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Generalization of the Kövesligethy equation for non-circular macroseismic fields

Posplošitev Kövesligethyjeve enačbe za nekrožna makroseizmična polja

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Abstract

In European countries the equation of Kövesligethy for macroseismic determination of focal depths is often used as the attenuation law for seismic hazard assessment. For non-circular fields the expression with an additional parameter – "the coefficient of non-circulatity" is much better model as the original form of the Kövesligethy equation. This coefficient is a function of azymuth and can also be a function of epicentral distance. The function of azymuth can be analytically simply expressed for elliptic macroseismic fields.

Kratka vsebina

Kövesligethyjeva enačba za makroseizmično določanje globine potresnih žarišč se v Evropi pogosto uporablja kot enačba pojemanje učinkov pri oceni potresne nevarnosti. Za krožna in približno krožna polja je enačba uporabna v svoji prvotni obliki. Za nekrožna polja pa daje ustreznejši model enačba, v kateri nastopa dodatna količina – »koeficient nekrožnosti«:

$$I = I_0 - 3\log\frac{\sqrt{r^2 + (k\bar{h})^2}}{k\bar{h}} - 1.3\,\frac{\bar{\alpha}}{k}\left(\sqrt{r^2 + (k\bar{h})^2} - k\bar{h}\right)$$

kjer je I_0 potresna stopnja v epicentru, r epicentralna razdalja, h žariščna globina, $\bar{\alpha}$ povprečni absorpcijski koeficient in k koeficient nekrožnosti. Ta koeficient je funkcija azimuta, lahko pa se tudi spreminja z oddaljenostjo od epicentra. Odvisnost od azimuta analitično enostavno izrazimo za eliptično polje.

The formula of Kövesligethy, Jánosi, and Gassmannn (Blake, 1941) or simply the equation of Kövesligethy (Sponheuer, 1960) has been often used for the determination of the focal depth and other macroseismic parameters of historic earthquakes from the available isoseismal maps. Its well known form is:

$$I = I_0 - 3\log\frac{\sqrt{\bar{r}^2 + \bar{h}^2}}{\bar{h}} - 1.3\,\bar{\alpha}\left(\sqrt{\bar{r}^2 + \bar{h}^2} - \bar{h}\right) \quad (1)$$

in which I_0 is the epicentral intensity, $\bar{r}$ is the radius of the area enclosed within the isoseismal I, $\bar{h}$ is the focal depth, and $\bar{\alpha}$ is the average value of the absorption coefficient.

The procedure of evaluation of macroseismic parameters consists of two steps:

1. the determination of $\bar{r}$ for all isoseismals of the given macroseismic field,
2. the calculation of parameters I_0, $\bar{h}$, and $\bar{\alpha}$ from the system of equations of the type (1), corresponding to different isoseismals of the given macroseismic field.

In European countries Kövesligethy equation is also very often applied as a macroseismic attenuation law $I = I(r, \Phi)$ in the form:

$$I = I_0 - 3 \log \frac{\sqrt{r^2 + \bar{h}^2}}{\bar{h}} - 1.3\, \alpha \left(\sqrt{r^2 + \bar{h}^2} - \bar{h}\right) \tag{2}$$

where r is the epicentral distance, $\alpha = \alpha(\Phi)$ is the absorption coefficient, and Φ is azimuth. It is presumed that the formula of Kövesligethy holds also for all and singular direction from the epicentre.

To use the equation (2) as the attenuation relation we must first evaluate macroseismic parameters I_0, $\bar{h}$, and $\alpha = \alpha(\Phi)$ for all earthquake sources from the available isoseismal maps. In the calculation procedure a system of equations of the type (2) for different directions Φ is used. This procedure works usually quite well for circular and near-circular macroseismic fields. In the case of non-circular fields the results of calculations are often great standard deviations. The more the macroseismic field deviates from the circular one the bigger are standard errors.

Obviously, the equation (2) is generally not appropriate for non-circular fields. If for instance, the fitness of the model to the observed field is good for the direction in which the field is prolongated, then for the direction in which the isoseismals are squeezed together is very bad, and vice versa. Farther, if the fitness is good for medium epicentral distances, then for small and large distances is very bad.

Main deficiency of the equation (2) lies in the fact that basically only the last part of the equation can be (through α) a function of azimuth. The influence of the last part of the equation (2) increases with increasing epicentral distance, but is negligiable in the vicinity of the epicentre. For this reason the isoseismals of the model field corresponding to the equation (2) are almost circular near the epicentre and become more and more non-circular with increasing epicentral distance (Fig. 1a). The shape of the isoseismals of an observed field has mostly just the oposite tendency (Fig. 1c).

To get a better expression for non-circular fields we may still rely on the formula of Kövesligethy. But we must consider its original purpose: the determination of the average characteristics of the given macroseismic field. Namely, Kövesligethy equation has been proved a good model for the equivalent circular fields of many non-circular fields. The radius $\bar{r}$ is actually the average epicentral distance to the intensity I isoseimal. It can be written as $\bar{r} = r/k$, where r is the epicentral distance to the intensity I isoseismal in an arbitrary direction Φ. Parameter k is a function of azimuth (and could be also a function of the epicentral distance). We can call it "the coefficient of non-circularity". Puting k into equation (1), we get the following expression:

$$I = I_0 - 3 \log \frac{\sqrt{r^2 + (k\bar{h})^2}}{k\bar{h}} - 1.3\, \frac{\bar{\alpha}}{k} \left(\sqrt{r^2 + (k\bar{h})^2} - k\bar{h}\right) \tag{3}$$

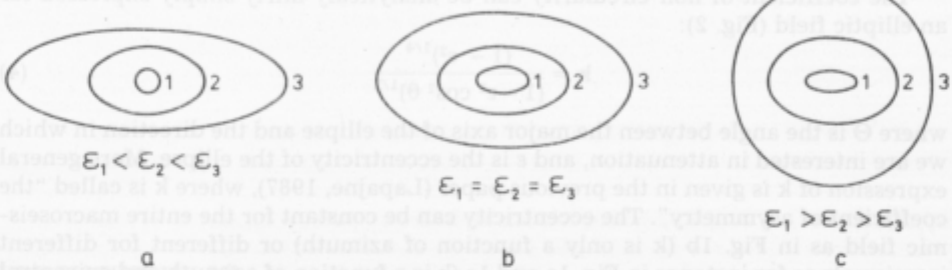

Fig. 1. Three macroseismic field models

Sl. 1. Trije modeli makroseizmičnega polja

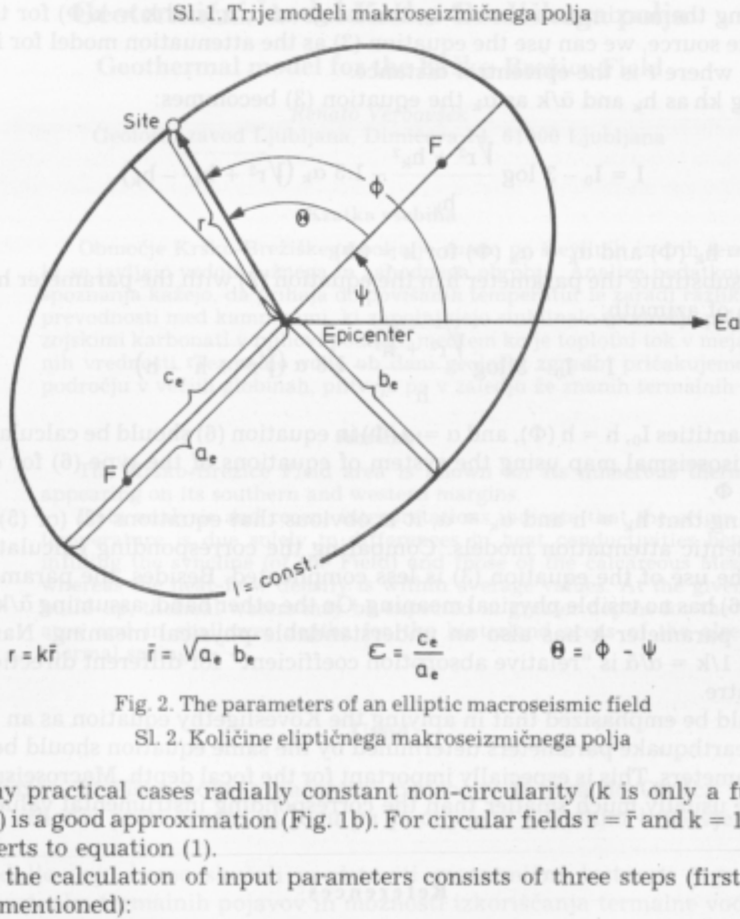

Fig. 2. The parameters of an elliptic macroseismic field

Sl. 2. Količine eliptičnega makroseizmičnega polja

For many practical cases radially constant non-circularity (k is only a function of azimuth) is a good approximation (Fig. 1b). For circular fields $r = \bar{r}$ and $k = 1$; equation (3) converts to equation (1).

Now the calculation of input parameters consists of three steps (first two were already mentioned):

1. the determination of $\bar{r}$ for all isoseismals of the given macroseismic field,
2. the calculation of I_0, $\bar{h}$ and $\bar{\alpha}$ from the system of equations of the type (1), corresponding to different isoseismals of the given macroseismic field,
3. the calculation of k as a function of azimuth from the system of equations $k_\Phi = r_\Phi / \bar{r}$, corresponding to different azimuths Φ.

The coefficient of non-circularity can be analyticaly fairly simply expressed for an elliptic field (Fig. 2):

$$k = \frac{(1 - \varepsilon^2)^{1/4}}{(1 - \varepsilon^2 \cos^2 \theta)^{1/2}} \tag{4}$$

where Θ is the angle between the major axis of the ellipse and the direction in which we are interested in attenuation, and ε is the eccentricity of the ellipse. More general expression of k is given in the previous paper (Lapajne, 1987), where k is called "the coefficient of asymmetry". The eccentricity can be constant for the entire macroseismic field as in Fig. 1b (k is only a function of azimuth) or different for different isoseismals as for instance in Fig. 1a and 1c (k is a function of azimuth and epicentral distance).

Knowing the maximum value of I_0, the range of $\bar{h}$, $\bar{\alpha}$, and $k = k(\Phi)$ for the given earthquake source, we can use the equation (3) as the attenuation model for intensity $I = I(r,\Phi)$, where r is the epicentral distance.

Writing $k\bar{h}$ as h_k and $\bar{\alpha}/k$ as α_k the equation (3) becommes:

$$I = I_0 - 3 \log \frac{\sqrt{r^2 + h_k^2}}{h_k} - 1.3\, \alpha_k \left(\sqrt{r^2 + h_k^2} - h_k\right) \tag{5}$$

where $h_k = h_k (\Phi)$ and $\alpha_k = \alpha_k (\Phi)$ for $k = (\Phi)$

Let us substitute the parameter $\bar{h}$ in the equation (2) with the parameter h beeging a function of azimuth:

$$I = I_0 - 3 \log \frac{\sqrt{r^2 + h^2}}{h} - 1.3\, \alpha \left(\sqrt{r^2 + h^2} - h\right) \tag{6}$$

The quantities I_0, $h = h(\Phi)$, and $\alpha = \alpha(\Phi)$ in equation (6) should be calculated from the given isoseismal map using the system of equations of the type (6) for different directions Φ.

Realizing that $h_k \equiv h$ and $\alpha_k \equiv \alpha$, it is obvious that equations (3) (or (5)) and (6) express identic attenuation models. Comparing the corresponding calculation procedures, the use of the equation (3) is less complicated. Besides, the parameter h in equation (6) has no visible physical meaning. On the other hand, assuming $\bar{\alpha}/k = \alpha$, the geometric parameter k has also an understandable physical meaning. Namely, its reciprocal $1/k = \alpha/\bar{\alpha}$ is "relative absorption coefficient" for different directions from the epicentre.

It should be emphasized that in aplying the Kövesligethy equation as an attenuation law, earthquake parameters determined by the same equation should be used as input parameters. This is especially important for the focal depth. Macroseismic focal depths are usually much smaller than the corresponding instrumental values.

References

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