Steepest Descent Optimisation in the Secondary Space of Redundant Manipulators

Jadran Lenarčič and Bojan Nemec
Jožef Stefan Institute, Jamova 39, 1111 Ljubljana, Slovenia
Phone: +386 61 1773 563, Fax: +386 61 1258 058
jadran.lenarcic@ijs.si

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To minimise a cost function in the secondary level of control of redundant manipulators, the gradient of the cost function is projected on the null space of the primary task. This projection, however, does not guarantee the maximum decrement of the cost function. In the present paper we rotate the null space of the primary task in order to find the maximum optimisation step. By this improvement, the optimisation procedure can provide the desired solution in much less iterations.

1 Introduction

One of the emerging issues of the new generation of manipulators, in particular service robots, is the kinematical redundancy. The mechanism of a redundant manipulator possesses superfluous degrees of freedom with respect to the motion constraints posed by the assigned primary task. Hence, as a secondary task, it can operate by simultaneously optimising a set of performance criteria or can avoid obstacles and ill-conditioned configurations. Assume that the primary task is to place the hand in a given spatial position and orientation. The rest of the mechanism will still be able to move (see Fig 1). This is the so-called self or secondary motion of the mechanism and is a typical property of kinematically redundant manipulators. From this viewpoint also the human arm contains a degree of redundancy. By rotating the elbow in a fixed pose of the hand, the human arm mechanism accommodate its static and dynamic characteristics.

A majority of efforts in treating redundancy have been concentrated at the kinematic level of control with respect to different type of criteria with robots moving in complex environments avoiding obstacles, as reported in [1], and undesired or ill-conditioned configurations, as reported in [2, 3]. Many authors based their approach on the utilisation of the pseudoinverse of the Jacobian matrix associated with the primary task, such as [4], which is a purely numeric approach. It has intrinsic inaccuracies and accumulates errors that become larger as the velocity increases. To overcome these difficulties, some authors developed symbolic solutions to this problem, such as [5]. It must be pointed out, however, that no symbolic solution can be developed for a general-type redundant manipulator unless certain conditions are met by the mechanism.

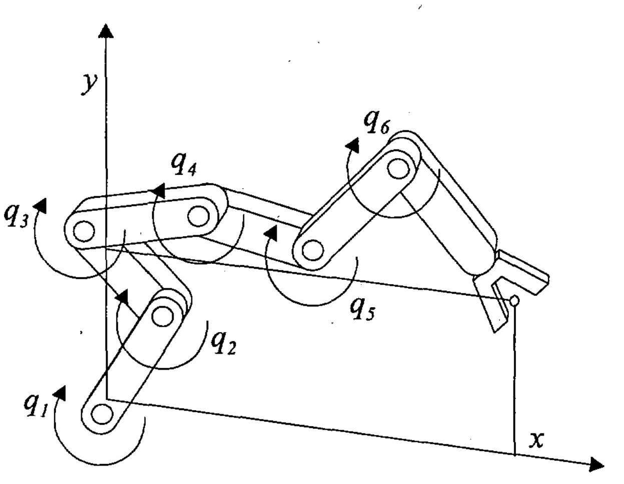

Figure 1: A 6-degree-of-freedom manipulator that positions end effector in x,y plane is kinematically redundant

The calculation schemes based on the pseudoinverse are procedures of local optimisation [6]. They implicitly minimise a norm of joint velocities. Yet, the central point and a distinctive property of various pseudoinverse-based methods is in the determination of the null space projection operator associated with the secondary task [7]. A proper selection of the null space projection operator in combination with the null space vector provides a secondary motion of the manipulator that respects different types of criteria, such as joint torque minimisation, obstacle avoidance, singular pose avoidance, etc.[8, 9]. [10, 11] reported some results obtaining global optima with integral type of criteria. [12] optimised the weighted null space projection operator in order to avoid instabilities. An alternative to the pseudoinverse-based methods is the extended Jacobian method introduced by [13]. [14] described a new formulation of this algorithm that is well suited for more general choices of the secondary criterion. Characteristics of different optimum solutions were studied in [15].

In a vast variety of applications, the secondary task of a redundant manipulator, formulated in terms of an optimisation, is based on a gradient-type procedure. Usually, not to disturb the primary task execution, this gradient is mapped in the null space of the Jacobian matrix pertinent to the primary task. Unfortunately, the projection in this null space may distort the gradient vector, so that in a final consequence it may not provide a decrease of the cost function and the optimisation procedure may become inefficient. In the present work, to overcome the difficulty, we rotate the null space operator in order to achieve the best direction and amplitude of the optimisation step in the domain of the secondary task. By this improvement, the optimisation procedure becomes more effective.

2 Mathematical background

Let the primary task of a redundant manipulator be to achieve some desired poses $\mathbf{p}$ (such as position and orientation of the hand) that are given as a function of joint coordinates $\mathbf{q}$ (angles or linear displacements in joints), where vector $\mathbf{p}$ is of dimension m and $\mathbf{q}$ of dimension n. Typically, vector $\mathbf{p}$ is a set of non-linear independent algebraic equations where $\mathbf{q}$ are arguments of trigonometric functions. It is expressed in the well know differential form [16] by

$$d\mathbf{p} - \mathbf{J}_\mathrm{P} d\mathbf{q} = 0, \tag{1}$$

where $\mathbf{J}_\mathrm{P}$ is a $m \times n$ Jacobian matrix that incorporates the derivatives $\partial\mathbf{p}/\partial q_\mathrm{i}$. We assume that $n > m$ and

$$\mathbf{J}_\mathrm{P}^+ d\mathbf{p} - d\mathbf{q} = 0. \tag{2}$$

If $\mathbf{J}_\mathrm{P}\mathbf{J}_\mathrm{P}{}^\mathrm{T}$ of dimension $m \times n$ isn't singular, the following

$$\mathbf{J}_\mathrm{P}^+ = \mathbf{J}_\mathrm{P}{}^\mathrm{T}(\mathbf{J}_\mathrm{P}\mathbf{J}_\mathrm{P}{}^\mathrm{T})^{-1} \tag{3}$$

is the so-called unweighted pseudoinverse of $\mathbf{J}_\mathrm{P}$ whose dimension is $m \times n$. The utilisation of the pseudoinverse in solving the task to move a redundant manipulator in a given $\mathbf{p}$ implicitly leads to a minimisation of joint velocities [16]. To satisfy other criteria, one can introduce additional optimisations that must not interfere with the primary task. Let the secondary task be expressed similarly to the primary task

$$d\mathbf{s} - \mathbf{J}_\mathrm{S} d\mathbf{q} = 0, \tag{4}$$

where the objective is to achieve a $\mathbf{q}$ that corresponds to given values of secondary task coordinates $\mathbf{s}$ (for instance joint torques), and $\mathbf{J}_\mathrm{S}$ is the Jacobian that includes derivatives $\partial\mathbf{s}/\partial q_\mathrm{i}$. If $\mathbf{s}$ is of dimension l, $\mathbf{J}_\mathrm{S}$ is of dimension $l \times n$. If $n > l$, we can take advantage of the $n \times l$ pseudoinverse

$$\mathbf{J}_\mathrm{S}^+ d\mathbf{s} - d\mathbf{q} = 0. \tag{5}$$

Arrange now the increment of joint coordinates into the primary $d\mathbf{q}_\mathrm{P}$ and the secondary $d\mathbf{q}_\mathrm{N}$ (subordinated) part, so that $d\mathbf{q}_\mathrm{N}$ does not produce any change of primary coordinates $\mathbf{p}$

$$d\mathbf{q} = d\mathbf{q}_\mathrm{P} + d\mathbf{q}_\mathrm{N}. \tag{6}$$

By multiplying with $\mathbf{J}_\mathrm{P}$ we get

$$d\mathbf{p} = \mathbf{J}_\mathrm{P} d\mathbf{q}_\mathrm{P} + \mathbf{J}_\mathrm{P} d\mathbf{q}_\mathrm{N}. \tag{7}$$

Since

$$\mathbf{J}_\mathrm{P} d\mathbf{q}_\mathrm{N} = 0, \tag{8}$$

we have

$$d\mathbf{p} = \mathbf{J}_\mathrm{P} d\mathbf{q}_\mathrm{P} \Rightarrow d\mathbf{q}_\mathrm{P} = \mathbf{J}_\mathrm{P}^+ d\mathbf{p} \tag{9}$$

Let

$$d\mathbf{q}_\mathrm{N} = \mathbf{N}_\mathrm{P} d\mathbf{q}_\mathrm{S} \tag{10}$$

where $d\mathbf{q}_\mathrm{S}$ is an arbitrary n-dimensional vector of joint increments associated with the secondary task, while $\mathbf{N}_\mathrm{P}$ is of dimension $n \times n$

$$\mathbf{J}_\mathrm{P}\mathbf{N}_\mathrm{P} = 0. \tag{11}$$

According to [17]

$$\mathbf{N}_\mathrm{P} = \mathbf{I} - \mathbf{J}_\mathrm{P}^+ \mathbf{J}_\mathrm{P} \tag{12}$$

lies in the null space of $\mathbf{J}_\mathrm{P}$ so that $d\mathbf{q}_\mathrm{N}$ and $d\mathbf{q}_\mathrm{P}$ are orthogonal. The null space projection operator $\mathbf{N}_\mathrm{P}$ is Hermitian and idempotent, $\mathbf{N}_\mathrm{P}{}^\mathrm{T} = \mathbf{N}_\mathrm{P}, \mathbf{N}_\mathrm{P}\mathbf{N}_\mathrm{P} = \mathbf{N}_\mathrm{P}$.

The kinematic redundancy, in our opinion, must be understood as an instantaneous property associated with the dimension of the null space of the primary task. We thus define the degree of redundancy as the rank of the null space projection operator

$$D = rank\,\{\mathbf{N}_\mathrm{P}\} = n - rank\,\{\mathbf{J}_\mathrm{P}\}\,. \qquad (13)$$

D is the achievable order of the secondary motion that can change throughout the workspace of the manipulator mechanism in dependence on $\mathbf{q}$. We assume in this work that $l \geq D$.

By substituting (10) into (6) and by multiplying with **JS** we have

$$\begin{aligned} d\mathbf{s} &= \mathbf{J}_\mathrm{S} d\mathbf{q}_\mathrm{P} + \mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P} d\mathbf{q}_\mathrm{S} \qquad (14)\\ \Rightarrow d\mathbf{q}_\mathrm{S} &= (\mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P})^{+}(d\mathbf{s} - \mathbf{J}_\mathrm{S} d\mathbf{q}_\mathrm{P}). \end{aligned}$$

The secondary task coordinates depend on $d\mathbf{q}_\mathrm{N}$ and $d\mathbf{q}_\mathrm{P}$. We denote

$$\begin{aligned} d\mathbf{s}_\mathrm{P} &= \mathbf{J}_\mathrm{S} d\mathbf{q}_\mathrm{P},\\ d\mathbf{s}_\mathrm{N} &= \mathbf{J}_\mathrm{S} d\mathbf{q}_\mathrm{N}\\ \Rightarrow d\mathbf{s} &= d\mathbf{s}_\mathrm{P} + d\mathbf{s}_\mathrm{N}. \qquad (15) \end{aligned}$$

If we take into account (6,10,14), a complete increment in joint coordinates can be written as

$$d\mathbf{q} = \mathbf{J}_\mathrm{P}{}^{+} d\mathbf{p} + \mathbf{N}_\mathrm{P}(\mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P})^{+}(d\mathbf{s} - \mathbf{J}_\mathrm{S}\mathbf{J}_\mathrm{P}^{+} d\mathbf{p}), \qquad (16)$$

which is the know task priority approach where the first part of the joint increment is the particular solution associated with the primary task. It is of a higher priority in comparison with the second part which is the homogeneous solution associated with the secondary task [1, 4].

3 Secondary domain

In this work, an increment in joint coordinates is referred to as the secondary motion (self-motion) of a redundant manipulator when it does not produce any increment in the primary task coordinates. In connection with the definition of the manipulability ellipsoids [18], a sphere $d\mathbf{q}_\mathrm{S}^\mathrm{T} d\mathbf{q}_\mathrm{S} = 1$ produces an ellipsoid in l-dimensional space of $d\mathbf{s}$ whose principal axes are the eigenvectors of $\mathbf{J}_\mathrm{S}\mathbf{J}_\mathrm{S}^\mathrm{T}$ and their lengths are the related singular values. It is clear, however, that only a part of elements $d\mathbf{s}$ in this ellipsoid can be accomplished by the secondary motion of the redundant manipulator. Note that there is some controversy in the definition and utilisation of manipulability when different types of task coordinates are treated simultaneously, such as linear and angular velocities. [19, 8] dealt with this problem.

The secondary motion can only be assembled in the null space of the Jacobian matrix $\mathbf{J}_\mathrm{P}$ where a vector $d\mathbf{q}_\mathrm{S}$ is projected through $\mathbf{N}_\mathrm{P}$ into $d\mathbf{q}_\mathrm{N}$. Thus, an element of a sphere $d\mathbf{q}_\mathrm{S}^\mathrm{T} d\mathbf{q}_\mathrm{S} = 1$ is transformed in the sphere $d\mathbf{q}_\mathrm{N}^\mathrm{T} d\mathbf{q}_\mathrm{N} \leq 1$. The null space of $\mathbf{J}_\mathrm{P}$ is spaned by the n-dimensional orthonormal eigenvectors of matrix $\mathbf{N}_\mathrm{P}\mathbf{N}_\mathrm{P}^\mathrm{T} = \mathbf{N}_\mathrm{P}$, denoted by $\mathbf{E}_\mathrm{P} = (\mathbf{e}_1, \mathbf{e}_2, \ldots.., \mathbf{e}_\mathrm{P})$. Thus, any $d\mathbf{q}_\mathrm{N}$ as a function of parameters $\gamma = (\gamma_1, \ldots.., \gamma_D)^\mathrm{T}$

$$d\mathbf{q}_\mathrm{N} = \mathbf{E}_\mathrm{P}\gamma \qquad (17)$$

is an element of a sphere if

$$d\mathbf{q}_\mathrm{N}^\mathrm{T} d\mathbf{q}_\mathrm{N} = \gamma_1^2 + \gamma_2^2 + \ldots. + \gamma_D^2 = 1. \qquad (18)$$

The values of $d\mathbf{s}_\mathrm{N}$ (15) that are functions of $d\mathbf{q}_\mathrm{N}$ constrained by (18) form that part of the vector space of $d\mathbf{s}$ that can be accomplished by the secondary motion of the manipulator. Vectors $d\mathbf{s}_\mathrm{N}$ produce an ellipsoid whose principal axes are the l-dimensional eigenvectors of matrix $\mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P}(\mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P})^\mathrm{T}$ (denoted by $\mathbf{u}_1, \mathbf{u}_2, \ldots..., \mathbf{u}_D$), while the corresponding singular values of $\mathbf{J}_\mathrm{S}\mathbf{N}_\mathrm{P}$ are the lengths of these principal axes. The secondary manipulability ellipsoid characterises the domain of the secondary task bounded by (18). It describes the potential of a redundant manipulator to solve the secondary task with no interference with the primary task in a given location determined by the joint coordinates $\mathbf{q}$ [20].

4 Best optimisation step

Assume that the secondary task of a redundant manipulator is to minimise a quadratic cost function (which is a very reasonable assumption for a vast variety of practical implementations, e.g. joint torque minimisation)

$$c = \mathbf{s}^\mathrm{T}\mathbf{W}\mathbf{s} \rightarrow min|_q \quad , \qquad (19)$$

where $\mathbf{W}$ is a diagonal $l \times l$ full-rank matrix of positive weights w_i, while $d\mathbf{q}_\mathrm{N}$ is constrained by the primary task. An iterative procedure is to provide a series of $d\mathbf{q}_\mathrm{N}$ that step by step minimise c and do not interfere with the primary task. In a commonly used gradient projection technique to minimise a scalar cost function [17, 8], the joint displacement is

$$d\mathbf{q} = k_\mathrm{P}\mathbf{J}_\mathrm{P}^{+} d\mathbf{p} - k_\mathrm{S}\mathbf{N}_\mathrm{P}\mathbf{J}_\mathrm{S}^{+}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\}. \qquad (20)$$

where k_S and k_P are selected in order to assure the convergence of the numerical procedure. The trouble with such an approach is that in general the projection of the gradient in the null space of $\mathbf{J}_\mathrm{P}$ may not provide the maximum decrease of the cost function. Hence, the iterative procedure may not be able to locate the desired minimum in an acceptable number of iterations. There are two remedies for it. One is to adapt the projection vector directly in the domain of the secondary task. This can be done by searching the optimum projection vector produced by joint increments that lie the secondary domain. The other way is to adequately rotate the null space of $\mathbf{J}_\mathrm{P}$. In the present paper we show only the second possibility.

It is well known [16] that a weighted pseudoinverse

$$\mathbf{J}_\mathrm{PA}^{+} = \mathbf{A}^{-1}\mathbf{J}_\mathrm{P}^\mathrm{T}(\mathbf{J}_\mathrm{P}\mathbf{A}^{-1}\mathbf{J}_\mathrm{P}^\mathrm{T})^{-1} \tag{21}$$

minimises a performance criterion of joint velocities in the form of $\dot{\mathbf{q}}^\mathrm{T}\mathbf{A}\dot{\mathbf{q}}$ where $\mathbf{A}$ is a positive definite weighting matrix. The weighted null space projection operator for the weighted pseudoinverse $\mathbf{J}_\mathrm{PA}^{+}$ is given by

$$\mathbf{N}_\mathrm{PA} = \mathbf{I} - \mathbf{J}_\mathrm{PA}^{+}\mathbf{J}_\mathrm{P}. \tag{22}$$

This null space projection operator is idempotent but in general is not Hermitian. It rotates the null space of the Jacobian in what was termed the effective null space by [12]. It is the central point of many attempts to resolve redundancy with solutions possessing different characteristics that assist to avoid obstacles and singularities or minimise a given criterion. In [7] one can find a variety of possibilities later implemented and improved by other authors. For instance, [10, 11] used this approach in order to incorporate some global properties in the optimisation of dynamic joint torques. [21] showed the limitations of these methods when they are implemented independently on a real mechanism without considering its kinetic behaviour. Lately, [12] reported a weighted null space projection operator suitable for a weighted joint torque optimisation with the aim to avoid instabilities that arise from unrealisable joint velocities.

We show here that it is possible to determine a weighted null space projection operator that provides a steepest descent minimisation of a given cost function. For this purpose we search for an optimum $d\mathbf{s}$ that minimises a quadratic form $c' = c(\mathbf{s} + d\mathbf{s})$ (where c is given in (19)) with respect to the primary task (1).

The associated Lagrangian, in which we substitute $d\mathbf{s}$ by $\mathbf{J}_\mathrm{S}d\mathbf{q}$, is as follows

$$\begin{aligned} L = \mathbf{s}^\mathrm{T}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\} + 2d\mathbf{q}^\mathrm{T}\mathbf{J}_\mathrm{S}^\mathrm{T}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\} + \\ + d\mathbf{q}^\mathrm{T}\mathbf{J}_\mathrm{S}^\mathrm{T}\mathbf{W}\mathbf{J}_\mathrm{S}d\mathbf{q} + \lambda^\mathrm{T}(d\mathbf{p} - \mathbf{J}_\mathrm{P}d\mathbf{q}) \end{aligned} \tag{23}$$

Here,λ is the vector of Lagrangian multipliers. The derivative of L with respect to $d\mathbf{q}$ must vanish

$$\left\{\frac{\partial L}{\partial dq_\mathrm{i}}\right\} = 2\mathbf{J}_\mathrm{S}^\mathrm{T}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\} + 2\mathbf{J}_\mathrm{S}^\mathrm{T}\mathbf{W}\mathbf{J}_\mathrm{S}d\mathbf{q} - \mathbf{J}_\mathrm{P}^\mathrm{T}\lambda = 0, \tag{24}$$

as well as its derivative with respect to λ so that (1) holds. As a result,

$$d\mathbf{q} = \frac{1}{2}\mathbf{A}^{-1}\mathbf{J}_\mathrm{P}^\mathrm{T}\lambda - \mathbf{A}^{-1}\mathbf{J}_\mathrm{S}^\mathrm{T}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\} \tag{25}$$

where we introduced the following notation

$$\mathbf{A} = \mathbf{J}_\mathrm{S}^\mathrm{T}\mathbf{W}\mathbf{J}_\mathrm{S}. \tag{26}$$

Since $\mathbf{A}$ is singular when $l > m$ and cannot directly be inverted, one can utilise either the damped least square solution

$$\mathbf{A}^{-1} = (\mathbf{J}_\mathrm{S}^\mathrm{T}\mathbf{W}\mathbf{J}_\mathrm{S} + \alpha\mathbf{I})^{-1} \tag{27}$$

or the formulation

$$\mathbf{A}^{-1} = \mathbf{J}_\mathrm{S}^{+}\mathbf{W}^{-1}(\mathbf{J}_\mathrm{S}^{+})^\mathrm{T}. \tag{28}$$

Multiplying (25) by $\mathbf{J}_\mathrm{P}$ gives

$$\frac{1}{2}\lambda = (\mathbf{J}_\mathrm{P}\mathbf{A}^{-1}\mathbf{J}_\mathrm{P}^\mathrm{T})^{-1}(d\mathbf{p} + \mathbf{J}_\mathrm{P}\mathbf{A}^{-1}\mathbf{J}_\mathrm{S}^\mathrm{T}\left\{\frac{\partial c}{\partial s_\mathrm{i}})\right\}. \tag{29}$$

Then by substituting λ again in (25), by taking into account (21,22), as well as $\mathbf{A}^{-1}\mathbf{J}_\mathrm{S}^\mathrm{T} = \mathbf{J}_\mathrm{S}^{+}\mathbf{W}^{-1}$, and by introducing scalar constants $k_\mathrm{P}, k_\mathrm{S}$ to control the iteration step we get

$$d\mathbf{q} = k_\mathrm{P}\mathbf{J}_\mathrm{PA}^{+}d\mathbf{p} - k_\mathrm{S}\mathbf{N}_\mathrm{PA}\mathbf{J}_\mathrm{S}^{+}\left\{\frac{\partial c}{\partial s_\mathrm{i}}\right\}, \tag{30}$$

which is analogous to the standard formula (20) with a weighted pseudoinverse and the corresponding null space projection operator, while the weighting matrix is given by (26) and its inverse by (27) or (28).

The approach (30) turns out to be numerically very effective if compared to (20). The necessary number of iterations to solve the primary and the secondary task can drastically be decreased. While in some cases the formulation (20) fails to carry out the imposed optimisation in the secondary level, the approach formulated in (30) guarantees the best solution to the primary and to the secondary task simultaneously.

5 Conclusions

The paper deals with a steepest descent local minimisation in the level of the secondary task of a redundant manipulator. In previously reported pseudoinverse-based optimisation techniques, the gradient of a given cost function is directly mapped in the null space of the primary task. The projection in the null space may distort the gradient and the optimisation procedure may be troubled. In our work, we overcome the problem by utilising an optimum weighted pseudoinverse. By this improvement, the optimisation procedure becomes more effective and numerically robust. It finds a solution in less iterations and its potential to locate the optimum solution is greater.

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