

# Determination of a Light Helicopter Flight Performance at the Preliminary Design Stage

Zlatko Petrović - Slobodan Stupar - Ivan Kostić\* - Aleksandar Simonović  
University of Belgrade, Faculty of Mechanical Engineering, Serbia

*Promising solutions for the problem of the extensively time-consuming modern urban transportation has been found in the use of light and very light helicopters. This paper presents a part of the preliminary design methodology, compiled at the Faculty of Mechanical Engineering, University of Belgrade and includes performance calculations of such helicopters. Due to limited budgets and an extremely demanding process of helicopter development, it is highly significant that during all development stages reliable performance estimates are obtained in order to ensure assigned operational requirements. The scope of this paper is confined to the preliminary design stage, where it is customary to substitute the very complex helicopter rotor dynamics with its averaged mechanical and aerodynamic characteristics and apply certain empirically verified simplifications. Based on this approach, the independent, efficient and reliable computer programs for the calculation of different performance characteristics have been developed. In addition to their application on an actual on-going project, they have also been applied on several existing helicopters of a similar class for a more accurate determination of the empirical input parameters. The applied methodology and obtained results have been presented, verifying the overall algorithm efficiency.*

©2010 Journal of Mechanical Engineering. All rights reserved.

**Keywords:** light helicopter, flight performance, preliminary design stage

## 0 INTRODUCTION

Proper evaluation of the most important helicopter design parameters and estimation of its basic flight performance in the initial design stages is highly significant for the overall project time and cost effectiveness. It assumes the application of mathematical models that are fairly simple, in conjunction with empirical coefficients and parameters derived from the previous successful designs. Such a requirement of simplicity is in contrast with the extremely complex helicopter rotor dynamics. During one revolution, a rotor blade in progressive flight is generally subjected to the pitching, flapping and leading - lagging motions, repeated several hundreds of times in a minute, while being exposed to the gravitational, centrifugal, inertial, and aerodynamic loads [1] and [2]. At higher progressive flight speeds, blades at the advancing azimuths are at very small pitch, with tips that might have local supersonic flow zones even in case of light helicopters, while the retreating blades are at incidences that are often beyond the static stall angle, with inner domains subjected to the reverse flow (velocity of flight is higher than local tangential velocities in this domain). The

aim of such cyclic blade motions is to keep the resultant rotor thrust acting in the plane of symmetry. Flapping motions of the blades enable tilting of the main rotor disc in certain directions when desired, to generate forward, backward or lateral thrust components. Due to all these factors, helicopter blades in progressive flight are subjected to very complex unsteady airflow patterns.

In spite of that, many years of industry experience has shown that quite good and very efficient preliminary estimates of helicopter flying characteristics can be obtained by averaging some aspects of the rotor and overall helicopter dynamics and aerodynamics. Many methods have been developed so far, varying in the level of complexity and accuracy of the obtained results. Their general aim is to bring the most important design parameters of a new helicopter close enough to their optimums, so that major changes are hopefully not necessary at higher design levels, which involve very complex computational and experimental methods that are expensive and time consuming. The best results in the initial design analyses can be obtained if calculations at the preliminary level are repeated

\*Corr. Author's Address: University of Belgrade, Faculty of Mechanical Engineering, Aeronautical Department, Kraljice Marije 16, 11120 Belgrade 35, Serbia, ikostic@mas.bg.ac.rs

in several refinement steps, with the complexity level increasing from one to another.

This paper presents some of the calculation procedures applied in the preliminary flight performance estimates of the light helicopter designs currently under development at the Aeronautical Institute of the Belgrade Faculty of Mechanical Engineering. The first one is a design ordered by a foreign partner (Fig. 1.), while the second one is aimed to be its simplified technology demonstrator version, with the take off mass limited to 650 kg, for which the results are presented in this paper. The design process, in accordance with Certification Specifications for Small Rotorcraft CS-27 and Certification Specifications for Very Light Rotorcraft CS-VLR, was initiated with certain performance requirements. Some of these requirements considered low gross weight, payload larger than 180 kg, range over 450 km, high value of hover ceiling and cruising speed at 1000 m ISA+15, higher than 160 km/h.

Applied calculation algorithms have been compiled with an aim to establish a proper balance between the required simplicity and time effectiveness on one hand, and the expected accuracy on the other. For the verification purposes, the same calculations have been applied on several existing light helicopters. Those results were then used to improve some of the empirical parameters initially applied in new helicopter calculations. The obtained results have proven to be very valuable inputs for the following higher level calculations. Considering the fact that this is an on-going project, only the results from the initial calculation stage will be presented in this paper.

## 1 CALCULATION PROCEDURES

For the here presented analyses, the take off mass of  $m = 650$  kg has been considered as a constant input value. For operational purposes, mass should be varied within the predefined range. Initially, an optimization procedure had been applied to determine the most relevant calculation inputs. Considering the main rotor, the most relevant parameters that were obtained are: number of blades  $n = 2$ , rotor radius  $R = 3.8$  m, blade chord length  $c = 0.205$  m, solidity factor of the rotor  $\sigma = (n \cdot c) / (R \cdot \pi) = 0.0343$ , rotor disc

area  $A = 45.96$  m<sup>2</sup>, number of revolutions per minute  $N = 440$  rpm = const. for all flight regimes, giving blade tip tangential velocity  $V_T = 175$  m/s = const.

The reciprocating power plant gives the maximum output of  $P_{0\max} = 147$  kW (200 HP) at the sea level. The engine power at other altitudes is estimated as:

$$P_H = P_{0\max} \cdot (1.11\rho / \rho_0 - 0.11), \quad (1)$$

where  $\rho_0 = 1.2255$  kg/m<sup>3</sup> represents the density of air at  $H = 0$  m, and  $\rho$  is density at a given altitude, defined by equation:

$$\rho = \rho_0 \frac{20000 - H}{20000 + H} \quad (2)$$

in which the altitude  $H$  is expressed in meters. Also, for the purpose of this paper, the optimum fuel/air mixture at all altitudes has been assumed. In operational design work, the actual engine characteristics for different altitudes should be used.

At this level of helicopter performance calculations, a standard approach is that the aerodynamic characteristics of the blades are averaged over the main rotor disc. All presented analyses, based on [1] to [3] have been done using a custom developed software for solving the sets of equation that will briefly be presented within the oncoming sections.

### 1.1 The Average Main Rotor Blade Lift Coefficient

The average blade lift coefficient is determined as:

$$C_L = 6C_T / \sigma, \quad (3)$$

while the thrust coefficient in hovering and level flight is given by:

$$C_T = \frac{T}{\rho \cdot R^2 \pi \cdot V_T^2} \quad (4)$$

and  $T = W = m \cdot g$ . For initial estimates, Eq. (4) can be used both for hovering and progressive flight. At higher calculation levels, this equation should be refined by including the disc slope angle and collective pitch for the given mass for example to determine a more accurate  $C_T$  for the given progressive flight regimes.

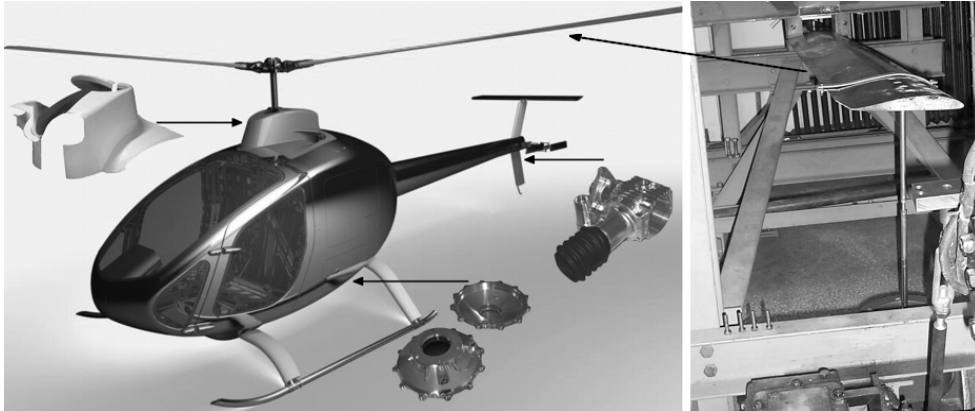


Fig. 1. Helicopter CAD model and some examples of the components manufactured for the laboratory tests and the production technology verifications (higher development stages of the on-going project)

## 1.2 Power Required for Hovering and Horizontal Flight

Total power which is required for hovering and horizontal flight of a helicopter can be obtained as the main rotor power increased by approximately 10% to take into account the required tail rotor power and transmission losses  $P = 1.1 \cdot C_p \rho A V_T^3$ . The main rotor power coefficient  $C_p$  in hovering is calculated from:

$$C_p = \kappa \lambda_h C_T + \frac{\sigma}{8} C_{D_0}, \quad (5)$$

while for horizontal flight it becomes:

$$C_p = \kappa \lambda_i C_T + \frac{\sigma}{8} C_{D_0} (1 + k \mu^2) + \mu \frac{R_D}{W} C_T. \quad (6)$$

In previous equations  $\kappa$  represents a coefficient which takes into account the induced velocity distribution irregularities over the main rotor disc. For hovering  $\kappa = 1.15$ , while for forward flight  $\kappa = 1.2$ ;  $\mu = V/V_T$ , where  $V$  is progressive flight velocity. Constant  $k = 4.65$  is an empirical value, used by the Westland Helicopters [3]. Drag of the helicopter, except the rotor, is  $R_D = 0.5 \cdot \rho \cdot C_D \cdot S \cdot V^2 = 0.5 \cdot \rho \cdot f_A \cdot V^2$ , where  $f_A \approx 1 \text{ m}^2$  is an estimate of the flat plate equivalent area, a usual rounded value applied in the initial step for small helicopter designs. Using the notation  $v_h$  and  $v_i$  for velocities induced by the main rotor in hovering and horizontal flight, the induced velocity coefficient in progressive flight  $\lambda_i = v_i/V_T$  is obtained from equations:

$$\lambda_i = \lambda_h (v_i/v_h) \quad (7)$$

and:

$$(v_i/v_h)^4 + (V/v_h)^2 (v_i/v_h)^2 - 1 = 0. \quad (8)$$

Parameter  $\lambda_h = v_h/V_T = \sqrt{C_T/2}$  represents the induced velocity coefficient in hovering.

For the calculated average rotor disk lift coefficients determined using Eq. (3), the averaged blade profile drag coefficients  $C_{D_0}$  for several characteristic altitudes have been obtained using the steady state polar curve shown in Fig. 2. Knowing that vibrations during the flight generally affect the boundary layer generation over the rotor blades [1], a dominant turbulent layer has been assumed, and standard roughness polar for Reynolds number  $1.8 \times 10^6$  has been used [4]. It should be noted that, according to Fig. 2, the average  $C_L$  for  $H = 5 \text{ km}$  is practically at the maximum the steady-state lift curve. On the other hand, the retreating blades under operational conditions will require higher local lift coefficient values. Since the maximum lift coefficient of an airfoil under dynamic flow conditions encountered on helicopter rotors is always higher than in steady flow [2], this value can also be used in formal averaged calculations. It should be noted that because of the actual engine operational restrictions, kinematic limitations of the main rotor (still not known at this design level) and similarly, the preliminary results considering the domain of the absolute ceiling must be taken with reservations as they might be overoptimistic to a certain extent.

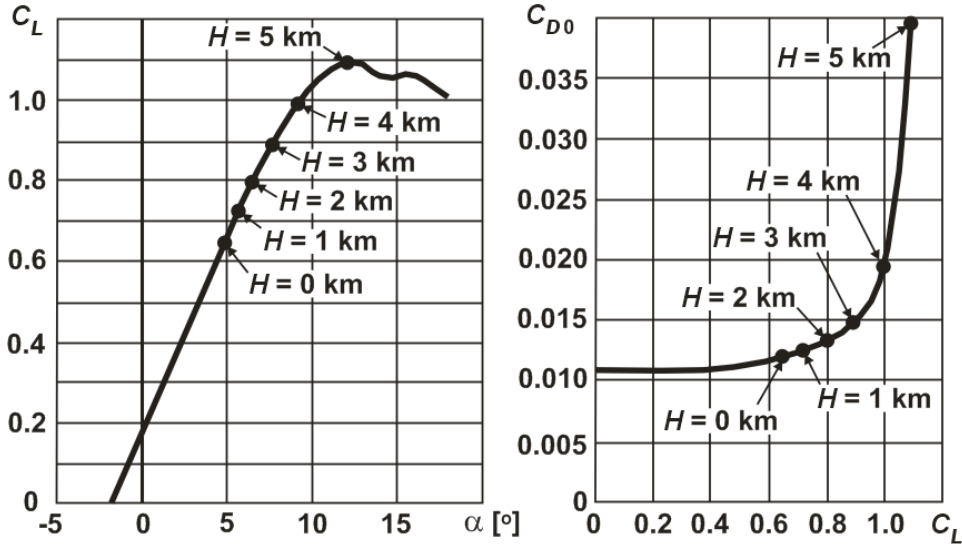


Fig. 2. Steady-state lift coefficient and standard roughness polar curve for the NACA 8-H-12 airfoil [4]

### 1.3 Rate of Climb in Progressive Flight

Main rotor power required for climbing is  $P = C_p \rho A V_T^3$ . In this case, the power coefficient is given by:

$$C_p = \kappa \lambda_i C_T + \frac{\sigma}{8} C_{D_0} (1 + k \mu^2) + \frac{1}{2} \mu^3 \frac{f_A}{A} + \xi \lambda_c C_T \quad (9)$$

In Eq. (9),  $\lambda_c = w/V_T$  is relative climbing velocity, where  $w$  is the actual rate of climb in meters per second. Parameter  $\xi = 1.3$  takes into account additional losses caused by the changes of relative flow direction in climb, while other parameters have the same meaning as already mentioned. The highest rates of climb at a given flight velocity  $V$  and altitude  $H$  can be reached when maximum available engine power  $P_{AV}$  for this altitude is applied. Since the main rotor receives approximately 90% of the total engine power, the equation for  $\lambda_c$  becomes:

$$\lambda_c = \frac{0.9 \cdot C_{P_{AV}}}{\xi C_T} - \kappa \frac{\lambda_i}{\xi} - \frac{\sigma}{8} \frac{C_{D_0}}{\xi C_T} (1 + k \mu^2) - \frac{\mu^3}{2 \xi C_T} \frac{f_A}{A} \quad (10)$$

### 1.4 Rate of Descent in Autorotation

Even in case of total engine power loss ( $P_{AV} = 0 kW$ ), helicopters are able to land the autorotation regime. For this case, Eq. (10) transforms into:

$$\lambda_d = -\kappa \frac{\lambda_i}{\xi} - \frac{\sigma}{8} \frac{C_{D_0}}{\xi C_T} (1 + k \mu^2) - \frac{\mu^3}{2 \xi C_T} \frac{f_A}{A} \quad (11)$$

where  $\lambda_d$  represents relative descending velocity. For this purpose, the value  $\xi = 1.0$  gives more reliable results. The rate of descent in autorotation  $w_{aut}$  for given  $V$  at  $H$  is then obtained as:

$$w_{aut} = -V_T \left( \kappa \lambda_i + \frac{\sigma}{8} \frac{C_{D_0}}{C_T} (1 + k \mu^2) \right) - V_T \left( \frac{1}{2} \frac{\mu^3}{C_T} \frac{f_A}{A} \right) \quad (12)$$

### 1.5 Height-Speed Envelope, Optimum Speeds and Maximum Rates of Climb in Progressive Flight

After calculating the powers required for progressive flight and the available powers for different altitudes using the presented algorithms, at their crossing points, the minimum ( $V_{min}$ ) and maximum ( $V_{max}$ ) flight speeds of flight have been determined. To achieve sufficiently small altitude

steps, required values of  $C_L$  and  $C_{D_0}$  have been interpolated or extrapolated using the data from Table 1.

At the points of maximum corresponding power differences, the velocities  $V_{opt}$  for given altitudes have been defined. Maximum rates of climb that correspond to the calculated optimum speeds are obtained as described in section 1.3.

### 1.6 Ground-Effect Influence on the Main Rotor Power Required for Hovering

In the proximity of ground, the power required for hovering becomes smaller. The induced velocity decreases progressively with the ground proximity according to the equation:

$$\frac{v_{hg}}{v_h} = 1 - \frac{0.5}{1 + 4\left(\frac{H}{R}\right)^2} = \xi, \quad (13)$$

where  $H$  represents the main rotor disc height from the ground, and  $\xi$  is the ground effect coefficient. The equation for the power coefficient for the calculation of the main rotor power (only) in the ground proximity  $P_G = C_{P_G} \rho AV_T^3$  is:

$$C_{P_G} = \xi \kappa \lambda_h C_T + \frac{\sigma}{8} C_{D_0}. \quad (14)$$

In case of light helicopters, at heights of the order of 15 m, Eq. (12) practically takes the form of Eq. (5). The total power required should include an additional 10% for the tail rotor and transmission losses.

### 1.7 Acceleration in Horizontal Flight

Expressions which define acceleration in horizontal flight are derived from the Second Law of Newton:

$$m \frac{dV}{dt} = X, \quad (15)$$

$$mV \frac{dV}{dt} = XV = P_{AV} - P. \quad (16)$$

from which the acceleration is:

$$a_{hor} = \frac{dV}{dt} = \frac{P_{AV} - P}{mV}. \quad (17)$$

## 2 RESULTS AND DISCUSSION

Figs. 3 to 7 show the results that were obtained in the initial stage of the preliminary analyses of the here presented helicopter project, using algorithms explained in previous sections. All diagrams, except Fig. 6, have been obtained using the induced velocity distribution coefficient  $\kappa=1.2$  for the progressive flight in the whole domain. On the other hand, as mentioned in section 1.2, for  $V = 0$  km/h the hovering value  $\kappa=1.15$  should be applied. In order to avoid a singularity jump at  $V = 0 + \text{km/h}$  in curves involving this coefficient, an interpolation should be made in latter refinement steps in the domain of small progressive flight speeds. For example, considering the power required at  $H = 0$  m, the difference between the application of the two values of the coefficient  $\kappa$  results in the difference of the order of about 2.5 kW at  $V = 0$  km/h. This could be verified using the main rotor power value  $P$  uncorrected for the ground influence in Fig. 6 (dashed line). To get the total power required, this value should be multiplied by factor 1.1, and then compared with the  $H = 0$  km curve in Fig. 3, which leads to the above mentioned value.

Considering the last diagram shown in Fig. 9, it is obvious that values of acceleration in horizontal flight at small velocities, when  $V \rightarrow 0$ , tend to infinitely large values. This is a natural consequence of the application of a simple approach described in section 1.7, which is good enough in preliminary analyses, but results for very small speeds of flight must be ignored.

Table 1. Definition of the averaged profile drag coefficient for some characteristic altitudes for NACA 8-H-12airfoil, derived from Fig. 2.

$H$	0 m	1000 m	2000 m	3000 m	4000 m	4500 m	5000 m
$C_L$	0.654	0.723	0.799	0.885	0.981	1.034	1.090
$C_{D_0}$	0.0120	0.0126	0.0134	0.0147	0.0181	0.0250	0.0393

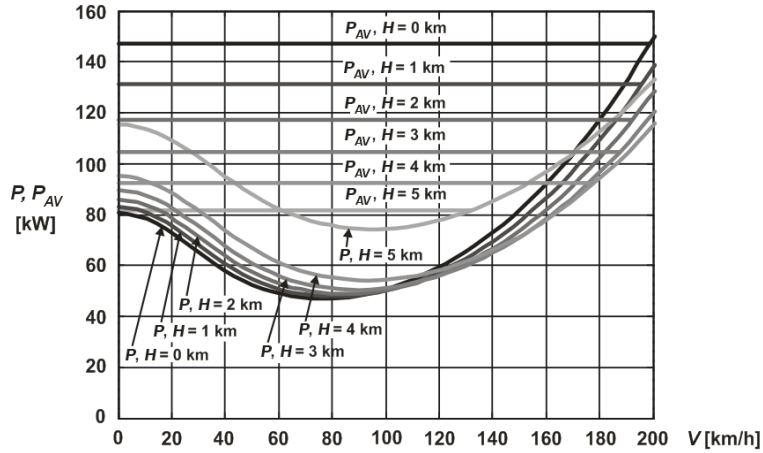


Fig. 3. Variation of the total power  $P$  for horizontal flight and the available power  $P_{AV}$ , with altitude

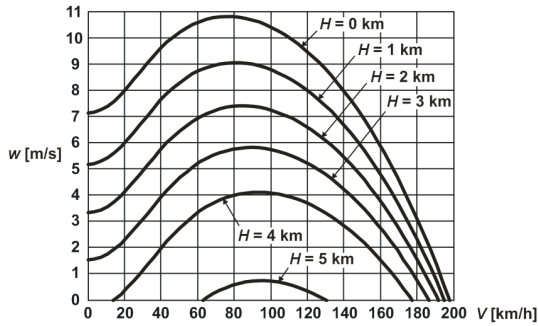


Fig. 4. Variation of the rate of climb with speed

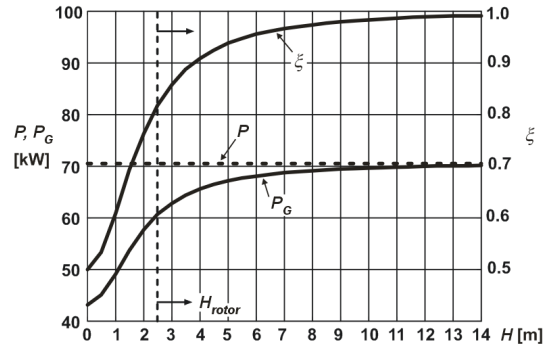


Fig. 6. Main rotor hovering power including ground effect

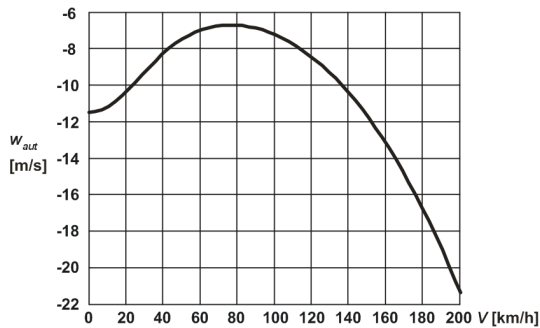


Fig. 5. Rate of descent in autorotation ( $H = 0$  m)

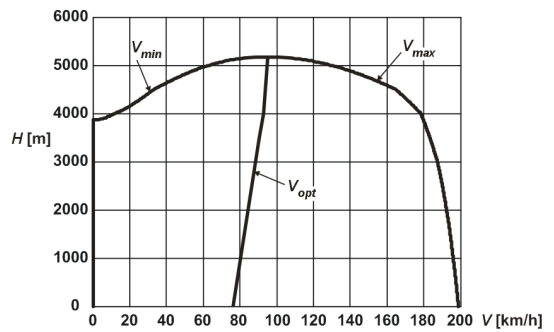


Fig. 7. Height-speed envelope and  $V_{opt}$

Algorithms which are applied in this paper are influenced by empirical factors. Thus, it is good engineering practice to verify them on several existing helicopters whose overall design characteristics are as close as possible to the category of the new model under development. Some results obtained for the Robinson R22 Beta

II are shown in Fig. 10 and in Table 2 (the R22 is undoubtedly one of the most successful and popular light helicopters ever built). Parameters [5] to [7] applied in the calculations simulating the preliminary design level of this helicopter were:  $m = 621$  kg,  $n = 2$ ,  $R = 3.85$  m,  $c = 0.18$  m, main rotor blade airfoil

NACA 63-015 [8] and [9], etc. Its Lycoming O320B2C engine whose nominal maximum power is 160 HP, is derated to the maximum of 131 HP at lower altitudes in order to extend the engine and transmission life time. For the here presented calculations it has been assumed that  $P = 131$  HP can be maintained constant up to the altitude of 1780 m, after which it begins to decrease (value obtained from Eq. (1), using  $P_{0_{\max}} = 160$  HP; the actual engine can develop 131 HP up to  $H \approx 2250$  m [7]).

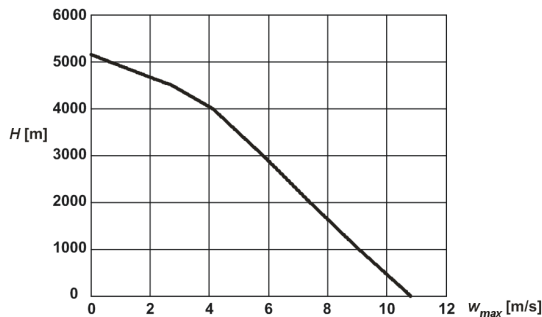


Fig. 8. Variation of the maximum rate of climb with altitude

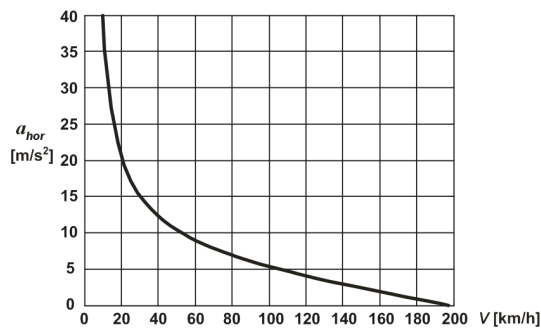


Fig. 9. Acceleration in horizontal flight at  $H = 0$  m

Keeping in mind that we are talking about the accuracy at the preliminary analyses level, agreements between the calculated values and the existing data are good. Again, the absolute ceiling is most probably overestimated to a certain extent, but this result can not be compared with the operational data. It is actually a theoretical value which is generally not flight-tested for helicopters because it could lead to a disaster. On the other hand, the operational ceiling can be assigned by the manufacturer only after a vast number of rigorous test flights. Still, Fig. 10A

leads to a conclusion that this helicopter would have a very reasonable speed range and power reserve for safe operations if flown at the altitude of 4270 m (oxygen system is not a part of the R22 standard equipment).

Performing such "reverse-engineering" analyses of the existing models, together with the calculations for a new helicopter, is very useful for fine adjustments of the applied empirical coefficients in calculation algorithms. For example, a proper match for the Robinson's maximum speed has been achieved using  $f_A = 0.8 \text{ m}^2$  instead of the  $1 \text{ m}^2$  value from section 1.2, which has been applied in the here presented initial calculations of the new helicopter. Therefore, in the following steps, the calculations had to be repeated with  $f_A \approx 0.8$  to  $0.9 \text{ m}^2$  for more realistic estimates of its fuselage drag of the here analyzed project. Such relatively simple calculation techniques are obviously extremely valuable for quick and efficient relative comparisons with the existing designs. In addition, the obtained results provide a good initial insight in the capability of the new design to satisfy certain requirements prescribed by air regulations for the given helicopter category.

Preliminary performance calculations, such as the ones presented in this paper, are most often done using the data for the original airfoil (or airfoils) applied in rotor design. On the other hand, it is known that for composite rotor blades, the airfoil must be modified to comprise a fixed flat tab along the whole trailing edge, primarily in order to enable proper merging of the upper and lower blade surface plies during the manufacturing. This modification can affect the airfoil profile drag [1] to a certain amount. For the here presented new helicopter, it has been shown [8] that the most unfavorable expected tab design from the aspect of drag increase should not absorb more than 1.5% of the maximum available engine power at  $H = 0$  m, so at the preliminary design level this influence can be ignored. On the other hand, if an asymmetrical airfoil is used in the blade design, even a small error in determining a proper angular position of the tab can seriously affect the moment about the aerodynamic center [8]. In case of helicopter blades this value must remain small enough, so this particular issue should also be very carefully considered at higher design levels.

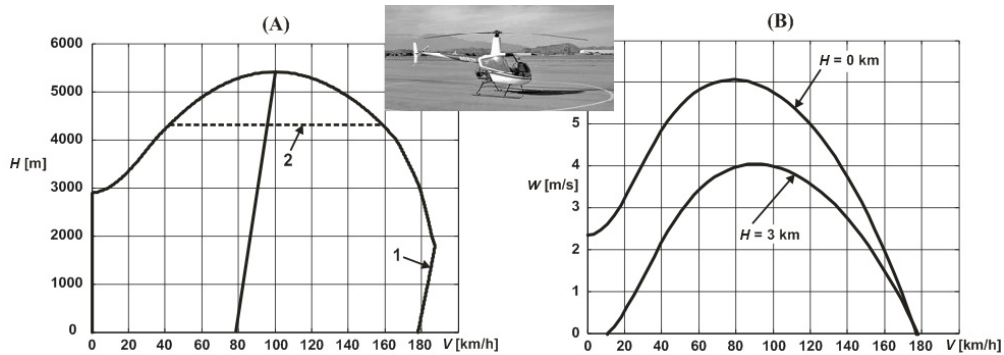


Fig. 10. Calculations for the Robinson R22 Beta II: A) height-speed envelope (1 - power derated to 131 HP = const. up to 1780 m; 2 - operational ceiling value taken from Table 2); B) variation of the rate of climb with speed

Table 2. Comparison of the existing and calculated data for Robinson R22 Beta II – hovering ceiling; 2 – absolute ceiling; 3 – maximum operating altitude (service ceiling); 4 – maximum rate of climb at sea level; 5 – maximum rate of climb at 3 km altitude; 6 – maximum speed in level flight

	1	2	3	4	5	6
Robinson R22 $m = 621$ kg	$H_{max\ hover}$ [M]	$H_{max}$ [M]	$H_{max\ oper}$ [M]	$w_{max}$ ( $H = 0$ km) [m/s]	$w_{max}$ ( $H = 3$ km) [m/s]	$V_{max}$ [km/h]
Existing data	2867* [5]	/	4270 [5]	> 5.1 [5]; 6.1 [6]	> 3.05 [5]	180 [7]
Calculations	2889	5433	/	6.07	4.03	178÷188**

\* in ground effect; out of ground effect, a bit smaller value would be obtained

\*\* for  $H = 0$  m to 1780 m, assuming that derated power of  $P = 131$  HP is kept constant in this altitude range

### 3 CONCLUSION

In this paper some of the most important issues considering a light helicopter flight performance, confined to the preliminary design level, have been analyzed. An approach common at this project level, based on the averaged rotor and helicopter dynamic and aerodynamic characteristics, has been applied. For the presented calculations of the power required for hovering and horizontal flight, the rate of climb in progressive flight and the rate of descent in autorotation, height-speed envelope, optimum speeds and maximum rates of climb, ground effect influence and acceleration in horizontal flight, sets of equations have been carefully selected to achieve proper balance between the simplicity, time effectiveness and the required accuracy at this design level. Custom developed software has shown the ability to perform

efficient analyses and enable variations of the design parameters in the required ranges, giving stable and smooth solutions. Knowing the fact that such calculations are based on certain empirical parameters, parallel calculations have been performed for several similar existing helicopter designs, and some of them for the Robinson R22 Beta have also been presented in this paper. This particular example has shown the need for certain adjustments of the fuselage drag calculations of the here presented helicopter project in the oncoming design steps. After verifying all other applied parameters for the given helicopter category in a similar manner, the presented calculations can provide extremely useful inputs for further, much more complex and time consuming analyses at higher design levels where detailed helicopter dynamics and aerodynamics must be taken into account. The presented approach in preliminary calculation



metodology can noticeably contribute to the overall project effectiveness.

#### 4 REFERENCES

- [1] Leishman, J.G. (2000). *Principles of Helicopter Aerodynamics*. Cambridge University Press, Cambridge.
- [2] Bramwell, A.R.S., Done, G., Balmford, D. (2001). *Bramwell's Helicopter Dynamics*, - 2<sup>nd</sup> ed.. Butterworth-Heinemann, Oxford.
- [3] Seddon, J. (1990). *Basic Helicopter Aerodynamics*. BSP Professional Books, Oxford.
- [4] Stivers, L.S., Rice, F.J. (1946). *Aerodynamic Characteristics of Four NACA Airfoil Sections Designed for Helicopter Rotor Blades*. NACA RB, No. L5K02.
- [5] Robinson Helicopter Company, USA, R22 Beta II Specifications and Dimensions, from <http://www.robinsonheli.com/r22/specs.htm>, accessed on 2007-22-11.
- [6] Robinson Helicopter Company, USA, R22 Pilot's Operating Handbook and FAA Approved Rotorcraft Flight Manual, p. 2-2, & p. 7-17., 1996; from Wikipedia, Internet encyclopedia, [http://en.wikipedia.org/wiki/Robinson\\_R22\\_2](http://en.wikipedia.org/wiki/Robinson_R22_2), accessed on 2007-22-11.
- [7] Airliners.net, Aircraft Technical Data & Specifications - Robinson R22, from <http://www.airliners.net/info/stats.main?id=339>, accessed on 2007-23-11.
- [8] Lednicer, D., Incomplete Guide to Airfoil Usage (aircraft-airfoil database under progressive development), Analytical Methods Inc., Redmond WA, from <http://amber.aae.uiuc.edu/~m-selig/ads/aircraft.html>, accessed on 2004-06-05.
- [9] Abbot, I.H., Von Doenhoff. A.E. (1959). *Theory of Wing Sections, including a summary of airfoil data*. Dover Publications Inc., New York.
- [10] Kostić, I. (2007). Numerical Evaluation of the Aerodynamic Influence of the Helicopter Composite Blade Trailing Edge Tabs, *Archive of Applied Mechanics*, Springer-Verlag, vol. 77, p. 893-909.