaker flow
absolutna vstopna in izstopna temperatura močnejšega pretoka	T_2', T_2''	K	absolute input and output temperatures of the stronger flow

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