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ADJUSTMENT OF OBSERVATIONS WITH SIMULTANEOUS COMPUTATION OF RESIDUALS AND UNKNOWNS

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Abstract

The paper presents the possibility of modifying standard adjustment algorithms: direct observations, indirect observations, combined direct and indirect observations, and indirect observations with constraints; residuals and unknowns are estimated simultaneously by solving appropriate systems of linear equations. Such a modification of standard adjustment algorithms corresponds to the use of modern personal computers and pocket calculators, since they support direct matrix algebra operations.
Keywords: *adjustment algorithms, systems of linear equation, personal computer*

1 INTRODUCTION

Modern personal computers equipped with program systems for table computations (Ingalsbe, 1988, Božić, 1994, Husnjak, 1994, Crnko et al., 1995) and pocket calculators (Sharp Corporation, 1986) enable direct matrix algebra operations. As a result of this, the use of adjustment algorithms in geodetic practice is nowadays more efficient than it used to be, since all modern adjustment algorithms are theoretically defined with the use of matrix algebra, while computers enable their practical use. Computational procedures are thus accelerated and simplified and the possibility of errors occurenceis reduced. Professionals therefore do not need to know more difficult procedures for the solving of geodetic tasks. Direct matrix algebra operations also needs certain modifications to adjustment algorithms, which can be further adopted to possibilities for the computers. The standard form used in several publications (Wolf, 1968, Bjerhammat, 1973, Mikhail, Ackerman, 1976) is due to tradition not best suited to these possibilities.

Nowadays the use of adjustment of indirect observations has advantages in the solving of various geodetic tasks (Caspary, 1988), while one of the possible modifications is an algorithm which is used for simultaneous computation of residuals and unknowns. In the standard algorithm for indirect observations these quantities are determined gradually, beginning with unknowns (by solving normal equations) and followed by residuals (by including unknowns into corresponding observation equations, Feil, 1989). A modified algorithm for indirect observations is presented in Hoepcke, 1980, but, if used in an appropriate way, it can also be used for direct observations, simultaneous adjustment of direct and indirect observations and the adjustment of indirect observations with constraints.

2 DIRECT OBSERVATIONS

A functional model of direct observations is determined by a system of observation equations (Feil, 1989):

$$\underset{n\times 1}{\mathbf{v}} = \underset{n\times 1}{\mathbf{e}}\,\underset{1\times 1}{x} - \underset{n\times 1}{\mathbf{l}}, \quad \underset{n\times n}{\mathbf{P}}, \tag{1}$$

where
n – number of observations
x – approximate value of unknown
$\mathbf{e}$ – unit vector
$\mathbf{l}$ – vector of reduced observations
v – vector of residuals
$\mathbf{P}$ – weight matrix.

Unique solution to this system is obtained with the use of the least squares principle:

$$\mathbf{v}^t\mathbf{P}\,\mathbf{v} = \text{minimum} \tag{2}$$

which also determines normal equations:

$$(\mathbf{e}^t\mathbf{P}\,\mathbf{e})\,x - \mathbf{e}^t\mathbf{P}\,\mathbf{l} = 0. \tag{3}$$

From the use of the least squares principle comes out the basic control for the checking of correctness of residuals:

$$\mathbf{e}^t\mathbf{P}\,\mathbf{v} = 0. \tag{4}$$

By multiplying equation (1) with the weight matrix $\mathbf{P}$ from the left side and by rearranging the equation, we obtain:

$$\mathbf{P}\,\mathbf{v} - \mathbf{P}\,\mathbf{e}\,x + \mathbf{P}\,\mathbf{l} = \mathbf{0}. \tag{5}$$

Equations (4) and (5) determine the following system of linear equations

$$\begin{bmatrix} \mathbf{P} & \mathbf{Pe} \\ \mathbf{e}^t\mathbf{P} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ -x \end{bmatrix} + \begin{bmatrix} \mathbf{Pl} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \tag{6}$$

in which the matrix of coefficients is a symmetrical matrix of formats (n + 1) x (n -1). It should be emphasised that this matrix has the property of positive definiteness, but it is regular and can be inverted through classical inversion. By inverting this matrix, the solution to the system of equations given in equation (6) is also determined:

$$\begin{bmatrix} v \\ -x \end{bmatrix} = \begin{bmatrix} P & Pe \\ e^tP & 0 \end{bmatrix}^{-1} \begin{bmatrix} -Pl \\ 0 \end{bmatrix} = \begin{bmatrix} Q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} -Pl \\ 0 \end{bmatrix}. \tag{7}$$

In this manner, the residuals and the unknowns are estimated simultaneously. In equation (7), submatrices and a subvector are defined by inversion:

$$\underset{1x1}{q_{22}} = -(e^tPe)^{-1} = -q_{xx}, \tag{8}$$

$$\underset{nx1}{q_{12}} = e\,q_{xx}, \tag{9}$$

$$\underset{nxn}{Q_{11}} = P^{-1} - eq_{xx}e^t, \tag{10}$$

where q_{xx} is a cofactor of the unknown.

Since a theoretical form of the system of equations (6) is important for the practical use of this algorithm, it must therefore be understood in an analogy with the standard adjustment algorithm for direct observations, along with the system of observation equations. A division of system (6) into submatrices and subvectors has no effect on the efficiency of practical computation, since on a personal computer or pocket calculator appropriate commands are used for inversion and multiplication. A comparison of this algorithm with a standard adjustment algorithm for direct observations shows that instead of solving one normal equation, equation (3), a system of (n+1) linear equations is solved, equation (6), for the determination of unknown quantities (the corrections and the unknown). The task is not difficult, since matrix algebra computational operations can be used directly.

The system of equations (6) and (7) also comprises the most general example of adjustment of direct observations, i.e. the adjustment of direct correlation observations. If adjustment of independent observations of different accuracy and direct independent observations of equal accuracy is performed, the algorithm is simplified, which depends on the form of the appertaining weight matrix **P**.

3 INDIRECT OBSERVATIONS

The functional model of indirect observations is also determined by a system of observation equations, but in contrast to direct observations it contains a greater number of unknowns:

$$\underset{nx1}{v} = \underset{nxu}{A}\,\underset{ux1}{x} - \underset{nx1}{l}, \quad \underset{nxn}{P}, \tag{11}$$

where

u – number of unknowns
A – matrix of the coefficient of observation equations.

The procedure for defining the adjustment algorithm with a simultaneous estimation of residuals and unknowns matches direct observations. From the use of the least squares principle, the control for checking the correctness of residuals is:

$$\mathbf{A}^t \mathbf{P} \mathbf{v} = \mathbf{0}. \tag{12}$$

By multiplying equation (11) with the weight matrix $\mathbf{P}$ from the left side and rearranging the equation, we obtain:

$$\mathbf{P} \mathbf{v} - \mathbf{P} \mathbf{A} \mathbf{x} + \mathbf{P} \mathbf{l} = \mathbf{0}. \tag{13}$$

Equations (12) and (13) determine a system of linear equations:

$$\begin{bmatrix} \mathbf{P} & \mathbf{PA} \\ \mathbf{A}^t\mathbf{P} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ -\mathbf{x} \end{bmatrix} + \begin{bmatrix} \mathbf{Pl} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \tag{14}$$

the solution of which is:

$$\begin{bmatrix} \mathbf{v} \\ -\mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{PA} \\ \mathbf{A}^t\mathbf{P} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} -\mathbf{Pl} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} \begin{bmatrix} -\mathbf{Pl} \\ \mathbf{0} \end{bmatrix}. \tag{15}$$

In this equation, the following submatrices are defined by inversion:

$$\underset{u \times u}{\mathbf{Q}_{22}} = -(\mathbf{A}^t\mathbf{PA})^{-1} = -\mathbf{Q}_{xx}, \tag{16}$$

$$\underset{n \times u}{\mathbf{Q}_{12}} = \mathbf{A}\mathbf{Q}_{xx}, \tag{17}$$

$$\underset{n \times n}{\mathbf{Q}_{11}} = \mathbf{P}^{-1} - \mathbf{A}\mathbf{Q}_{xx}\mathbf{A}^t, \tag{18}$$

where $\mathbf{Q}_{xx}$ is a matrix of the cofactor of unknowns.

Equations (10) and (18) also include the matrix of cofactors of adjusted observations $\overline{\mathbf{Q}}$, i.e. in direct observations:

$$\underset{n \times n}{\overline{\mathbf{Q}}} = \mathbf{e} q_{xx} \mathbf{e}^t \tag{19}$$

and in indirect observations:

$$\underset{n \times n}{\overline{\mathbf{Q}}} = \mathbf{A}\mathbf{Q}_{xx}\mathbf{A}^t. \tag{20}$$

A comparison of this algorithm with the standard algorithm for adjustment of indirect observations shows that instead of (u) normal equations, a system of (n +u) linear equations is solved to estimate unknown quantities (residuals and unknowns).

4 COMBINED DIRECT AND INDIRECT OBSERVATIONS

In the combined form of adjustment of direct and indirect observations, a functional model is determined with error equations of direct observations:

$$\underset{n_1 x1}{v_1} = \underset{n_1 xu}{A_1}\ \underset{uxl}{x} - \underset{n_1 xl}{l_1},\quad \underset{n_1 xn_1}{P_1} \tag{21}$$

and observation equations of indirect observations:

$$\underset{n_2 x1}{v_2} = \underset{n_2 xu}{A_2}\ \underset{uxl}{x} - \underset{n_2 xl}{l_2},\quad \underset{n_2 xn_2}{P_2}\ . \tag{22}$$

Unique solutions of the functional model is obtained with the use of the least squares principle in accordance with equation (2):

$$v^t P v = v_1^t P_1 v_1 + v_2^t P_2 v_2 = \text{minimum}, \tag{23}$$

such that the basic control for checking the correctness of residuals is:

$$A^t P v = A_1^t P_1 v_1 + A_2^t P_2 v_2 = 0. \tag{24}$$

Taking into account (21) and (22) and multiplying them with appertaining weight matrices from the left side, and equation (24), a system of linear equations is determined:

$$\begin{bmatrix} P_1 & 0 & P_1 A_1 \\ 0 & P_2 & P_2 A_2 \\ A_1^t P_1 & A_2^t P_2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ -x \end{bmatrix} + \begin{bmatrix} P_1 l_1 \\ P_2 l_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{25}$$

By the solution of the coefficient of this system, a vector of unknown quantities is determined, i.e. the vector of residuals of direct observations v_1, vector of residuals of indirect observations v_2 and the vector of approximate values of unknowns x at the same time:

$$\begin{bmatrix} v_1 \\ v_2 \\ -x \end{bmatrix} = \begin{bmatrix} P_1 & 0 & P_1 A_1 \\ 0 & P_2 & P_2 A_2 \\ A_1^t P_1 & A_2^t P_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -P_1 l_1 \\ -P_2 l_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \begin{bmatrix} -P_1 l_1 \\ -P_2 l_2 \\ 0 \end{bmatrix} \tag{26}$$

In this equation, the following matrices are defined by inversion:

$$\underset{uxu}{Q_{33}} = -(A_1^t P_1 A_1 + A_2^t P_2 A_2)^{-1} = -N^{-1} = -Q_{xx}, \tag{27}$$

$$\underset{n_2 xu}{Q_{23}} = A_2 Q_{xx}\ , \tag{28}$$

$$\underset{n_2 xn_2}{Q_{22}} = P_2^{-1} - A_2 Q_{xx} A_2^t\ , \tag{29}$$

$$\underset{n_1 x u}{Q_{13}} = A_1 Q_{xx}, \tag{30}$$

$$\underset{n_1 x n_2}{Q_{12}} = -A_1 Q_{xx} A_2^t, \tag{31}$$

$$\underset{n_1 x n_1}{Q_{11}} = P_1^{-1} - A_1 Q_{xx} A_1^t, \tag{32}$$

where Q_{xx} is the matrix of the cofactor of unknowns.

In this case, in contrast to the standard algorithm, a system of linear equations $(n_1 + n_2 + u)$ must be solved.

5 INDIRECT OBSERVATIONS WITH CONSTRAINTS

In the combined form of indirect observations with constraints, the functional model is determined with observation equations of indirect observations:

$$\underset{nx1}{v} = \underset{nxu}{A}\,\underset{ux1}{x} - \underset{nx1}{l}, \quad \underset{nxn}{P} \tag{33}$$

and with constraint equations:

$$\underset{rxu}{B^t}\,\underset{ux1}{x} + \underset{rx1}{\varpi} = \underset{rx1}{0}, \tag{34}$$

where:
r – number of constraints
ϖ – vector of misclosure
B – matrix of coefficients of constraint equations.

The basic check for the computation of residuals is determined with the use of the least squares principle:

$$A^t P v + B k = 0, \tag{35}$$

where k is the vector of the correlate.

Equations (33) and (34) and equation (32) determine the system of linear equations after their multiplication with the weight matrix P on the left side:

$$\begin{bmatrix} P & PA & 0 \\ A^t P & 0 & B \\ 0 & B^t & 0 \end{bmatrix} \begin{bmatrix} v \\ -x \\ k \end{bmatrix} + \begin{bmatrix} Pl \\ 0 \\ -\varpi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{36}$$

By the solution of this system, the vector of unknown quantities is determined, i.e. the vector of corrections of indirect observations v, the vector of approximate values of unknowns x and the vector of the correlate k, are determined simultaneously:

$$\begin{bmatrix} v \\ -x \\ k \end{bmatrix} = \begin{bmatrix} P & PA & 0 \\ A^tP & 0 & B \\ 0 & B^t & 0 \end{bmatrix}^{-1} \begin{bmatrix} -Pl \\ 0 \\ \varpi \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \begin{bmatrix} -Pl \\ 0 \\ \varpi \end{bmatrix}. \tag{37}$$

In this equation, matrices are defined by inversion:

$$\underset{rxr}{Q_{33}} = (B^tN^{-1}B)^{-1} = Q_{kk}, \tag{38}$$

$$\underset{uxr}{Q_{23}} = N^{-1}BQ_{kk}, \tag{39}$$

$$\underset{uxu}{Q_{22}} = N^{-1}BQ_{kk}\,B^tN^{-1} - N^{-1} = -Q_{xx}, \tag{40}$$

$$\underset{nxr}{Q_{13}} = -AN^{-1}BQ_{kk}, \tag{41}$$

$$\underset{nxu}{Q_{12}} = AQ_{xx}, \tag{42}$$

$$\underset{nxn}{Q_{11}} = P^{-1} - AQ_{xx}A^t, \tag{43}$$

where N is the matrix of coefficients of normal equations of indirect observations:

$$N = A^tPA \tag{44}$$

and Q_{xx} is the matrix of the cofactor of unknowns. The number of linear equations which must be solved increases from (u + r) to (n + u + r).

6 CONCLUSION

Modern pocket calculators and personal computers considerably increase the efficiency of the use of adjustment algorithms in solving different geodetic tasks. Their efficiency is shown at the elementary level of use, above all in the possibility of direct performance of matrix computational operations, so that there is no discrepancy between the theoretical presentation of adjustment algorithms on one hand and practical computations on the other. In comparison with classical algebra, each matrix computational operation is more difficult and generally consists of a series of elementary operations. By direct use of matrix operations integrated in the computer/calculator, the computational procedure is accelerated and, simultaneously, the possibility of appearance of computational errors is reduced.

The use of inversion, i.e. a command incorporated in the computer or calculator, also solves one of the problems which had an influence on the development and use of adjustment procedures, that is the solving of normal equations. In practice, classical methods (or their partially modernised versions) are still present due to tradition (Burmistrov, 1963, Čubranić, 1980, Klak 1982). As a result of direct inversion of the matrix of the coefficient and the use of indeterminate methods in their solution, these methods have become inefficient. The solving of a system of linear equations, the extent of which exceeds the number of normal equations in solving of the same geodetic task, is not difficult at all. This is also the conclusion of this paper, in which the adjustment algorithms for simultaneous calculation of residuals and unknowns are based on the solving of a system of linear equations which are increased with regard to the corresponding normal equations in the standard algorithm by the number of observations. It must be emphasised that the direct use of computational operations of matrix algebra in modern computers and calculators also has an influence of increasing the efficiency of practical computations, while the functional model or its coefficients are defined with regard to the geodetic task. The functional model can be determined in the classical way or by writing a special program in one of higher program languages (Rožić, 1992).

In comparison with standard algorithms, the presented adjustment algorithms with simultaneous determination of residuals and unknowns mainly use the advantages of direct matrix algebra operations. They are therefore appropriate for practical use in solving all standard geodetic tasks in daily practice which are based on the use of adjustment algorithms. The use of modern computers and calculators for the previously described algorithms was very rare, but expert programs should be written in higher program languages. On the other hand, the production, standardisation, verification and licensing of such programs present a special problem which is not discussed in this paper.

It can be established on the basis of the above that the most basic computational accessories of a modern geodetic professional in the use of adjustment algorithms are pocket calculators with the possibility of performing matrix algebra computational operations. Their use makes practical computations simpler and more efficient without the knowledge of programming, with the use of the modification of standard algorithms presented in this paper. Adjustment algorithms for conditional observations (Hoepcke, 1980) can be modified in a similar way, as well adjustment algorithms for conditional observations with unknowns.

Example:

Comparison of practical computation in the use of a standard adjustment algorithm and an adjustment algorithm with simultaneous calculation of corrections and unknowns in indirect observations (computation was performed on a SHARP PC-1403 pocket calculator). The coordinates of points T_1, T_2, T_3 and T_4 are given and the approximate coordinates of point T (x_o= 117.00 m, y_o= 145.00 m), the position of which is unknown. On the basis of measured lengths s_i (i=1, 2,..., 4), the adjusted coordinates of point T must be determined (section of an arc). This example is from Rožić, 1993, exercise 3.1.12.

Measured lengths	Coordinates of points	
$TT_1 = s_1 =$ 105.60 m,	T_1, $y_1 =$ 54.80 m,	$x_1 =$ 172.94 m
$TT_2 = s_2 =$ 107.60 m,	T_2, $y_2 =$ 233.65 m,	$x_2 =$ 177.55 m
$TT_3 = s_3 =$ 109.30 m,	T_3, $y_3 =$ 237.50 m,	$x_3 =$ 59.76 m
$TT_4 = s_4 =$ 103.10 m,	T_4, $y_4 =$ 57.38 m,	$x_4 =$ 65.33 m

Observation equations: $\mathbf{v} = \mathbf{A}\,\mathbf{x} - \mathbf{l}$, $\mathbf{P} = \mathbf{E}$

$$\begin{array}{cccc} \mathbf{A} & \mathbf{Ae} & -\mathbf{l} & \mathbf{s} \\ \begin{bmatrix} -0.53 & 0.85 \\ -0.56 & -0.83 \\ 0.53 & -0.85 \\ 0.51 & 0.86 \end{bmatrix} & \begin{bmatrix} 0.32 \\ -1.39 \\ -0.32 \\ 1.37 \end{bmatrix} & \begin{bmatrix} 0.54 \\ -0.24 \\ -0.52 \\ -1.38 \end{bmatrix} & \begin{bmatrix} 0.86 \\ -1.63 \\ -0.85 \\ -0.01 \end{bmatrix} \end{array}$$

A) Standard algorithm

Normal equations: $\mathbf{N}\,\mathbf{x} - \mathbf{n} = \mathbf{0}$

$$\begin{array}{cccc} \mathbf{N} & \mathbf{Ne} & -\mathbf{n} & \mathbf{A}^t\mathbf{s} \\ \begin{bmatrix} 1.1308 & 0.0079 \\ 0.0079 & 2.8692 \end{bmatrix} & \begin{bmatrix} 1.1387 \\ 2.8771 \end{bmatrix} & \begin{bmatrix} -1.1210 \\ -0.0849 \end{bmatrix} & \begin{bmatrix} 0.0178 \\ 2.7922 \end{bmatrix} \end{array}$$

Solving of normal equations with Choleski's algorithm:

$$\begin{array}{ccccc} \mathbf{C} & -(\mathbf{C}^t)^{-1}\mathbf{n} & (\mathbf{C}^t)^{-1} & (\mathbf{C}^t)^{-1}\mathbf{s} & \sigma \\ \begin{bmatrix} 1.06340 & 0.00745 \\ & 1.69385 \end{bmatrix} & \begin{bmatrix} -1.05413 \\ -0.04547 \end{bmatrix} & \begin{bmatrix} 0.94038 & \\ -0.00414 & 0.59037 \end{bmatrix} & \begin{bmatrix} 0.95710 \\ 2.23462 \end{bmatrix} & \begin{bmatrix} 0.95710 \\ 2.23462 \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} \mathbf{Q}_{xx} & \mathbf{Q}_{xx}\mathbf{e} & \mathbf{x} \\ \begin{bmatrix} 0.88434 & -0.00244 \\ -0.00244 & 0.34854 \end{bmatrix} & \begin{bmatrix} 0.88189 \\ 0.34610 \end{bmatrix} & \begin{bmatrix} 0.991 \\ 0.027 \end{bmatrix} \end{array}$$

Calculation of residuals: $\mathbf{v} = \mathbf{A}\,\mathbf{x} - \mathbf{l}$

$$\begin{array}{cccc} \mathbf{Ax} & -\mathbf{l} & & \mathbf{v} \\ \begin{bmatrix} -0.500 \\ -0.581 \\ 0.499 \\ 0.527 \end{bmatrix} & \begin{bmatrix} 0.54 \\ -0.24 \\ -0.52 \\ -1.38 \end{bmatrix} & = & \begin{bmatrix} 0.039 \\ -0.826 \\ -0.023 \\ -0.853 \end{bmatrix} \end{array}$$

B) Simultaneous calculation of residuals and unknowns

Defining the coefficients of systems of linear equations according to equation (14):

$$\begin{bmatrix} \mathbf{P} & \mathbf{A} \\ \mathbf{A} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 1.00 & & & & -0.53 & 0.85 \\ & 1.00 & & & -0.56 & -0.83 \\ & & 1.00 & & 0.53 & -085 \\ & & & 1.00 & 0.51 & 0.86 \\ -0.53 & -0.56 & 0.53 & 0.51 & 0.00 & 0.00 \\ 0.85 & -0.83 & -0.85 & 0.86 & 0.00 & 0.00 \end{bmatrix}, \quad \begin{bmatrix} -\mathbf{l} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0.54 \\ -0.24 \\ -0.52 \\ -1.38 \\ 0.00 \\ 0.00 \end{bmatrix}.$$

Solving of the system of equations according to equation (15):

$$\begin{bmatrix} P & A \\ A^t & 0 \end{bmatrix}^{-1} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \qquad \begin{bmatrix} v \\ -x \end{bmatrix}$$

$$\begin{bmatrix} 0.50044 & -0.01840 & 0.49932 & -0.01844 & -0.46816 & 0.29749 \\ -0.01840 & 0.48329 & 0.01783 & 0.49906 & -0.49676 & -0.28643 \\ 0.49932 & 0.01783 & 0.50092 & 0.01897 & 0.46742 & -0.29767 \\ -0.01844 & 0.49906 & 0.01897 & 0.51535 & 0.44710 & 0.29898 \\ -0.46816 & -0.49676 & 0.46742 & 0.44710 & -0.88434 & 0.00244 \\ 0.29749 & -0.28643 & -0.29767 & 0.29898 & 0.00244 & -0.34854 \end{bmatrix} \begin{bmatrix} 0.039 \\ -0.826 \\ -0.023 \\ -0.853 \\ -0.991 \\ -0.027 \end{bmatrix}$$

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