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MECHANICAL EFFICIENCY, WORK AND HEAT OUTPUT IN RUNNING UPHILL OR DOWNHILL

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ABSTRACT

Heat output in running, per unit distance and body mass, was evaluated from published data on the corresponding energy cost (Cr). Cr is independent of the speed and is a function of the incline (i); between i = –0.45 and +0.45, it is described by $Cr = 155.4\,i^5 - 30.4\,i^4 - 43.3\,i^3 + 46.3\,i^2 + 19.5\,i + 3.6$*, where 3.6 J/(kg m) is the cost on flat terrain. Since the mechanical work performed against gravity is proportional to the incline (i), this equation allows one to calculate the efficiency (η) of work performance against gravity: η increases with i to attain a value of about 0.23 for* $i \geq 0.25$*. When running downhill, η becomes negative to attain a value of about – 1.0 for i = –0.25 or steeper. Cr is transformed into mechanical work (w) and/or dissipated as heat (h): Cr = w + h. Since η = w/Cr, h, per unit mass and distance can be calculated for any given slope and speed (h = Cr – w = Cr (1 – η)). The minimum Cr (2.28 J/(kg m)) is attained for* $i \approx -20\%$*, whereas the minimum h (3.53 J/(kg m)) for* $i \approx -8\%$*. Furthermore, since both Cr and h are independent of the speed, the ratio h/Cr, which ranges from about 2 (for i = –0.40) to 0.77 (for i = +0.40), at any given speed is equal to the ratio of heat output to metabolic power rates.*

Keywords: running; energy cost; mechanical efficiency; heat output; uphill slopes; downhill slopes

MEHANSKA UČINKOVITOST, DELO IN PROIZVODNJA TOPLOTE PRI TEKU NAVKREBER ALI NAVZDOL

IZVLEČEK

Proizvodnja toplote na enoto razdalje in telesne mase je bila ocenjena iz objavljenih podatkov o energetski porabi (Cr). Cr je neodvisna od hitrosti in je funkcija naklona (i); med i = –0.45 in +0.45 jo opišemo z enačbo Cr = 155.4 i^5 – 30.4 i^4 – 43.3 i^3 + 46.3 i^2 + 19.5 i + 3.6, po kateri je na ravni podlagi poraba približno 3.9 J/(kg m). Ker je mehansko delo, opravljeno proti težnosti, sorazmerno z naklonom (i), nam ta enačba omogoča računati mehansko učinkovitost (η) dela proti težnosti: η se povečuje z i do vrednosti okrog 0.23 za i ≥ 0.25. Ob teku navzdol η postaja negativna do vrednosti –1.0 za i = –0.25 ali bolj strm. Cr se spremeni v mehansko delo ali/in se sprosti kot toplota (h): Cr = w + h. Ker velja η = w/Cr se lahko izračuna h na enoto mase in razdalje za katerikoli dani naklon in hitrost (h = Cr – w = Cr (1 – η)). Najnižjo Cr (2.28 J/(kg m)) dosežemo za i ≈ – 20 %, medtem ko je najnižja h (3.53 J/(kg m)) pri i ≈ –8 %. Ker sta tako Cr kot h neodvisni od hitrosti, je razmerje h/Cr, ki sega med približno 2 (za i = –0.40) in 0.77 (za i = +0.40), pri katerikoli dani hitrosti enako razmerju med proizvedeno toploto in metabolno močjo.

Ključne besede: tek, energijska poraba, mehanska učinkovitost, proizvedena toplota, naklon navzgor, naklon navzdol

INTRODUCTION

One of the great achievements of classical thermodynamics (among the many) was showing that the efficiency of a heat engine (η) is set by the ratio of the temperature difference between the heat source and the heat sink to the temperature of the heat source:

$$\eta = (Th - Tc)/Th \qquad (1)$$

where *Th* and *Tc* are absolute temperatures (°K) of the heat source (*Th*) and sink (*Tc*) (Klotz, 1964). Equation 1 shows that to approach an efficiency of 1, the temperature of the heat sink should be negligibly small (i.e. close to 0 °K), a feat unattainable by any real physical engine. In addition, this same equation shows that *the muscle cannot possibly be a heat engine*. Indeed, were this the case and since *Tc* cannot be much different than 310 °K (37 °C) and the efficiency of muscle contraction under ideal isotonic conditions on the order of 0.25–0.30, *Th* should amount to 140–170 ° C, an obviously nonsensical conclusion.

Indeed, as shown by Carlo Reggiani elsewhere in this same issue, *the muscle is a chemical engine* which transforms part of the free energy (ΔG) of ATP hydrolysis into mechanical work performed on the surroundings (*w*), the remaining fraction being dissipated as heat (*h*). In turn, ATP is reconstituted at the expense of several pathways all finally depending on oxygen consumption (VO2). Thus:

$$VO2 = E = h + w \qquad (2)$$

where *E* defines the overall energy output.

The energetics of running

The energetic analysis of running, as well as of any other form of locomotion, is crucially dependent on the concept of energy cost (Cr), i.e. the amount of energy spent per unit of distance travelled. To compare subjects of different sizes, Cr is generally expressed per kg of body mass and is expressed in J/(kg m), even if it is not unusual to express it in kcal/(kg km) or in Litres O2/(kg km). These units can be easily transformed into one another, considering that the consumption of 1 L of O2 in the human body yields an amount of energy on the order of 5 kcal (≈ 21 kJ), the precise value depending on the Respiratory Quotient (RQ) and ranging from 19.55 kJ/L for the oxidation of pure lipids (RQ = 0.71) to 21.14 kJ/L for the oxidation of pure carbohydrates (RQ = 1.00).

The first data on the energy cost of running date back to the second half of the nineteenth century and are astonishingly close to the values that are presently considered to be correct. However, the first comprehensive data on the energetics of running on flat terrain, as well as uphill or downhill, were published by Rodolfo Margaria in 1938. These data did show that the energy cost of running (Cr), in all conditions (flat terrain, uphill or downhill) is independent of the speed, a fact that was confirmed by more recent studies (e.g. see Figure 1). Indeed, the data obtained by Minetti et al. in 2002, besides supporting the data obtained previously by this same group (1994) and by Margaria et al. (1938, 1963) greatly extend the range of the investigated slopes. In addition Minetti et al. show that the energy cost of running (Cr, J/(kg m)) in the investigated range of slopes (i.e. from –0.45 to +0.45) is a polynomial function of the incline (i), as described by:

$$Cr = 155.4\, i^5 - 30.4\, i^4 - 43.3\, i^3 + 46.3\, i^2 + 19.5\, i + 3.6 \qquad (3)$$

where the last term is the energy cost of running on flat terrain.

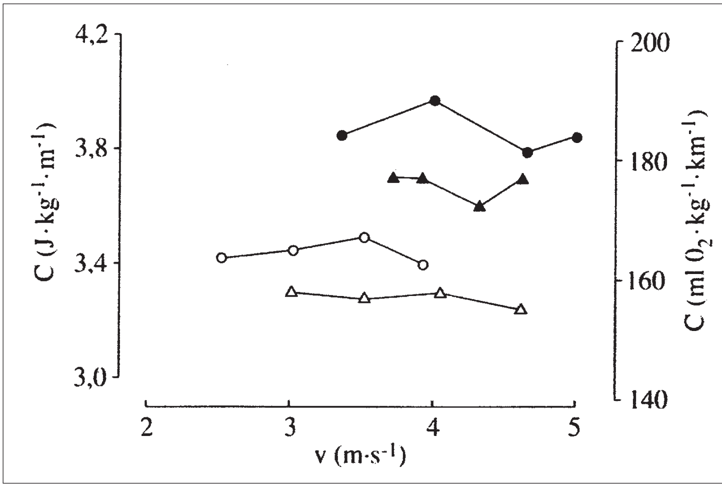

Figure 1a – Energy cost of running at constant speed on flat terrain (Cr, J/(kg m) or ml O2/(kg km)) as a function of the speed (v, m/s). Filled symbols refer to the two less economical and open symbols to the two most economical among 36 subjects taking part in the "Marathon International de Genève". (Data from P.E. di Prampero et al., 1986).

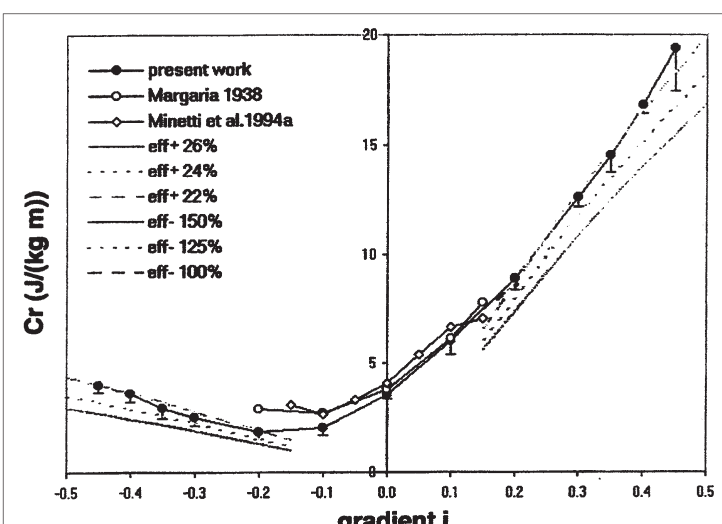

Figure 1b – Energy cost of running along the direction of motion (Cr, J/(kg/m)), as a function of the incline of the terrain. As was the case on flat terrain, Cr is independent of the speed even when running up or downhill. (From Minetti et al., 2002).

The efficiency of work against gravity

In equation 3, Cr is expressed per unit body mass (M) and distance (d) along the direction of motion. When the subject is running uphill, the corresponding mechanical work performed against gravity, per unit mass and distance, is given by:

$$w = (M\,g\,h)/(M\,d) = (M\,g \sin \alpha)/M = g \sin \alpha \qquad (4)$$

where g is the acceleration of gravity and α is the angle of the terrain with the horizontal.

In equation 3, the slope of the terrain is expressed by the tangent of the angle α ($i = \tan \alpha$). Therefore:

$$w = g \sin \alpha = g \sin (\arctan i) \qquad (5)$$

Hence the efficiency (η) of the work performed against gravity when the subject is running uphill at constant speed can be expressed as:

$$\eta = w/Cr = g\,[\sin (\arctan i)]/Cr \qquad (6)$$

Equation 3 by Minetti et al., describing, as it does, the relationship between Cr and i allows one to make explicit the efficiency for any given slope. It also goes without saying that when running downhill, work is done by gravity on the subject. As a consequence, the corresponding efficiency becomes negative. The corresponding values, calculated as described above, attain a maximal positive value of about +0.23 for slopes i = +0.20 or greater and a maximal negative value of about –1.0 for slopes i = –0.20 or steeper (Figure 2).

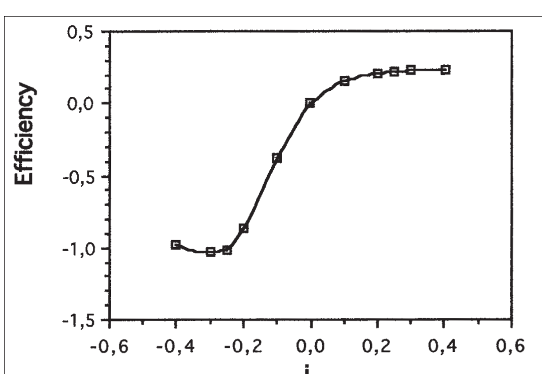

Figure 2 – The efficiency of work performance against gravity when running uphill or downhill is plotted as a function of the incline (i), as from the data by Minetti et al. (2002) (see text for details).

The heat output

The equation of Minetti et al. also allows one to calculate the heat output per unit body mass and distance. Indeed, from equation 6:

$$w = \eta \ Cr \qquad (7)$$

and since Cr = h + w = E (see equation 2), from equation 2 and 7 one obtains:

$$h = Cr - \eta \ Cr = Cr \ (1 - \eta) \qquad (8)$$

The energy cost and the heat output per unit body mass and distance, both expressed in J/(kg m), are reported in Figure 3 as a function of the incline of the terrain. It becomes immediately apparent that when running on flat terrain (i = 0), the overall energy expenditure appears as heat (i.e. Cr = h). However, when running uphill, since a fraction of Cr is converted into mechanical work against gravity, Cr > h, the difference between the two quantities being greater the greater the mechanical efficiency. On the contrary, when running downhill the heat dissipated is the sum of the metabolic energy expenditure and of the mechanical work done by gravity on the subject, once again the difference between the two quantities increasing with the /absolute/ efficiency.

Furthermore, the rate of heat output and the metabolic power, expressed in W/kg, are given by the product of Cr (or h) and the speed. Therefore, since both Cr and h are independent of the speed, the ratio h/Cr is also equal for any given speed, to the ratio between the rate of heat output and metabolic power. It can be calculated from Figure 3 that the ratio h/Cr ranges from about 2.0 (for i = –0.40) to 0.77 for (i = + 0.40) and is obviously equal to 1.0 on flat terrain, i.e. when the overall energy expenditure is dissipated as heat, since the mechanical work performance is nil.

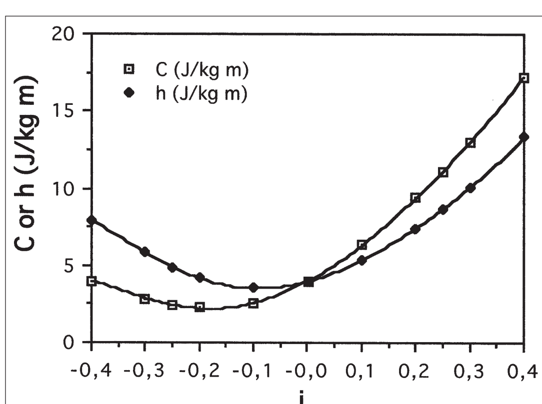

Figure 3 – Energy cost (C, J/(kg m), black) and heat output (h, J/(kg m), blue) when running uphill or downhill are plotted as a function of the incline (see text for details).

Metabolic power and rate of heat output

The data reported in Figure 3 allow one to calculate the metabolic power (E' = Cr * v) and the rate of heat output (h' = h * v) for any given incline and speed (v). It is interesting to note that it is possible to select two slopes (an uphill and a downhill one) at which the two heat outputs are about equal whereas the two metabolic powers are widely different. This can be seen in Figure 4 wherein h' and E' are plotted as a function of the speed for two widely different slopes (i.e. +0.10 vs. –0.25). Indeed, whereas the two rates of heat output are very close, the metabolic power is about 2.6 times larger when running uphill.

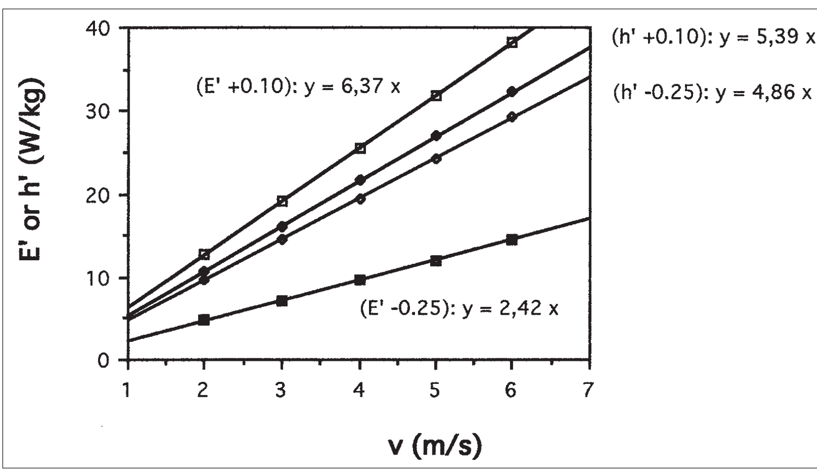

Figure 4 – Metabolic power (E', W/kg) and rate of heat output (h', W/kg) as a function of the speed (v, m/s) for two different up or down slopes (+0.10 vs. –0.25). When running uphill, h' is only about 10 % larger, as compared to running downhill (see green and blue straight lines), whereas E' is about 2.6 times larger (see black upper and red lowest lines).

Finally, it is possible to calculate for any given absolute slope (/i/), the down to up-hill speed ratios (v^-/v^+) that bring about equal metabolic power values ($E'^- = E'^+$) or equal rates of heat output ($h'^- = h'^+$). These speed ratios are shown in Figure 5 as a function of the absolute slope. This figure shows that the down to up-hill speed ratios (v^-/v^+) which bring about equal metabolic power values ($E'^- = E'^+$) increase rapidly with the slope to become about 4.5 for /i/ ≥ 0.25, whereas those which bring about equal values of heat output ($h'^- = h'^+$) are much smaller attaining about 1.8 for /i/ ≥ 0.25.

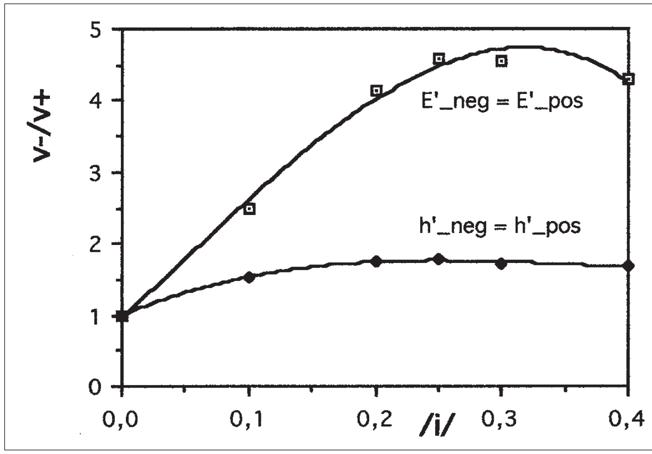

Figure 5 – The ratios of the down- to the up-hill speeds (v-/v+) which bring about equal metabolic power values (E'– = E'+, black) or equal heat output rates (h'– = h'+) are plotted as a function of the absolute incline (/i/).

The "human machine"

The preceding sections have outlined the role of the incline of the terrain and of the speed on heat and metabolic power outputs in running. So far, however, the physiological constraints specifically imposed upon the "human machine" by thermoregulation have been neglected. These are briefly discussed below.

Consider an average subject with a body mass of 70 kg running downhill at an incline of –40 % and a speed of 4 m/s. The corresponding metabolic power requirement is on the order of 16 W/kg (Fig. 3), equal to an O2 consumption above resting of 46 ml/(kg min), or 3.22 l/min, a relatively moderate intensity for young trained runners. The heat output, however, amounts to about twice as much (i.e. 32 W/kg or 2.24 kW) (Fig. 3 and 5).

Considering now the average specific heat of the human body (0.86 kcal/(kg °C) = 3.6 kJ/(kg °C) or 252 kJ/°C for a 70 kg body mass) it can be calculated that, were the whole heat produced accumulating in the body, the subject's core temperature would climb at a rate of 0.009 °C/s, i.e. of 0.54 °C/min. To avoid such an untenable situation, the heat produced must be eliminated via the four physical routes available for thermoregulation: conduction, convection, radiation and evaporation, evaporation playing the major role during exercise in normal environments.

The evaporation heat of water is 0.58 kcal/g (2.43 kJ/g); thus, the elimination of 2.24 kW of heat requires the evaporation of (2.24/2.43 =) 0.92 g/s or 55 g of water per minute (3.30 l/hour), on the assumption that the entire sweat output can evaporate, a rather unlikely situation. Thus the scenario described above (running downhill at an incline of –40 % at 4 m/s), albeit metabolically feasible in principle, would be sustainable in practice only under extremely favourable weather conditions, and only for a few minutes.

It can be concluded that when dealing with real world situations, in addition to the "in vitro" calculations reported in the preceding paragraphs, the real "efficacy" of the "human machine" needs thorough consideration

CONCLUSIONS

These data highlight: i) the role of the incline of the terrain in the transformation of the metabolic energy into work and/or heat; ii) the fact that consistently with the shape of the force velocity relationship described by A.V. Hill in 1938, the muscle is much more efficient in dissipating than in generating mechanical work; iii) finally, these may provide useful information for the design and characteristics of garments to be used when running outdoors in mountainous terrains, such as in "Sky Running" events.

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