

Also available at <http://amc-journal.eu>
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
Ars Mathematica Contemporanea Volume 2, Issue 2, Year 2009, Pages 191-205

On Cartesian skeletons of graphs

Richard H. Hammack, Wilfried Imrich

Abstract

Under suitable conditions of connectivity or non-bipartiteness, each of the three standard graph products (the Cartesian product, the direct product and the strong product) satisfies the unique prime factorization property, and there are polynomial algorithms to determine the prime factors. This is most easily proved for the Cartesian product. For the other products, current proofs involve a notion of a *Cartesian skeleton* which transfers their multiplication properties to the Cartesian product.

The present article introduces simplified definitions of Cartesian skeletons for the direct and strong products, and provides new, fast and transparent algorithms for their construction. Since the complexity of the prime factorization of the direct and the strong product is determined by the complexity of the construction of the Cartesian skeleton, the new algorithms also improve the complexity of the prime factorizations of graphs with respect to the direct and the strong product.

We indicate how these simplifications fit into the existing literature.

Keywords: Graph product, Cartesian skeleton, prime factorization of graphs, graph algorithms.

Math. Subj. Class.: 05C85, 05C99

Math Sci Net: [05C76 \(05C85\)](#)

O kartezičnih skeletih grafov

Povzetek

Pod primernimi pogoji (povezanosti ali ne-dvodelnosti) vsak od treh standardnih grafovskih produktov (kartezični produkt, direktni produkt in krepki produkt) zadošča lastnosti enolične faktorizacije, za določitev prafaktorjev pa obstajajo polinomski algoritmi. To najlažje dokažemo za kartezični produkt. V dokazih za druga dva produkta je uporabljan pojem *kartezičnega skeleta*, ki prenese njune multiplikativne lastosti na kartezični produkt.

Ta članek vpelje poenostavljeni definiciji kartezičnih skeletov direktnega in krepkega produkta in predstavi nove, hitre in transparentne algoritme za njihovo konstrukcijo. Ker je kompleksnost razcepa na prafaktorje direktnega in krepkega produkta določena s kompleksnostjo konstrukcije kartezičnega skeleta, novi algoritmi izboljšujejo tudi kompleksnost razcepa grafov na prafaktorje glede na direktni in krepki produkt.

Pokažemo tudi, kako se te poenostavitve vklapljajo v obstoječo literaturo.

Ključne besede: Produkt grafov, kartezični skelet, praštevilska faktorizacija grafov, grafovski algoritmi.