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Generation Simulation of Involute Spur Gears Machined by Pinion-Type Shaper Cutters

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A pinion-type shaper cutter is considered as the generating tool for the generation of the gear, and a mathematical model of spur gears with asymmetric involute teeth is given according to the gearing theory. The equations of the profile of the cutter, the principle of coordinate transformation, the theory of differential geometry, and the theory of gearing are applied for describing generating and generated surfaces. Trochoidal envelope traced by cutter during the generating process is also investigated. Trochoidal curves of the cutter depends on the type of tip rounding. Computer graphs of involute spur gears are presented based on the given model. In addition, generation simulation is illustrated. The simulated motion path of the cutter can be used to determine chip geometry for further analysis. The results of this research should be helpful in the design and manufacture of spur gears.

Keywords: asymmetric involute teeth, gear design, gear shaper, generation simulation, spur gears, trochoidal fillet

0 INTRODUCTION

Involute gears are widely used because of their advantages, such as, simple geometry, ease of manufacture, and constant gear ratio even if the centre distance changes. As modern applications demand higher power density gears, accurate numerical tools to predict gear stress are required. Computer simulations performed during the design process reduce the cost and time required to simulate many possible transmission designs. A mathematical model which defines all generating and generated tooth surfaces is the basis for accurate numerical tools and tooth contact analysis. Therefore, a good knowledge of the gear geometry is required [1] and [2].

In practice, cylindrical gears are produced by generation cutting using rack- and pinion-type cutters. In the generation cutting process, gear teeth are generated as a result of relative motion between the gear blank and the cutter. Gear shaping with pinion–type cutter is a versatile and accurate means of manufacturing spur and helical gears and internal gears. Equations that determine the gear tooth profile manufactured by generating-type cutters have been studied by various authors [3] to [10]. Chang and Tsay proposed a complete mathematical model of involute-shaped shaper cutters [7]. Chen and Tsay developed the mathematical models of helical gear sets with small numbers of teeth manufactured by modified rack- and pinion-type cutters [8]. Yang proposed the mathematical models of asymmetric helical external gears generated by rack-type cutters and internal gears generated by shaper cutters [9] and [10]. Asymmetric involute teeth have been studied as a promising design alternative to the standard involute for increasing the load carrying capacity of gear mechanisms [9] to [11].

The gear tooth profile can be split into three distinct regions, each generated by a different part of the cutter. The first part of the gear profile, the involute edge, is cut by the involute flank of the cutter. It is the rounded edge of the cutter which cuts the second section, the trochoidal root fillet. The final portion, the bottomland is generated by the topland of the cutter. The shape of the fillet depends upon the form of generating tool and the method used to produce this gear. In gears machined by generation a tooth fillet arises as an envelope of successive rolling locations of the tool tip. The tip of the tooth of the pinion-shaped cutter is rounded to give a larger radius of fillet or to produce a full-rounded-root form to reduce the stress concentration at the root section of the tooth on critical and heavily loaded gear drives. Procedures for computing a form of root fillet of spur or helical gear produced by different generation methods are available in literature [3] to [10] and [12]. In addition, the machining

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parameters of the generating cutter have been given for rack-type and pinion-type cutters with two round edges or single round edge [12]. Alipiev [13] investigated geometric varieties of the rounded corner of rack-type cutter tooth for generating symmetric and asymmetric involute gear teeth profiles. Su and Houser studied the application of trochoids to find exact fillet shapes generated by rack-type cutters [14]. Fetvaci and Imrak [15] have adapted the equations of trochoids given by Su and Houser to Yang's mathematical model for spur gears with asymmetric involute teeth. In addition, simulated motion path of the generating cutter has been illustrated.

The aim of this paper is to present a method for computerized tooth profile generation of symmetric and asymmetric involute gears manufactured by pinion-type shapers. A mathematical model available in literature [7] is adopted for generating cutter surfaces. Different coordinate systems and transformations are used for generated external tooth surfaces. In addition to mathematical models of tool and generated gear, trochoidal paths traced by the tool tip are investigated. The following Section 1 provides mathematical models of the shaper cutter surfaces based on Litvin's vector approach and Chang and Tsay's application [6] and [7]. The mathematical models: the locus of the cutter surfaces, the equation of meshing and the generated gear tooth surface are given in Section 2. Trochoidal envelope of the rounded tip of the generating cutter is studied in Section 3. Based on the mathematical models given in Sections 1 to 3, a computer simulation of the generating process is presented in Section 4. The relative positions of the cutter with respect to the gear during the generation process are also illustrated. The effects of tool settings on generated surfaces are shown. Finally, a conclusive summary of this study is given in Section 5.

1 MATHEMATICAL MODEL OF THE GEAR CUTTER

The mathematical model of the gear cutter is adopted from [7] to asymmetric gearing. Contrary to symmetric gearing, left and right sides of the cutter have different pressure angles. A shaper cutter of asymmetric involute teeth that composes six curves is depicted in Fig. 1. Regions 1 and 6 of the cutter are involute-shaped curves, regions 2 and 5 are circular arcs with centers at E and F, and regions 3 and 4 are straight lines. For simplicity, only the parameters of the left side regions are indicated in Fig. 1.

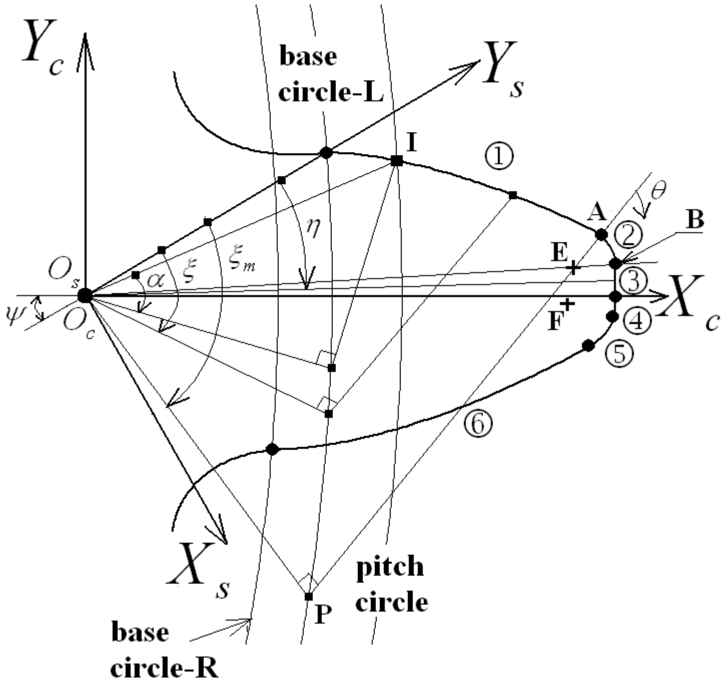

Fig. 1. *Geometry of the shaper cutter with asymmetric involute teeth (adapted from [7])*

Coordinate systems $S_s(X_s, Y_s)$ and $S_c(X_c, Y_c)$ represent the reference and the shaper cutter coordinate systems, respectively. According to the relationship between coordinate systems S_s and S_c, the position vector of region i can be transformed from coordinate systems S_s to S_c by applying the following coordinate transformation [6] and [7] :

$$\boldsymbol{R}_c^i = \begin{Bmatrix} x_c^i \\ y_c^i \end{Bmatrix} = \begin{bmatrix} \sin\psi & -\cos\psi \\ \cos\psi & \sin\psi \end{bmatrix} \begin{Bmatrix} x_s^i \\ y_s^i \end{Bmatrix}, \qquad (1)$$

where $\psi = \pi / 2N_s + \tan\alpha - \alpha$ determines half the tooth width of the shaper on the base circle, N_s is the number of shaper cutter teeth and α is the pressure angle of the cutter at the pitch point, as depicted in Fig. 1. Supercript i represents regions 1 to 6.

As shown in Fig. 1, the regions 1 and 6 of the shaper cutter are used to generate the opposite sides of the working tooth surfaces of involute spur gears. ξ is the curve variable of the left side cutter surface which determines the location of points on the involute region and $0 \le \xi \le \xi_m$. The position vector of region 1 is represented in the coordinate system S_s as follows [7] :

$$\boldsymbol{R}_s^1 = \begin{Bmatrix} x_s^1 \\ y_s^1 \end{Bmatrix} = \begin{Bmatrix} r_b \sin\xi - r_b\xi\cos\xi \\ r_b \cos\xi + r_b\xi\sin\xi \end{Bmatrix}, \qquad (2)$$

where is the radius of base circle. Substituting Eq. (2) into Eq. (1) yields the position vector of region 1 represented in coordinate system as follows [7]:

$$\boldsymbol{R}_c^1 = \begin{Bmatrix} r_b\cos(\xi-\psi) + r_b\xi\sin(\xi-\psi) \\ -r_b\sin(\xi-\psi) + r_b\xi\cos(\xi-\psi) \end{Bmatrix}. \qquad (3)$$

Regions 2 and 5 of the shaper cutter generate different sides of the fillet surfaces of spur gears. As indicated in Fig. 1, parameter θ of the cutter surface determines the location of points on the fillet region and it is limited by $0 \le \theta \le \pi/2 - \tan^{-1}(\xi_m - (\rho_f / r_b))$. The position vector of region 2 is represented in the coordinate system S_s as follows [7]:

$$\boldsymbol{R}_s^2 = \begin{Bmatrix} r_b\sin\xi_m - r_b\xi_m\cos\xi_m \\ r_b\cos\xi_m + r_b\xi_m\sin\xi \\ +\rho_f\cos\xi_m + \rho_f\cos(\theta+\xi_m) \\ -\rho_f\sin\xi_m + \rho_f\sin(\theta+\xi_m) \end{Bmatrix}, \qquad (4)$$

where ρ_f is the radius of tip fillet of the generating cutter, and ξ_m is the maximum extension angle of the involute curve at point A. The position vector of region 2 can be represented in coordinate system S_c as follows [7]:

$$\boldsymbol{R}_s^2 = \begin{Bmatrix} r_b\cos(\xi_m-\psi) + r_b\xi_m\sin(\xi_m-\psi) \\ -r_b\sin(\xi_m-\psi) + r_b\xi_m\cos(\xi_m-\psi) \\ -\rho_f\sin(\xi_m-\psi) + \rho_f\sin(\theta+\xi_m-\psi) \\ -\rho_f\cos(\xi_m-\psi) + \rho_f\cos(\theta+\xi_m-\psi \end{Bmatrix}. \qquad (5)$$

As depicted in Fig. 1, the regions 3 and 4 are used to generate the bottomland of the machined gear. η is a linear parameter on the cutter topland and its range is $\xi_m + \beta - \pi/2 \le \eta \le \tan\alpha - \alpha + \pi/2N_s$. Based on the cutter geometry, equation of region 3, represented in coordinate system S_s, can be expressed as [7]:

$$\boldsymbol{R}_s^3 = \begin{Bmatrix} x_s^3 \\ y_s^3 \end{Bmatrix} = \begin{Bmatrix} r_a\sin\eta \\ r_a\cos\eta \end{Bmatrix}, \qquad (6)$$

where $r_a = \sqrt{r_b^2 + (r_b\,\xi_m - \rho_f)^2} + \rho_f$ is the radius of the tip circle of the cutter and $\beta = \pi/2 - \tan^{-1}(\xi_m - (\rho_f / r_b))$. Substituting Eq. (6) into Eq. (1) yields the position vector of region 3 represented in coordinate system S_c as follows [7]:

$$\boldsymbol{R}_c = \begin{Bmatrix} x_c \\ y_c \end{Bmatrix} = \begin{Bmatrix} r_a\sin(\eta-\psi) \\ -r_a\cos(\eta-\psi) \end{Bmatrix}. \qquad (7)$$

Based on the differential geometry, the unit normal vectors of the above mentioned shaper cutter surface represented in coordinate system S_c are [6] and [7] :

$$\boldsymbol{n}_c = \frac{\dfrac{d\boldsymbol{R}_c^i}{dl_j} \times \boldsymbol{k}_c}{\left| \dfrac{d\boldsymbol{R}_c^i}{dl_j} \times \boldsymbol{k}_c \right|}, \qquad (8)$$

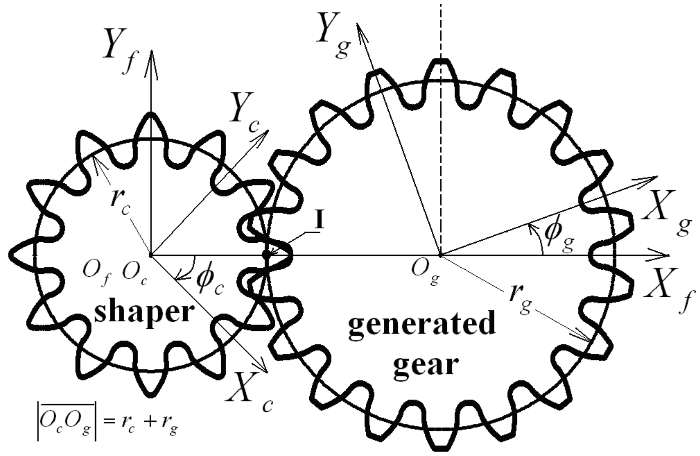

Fig. 2. *Kinematic relationship between the shaper cutter and the generated gear*

where k_c is the unit vector of the Z_c-axis. Parameter l_j represents ξ, θ and η, respectively. By substituting Eq. (3) in Eq. (8), the unit normal vector of region 1 can be obtained as follows [7]:

$$\boldsymbol{n}_c^i = \begin{Bmatrix} n_{xc}^i \\ n_{yc}^i \end{Bmatrix} = \begin{Bmatrix} -\sin(\xi - \psi) \\ -\cos(\xi - \psi) \end{Bmatrix}. \tag{9}$$

By substituting Eq. (5) in Eq. (8), the unit normal vector of region 2 can be obtained as follows [7]:

$$\boldsymbol{n}_c^i = \begin{Bmatrix} n_{xc}^i \\ n_{yc}^i \end{Bmatrix} = \begin{Bmatrix} -\sin(\theta + \xi_m - \psi) \\ -\cos(\theta + \xi_m - \psi) \end{Bmatrix}. \tag{10}$$

By substituting Eq. (7) in Eq. (8), the unit normal vector of region 3 can be obtained as follows [7] :

$$\boldsymbol{n}_c^i = \begin{Bmatrix} n_{xc}^i \\ n_{yc}^i \end{Bmatrix} = \begin{Bmatrix} -\cos(\eta - \psi) \\ \sin(\eta - \psi) \end{Bmatrix}. \tag{11}$$

The equations for the right side of the cutter are similar to those of the left provided that the parameters are calculated according to the corresponding pressure angle, and all equations corresponding to X_c coordinate are assigned an appropriate sign.

2 GENERATED GEAR TOOTH SURFACES

Fig. 2 illustrates the relationship between shaper cutter and generated gear of the gear generation mechanism. The right-handed coordinate systems are considered. The coordinate system $S_f(X_f, Y_f)$ is the reference coordinate system, the coordinate system $S_g(X_g, Y_g)$ denotes the gear blank coordinate system, and the coordinate system $S_c(X_c, Y_c)$ represents the shaper cutter coordinate system. On the basis of the gear theory, the cutter rotates through an angle ϕ_c while the gear blank rotates through an angle ϕ_g. Based on the above idea, the coordinate transformation matrix from S_c to S_g can be represented as [6]

$$[M_{gc}] = \begin{bmatrix} \cos(\phi_c + \phi_g) & \sin(\phi_c + \phi_g) & -(r_c + r_g)\cos\phi_g \\ -\sin(\phi_c + \phi_g) & \cos(\phi_c + \phi_g) & (r_c + r_g)\sin\phi_g \\ 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

The relationship between the angles ϕ_g and ϕ_c is $\phi_g = (N_c / N_g)\phi_c$ where N_c is the number of the teeth of the cutter and N_g denotes the number of the teeth of the generated gear. Point I is the instantaneous center of rotation and r_c and r_g are the standard pitch radii of the shaper cutter and the gear, respectively.

According to the theory of gearing [6], the mathematical model of the generated gear tooth surface is a combination of the meshing equation and the locus of the rack cutter surfaces. The locus of the shaper cutter surface, expressed in coordinate system S_g, can be determined as follows [6]:

$$\mathbf{R}_g^i = [M_{gc}]\mathbf{R}_c^i \quad , \ (i = 1,...,6) \ . \tag{13}$$

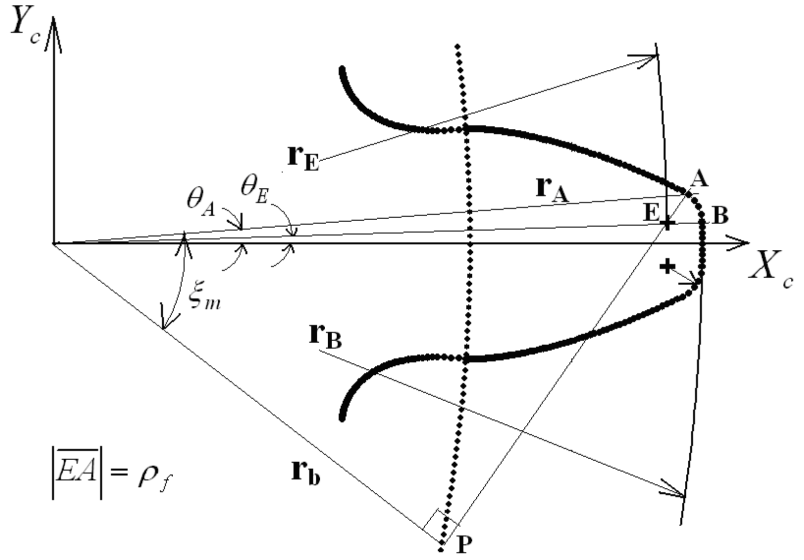

Fig. 3. *Gear cutter tip geometry*

When two gear surfaces are meshing, both meshing surfaces should remain in tangency throughout the contact under ideal contact conditions. Conjugate tooth profiles have a common surface normal vector at the contact point which intersects the instantaneous axis of rotation (pitch point I) for a parallel axis gear pair. Therefore, the equation of meshing can be represented using the coordinate system $S_c(X_c, Y_c, Z_c)$ as follows [6] :

$$\frac{X_c - x_c^i}{n_{cx}^i} = \frac{Y_c - y_c^i}{n_{cy}^i}, \tag{14}$$

where $X_c = r_c \cos\phi_c$ and $Y_c = r_c \sin\phi_c$ are coordinates of the pitch point I represented in coordinate system S_c; x_c^i and y_c^i are the surface coordinates of the shaper cutter; symbols n_{cx}^i and n_{cy}^i symbolize the components of the common unit normal represented in coordinate system S_c. In Eqs. (13) and (14), supercript i represents regions 1 through 6 of the corresponding shaper cutter surfaces.

The mathematical model of the generated gear tooth surfaces is a combination of the meshing equation and the locus of the rack cutter surfaces according to the gearing theory. Hence, the mathematical model of the gear tooth surfaces can be obtained by simultaneously considering Eqs. (13) and (14).

3 EPITROCHOIDAL PATH OF THE GENERATING CUTTER

The tooth fillet resulting from gear generation is in fact a trochoid which is created by the tool tip in its rolling movement [2]. An epitrochoid curve determines the shape of the fillet of the generated external gear tooth as a result of the generation process by pinion-type shaper cutters. An epitrochoid is a curve traced by a point attached to a circle of radius r rolling around the outside of a fixed circle of radius R, where the point is a distance d from the center of the exterior circle. According to the analytical mechanics of gears, the rolling circle is the pitch circle of the generating shaper cutter, the fixed circle is the pitch circle of the machined gear and the distance d is measured from the origin of the cutter to the center of its rounded corner at the tip (point E).

During the generating process of spur gear tooth presented in this paper, the center of the rounded corner at the tip traces out a trochoid. An equidistant curve with a distance of ρ_f defines the gear tooth root fillet. As depicted in Figs. (1) and (3), the rounded edge of the cutter is a circular arc and its center is located at point E. To ensure the tangents of the involute curve and circular arc at point A are the same and continuous, point E should be on the line $\overline{PA}$. It is first necessary to find the coordinates of points A and E [5].

The maximum involute extension angle at point A, denoted as ξ_m, can be evaluated from the following equation when the radius of tip circle r_B is given.

$$r_b \tan \xi_m = \sqrt{(r_B - \rho_f)^2 - r_b^2} + \rho_f . \tag{15}$$

According to involute geometry, the polar coordinates of point $A(r_A, \theta_A)$ are given by ,

$$\begin{cases} r_A = r_b / \cos \xi_m \\ \theta_A = \pi / 2N_s + inv\alpha - inv\xi_m \end{cases} . \tag{16}$$

The rectangular coordinates of point E can then be expressed in terms of x_E and y_E,

$$\begin{cases} x_E = r_A \cos \theta_A - \rho_f \sin(\xi_m - \theta_A) \\ y_E = r_A \sin \theta_A - \rho_f \cos(\xi_m - \theta_A) . \\ \theta_E = \tan^{-1}(y_E / x_E) \end{cases} \tag{17}$$

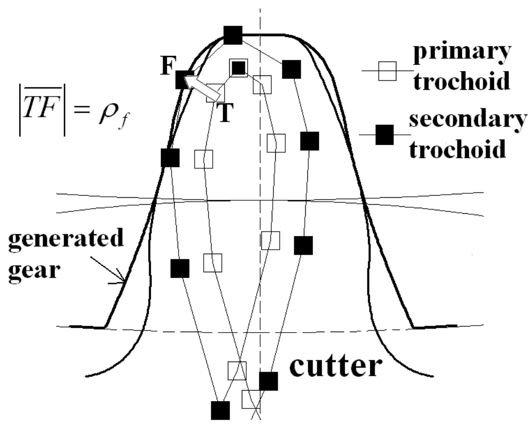

Fig. 4. *Primary and secondary trochoids of the cutter with rounded tip corners*

Fig. 4 displays a general point on the primary trochoid which is the envelope of the center of round tip. Applying the homogeneous coordinate transformation matrix given in Eq. (12), the equation of the primary trochoid (epitrochoid curve) can be written as follows:

$$\begin{Bmatrix} x_T \\ y_T \end{Bmatrix} = \begin{Bmatrix} x_E \cos(\phi_c + \phi_g) \\ - x_E \sin(\phi_c + \phi_g) \\ + y_E \sin(\phi_c + \phi_g) - (r_c + r_g)\cos\phi_g \\ + y_E \cos(\phi_c + \phi_g) + (r_c + r_g)\sin\phi_g \end{Bmatrix} , \tag{18}$$

where (x_E, y_E) is the coordinate of point E, ϕ_c and ϕ_g are the rolling parameters, r_c and r_g are the pitch circle radius of the shaper and the machined gear, respectively.

The envelope of the path of a series of circles with their geometric centers on the primary trochoidal path determine the actual form of spur gear tooth fillet. This new path is called the secondary trochoid which is the paralel curve of the primary trochoid. As a result, the coordinate of the corresponding point F on the secondary trochoid can be expressed as:

$$\begin{cases} x_F = x_T + \dfrac{\rho_f y'_T}{\sqrt{x'^2_T + y'^2_T}} \\ y_F = y_T - \dfrac{\rho_f x'_T}{\sqrt{x'^2_T + y'^2_T}} \end{cases} , \tag{19}$$

where ρ_f denotes the tip rounding radius of the shaper cutter, $x'_T = dx_T/d\phi_c$ and $y'_T = dy_T/d\phi_c$.

4 COMPUTER IMPLEMENTATIONS

A computer program has also been developed to compute coordinates of the gear tooth profile generated by different shaper cutters with partially round and full round tip. The computer graphs of generating cutter and generated gear can be obtained. Furthermore, considering appropriate limits of the rolling parameter, the simulated motion path of the cutting tool during the generation process can also be visualized. The effect of tool parameters on the generated tooth profile can be investigated before it is manufactured. Illustrative examples investigating the effect of tool tip radius follow:

Cutters with rounded tip corners are widely used in manufacturing. The centers of the roundings are not on the center line of the tooth. In asymmetric involute gearing, the left and the right sides of cutter are designed with different pressure angles. In this case, the left and right secondary trochoids are the offset curves of their corresponding primary ones with distances ρ_f and ρ_{f2}, respectively. The radius of the tip fillet surface at left side of the cutter is ρ_f. Similarly, the radius of the tip fillet surface at the right side is ρ_{f2}. Fig. 5 displays the shaper with asymmetric involute teeth, trochoidal paths of the centers of left and right rounded corners and the generated profile.

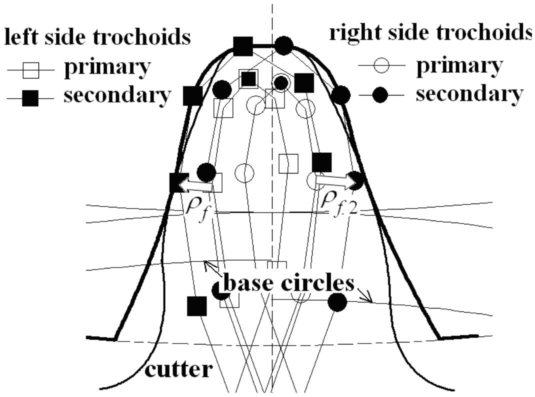

Fig. 5. *Trochoids of the cutter with rounded tip corners*

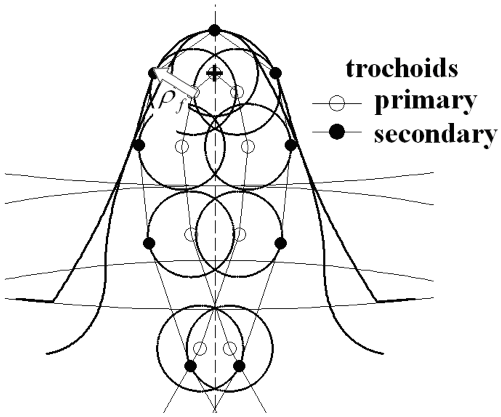

Fig. 6. *Trochoids of the symmetric cutter with a full radius tip*

In symmetric involute gearing, the location of the center of the tool's fully-rounded tip is on the center line of the tool tooth. The trochoid of the center of the rounding, the secondary trochoid, the generating and generated surfaces are shown in Fig. 6.

Fig. 7 displays the asymmetric involute-shaped shaper cutter with a full rounded tip and trochoidal paths of the centers of the left- and right sides of tip fillet. The centers of the rounded tip are at the center line of the cutter tooth. For visual clarity, only the corresponding halves (of secondary trochoids) that contribute to final formation of the generated tooth shape are shown.

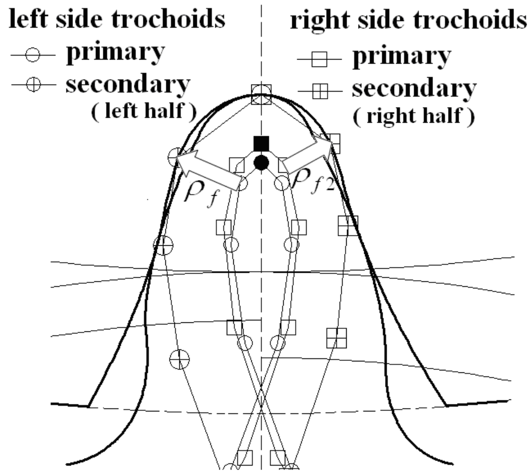

Fig. 7. *Trochoids of the asymmetric cutter with a full radius tip*

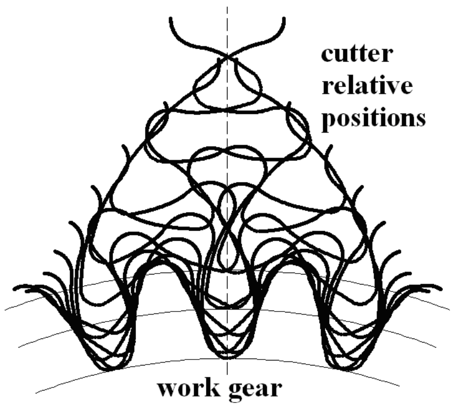

Fig. 8. *Generation simulation of gear blank*

Fig. 8 displays the work gear and simulated motion path of the generating cutter. Each gear gap is produced through successive penetrations of the tool teeth into the workpiece, in the individual generating positions. This simulation can be used to determine the chip geometry.

5 CONCLUSIONS

In this paper, computerized tooth profile generation of involute gears manufactured by pinion-type cutters is studied based on the Litvin's vector method. The asymmetric involute teeth are adopted to Chang and Tsay's application. The developed computer program provides an investigation of the effect of tool parameters on the generated tool profile before manufacture. Trochoidal paths traced by the generating tool tip are investigated. It has been noticed that geometric varieties of the rounded corner of the pinion-type cutter determine the position of trochoidal paths relative to the center line of the tooth space of the generated gear. The relative position of the cutter to the workpiece has been illustrated. The simulation of shaper cutting action can be used to determine the chip geometry for further analysis about tool wear and tool life.

The optimisation of the generating shaping process to increase productivity and cost efficiency, enhance gear quality is beyond the scope of this paper. However, related studies [16] and [17] should be mentioned here for further studies. Bouzakis et al. developed methods describing the chip geometry and predicting the tool life and cutting forces for manufacturing cylindrical gears by generating cutters. It has been found that chip formation and chip flow have a varying influence on wear behavior depending on the width of land at the tip of shaper cutter tooth. As a result it has been stated that while designing the geometry of the cutter the chip formation should be considered to improve the productivity [16]. Recently, Pedersen has introduced a shape optimization method based on rack-cutter tooth geometrical parameterization for improving bending stresses in spur gears with asymmetric teeth [17]. As a result, the two new standard rack cutters that can reduce the bending stress rather significantly, have been introduced [17]. In the present paper, shaper cutters with rounded tips are studied. The methods proposed for optimisitions [16] and [17] can be applied to symmetric and asymmetric involute spur gears manufactured by the shaper cutters with rounded tips for further investigations.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

[1] Kawalec, A., Wiktor, J., Ceglarek, D. (2006). Comparative analysis of tooth-root strength using ISO and AGMA standards in spur and helical gears with FEM-based verification. *Journal of Mechanical Design*, vol. 128, p. 1141-1158.

[2] Kawalec, A., Wiktor, J. (2004). Tooth-root stress calculation of internal spur gears. *Journal of Engineering Manufacture*, vol. 218, no. 9, p. 1153-1166.

[3] Buckingham, E. (1988). *Analytical Mechanics of Gears*, McGraw-Hill, New York.

[4] Salamoun, C., Suchy, M. (1973). Computation of helical or spur gear fillets. *Mechanism and Machine Theory*, vol. 8, p. 305-323.

[5] Colbourne, J.R. (1987). *The Geometry of Involute Gears*, Springer-Verlag, New Jersey.

[6] Litvin, F.L. (1994). *Gear Geometry and Applied Theory*. Prentice Hall, New Jersey.

[7] Chang, S.-L., Tsay, C.-B. (1998). Computerized tooth profile generation and undercut analysis of noncircular gears manufactured with shaper cutters. *Journal of Mechanical Design*, vol. 120, p. 92-99.

[8] Chen, C.-F., Tsay, C.-B. (2005). Tooth profile design for the manufacture of helical gear sets with small numbers of teeth. *International Journal of Machine Tools and Manufacture*, vol. 45, p. 1531-1541.

[9] Yang, S.-C. (2005). Mathematical model of a helical gear with asymmetric involute teeth and its analysis. *International Journal of Advanced Manufacturing Technology*, vol. 26, p. 448-456.

[10] Yang, S.-C. (2006). Study on an internal gear with asymmetric involute teeth. *Mechanism and Machine Theory,* vol. 42, no. 8, p. 977-994.

[11] Kapelevich, A. L., Shekhtman, Y.V. (2009). Tooth fillet profile optimization for gears

with symmetric and asymmetric teeth. *Gear Technology*, vol. 26, no. 7, p. 73-79.

[12] Lin, T., Ou, H., Li, R. (2007). A finite element method for 3-D static and dynamic contact/impact analysis of gear drives. *Computer Methods in Applied Mechanics and Engineering*, vol. 196, p. 1716-1728.

[13] Alipiev, O. (2009). Geometric synthesis of symmetric and asymmetric involute meshing using the method of realized potential. *General Machine Design Conference*, Ruse – Bulgaria, p. 43-50.

[14] Su, X., Houser, D.R. (2000). Characteristics of trochoids and their application to determining gear teeth fillet shapes. *Mechanism and Machine Theory*, vol. 35, p. 291-304.

[15] Fetvaci, C., Imrak, C. (2008). Mathematical model of a spur gear with asymmetric involute teeth and its cutting simulation. *Mechanics Based Design of Structures and Machines*, vol. 36, p. 34-46.

[16] Bouzakis, K.-D., Lili, E., Michailidis, N., Friderik, O. (2008). Manufacturing of cylindrical gears by generating cutting processes: A critical synthesis of analysis methods. *CIRP Annals - Manufacturing Technology*, vol. 57, p. 676-696.

[17] Pedersen, N.L. (2010). Improving bending stress in spur gears using asymmetric gears and shape optimization. *Mechanism and Machine Theory*, in press.