ELEMENTS OF METAMATHEMATICAL AND INFORMATIONAL CALCULUS

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This article deals with problems pertaining to the elements of axiomatics in traditional (mathematical, symbolic, philosophical) logic and to the problems of newly emerging axiomatics in informational logic. Informational axioms can be derived from propositional and predicate axioms and then, particularized and universalized within the informational domain. Traditional axiomatic formulas of the propositional and predicate calculus can become a rebounding cause for the construction of essentially different axioms in general informational theory. It is shown how propositional and predicate axioms and rules can be informationally extended for the needs of the general informational theory.

1 Introduction

Which are the basic and inferential informational axioms[1] which govern the derivation of formulas (theorems, consequences), that is, their proving procedures (proofs) in informational theories? This question is significant for the consciousness of that what scientific theories do outside of theories themselves, in their—from theories themselves separated—metatheories. On the other hand, informational theories are always unions of object theories and metatheories, where the last have the role of being the producers of theories in the sense of informational arising, that is, spontaneity and circularity.

2 Fundamental Figures of Syllogistic Inference

Syllogistic inference can be interpreted in different ways, for instance, in the scholastic (philosophical), informational and traditional-mathematical manner.

2.1 An Informational Interpretation of Syllogism

Syllogism (συλλογή, in Greek, means gathering, collecting, assembly, concourse and συλλογίζομαι means to reckon, consider, think, reflect; to infer, conclude) (in German, das Zusammenrechnen, der Schluß, die Folgerung) is a valid (e.g., true) inference in a syllogistic form.

Syllogistics (in Greek, συλλογιστικῆ τέχνη the art of inferring, concluding) was founded by Aristotle and developed in scholasticism as teachings of (correct) inference in syllogistic form. This technique became the keystone of the traditional logic ([4], pp. 407–409).

The inferring in a syllogistic form proceeds from two premises to one conclusion. Thus, the valid inference, the so-called syllogism, with true premises, delivers a true conclusion. In parallel to the traditional logic, as premises of syllogisms, only the following forms of 'equivalent' informational formulas are allowed:

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1. "All x are $F(x)$." Formula
$$x \models_{\forall x} F(x)$$
is a universal affirmative judgement and reads, informationally, x informs for all x the property (entity) $F(x)$.

2. "No x is $F(x)$." Formula
$$x \not\models_{\forall x} F(x)$$
is a universal negatory judgement and reads, informationally, x does not inform for all x the property (entity) $F(x)$.

3. "Some x are $F(x)$." Formula
$$x \models_{\exists x} F(x)$$
is a partial affirmative judgement and reads, informationally, x informs for some x the property $F(x)$.

4. "Some x are not $F(x)$." Formula
$$x \not\models_{\exists x} F(x)$$
is a partial negatory judgement and reads, informationally, x does not inform for some x the property (entity) $F(x)$.

The connections among these four formulas can be presented by the logical quadrate in Fig. 1.

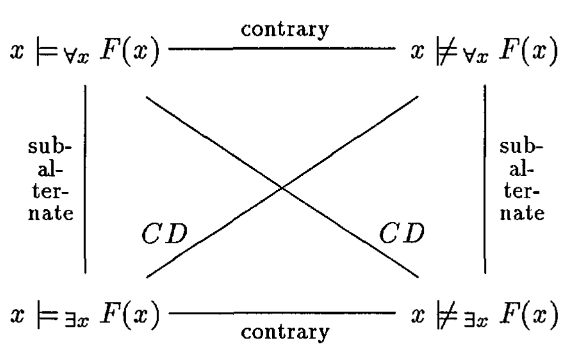

Figure 1: *The logical quadrate, drawn according to the informational formulas of syllogistics.* CD *means contradictory.*

Four fundamental figures of the syllogistic inference can be distinguished, where $\models_{\mathfrak{a}}$, $\models_{\mathfrak{b}}$ and $\models_{\mathfrak{c}}$ are determined by

$$\models_{\mathfrak{a}}, \models_{\mathfrak{b}}, \models_{\mathfrak{c}} \in \{\models_{\forall x}, \not\models_{\forall x}, \models_{\exists x}, \not\models_{\exists x}\}$$

The position of the so-called middle entity μ is decisive and it must appear in both premises in the following manner:

$$(\text{FF1}) \quad \frac{\begin{pmatrix} \mu \models_{\mathfrak{a}} \pi; \\ \sigma \models_{\mathfrak{b}} \mu \end{pmatrix}}{\sigma \models_{\mathfrak{c}} \pi}; \qquad (\text{FF2}) \quad \frac{\begin{pmatrix} \pi \models_{\mathfrak{a}} \mu; \\ \sigma \models_{\mathfrak{b}} \mu \end{pmatrix}}{\sigma \models_{\mathfrak{c}} \pi};$$

$$(\text{FF3}) \quad \frac{\begin{pmatrix} \mu \models_{\mathfrak{a}} \pi; \\ \mu \models_{\mathfrak{b}} \sigma \end{pmatrix}}{\sigma \models_{\mathfrak{c}} \pi}; \qquad (\text{FF4}) \quad \frac{\begin{pmatrix} \pi \models_{\mathfrak{a}} \mu; \\ \mu \models_{\mathfrak{b}} \sigma \end{pmatrix}}{\sigma \models_{\mathfrak{c}} \pi}$$

If operators $\models_{\mathfrak{a}}$, $\models_{\mathfrak{b}}$ and $\models_{\mathfrak{c}}$ are replaced through concrete (particularized) operators $\models_{\forall x}$, $\not\models_{\forall x}$, $\models_{\exists x}$ and $\not\models_{\exists x}$, for every case an inference modus is obtained. In all, there are $4^3 = 64$ modi for each fundamental inference figure, that is, 256 figures. Only 24 of them are valid, that is, syllogisms (6 for each fundamental form, and for some additional suppositions have to be introduced). A modus is uniquely determined if the fundamental figure and the operators $\models_{\mathfrak{a}}$, $\models_{\mathfrak{b}}$ and $\models_{\mathfrak{c}}$ are known.

2.2 A Mathematical Interpretation of Syllogism

In modern logic, syllogism is treated in the framework of the first order predicate logic. The so-called universal and existential qauntifier, $\forall$ and $\exists$, can be represented with conjunctive ($\wedge$) and disjunctive ($\vee$) logical connectives within a certain domain (set) D of elements. E.g., the scholastic modi "B*a*rb*a*r*a*" and "F*e*l*a*pt*o*n", where the emphasized *a* stands "for all", *e* for "no", and *o* for "some ... are not", become

$$\frac{\begin{pmatrix} \bigwedge_{x\in D} M(x) \to P(x) \\ \bigwedge_{x\in D} S(x) \to M(x) \end{pmatrix}}{\bigwedge_{x\in D} S(x) \to P(x)} \quad \text{and}$$

$$\frac{\begin{pmatrix} \bigwedge_{x\in D} M(x) \to \overline{P(x)} \\ \bigwedge_{x\in D} M(x) \to S(x) \end{pmatrix}}{\bigvee_{x\in D} S(x) \wedge \overline{P(x)}}$$

respectively. The premise $\bigwedge_{x\in D} M(x)$ in Felapton must be completed. Predicate $P(x)$ corresponds to the middle entity μ in fundamental formulas (FF1–4) and, adequately, predicates $P(x)$ and $S(x)$ correspond to entities π and σ in (FF1–4).

The reader can find the logic quadrate for formulas, together with negatory cases, using universal and existential quantifiers and predicate $F(x)$ in Fig. 2.

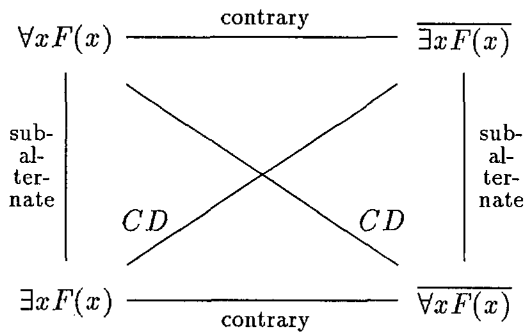

Figure 2: *The logical quadrate, drawn according to the predicate formulas of syllogistics using quantifiers* $\forall$ *and* $\exists$. CD *means contradictory.*

In Fig. 2 the contrary, contradictory and subalternate cases are shown in a clear, that is, negatory manner. There is, certainly,

$$\overline{\forall x F(x)} \equiv \exists x \overline{F(x)} \quad \text{and} \quad \overline{\exists x F(x)} \equiv \forall x \overline{F(x)}$$

and this equivalences (see Subsection 3.7) can be used as interpretations in Fig. 3.

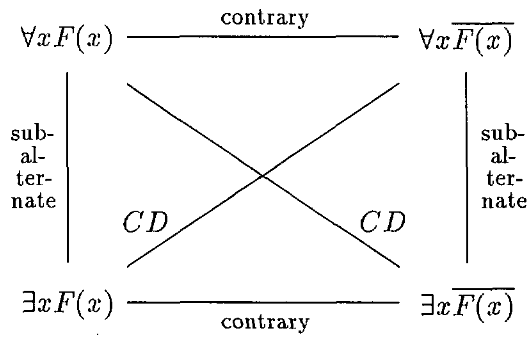

Figure 3: *The logical quadrate, drawn according to formulas in the previous picture with the resolved negations of the quantified formulas.* CD *means contradictory.*

3 A Short Overview of Some Mathematical Axioms

> In der Aussage: „Der Hammer ist zu schwer“ ist das für die Sicht Entdeckte kein „Sinn“, sondern ein Seiendes in der Weise seiner Zuhandenheit. ...Aussage besagt soviel wie *Prädikation.* Von einem „Subjekt“ wird ein „Prädikat“ „ausgesagt“, jenes wird durch dieses *bestimmt.* Das Ausgesagte in dieser Bedeutung von Aussage ist nicht etwa das Prädikat, sondern „der Hammer selbst“.
>
> Martin Heidegger [2] 154

3.1 Introduction

What is the nature of mathematical axioms and which kind of axioms are significant for our discussion? As we shall see, propositional axioms can serve as an outlook to the axioms used in the predicate calculus and also in the informational approach. The beginning of the general informational theory (GIT) has to be founded in logical axioms by which the informational phenomenalism (externalism, internalism, metaphysicalism) becomes a consequence of the very first assumption, that is, of the informational entity.

3.2 Axioms of the Propositional Calculus

> Aussage bedeütet primär *Aufzeigung.* ...Aussage bedeutet *Mitteilung,* Herausage. Als diese hat sie direkten Bezug zur Aussage in der ersten und zweiten Beduetung. Sie ist Mitsehenlassen des in der Weise des Bestimmens Aufgezeigten.
>
> Martin Heidegger [2] 154–155

Axioms of propositional calculus are fundamental for (the entire) mathematics (metamathematics) so that they can be reasonably extended, for example, to different predicate calculuses, arithmetic and mathematical logic in general. There are several "systems of axioms" which differ, through time, from one author to another. In our approach, the axiomatic systems of Hilbert [3] and Novikov [5] were chosen.

In propositional calculus, axioms can be grouped in a traditional way (e.g., following Hilbert and Bernays [3] and Novikov [5]). According to [3], p.66, the initial axiomatic formulas can be grouped and, roughly, written in the form of axiomatic rules:

I. Axioms of implication
1) $A \rightarrow (B \rightarrow A)$,
2) $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$,
3) $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$

II. Axioms of conjunction
1) $A \wedge B \rightarrow A$,
2) $A \wedge B \rightarrow B$,
3) $(A \rightarrow B) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B \wedge C))$

III. Axioms of disjunction
1) $A \rightarrow A \vee B$,
2) $B \rightarrow A \vee B$,
3) $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$

IV. Axioms of equivalence
1) $(A \equiv B) \rightarrow (A \rightarrow B)$,
2) $(A \equiv B) \rightarrow (B \rightarrow A)$,
3) $(A \rightarrow B) \rightarrow ((B \rightarrow A) \rightarrow (A \equiv B))$

V. Axioms of negation
1) $(A \rightarrow B) \rightarrow (\overline{B} \rightarrow \overline{A})$,
2) $A \rightarrow \overline{\overline{A}}$,
3) $\overline{\overline{A}} \rightarrow A$

The presented system of propositional axioms is not the only possible. For instance, Novikov [5], p. 75, chooses the group I of implication axioms in the form

1) $A \rightarrow (B \rightarrow A)$,
2) $(A \rightarrow (A \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

Various axioms can be useful for the interpretation of basic informational cases of phenomenalism, as we shall see in one of following subsections.

3.3 Inference Rules of the Propositional Calculus

Inference rules of propositional calculus are constructed on the basis of propositional axioms in the preceding subsection. It is hard to say which "rules" are the primary, the axiomatic or the inferential. However, it is clear that inferential rules must strictly consider the primitive axioms of propositional calculus which are, for example, implicative, conjuctive, disjunctive, equivalent and negatory.

Which is the general philosophy of making (constructing) a rule, precisely, the inference rule? Irrespective of the theory, for which rules are constructed, these are always implicative, although they express something more than a pure implication, because they concern the so-called derivation procedure. The role of a rule in the derivation procedure is the following: taking a rule in which several premises are logically connected in one or another way, some of them can be detached in the form of the so-called conclusion and, thus, between the respective premises of the rule and its conclusion a special operator, marked by $\vdash$ is used in propositional and predicate calculus while in the informational calculus we use operator $\twoheadrightarrow$ for marking derivation and an operator $|\!\!-$ for marking the circular form of informing.

Let us settle the general form of an inferential rule for deriving propositional formulas from axioms or from already derived formulas.

The first rule is substitution. Let us mark propositional formulas which depend on various propositional variables by the capital Fraktur letters, for example, as $\mathfrak{A}$ or, in more detail, $\mathfrak{A}(A_1, A_2, \cdots, A_n)$. Let $\mathbb{S}$ mark the operator of substitution (in informational terms, the function of substitution). Let us introduce

$$\mathbb{S}_{A_1, A_2, \cdots, A_n}^{\mathfrak{B}_1, \mathfrak{B}_2, \cdots, \mathfrak{B}_n} \mathfrak{A}(A_1, A_2, \cdots, A_n) = \mathbb{S}_{A_n}^{\mathfrak{B}_n} \left(\cdots \mathbb{S}_{A_2}^{\mathfrak{B}_2} \left(\mathbb{S}_{A_1}^{\mathfrak{B}_1} \mathfrak{A}(A_1, A_2, \cdots, A_n) \right) \cdots \right)$$

where $\mathfrak{B}_1, \mathfrak{B}_2, \cdots, \mathfrak{B}_n$ are propositional (identically true) formulas. Thus, the substitution rule has the form

$$\frac{\mathfrak{A}(A_1, A_2, \cdots, A_n)}{\mathbb{S}_{A_1, A_2, \cdots, A_n}^{\mathfrak{B}_1, \mathfrak{B}_2, \cdots, \mathfrak{B}_n} \mathfrak{A}(A_1, A_2, \cdots, A_n)}$$

The second production rule is applied to a formula which is structured as a parenthesized sequence of implications, that is,

$$\mathfrak{A}_1 \rightarrow (\mathfrak{A}_2 \rightarrow (\cdots (\mathfrak{A}_{n-1} \rightarrow \mathfrak{A}_n) \cdots))$$

and is expressed in the following manner: *if formulas*

$$\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_{n-1} \quad \text{and}$$
$$\mathfrak{A}_1 \rightarrow (\mathfrak{A}_2 \rightarrow (\cdots(\mathfrak{A}_{n-1} \rightarrow \mathfrak{A}_n)\cdots))$$

are true then formula $\mathfrak{A}_n$ is true in the propositional calculus. The complex rule of inference (modus ponens) is

$$\frac{\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_{n-1}, \quad \mathfrak{A}_1 \rightarrow (\mathfrak{A}_2 \rightarrow (\cdots(\mathfrak{A}_{n-1} \rightarrow \mathfrak{A}_n)\cdots))}{\mathfrak{A}_n}$$

In general, we have the following scheme of inference, where $\mathfrak{P}_i$ are true premises and $\mathfrak{C}_j$ are true conclusions too:

$$\frac{\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_m}{\mathfrak{C}_1, \mathfrak{C}_2, \cdots, \mathfrak{C}_n}$$

In case $m = n$, the senseful inference scheme becomes

$$\frac{\mathfrak{P}_1(\mathfrak{Q}_1, \mathfrak{C}_1), \mathfrak{P}_2(\mathfrak{Q}_2, \mathfrak{C}_2), \cdots, \mathfrak{P}_n(\mathfrak{Q}_n, \mathfrak{C}_n)}{\mathfrak{C}_1, \mathfrak{C}_2, \cdots, \mathfrak{C}_n}$$

where conclusions

$$\mathfrak{C}_1, \mathfrak{C}_2, \cdots, \mathfrak{C}_n$$

are detached from premises

$$\mathfrak{P}_1(\mathfrak{Q}_1, \mathfrak{C}_1), \mathfrak{P}_2(\mathfrak{Q}_2, \mathfrak{C}_2), \cdots, \mathfrak{P}_n(\mathfrak{Q}_n, \mathfrak{C}_n)$$

and, according to the convention of derivation formulas, there is

$$\mathfrak{Q}_1 \vdash \mathfrak{C}_1, \mathfrak{Q}_2 \vdash \mathfrak{C}_2, \cdots, \mathfrak{Q}_n \vdash \mathfrak{C}_n$$

or, even more generally,

$$\mathfrak{Q}_1, \mathfrak{Q}_2, \cdots, \mathfrak{Q}_n \vdash \mathfrak{C}_1, \mathfrak{C}_2, \cdots, \mathfrak{C}_n$$

3.4 A Theorem of Deduction within the Propositional Calculus

Formula $\mathfrak{B}$ is derivable from formulas $\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_n$, that is,

$$\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_n \vdash \mathfrak{B}$$

if it is possible to derive formula $\mathfrak{B}$ merely by means of inference rules, using the initial formulas $\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_n$ and any true formulas within the propositional calculus.

Deduction Theorem. *If formula $\mathfrak{B}$ is derivable from formulas $\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_n$, then*

$$\mathfrak{A}_1 \rightarrow (\mathfrak{A}_2 \rightarrow (\cdots(\mathfrak{A}_n \rightarrow \mathfrak{B})\cdots))$$

is a true formula. □

By induction it is possible to prove the following: if

$$\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_{n-1}, \mathfrak{A}_n \vdash \mathfrak{B}$$

then

$$\mathfrak{A}_1, \mathfrak{A}_2, \cdots, \mathfrak{A}_{n-1} \vdash \mathfrak{A}_n \rightarrow \mathfrak{B}$$

where symbol $\vdash$ is used for marking a derived formula within the calculus.

3.5 Axioms of the Predicate Calculus

If $\mathfrak{M}$ is a set of objects and a, b, c, d are elements of $\mathfrak{M}$, then $P(a)$, $Q(b)$, $R(c, d)$, etc. are propositions concerning objects a, b, c, d. These propositions can be true as well as false.

Expressions $F(x)$, $G(x, y)$, $H(x_1, \cdots, x_n)$, $I(x, x)$, etc. are predicates, that is, functions of arguments belonging to a domain (field) $\mathfrak{M}$. Further, formula $\forall x F(x)$ is a proposition being true if $F(x)$ is true for each element of field $\mathfrak{M}$ and being false in the opposite case. Formula $\exists x F(x)$ is a proposition being true if there exists an element of field $\mathfrak{M}$ for which $F(x)$ is true and being false in the opposite case. $F(x)$ can also be a propositional formula of predicates.

A system of axioms for the predicate calculus is obtained, if to the axiom system I, II, ..., V of subsection 3.2 the following group of axioms is added:

VI. Predicate axioms
1) $\forall x F(x) \rightarrow F(y)$,
2) $F(y) \rightarrow \exists x F(x)$

3.6 Inferential Axioms of the Predicate Calculus

The question is how the inferential axioms of the predicate calculus can impact the construction of axioms in the informational calculus. In which respect could the informational entities "for all" and "exists", which belong to quantifiers $\forall$ and $\exists$, respectively, be important at all within the informational calculus? So, let us make merely a short overview to the matter which is treated in detail in [5].

In principle, the inference philosophy of the predicate calculus does not differ essentially from that of the propositional calculus although the situation with quantifiers brings differences which must be considered with special care. As within the propositional calculus, the rules of substitution and inference keep their central roles in the predicate calculus too. To make the difference clear, let us introduce the operator $\leftrightarrows$ (for 'replace') instead of $\mathbb{S}$ (for 'substitute').

In a predicate formula $\mathfrak{A}$ (formula with predicates), a proposition A or a predicate $F(\cdots)$ can be replaced (substituted) by formula $\mathfrak{B}$. In this case, the substitution is marked by

$$\leftrightarrows_{A}^{\mathfrak{B}}(\mathfrak{A}) \quad \text{or} \quad \leftrightarrows_{F(\cdots)}^{\mathfrak{B}(t_1,\cdots,t_n)}(\mathfrak{A})$$

respectively, where A is a variable proposition, F a variable predicate of n variables; formula $\mathfrak{B}(t_1, \cdots, t_n)$ includes among their free variables specially marked variables $t_1, \cdots, t_n$, the number of which is equal to the number of variables of predicate F, that is, n.

Rules connected with quantifiers are the following:

1. *If $\mathfrak{B} \rightarrow \mathfrak{A}(x)$ is a true formula and $\mathfrak{B}$ does not include variable x, then $\mathfrak{B} \rightarrow \forall x\, \mathfrak{A}(x)$ is a true formula too.*

2. *If $\mathfrak{A}(x) \rightarrow \mathfrak{B}$ is a true formula and $\mathfrak{B}$ does not include variable x, then $\exists x\, \mathfrak{A}(x) \rightarrow \mathfrak{B}$ is a true formula too.*

The basic inference scheme (rule of modus ponens) of the predicate calculus remains

$$\frac{\begin{array}{c}\mathfrak{A}\\ \mathfrak{A} \rightarrow \mathfrak{B}\end{array}}{\mathfrak{B}}$$

According to [3], various rules for the predicate calculus can be derived by means of the substitution and inference rule considering the basic axioms. Such a scheme is, for instance,

$$\frac{\begin{array}{c}\mathfrak{A} \rightarrow \mathfrak{B}\\ \mathfrak{A} \rightarrow \mathfrak{C}\end{array}}{\mathfrak{A} \rightarrow \mathfrak{B} \wedge \mathfrak{C}}$$

or, in a general form, at given variables $\mathfrak{a}, \mathfrak{b}, \cdots, \mathfrak{k}$

$$\frac{\begin{array}{c}\mathfrak{A} \rightarrow \mathfrak{B}(\mathfrak{a})\\ \mathfrak{A} \rightarrow \mathfrak{B}(\mathfrak{b})\\ \vdots\\ \mathfrak{A} \rightarrow \mathfrak{B}(\mathfrak{k})\end{array}}{\mathfrak{A} \rightarrow \mathfrak{B}(\mathfrak{a}) \wedge \mathfrak{B}(\mathfrak{b}) \wedge \cdots \wedge \mathfrak{B}(\mathfrak{k})}$$

Another inference scheme, with the universal quantifier, being important for a later consideration could be

$$\frac{\mathfrak{A} \rightarrow \mathfrak{B}(a)}{\mathfrak{A} \rightarrow \forall x\, \mathfrak{B}(x)}$$

where x must not appear in $\mathfrak{B}(a)$. The next inference scheme which comes out is, for instance,

$$\frac{\begin{array}{c}\mathfrak{A} \rightarrow (\mathfrak{B} \rightarrow \mathfrak{U})\\ \mathfrak{A} \rightarrow (\mathfrak{B} \rightarrow \mathfrak{V})\end{array}}{\mathfrak{A} \rightarrow (\mathfrak{B} \rightarrow \mathfrak{U} \wedge \mathfrak{V})}$$

This scheme can be extended by a transition from a two-part conjunction to the value domain of a variable a, that is,

$$\frac{\mathfrak{A} \rightarrow (\mathfrak{B} \rightarrow \mathfrak{C}(a))}{\mathfrak{A} \rightarrow (\mathfrak{B} \rightarrow \forall x\, \mathfrak{C}(x))}$$

where in the premise x must not appear.

Analogously, for the existential quantifier, the scheme

$$\frac{\mathfrak{B}(a) \rightarrow \mathfrak{A}}{\exists x\, \mathfrak{B}(x) \rightarrow \mathfrak{A}}$$

can be derived. This scheme corresponds to the disjunction scheme

$$\frac{\begin{array}{c}\mathfrak{A} \rightarrow \mathfrak{C}\\ \mathfrak{B} \rightarrow \mathfrak{C}\end{array}}{\mathfrak{A} \vee \mathfrak{B} \rightarrow \mathfrak{C}}$$

etc. The listed inference schemes will be used for comparison with schemes of informational calculus.

3.7 Theorems of Deduction within the Predicate Calculus

The basic theorem of deduction in the predicate calculus is similar to the theorem in the propositional calculus. But there are several theorems concerning formulas with quantifiers.

Deduction Theorems for Predicates. *If formula $\mathfrak{B}$ is derivable from formula $\mathfrak{A}$, then formula*

$$\mathfrak{A} \to \mathfrak{B}$$

is derivable in the predicate calculus, that is,

$$\vdash \mathfrak{A} \to \mathfrak{B}$$

Further, for formulas including universal and existential quantifiers, there is

$$\begin{array}{l}
\vdash \forall x F(x) \to \exists x F(x), \\
\forall x \forall y F(x,y) \equiv \forall y \forall x F(x,y), \\
\exists x \forall y F(x,y) \to \forall y \exists x F(x,y), \\
\vdash \forall x (F(x) \to G(x)) \to (\forall x F(x) \to \forall x G(x)), \\
\vdash \forall x (F(x) \to G(x)) \to (\exists x F(x) \to \exists x G(x)), \\
\vdash \forall x (F(x) \equiv G(x)) \to (\forall x F(x) \equiv \forall x G(x)), \\
\exists x F(x) \equiv \overline{\forall x \overline{F(x)}},\ \exists x \overline{F(x)} \equiv \overline{\forall x F(x)}, \\
\overline{\exists x F(x)} \equiv \forall x \overline{F(x)},\ \overline{\exists x \overline{F(x)}} \equiv \forall x F(x)
\end{array}$$

where $\mathfrak{A} \equiv \mathfrak{B}$ represents the formula

$$(\mathfrak{A} \to \mathfrak{B}) \wedge (\mathfrak{B} \to \mathfrak{A})$$

□

Proofs for the listed theorems can be found in [5].

4 Axioms within an Informational Theory

4.1 Introduction

The question is how could the axioms of the propositional and predicate calculus be turned over to informational calculus. The main difference seems to be between the realm of truth in the traditional logic and realm of information in the informational logic. The informational realm is substantially broader and within it the truth appears only as a very particular case. Instead to say that formulas in the traditional logic are true or false, in the informational logic formulas can inform in one or another way, truly and falsely, particularly and universally, in parallel and serially, straightforwardly and circularly, algorithmically and spontaneously, programmingly and intelligently, routinely and creatively, logically (consistently) and controversially, etc. Informational logic becomes an active part of any informational system, the theoretical and the practical one.

4.2 Axioms of the Informational Calculus

Axioms of informational calculus can find a logical support in axioms of propositional and predicate calculus (Subsections 3.2 and 3.5, respectively). At the first glance, the axiom designer can behave in a withholding way, considering the traditional axioms as much as possible. But, thereupon, when getting the appropriate experience, the designer of an informational theory can go his/her own way, considering the entirety of the possible informational realm. Within this general scope, the true as a particular situation can be replaced by the informational as an extreme attitude of the informationally possible. The discussion in this section will follow the traditional way of axiom construction as much as possible. In the next section the most general and informationally open way will replace the traditional thinking.

First, let us classify the informational axioms according to the tradition in the propositional and predicate calculus. This way of interpretation will give us the necessary feeling of difference and informational generality in respect to the usual understanding of "logical" axioms. It will be possible to recognize the essential difference which governs the informational realm in its universality in comparison to the classical logical (propositional, predicate) realm. It certainly does not mean that the particular axiom situation in logical calculuses does not fit the informational principles—it fits them in a particular manner.

4.2.1 "Implication" Axioms of the Informational Calculus

In case of implicational axioms, we are concerned with two basic possibilities. At the beginning of the axiomatizing process we are concerned with the so-called informational phenomenalism for which we have to design particular axioms giving us the certainty of our initial steps into a general theory of the informational. This means, we have to explain in a formally consistent way the arising of initial axioms themselves. For instance, we must induce the primitive axioms of externalism, internalism and metaphysicalism, which constitute the very general axiom of informational phenomenalism.

In Subsection 3.2 we listed the group of axioms of implication. In informational calculus, we have a broader definition of the so-called informational implication, discussed in [10]. The question is where to begin the process of axiomatization in informational calculus. We are confronted with the principal difference existing between the truth in logical calculuses and the informing in informational calculus.

In the traditional logic, propositions and predicates inform in a true or false manner. In the informational calculus, informational entities—precisely informational operands—inform and are informed. Thus, we can choose the operand—informational entity—as the point from which one can begin the process of axiomatization. We shall see how different *initial* axioms will become possible and how they will be circularly interweaved. This axiomatic analysis will deepen the understanding of the informational phenomenalism, that is, the phenomenalism of the informational entity.

According to Subsection 3.2, axioms I. 1–3, in informational case, the following is obtained:

I. Axioms of informational implication

1) $\alpha \Longrightarrow (\beta \Longrightarrow \alpha)$,
2) $(\alpha \Longrightarrow (\alpha \Longrightarrow \beta)) \Longrightarrow (\alpha \Longrightarrow \beta)$,
3) $(\alpha \Longrightarrow \beta) \Longrightarrow ((\beta \Longrightarrow \gamma) \Longrightarrow (\alpha \Longrightarrow \gamma))$

where α and β are informational operands (entities) and $\Longrightarrow$ represents the operator of informational implication (in the most general and complex form [10]).

Prior to the informational systemization of implication axioms, let us look at examples which bring to the surface the logical sense of the listed implication axioms within the informational realm.

An axiom of the form (I.1)

$$\alpha \Longrightarrow ((\alpha \models) \Longrightarrow \alpha) \qquad (1)$$

seems to be regular. It means that an informational entity α simply implies that entity α is implied by informing of the entity, that is, by $\alpha \models$. Theoretically, it seems to be impossible to oppose such an initial principle because the informational nature of an entity is circular in respect to itself and its informing. If introducing α's informing in the form $\mathfrak{I}_\alpha$, the last axiom can be expressed in the form

$$\alpha \Longrightarrow (\mathfrak{I}_\alpha \Longrightarrow \alpha) \qquad (2)$$

This form of the axiom is more general then the preceding one since it presupposes also the phenomenon of informational internalism, that is,

$$\alpha \Longrightarrow ((\models \alpha) \Longrightarrow \alpha) \qquad (3)$$

However, there is not an equivalence between the first (1) and the third (3) axiom on one side and the second (2) axiom on the other side.

A resulting system of axioms concerning informational phenomenalism in the sense of the propositional axiom (I.1) in Subsection 3.2 is

$$\alpha \Longrightarrow (\beta \Longrightarrow \alpha);$$
$$\beta \in \{\alpha \models, \models \alpha, \alpha \models \alpha, (\alpha \models; \models \alpha)\}$$

or informationally explicitly

$$\alpha \Longrightarrow \left(\left(\beta \in \left\{\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right\}\right) \Longrightarrow \alpha\right)$$

or simply and evidently

$$\alpha \Longrightarrow \left(\left(\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right) \Longrightarrow \alpha\right)$$

In this formula, the essential difference between "informational operators" *comma* and *semicolon* must be distinguished (the alternative and the parallel operator, respectively).

An extension of the discussed axiom system is evidently the following:

$$\left(\begin{matrix}\alpha; \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right) \Longrightarrow \left(\left(\begin{matrix}\alpha; \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right) \Longrightarrow \left(\begin{matrix}\alpha; \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right)\right)$$

An example of this system is not only the axiom

$$(\alpha \models) \Longrightarrow ((\models \alpha) \Longrightarrow (\alpha \models))$$

which fits the general axiomatic scheme (I.1), but also

$$(\alpha \models) \Longrightarrow ((\models \alpha) \Longrightarrow (\alpha \models \alpha))$$

if arbitrary items of alternative array (α, $\alpha \models$, $\models \alpha$, $\alpha \models \alpha$, $(\alpha \models; \models \alpha)$) are chosen. It is to stress that cases of non-informing, that is, $\alpha \not\models$, $\not\models \alpha$, $\alpha \not\models \alpha$, etc. are understood as particular cases of informing.

Informational axiom (I.2) offers other significant formulas (basic informational implications) which are used, for instance, in informational modus ponens and modus tollens.

One of the most important informational axioms, following the scheme (I.2), seems to be

$$(\alpha \Longrightarrow (\alpha \Longrightarrow (\alpha \models))) \Longrightarrow (\alpha \Longrightarrow (\alpha \models))$$

which delivers the necessary conclusion $\alpha \Longrightarrow (\alpha \models)$ needed in various rules of inference.

A resulting system of axioms concerning informational phenomenalism in the sense of the propositional axiom (I.2) in Subsection 3.2 is

$$\begin{array}{l}(\alpha \Longrightarrow (\alpha \Longrightarrow \beta)) \Longrightarrow (\alpha \Longrightarrow \beta);\\ \beta \in \{\alpha \models, \models \alpha, \alpha \models \alpha, (\alpha \models; \models \alpha)\}\end{array}$$

or informationally explicitly

$$\left(\alpha \Longrightarrow \left(\alpha \Longrightarrow \left(\beta \in \left\{\begin{array}{l}\alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right\}\right)\right)\right) \Longrightarrow$$

$$\left(\alpha \Longrightarrow \left(\beta \in \left\{\begin{array}{l}\alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right\}\right)\right)$$

One can imagine what happens if the first β and the second β in the last formula are chosen as different entities which, in the last scheme, is possible in several ways. Appearances of β within the alternative scheme (set) are legal independently of the randomly chosen element in each concrete case. It is to mention that some alternative informational schemes concerning the axiomatic attitude will also be discussed in Subsection 4.2.2.

This particular axiomatic situation can now be transformed simply and evidently into a more general axiomatic scheme of the form

$$\left(\alpha \Longrightarrow \left(\alpha \Longrightarrow \left(\begin{array}{l}\alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right)\right)\right) \Longrightarrow$$

$$\left(\alpha \Longrightarrow \left(\begin{array}{l}\alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right)\right)$$

The most general extension of the discussed axiom system could evidently be the following:

$$\left(\left(\begin{array}{l}\alpha;\\ \alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right) \Longrightarrow \left(\left(\begin{array}{l}\alpha;\\ \alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right) \Longrightarrow \left(\begin{array}{l}\alpha;\\ \alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right)\right)\right)$$

$$\Longrightarrow \left(\left(\begin{array}{l}\alpha;\\ \alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right) \Longrightarrow \left(\begin{array}{l}\alpha,\\ \alpha \models,\\ \models \alpha,\\ \alpha \models \alpha,\\ \left(\begin{array}{l}\alpha \models;\\ \models \alpha\end{array}\right)\end{array}\right)\right)$$

A characteristic and progressively diverse form of the last axiom system would be, for example,

$$\begin{array}{l}(\alpha \Longrightarrow ((\alpha \models) \Longrightarrow (\models \alpha))) \Longrightarrow\\ ((\alpha \models \alpha) \Longrightarrow (\alpha \models; \models \alpha))\end{array}$$

which shows a phenomenalistic sequence extending in a straight implicative manner from informational entity α over its externalism $\alpha \models$ and internalism $\models \alpha$ to its metaphysicalism $\alpha \models \alpha$ and phenomenalism $\alpha \models; \models \alpha$.

Axiomatic scheme (I.3) delivers a very fundamental property of informing of two entities concerning the third entity. Such axiom is, for instance,

$$\begin{array}{l}(\alpha \Longrightarrow (\alpha \models)) \Longrightarrow\\ (((\alpha \models) \Longrightarrow (\models \alpha)) \Longrightarrow (\alpha \Longrightarrow (\models \alpha)))\end{array}$$

or also the pair of axioms

$$\begin{array}{l}((\alpha \models) \Longrightarrow (\models \alpha)) \Longrightarrow\\ (((\models \alpha) \Longrightarrow (\alpha \models \alpha)) \Longrightarrow ((\alpha \models) \Longrightarrow (\alpha \models \alpha)));\\ ((\alpha \models) \Longrightarrow (\models \alpha)) \Longrightarrow\\ (((\models \alpha) \Longrightarrow (\alpha \models; \models \alpha)) \Longrightarrow ((\alpha \models) \Longrightarrow\\ (\alpha \models; \models \alpha)))\end{array}$$

The general scheme for the implicative rule (I.3) becomes

$$(\alpha \Longrightarrow \beta) \Longrightarrow ((\beta \Longrightarrow \gamma) \Longrightarrow (\alpha \Longrightarrow \gamma));$$
$$\beta, \gamma \in \{\alpha \models, \models \alpha, \alpha \models \alpha, (\alpha \models; \models \alpha)\}$$

etc., in the sense of the previous examples (I.1) and (I.2). Thus, the informational version of the last system becomes

$$\left(\alpha \Longrightarrow \left(\beta \in \left\{\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right\}\right)\right) \Longrightarrow$$

$$\left(\left(\left(\beta \in \left\{\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right\}\right) \Longrightarrow \left(\gamma \in \left\{\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right\}\right)\right)\right.$$

$$\left.\Longrightarrow \left(\alpha \Longrightarrow \left(\gamma \in \left\{\begin{matrix}\alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{matrix}\right\}\right)\right)\right)$$

The most general axiomatic version of the last system would be

$$\left(\begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix} \Longrightarrow \begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix}\right) \Longrightarrow$$

$$\left(\left(\left(\begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix} \Longrightarrow \begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix}\right) \Longrightarrow\right.\right.$$

$$\left.\left.\left(\begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix} \Longrightarrow \begin{pmatrix}\alpha, \\ \alpha \models, \\ \models \alpha, \\ \alpha \models \alpha, \\ \begin{pmatrix}\alpha \models; \\ \models \alpha\end{pmatrix}\end{pmatrix}\right)\right)\right)$$

Evidently, from this axiomatic scheme, a "complete" implicative circular formula proceeds:

$$(\alpha \Longrightarrow (\alpha \models)) \Longrightarrow$$
$$(((\models \alpha) \Longrightarrow (\alpha \models \alpha)) \Longrightarrow ((\alpha \models; \models \alpha) \Longrightarrow \alpha))$$

This formula is in no way an impossible speculation since all operands are only different phenomenal forms of one and the same informational entity α (with exception of the first and the last operand which are equal).

4.2.2 Another Form of "Implication" Axioms of the Informational Calculus

Informational calculus bases on the informing of entities and not solely on the logical truth. As the reader can observe, the discussed propositional and predicate axioms are always identically true logical formulas. We can show how other informational axioms which do not base on propositional and predicate axioms can be derived intuitively, trivially and formally in the same manner as the preceding informational axioms.

So let us take the syntactically rearranged axiom (I.1) in the form

$$(\alpha \Longrightarrow \beta) \Longrightarrow \alpha$$

instead of $\alpha \Longrightarrow (\beta \Longrightarrow \alpha)$. Propositional formula $(A \rightarrow B) \rightarrow A$ is not an identically true formula and its value is A. Axiom (I.1) is called *the axiom of the consequent determination.* What could the rearranged axiom mean at all? Let us check its meaning by some basic examples.

Let us look the semantic difference between formulas

$$\alpha \Longrightarrow ((\alpha \models) \Longrightarrow \alpha) \quad \text{and} \quad (\alpha \Longrightarrow (\alpha \models)) \Longrightarrow \alpha$$

If the first formula says that any informational entity α implies its externalistic informing $\alpha \models$ as a reason of itself, the second formula stresses that any informational entity α is implied by an implication in which entity α implies its externalistic informing $\alpha \models$. The reader will agree that it is practically impossible to argue against this argumentation. In informational cases we do strictly distinguish between circular and serially non-circular cases. As soon as $\alpha \models$ appears within the cycle $\alpha \Longrightarrow ((\alpha \models) \Longrightarrow \alpha)$, the transition $\alpha \Longrightarrow (\alpha \models)$ represents a part of the "whole history" of the cycle and, therefore, can or must be considered within the form $(\alpha \Longrightarrow (\alpha \models)) \Longrightarrow \alpha$. Thus, another substantial implication comes to the surface:

$$(\alpha \Longrightarrow ((\alpha \models) \Longrightarrow \alpha)) \Longrightarrow ((\alpha \Longrightarrow (\alpha \models)) \Longrightarrow \alpha)$$

Possibly, this fact becomes evident by the formula

$$(\alpha \Longrightarrow \mathfrak{I}_\alpha) \Longrightarrow \alpha$$

where $\mathfrak{I}_\alpha$ expresses the entire phenomenalistic nature of α's informing, that is, its hermeneutics (a kind of regular interpretation of α's history) which considers not only the history of both appearing components α and $\mathfrak{I}_\alpha$, but also transition $\alpha \Longrightarrow \mathfrak{I}_\alpha$ (or $\mathfrak{I}_\alpha \Longrightarrow \alpha$ in the first case).

4.2.3 "Conjunction" Axioms of the Informational Calculus

Let us draw an informational parallel to the propositional conjunction axioms (II.1–3) in Subsection 3.2.

What could be the conjunction in an informational sense? How could it be generalized?

The logical "and", represented by operator $\wedge$, means also "and simultaneously" or "in parallel". Informational operator of parallelism is $\parallel\!\models$ or, commonly, semicolon ';'. Thus, for the first axiom of conjunction (II.1) there is, informationally,

$$(\alpha \parallel\!\models \beta) \Longrightarrow \alpha \quad \text{or} \quad \begin{pmatrix} \alpha; \\ \beta \end{pmatrix} \Longrightarrow \alpha \ .$$

and for the second axiom of conjunction (II.2),

$$(\alpha \parallel\!\models \beta) \Longrightarrow \beta \quad \text{or} \quad \begin{pmatrix} \alpha; \\ \beta \end{pmatrix} \Longrightarrow \beta$$

For the third axiom of conjunction (II.3) there is

$$(\alpha \Longrightarrow \beta) \Longrightarrow ((\alpha \Longrightarrow \gamma) \Longrightarrow (\alpha \Longrightarrow (\beta \parallel\!\models \gamma)))$$

or, in a common informational form,

$$(\alpha \Longrightarrow \beta) \Longrightarrow \left((\alpha \Longrightarrow \gamma) \Longrightarrow \left(\alpha \Longrightarrow \begin{pmatrix} \beta; \\ \gamma \end{pmatrix}\right)\right)$$

The sense of parallelism axioms is significant in cases of the so-called informational decomposition (see Section 7). Two characteristic cases are, for example,

$$\begin{pmatrix} \alpha; \\ \alpha \models \end{pmatrix} \Longrightarrow \alpha; \quad \begin{pmatrix} \alpha; \\ \alpha \models \end{pmatrix} \Longrightarrow (\alpha \models)$$

4.2.4 "Disjunction" Axioms of the Informational Calculus

Disjunction axioms (III.1–3) in Subsection 3.2 introduce the meaning of informational formula $\alpha_1, \alpha_2, \cdots, \alpha_n$ where commas are used instead of semicolons. What does, in an informational formula, a comma mean at all?

Informationally, propositional axiom (III.1) can be interpreted as

$$\alpha \Longrightarrow (\alpha, \beta)$$

Instead of propositional formula $A \vee B$ there is informationally, simply α, β where the last formula just lists two alternatives which are α and β. Alternatives are informational entities which can be chosen by an informational system.

From which point of the philosophy could the discussed alternativeness come from? In an externalistic case, $\alpha \models$, intuitively, a presumption $\alpha \models \beta$ exists, otherwise the externalism of α would not perform meaningfully. One can agree that from the process represented by formula $\alpha \models \beta$ operands α and β can be listed. Thus,

$$(\alpha \models \beta) \Longrightarrow (\alpha, \beta) \quad \text{and} \quad (\alpha \models) \Longrightarrow (\alpha, \beta)$$

Through this demonstration the step towards the "disjunction" axiom in informational calculus becomes evident.

According to propositional axiom (III.2), there is

$$\beta \Longrightarrow (\alpha, \beta)$$

which is another form of the first disjunction informational axiom. It is not meant that the list α, β is ordered.

Finally, according to propositional axiom (III.3), we have

$$(\alpha \Longrightarrow \gamma) \Longrightarrow ((\beta \Longrightarrow \gamma) \Longrightarrow (\alpha, \beta \Longrightarrow \gamma))$$

In the last axiom we have to explain formula $\alpha, \beta \Longrightarrow \gamma$ additionally; the meaning is the following:

$$(\alpha, \beta \Longrightarrow \gamma) \rightleftharpoons \begin{pmatrix} \alpha \Longrightarrow \gamma; \\ \beta \Longrightarrow \gamma \end{pmatrix} \quad \text{and}$$

$$(\alpha, \beta) \rightleftharpoons \{\alpha, \beta\}$$

Thus, a list α, β connected with an operator in a formula results into a parallel system. In general,

$$(\alpha, \beta \models \gamma) \rightleftharpoons \begin{pmatrix} \alpha \models \gamma; \\ \beta \models \gamma \end{pmatrix} \quad \text{and}$$

$$(\alpha, \beta \models) \rightleftharpoons \begin{pmatrix} \alpha \models; \\ \beta \models \end{pmatrix}$$

This short discussion rounds up the possibilities of construction to the propositional disjunctive axioms parallel informational axioms.

4.2.5 "Equivalence" Axioms of the Informational Calculus

The reader might observe that we have never defined a rigorous informational formula by which informational operator of implication, $\Longrightarrow$, would be defined once for all. In [10], only verbal possibilities of the informational contents of implication have been listed. On the other hand, propositional implication, $\rightarrow$, is defined by a kind of logical table once for all. Does it mean that the contents (not a rough definition itself) of informational implication changes (arises) from case to case? The answer might be the following: operator $\Longrightarrow$ is a regular informational operator which, from case to case, can (must) be particularized and universalized, according to the involved operands. So, it must be interpreted only to the sufficient informational extent. The reader can imagine how complex and never ending interpretation of informational implication would proceed from the basis of its verbal (dictionary-like) determinations.

This fact does not restrict the discussion of informational equivalence in which informational implication plays an essential role. Thus, we can put the question of informational equivalence in parallel to the existing notion of propositional calculus.

According to the axiom group (IV.1–3) in Subsection 3.2, the adequate informational axioms concerning informational equivalence are the following:

$$\begin{array}{l} (\alpha \Longleftrightarrow \beta) \Longrightarrow (\alpha \Longrightarrow \beta); \\ (\alpha \Longleftrightarrow \beta) \Longrightarrow (\beta \Longrightarrow \alpha); \\ (\alpha \Longrightarrow \beta) \Longrightarrow ((\beta \Longrightarrow \alpha) \Longrightarrow (\alpha \Longleftrightarrow \beta)) \end{array}$$

Informational operator of equivalence, $\Longleftrightarrow$, can be defined dependently on informational implication, that is, according to deduction theorems of predicate calculus in Subsection 3.7. In this sense,

$$(\alpha \Longleftrightarrow \beta) \rightleftharpoons_{\mathrm{Def}} ((\alpha \Longrightarrow \beta; \beta \Longrightarrow \alpha))$$

Thus, in each particular case, when particularizing operator $\Longleftrightarrow$, this particularization depends on the particularization of implicative operator $\Longrightarrow$.

4.2.6 "Negation" Axioms of the Informational Calculus

The purpose of the negation of a statement in propositional calculus is to obtain the true negated statement, that is, the true otherwise false statement. In informational calculus, an entity informs or can be informed in a certain manner, but not in another one. This last situation can be discussed in the framework of an entity's non-informing.

Thus, an informational equivalent to the propositional situation $\overline{A}$ is, for example, $\alpha \not\models$ and $\not\models \alpha$ which reads α does not inform (in a certain manner) and α is not informed (in a certain manner), respectively. Additionally, operator $\not\models$ represents only a particular case of operator $\models$.

For a double negation $\overline{\overline{A}}$ it is possible to distinguish different cases of double non-informing, for instance,

$$(\alpha \not\models) \not\models; \quad \not\models (\alpha \not\models); \quad (\not\models \alpha) \not\models; \quad \not\models (\not\models \alpha)$$

Considering the axiom group of negation (V.1–3) in propositional calculus (Subsection 3.2), there is, informationally,

$$\begin{array}{l} (\alpha \Longrightarrow \beta) \Longrightarrow ((\beta \not\models; \not\models \beta) \Longrightarrow (\alpha \not\models; \not\models \alpha)); \\ \alpha \Longrightarrow ((\alpha \not\models; \not\models \alpha) \not\models; \not\models (\alpha \not\models; \not\models \alpha)); \\ ((\alpha \not\models; \not\models \alpha) \not\models; \not\models (\alpha \not\models; \not\models \alpha)) \Longrightarrow \alpha \end{array}$$

However, to these axioms, characteristic informational axioms concerning the phenomenon of non-informing can be adopted. For example,

$$\begin{array}{l} (\alpha \Longrightarrow (\alpha \models)) \Longrightarrow (\alpha \Longrightarrow (\alpha \not\models)); \\ (\alpha \Longrightarrow (\models \alpha)) \Longrightarrow (\alpha \Longrightarrow (\not\models \alpha)); \\ (\alpha \Longrightarrow (\alpha \models \alpha)) \Longrightarrow (\alpha \Longrightarrow (\alpha \not\models \alpha)); \\ (\alpha \Longrightarrow (\alpha \models; \models \alpha)) \Longrightarrow (\alpha \Longrightarrow (\alpha \not\models; \not\models \alpha)) \end{array}$$

etc. concern operator particularization in the processes of decomposition.

4.2.7 "Predicate" Axioms and Theorems of the Informational Calculus

The universal and existential quantifier do not play a significant role in the informational calculus. They are rather very uncommon entities within the informational realm. For instance, operator $\models_{\forall\alpha}$ reads *informs (is informed) for all α's* and formulas

$$\begin{array}{lll} \alpha \models_{\mathrm{for\_all\_}\alpha} & \text{or} & \alpha \models_{\forall\alpha}; \\ \models_{\mathrm{for\_all\_}\alpha} \alpha & \text{or} & \models_{\forall\alpha} \alpha; \\ \alpha \models_{\mathrm{for\_all\_}\alpha} \alpha & \text{or} & \alpha \models_{\forall\alpha} \alpha; \\ \alpha \models_{\mathrm{for\_all\_}\alpha}; \models_{\mathrm{for\_all\_}\alpha} \alpha & \text{or} & \alpha \models_{\forall\alpha}; \models_{\forall\alpha} \alpha \end{array}$$

are very special cases since it is not clear to which informational realm the 'all' could refer. A similar, problematic situation occurs in the case of the existential quantifier where formulas

$$\begin{array}{lll} \alpha \models_{\mathrm{exist\_}\alpha} & \text{or} & \alpha \models_{\exists\alpha}; \\ \models_{\mathrm{exist\_}\alpha} \alpha & \text{or} & \models_{\exists\alpha} \alpha; \\ \alpha \models_{\mathrm{exist\_}\alpha} \alpha & \text{or} & \alpha \models_{\exists\alpha} \alpha; \\ \alpha \models_{\mathrm{exist\_}\alpha}; \models_{\mathrm{exist\_}\alpha} \alpha & \text{or} & \alpha \models_{\exists\alpha}; \models_{\exists\alpha} \alpha \end{array}$$

Analogously to the predicate axiom group (VI.1–2) in Subsection 3.5, the adequate informational axioms could take the form

$$(\alpha \models_{\forall\alpha} \varphi(\alpha)) \Longrightarrow \varphi(\beta);$$
$$\varphi(\beta) \Longrightarrow (\alpha \models_{\exists\alpha} \varphi(\alpha))$$

The reader can imagine how the operator attribute *for all α's* in an informational realm causes that the informational function φ is informationally impacted by β which is within the realm of *all* α, etc. On the other hand, informational function $\varphi(\beta)$ (as determined in [9]) implies the existence of function φ being dependent on β, etc.

In parallel to the deduction theorems for predicates in Subsection 3.7 it is possible to derive "similar" informational theorems for informational entities.

Decomposition Theorems of Informational Entities. *If formula β is informationally derivable from formula α, then formula*

$$\alpha \Longrightarrow \beta$$

is derivable in the informational calculus, that is,

$$\twoheadrightarrow(\alpha \Longrightarrow \beta)$$

The theorems of predicate calculus in Subsection 3.7 become, informationally,

$$\begin{array}{l}
\twoheadrightarrow((\alpha \models_{\forall\alpha} \varphi(\alpha)) \Longrightarrow (\alpha \models_{\exists\alpha} \varphi(\alpha)));\\
(\alpha \models_{\forall\alpha} (\beta \models_{\forall\beta} \varphi(\alpha,\beta))) \Longleftrightarrow\\
\qquad (\beta \models_{\forall\beta} (\alpha \models_{\forall\alpha} \varphi(\alpha,\beta)));\\
(\alpha \models_{\exists\alpha} (\beta \models_{\forall\beta} \varphi(\alpha,\beta))) \Longleftrightarrow\\
\qquad (\beta \models_{\forall\beta} (\alpha \models_{\exists\alpha} \varphi(\alpha,\beta)));\\
\twoheadrightarrow((\alpha \models_{\forall\alpha} (\varphi(\alpha) \Longrightarrow \psi(\alpha))) \Longrightarrow\\
\qquad ((\alpha \models_{\forall\alpha} \varphi(\alpha)) \Longrightarrow (\alpha \models_{\forall\alpha} \psi(\alpha))));\\
\twoheadrightarrow((\alpha \models_{\forall\alpha} (\varphi(\alpha) \Longrightarrow \psi(\alpha))) \Longrightarrow\\
\qquad ((\alpha \models_{\exists\alpha} \varphi(\alpha)) \Longrightarrow (\alpha \models_{\exists\alpha} \psi(\alpha))));\\
\twoheadrightarrow((\alpha \models_{\forall\alpha} (\varphi(\alpha) \Longleftrightarrow \psi(\alpha))) \Longrightarrow\\
\qquad ((\alpha \models_{\forall\alpha} \varphi(\alpha)) \Longleftrightarrow (\alpha \models_{\forall\alpha} \psi(\alpha))));\\
(\alpha \models_{\exists\alpha} \varphi(\alpha)) \Longleftrightarrow (\alpha \not\models_{\forall\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha)));\\
(\alpha \models_{\exists\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha))) \Longleftrightarrow\\
\qquad (\alpha \not\models_{\forall\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha)));\\
(\alpha \models_{\exists\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha))) \Longleftrightarrow\\
\qquad (\alpha \models_{\forall\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha)));\\
(\alpha \models_{\exists\alpha} (\varphi(\alpha) \not\models; \not\models \varphi(\alpha))) \Longleftrightarrow (\alpha \models_{\forall\alpha} \varphi(\alpha))
\end{array}$$

□

These theorems can cause a retrograde and logically more critical understanding of the predicate theorems listed in Subsection 3.7.

4.3 The Variety of Informational Axioms of Inference

One of the basic questions concerning the inference schemes in different calculuses is in which manner inference rules are constructed and accepted by scientific communities. Obviously, the way to various inference rules leads through the understanding of the basic axioms treated in the preceding subsections. An inference rule is nothing else than a means for the derivation (deduction) process where it is used for a constructive transformation of theory-legal formulas into new formulas. The next step concerning axioms and inference rules is the introduction of the so-called replacement rules by which formulas can be transformed automatically, e.g. in a machine-like fashion.

Informational rules of inference function like axioms and only by them it is permitted to conclude in a theory-consistent manner. E.g., rules of substitution and modus ponens belong to the most obvious and widely accepted rules for formula transformation within a deduction process. On the other hand, informational inference rules can cover informationally unlimited possibilities and they can come into existence together with specific and particular informational problems.

The so-called modi informationis [6] can be grouped and expressed in several possible ways. We have the following possibilities:

1. *Modus ponens* is the most obvious inference rule in mathematics in the realm of truth. Informational modus ponens follows the general (phenomenalistic) principle of informing where the mathematical *to be true* is replaced by the informational *to inform*. This does not mean that the true is excluded, it only appears as a particular case of the informational. In this sense, informational modus ponens as an inference rule retains the basic logical form which is

$$\frac{\alpha; (\alpha \Longrightarrow \beta)}{\beta}$$

and where the implicational operator $\Longrightarrow$ has a broadened meaning in comparison with the logical implication operator $\rightarrow$, determined, for example, by a logical table. Implicit informational operator of parallelism ';' which simultaneously performs as a formula separator replaces the logical operator $\wedge$ (also, a comma in the traditional inference rule).

Particularizations of modus ponens can appear as special rules called, for example, modus procedendi, modus operandi and others.

2. *Modus tollens* is a rule of a reverse inferring in respect to the implication formula of its premise. There is a truly substantial difference between the mathematical and informational modus tollens. In the domain of truth, the falsity is the only essential counterpart to the truth and, so, the operator of negation of a formula becomes truly essential. Through the introduction of negation it becomes possible to express the falsity of a statement as its truth (e.g., identically true falsity).

In informational modus tollens the negation is replaced by the operator of noninforming (in a certain way), denoted by $\not\models$. The reader can now feel the dramatic difference existing between a static state of negation and the dynamic state of noninforming concerning a logical and an informational entity, respectively. Thus, the informational modus tollens in comparison with the logical one keeps the form

$$\frac{(\alpha \Longrightarrow \beta); (\beta \not\models; \not\models \beta)}{\alpha \not\models; \not\models \alpha}$$

with already mentioned differences concerning the modus ponens.

3. *Modus rectus* is not a mathematical inference rule and its origin can be searched in the analysis of the Latin speech. The aim of this rule is the filtering-out (detaching) of the intention marked by $\iota_{\text{intention}}(\alpha)$, hidden in entity represented by α through the informing of α, e.g. to an entity represented by β. It is understood that this intention is an informational function of α which is reflected in β to which α informs intentionally (or in an intending manner, that is, 'intendingly'). In this sense, a possibility of informational modus rectus becomes

$$\frac{\alpha; ((\alpha \models_{\text{intend}} \beta) \Longrightarrow \iota_{\text{intention}}(\alpha))}{(\iota_{\text{intention}}(\alpha) \models; \models \iota_{\text{intention}}(\alpha)) \subset \beta}$$

One can certainly construct 'intentional' rules where the intention or an intention-like entity (functional operand) performs in a certain manner, so, its detachment becomes possible.

4. *Modus obliquus* belongs to the most contentious inference rules because it usually proceeds from an absurd situation where the inference from a contradictory situation suggest to use the rule for an achievement of the logically consistent result. From a deceitful situation just the absurd inference should help to reach a firm conclusion. In fact, modus obliquus belongs to the so-called discursive informing where through a discourse (as communication, informing, reasoning) by all possible logical tricks, including absurd, contradiction, controversial informing, a valuable conclusion should be detached. In this sense, modus obliquus becomes a discursive filter delivering a useful result. In this sense, one of its possible schemes could be

$$\frac{\begin{array}{l}\alpha_{\text{absurd}}(\alpha) \subset \alpha; \\ (\alpha_{\text{absurd}}(\alpha) \models; \models \alpha_{\text{absurd}}(\alpha)) \Longrightarrow \beta)\end{array}}{(\alpha_{\text{absurd}}(\alpha) \not\models; \not\models \alpha_{\text{absurd}}(\alpha)) \subset \beta}$$

where $\alpha_{\text{absurd}}(\alpha)$ marks an absurd component of entity α hidden in α which phenomenalistically informs β, so that its informational phenomenalism can be identified in β as the observer.

The presented example of modus obliquus shows how other inference schemes would be possible and how, by further decomposition of the rule and its components, more complex and semantically concrete inferential schemes could be developed.

5. *Modus procedendi* is a mood by which a goal informational entity $\mathfrak{g}_{\mathrm{goal}}(\alpha)$ will be detached from the informing entity α. An example of modus procedendi is

$$\frac{(\mathfrak{g}_{\mathrm{goal}} \subset_{\mathrm{goal}} \alpha); (\alpha \Longrightarrow_{\mathrm{goal}} \mathfrak{g}_{\mathrm{goal}})}{\alpha \models_{\mathrm{goal}} (\mathfrak{g}_{\mathrm{goal}} \models_{\mathrm{goal}} \alpha)};$$

where implication operator $\Longrightarrow$ is particularized to $\Longrightarrow_{\mathrm{goal}}$, causing a consequent goal-like informing (operators $\subset_{\mathrm{goal}}$ and $\models_{\mathrm{goal}}$). The conclusion of the inference rule can certainly be extended, bringing other components of the goal structure to the parallel interpretative surface, for instance,

$$\begin{array}{l} \alpha \models_{\mathrm{goal}} (\mathfrak{g}_{\mathrm{goal}} \models_{\mathrm{goal}} \alpha); \\ (\alpha \models_{\mathrm{goal}} \mathfrak{g}_{\mathrm{goal}}) \models_{\mathrm{goal}} \alpha; \\ \mathfrak{g}_{\mathrm{goal}} \models (\mathfrak{G}_{\mathrm{goal}} \models \mathfrak{g}_{\mathrm{goal}}); \\ (\mathfrak{g}_{\mathrm{goal}} \models \mathfrak{G}_{\mathrm{goal}}) \models \mathfrak{g}_{\mathrm{goal}}; \\ \vdots \end{array}$$

To this goal system within entity α other interpretative formulas can be added.

6. *Modus operandi* discovers the inside of an entity, its modus operandi. The inside structure of an entity α is its metaphysicalism in the form of informing $\mathfrak{I}_\alpha$, counterinforming $\mathfrak{C}_\alpha$ and informational embedding $\mathfrak{E}_\alpha$, together with counterinformational entity $\mathfrak{c}_\alpha$ and embedding entity $\mathfrak{e}_\alpha$. Thus, the consequent inference rules are nothing else than circularly structured modi ponens, for example,

$$\frac{\alpha; (\alpha \Longrightarrow \mathfrak{I}_\alpha)}{\mathfrak{I}_\alpha}; \quad \frac{\mathfrak{I}_\alpha; (\mathfrak{I}_\alpha \Longrightarrow \mathfrak{C}_\alpha)}{\mathfrak{C}_\alpha};$$

$$\frac{\mathfrak{C}_\alpha; (\mathfrak{C}_\alpha \Longrightarrow \mathfrak{c}_\alpha)}{\mathfrak{c}_\alpha}; \quad \frac{\mathfrak{c}_\alpha; (\mathfrak{c}_\alpha \Longrightarrow \mathfrak{E}_\alpha)}{\mathfrak{E}_\alpha};$$

$$\frac{\mathfrak{E}_\alpha; (\mathfrak{E}_\alpha \Longrightarrow \mathfrak{e}_\alpha)}{\mathfrak{e}_\alpha}; \quad \frac{\mathfrak{e}_\alpha; (\mathfrak{e}_\alpha \Longrightarrow \alpha)}{\alpha}$$

An additional inference rule (modus operandi) could be

$$\frac{\alpha; (\alpha \Longrightarrow ((((\mathfrak{e}_\alpha \subset \mathfrak{E}_\alpha) \subset \mathfrak{c}_\alpha) \subset \mathfrak{C}_\alpha) \subset \mathfrak{I}_\alpha) \subset \alpha)}{\mathfrak{e}_\alpha; \mathfrak{E}_\alpha; \mathfrak{c}_\alpha; \mathfrak{C}_\alpha; \mathfrak{I}_\alpha}$$

7. *Modus vivendi* is certainly an informational case, for instance, how to infer in an (extremely) critical situation. Modus vivendi does not necessarily consider an extremely logical intention, so it can extend from an uncertain situation, e.g., modus possibilitatis, modus obliquus to modus ponens, as the most certain, approved and standard rule of inference.

Informational modus vivendi of an informational realm concerns inference in situations of life and surviving compromises happening under entity's environment, individual, populational and social circumstances [6].

Let the sensory information $\sigma_\alpha(\beta)$ of entity α observing entity β inform the α's metaphysicalistic structure, marked by $\alpha \models_{\mathrm{metaphysicalistically}} \alpha$, where the metaphysicalistic structure of α is something represented by the circular formula

$$\alpha \models (\mathfrak{I}_\alpha \models (\mathfrak{C}_\alpha \models (\mathfrak{c}_\alpha \models (\mathfrak{E}_\alpha \models (\mathfrak{e}_\alpha \models \alpha)))))$$

and other formulas belonging to the α's metaphysicalistic gestalt [9] of length $\ell = 6$. Thus,

$$\beta \models (\sigma_\alpha(\beta) \models (\alpha \models_{\mathrm{metaphysicalistically}} \alpha))$$

The last formula is nothing else than an informational decomposition of transition $\beta \models \alpha$ where the essential operator frames [9] in this formula are

$$\beta \boxed{\models (\sigma_\alpha(\beta) \models} (\alpha \models_{\mathrm{metaphysicalistically}} \alpha) \boxed{)} \text{ and}$$

$$\beta \boxed{\models (\sigma_\alpha(\beta) \models (\alpha \models_{\mathrm{metaphysicalistically}}} \alpha \boxed{))}$$

These cases of transition $\beta \models \alpha$ can be interpreted in the form of the so-called operator composition $\beta \models_\beta \circ \models_\alpha \alpha$, that is,

$$\beta \models_\beta \circ \models_{\sigma_\alpha(\beta)} (\alpha \models_{\mathrm{metaphysicalistically}} \alpha) \quad \text{and}$$

$$\beta \models_\beta \circ \models_{\sigma_\alpha(\beta) \models_{\mathrm{metaphysicalistically}} \alpha} \alpha$$

By observing of β by α, the so-called behavioral information $\beta_{\mathrm{behavior}}(\alpha, \beta)$ produced by the α's metaphysicalistic structure, through its sensory information $\sigma_\alpha(\beta)$ and, dependent on environment β, has to be detached, where $\beta_{\mathrm{behavior}}(\alpha, \beta)$ is on the way to assure the survival of α depending on environmental information β (in the sense of modus vivendi). Within this discourse, the following rule of modus vivendi arises:

$$\frac{\begin{pmatrix}(\sigma_\alpha(\beta) \subset (\beta \models \alpha)); \\ ((\sigma_\alpha(\beta) \models (\alpha \models_{\text{metaphysicalistically}} \alpha)) \Longrightarrow \\ \beta_{\text{behavior}}(\alpha, \beta))\end{pmatrix}}{\beta_{\text{behavior}}(\alpha, \beta)}$$

Senseful other examples of informational modus vivendi can come to the surface in more concrete informational situations and attitudes.

8. *Modus possibilitatis* roots in the philosophy of modal logic [1]. Instead of $\Diamond A$ which means *possibly* A (exactly, A is possibly true), we can introduce $\alpha \models_{\text{possibly}}$ which reads α informs possibly. A shortcut would be $\alpha\Diamond$. Thus, $\alpha\Diamond\beta$ is read *α possibly informs β* or also *β is possibly informed by α*. Sometimes, $\alpha \models_\Diamond \beta$ is an appropriate notation for a case of possible informational transition.

The question is, for instance, which are the possibilities of an informational entity α's arising. How can α possibly arise? The possibility of α's arising depends on the possible informing of α itself and on informing of an exterior entity, say β, which could possibly impact α informationally. Thus, the basic situation is a double transition $\alpha, \beta \models_\Diamond \alpha$ with the meaning

$$(\alpha, \beta \models_\Diamond \alpha) \rightleftharpoons \begin{pmatrix}\alpha \models_\Diamond \alpha; \\ \beta \models_\Diamond \alpha\end{pmatrix}$$

Let $\pi_\Diamond$ mark a structure of informational possibilities, where $\pi_1, \pi_2, \cdots, \pi_n \subset \pi_\Diamond$. These components can be detached transitionally by modus possibilitatis in the form

$$\frac{(\alpha, \beta, \pi_\Diamond); ((\alpha, \beta \models_\Diamond \alpha) \Longrightarrow \pi_\Diamond)}{\pi_1, \pi_2, \cdots, \pi_n \subset \pi_\Diamond}$$

Transitionally means, that the conclusion has to be unfolded in the sense of the operator $\subset$ definition [8]. In general, by informational modus possibilitatis it is possible to infer about possibilities of informing of interrelated entities.

9. *Modus necessitatis*, also, roots in the philosophy of modal logic [1]. Instead of $\Box A$ which means *necessarily* A (exactly, A is necessarily true), we can introduce $\alpha \models_{\text{necessarily}}$ which reads α informs necessarily. A shortcut would be $\alpha\Box$. Thus, $\alpha\Box\beta$ is read *α necessarily informs β* or also *β is necessarily informed by α*. Sometimes, $\alpha \models_\Box \beta$ is an appropriate notation for a case of possible informational transition. In modal logic, the interplay of possibility and necessity becomes the essential point of formula derivation. Thus, in modal logic, a schema embodying the idea of that *what is possible is just what is not-necessarily-not* is given by the formula

$$\Diamond A \leftrightarrow \neg\Box\neg A \quad (\text{or } \overline{\Box\overline{A}})$$

An informational transcription of this modal formula would be

$$(\alpha \models_\Diamond) \Longleftrightarrow (\alpha \not\models) \not\models_\Box$$

with the meaning 'α informs possibly' is informationally equivalent to 'α does not inform, does not inform necessarily'.

The question is again, which are the necessities of an informational entity α's arising. How can α necessarily arise? The necessity of α's arising depends on the necessary informing of α itself and on informing of an exterior entity, say β, which could necessarily impact α informationally. Thus, the basic situation is a double transition $\alpha, \beta \models_\Box \alpha$ with the meaning

$$(\alpha, \beta \models_\Box \alpha) \rightleftharpoons \begin{pmatrix}\alpha \models_\Box \alpha; \\ \beta \models_\Box \alpha\end{pmatrix}$$

Let $\nu_\Box$ mark a structure of informational necessities, where $\nu_1, \nu_2, \cdots, \nu_n \subset \nu_\Box$. These components can be detached transitionally by modus necessitatis in the form

$$\frac{(\alpha, \beta, \nu_\Box); ((\alpha, \beta \models_\Box \alpha) \Longrightarrow \nu_\Box)}{\nu_1, \nu_2, \cdots, \nu_n \subset \nu_\Box}$$

Transitionally means, that the conclusion has to be unfolded in the sense of the operator $\subset$ definition [8]. In general, by informational modus necessitatis it is possible to infer about necessities of informing of interrelated entities.

5 General Schemes of Informational Axioms

Up to now we have discussed the nature of informational axioms rooting to some extent in the tradition of mathematical logic. This tradition has its foundation in the groups I–V of propositional axioms dealt with in Subsection 3.2 and the two predicate axioms in the group VI in Subsection 3.5. The critical question is if these axioms can be generalized in a more radical way as they have been changed in an informational manner through a slight universalization of propositional operators when replacing implication $\rightarrow$, conjunction $\wedge$, disjunction $\vee$, equivalence $\equiv$ and negation $^{-}$ by informational implication $\Longrightarrow$, parallel operator ';' (semicolon), operand separator ',' (comma), informational equivalence $\Longleftrightarrow$ and operator of a (certain) noninforming $\not\models$, respectively. In case of predicates, the predicate expressions $\forall\alpha$ and $\exists\alpha$ representing the application of the universal and existential quantifier, must be replaced by informational operators of the form $\models_{\forall\alpha}$ and $\models_{\exists\alpha}$, respectively, where the concerned entity α had appeared in the operator subscript. At this occasion, the question of the informational nature of logical (predicate) quantifiers had come to the surface as a reflection which, possibly, may demand a deepened (more critical) informational treatise.

5.1 Informing versus Informational Implication

Implication belongs to the most significant traditional logical concepts. In this respect, the role of implication is revealed or hidden practically in any logical axiom and inference rule. Implication is the basis of the traditional logical calculus.

In informational calculus, the role of implication can be generalized. By generalization, the operator of informational implication, $\Longrightarrow$, is replaced by the most general informational operator (joker), $\models$. In this case, the concluding from one true situation to the other can be replaced by informing from one informing situation to the other. In the uttermost situation, this kind of the concluding informing can be comprehended as a general type of inference in which the traditional if-thenism (implicativeness, causality, consequentiality) is replaced by the most general informational impacting in the sense of informing (informational externalism) and observing (informational internalism), that is, in a form of informational transition occurring (happening) between the informing entity (emitter) on one side and the observing entity (receiver) on the other side. The informational transition between the informer and observer can be discussed in the form of an informational operator decomposition where one part of this decomposition belongs to the informer and the other part of decomposition to the observer. This kind of transitional decomposition between the informer and observer can be understood as the operator composition between two operands.

Let us discuss the axiom groups I–V in Subsection 3.2 and the group VI in Subsection 3.5 under the most general informational circumstances. There is:

I. General axioms of informing
1) $\alpha \models (\beta \models \alpha)$;
2) $(\alpha \models (\alpha \models \beta)) \models (\alpha \models \beta)$;
3) $(\alpha \models \beta) \models ((\beta \models \gamma) \models (\alpha \models \gamma))$

II. General axioms of parallelism
1) $(\alpha; \beta) \models \alpha$;
2) $(\alpha; \beta) \models \beta$;
3) $(\alpha \models \beta) \models$
$((\alpha \models \gamma) \models (\alpha \models (\beta; \gamma))$

III. General axioms of alternativeness
1) $\alpha \models \alpha, \beta$;
2) $\beta \models \alpha, \beta$;
3) $(\alpha \models \beta) \models$
$((\beta \models \gamma) \models (\alpha, \beta \models \gamma))$

IV. General axioms of equivalence
1) $(\alpha \models_{\equiv} \beta) \models (\alpha \models \beta)$;
2) $(\alpha \models_{\equiv} \beta) \models (\beta \models \alpha)$;
3) $(\alpha \models \beta) \models ((\beta \models \alpha) \models (\alpha \models_{\equiv} \beta))$

V. General axioms of noninforming
1) $(\alpha \models \beta) \models$
$((\beta \not\models; \not\models \beta) \models (\alpha \not\models; \not\models \alpha))$;
2) $\alpha \models ((\alpha \not\models; \not\models \alpha) \not\models; \not\models (\alpha \not\models; \not\models \alpha))$
3) $((\alpha \not\models; \not\models \alpha) \not\models; \not\models (\alpha \not\models; \not\models \alpha)) \models \alpha$

VI. General functional axioms
1) $(\alpha \models_{\forall\alpha} \varphi(\alpha)) \models \varphi(\beta)$;
2) $\varphi(\beta) \models (\alpha \models_{\exists\alpha} \varphi(\alpha))$

These axioms can be commented in several ways. Axiom (I.1) is evidently a satisfactory one. It says that entity α has, in general, its informer β and that this fact is informed by α itself. If one takes data δ, (I.1) becomes

$$\delta \models (\beta \not\models \delta)$$

Data informs as a fact and cannot be informed. Operator $\not\models$ is nothing else than a particular informing, that is, non-informing. Comments to other axioms can become not only very challenging but also provocative, controversial and novel for the usual understanding.

5.2 Generalized Initial Informational Axioms

The very initial informational axioms will always remain within the scope of the informationally possible (the informationally arising). Which could be the most universal (initial, original) informational axioms? To answer this question one must certainly consider the specific informational phenomenalism with its four basic phenomenal forms which are the externalism, internalism, metaphysicalism and phenomenalism. These axioms have not the usual implicative form but the universally informing one.

Some of the most general informational axioms are those concerning an informational entity's basic phenomenalism. For instance, axiom

$$\alpha \models ((\alpha \models) \models \alpha)$$

can be read in the following way: "Operand α informs in such a way that it is informed how does it inform." Axiomatic formula

$$\alpha \models ((\models \alpha) \models \alpha)$$

can be interpreted as: "Operand α informs in such a way that it is informed how it is being informed." Axiomatic formula

$$\alpha \models ((\alpha \models \alpha) \models \alpha)$$

can be read as: "Operand α informs in such a way that it is informed how does it inform and how is it informed within itself." At last, axiomatic formula

$$\alpha \models ((\alpha \models; \models \alpha) \models \alpha)$$

can mean: "Operand α informs in such a way that it is informed how does it inform and how is it informed as such."

The listed four general initial axioms, proceeding from (I.1), are structured in a strictly observer situation when the main operator $\models$ directly follows the leftmost operand α. The other possibility would be a strictly informer situation when the main operators precedes the rightmost operand. For this case, the listed four general axioms become

$$\begin{array}{l}(\alpha \models (\alpha \models)) \models \alpha; \\ (\alpha \models (\models \alpha)) \models \alpha; \\ (\alpha \models (\alpha \models \alpha)) \models \alpha; \\ (\alpha \models (\alpha \models; \models \alpha)) \models \alpha\end{array}$$

The point of informer philosophy is that, within the informational transition between α as informer and α as observer (an α-circular case), the entire additional information $[\alpha \models, \models \alpha, \alpha \models \alpha$, and $(\alpha \models; \models \alpha)]$, is on the informer side.

5.3 Generalized Inferential Informational Axioms

In a similar way as the basic axioms, it is possible to generalize the well-known inferential axioms, e.g., modus ponens, modus tollens, modus rectus, etc. A list of such generalizations of informational inference axioms would be the following:

GMP. $(\alpha; \alpha \models \beta) \models \beta;$

GMT. $(\alpha \models \beta; (\beta \not\models; \not\models \beta)) \models (\alpha \not\models; \not\models \alpha);$

GMR. $(\alpha; (\alpha \models_{\text{intend}} \beta) \models \iota_{\text{intention}}(\alpha))) \models ((\iota_{\text{intention}}(\alpha) \models; \models \iota_{\text{intention}}(\alpha)) \subset \beta);$

GMO. $\begin{pmatrix} \alpha_{\text{absurd}}(\alpha) \subset \alpha; \\ (\alpha_{\text{absurd}}(\alpha) \models; \models \alpha_{\text{absurd}}(\alpha)) \models \beta \end{pmatrix} \models (\alpha_{\text{absurd}}(\alpha) \not\models; \not\models \alpha_{\text{absurd}}(\alpha)) \subset \beta;$

GMPr. $(\mathfrak{g}_{\text{goal}} \subset \alpha; \alpha \models \mathfrak{g}_{\text{goal}}) \models (\alpha \models (\mathfrak{g}_{\text{goal}} \models \alpha));$

GMOp. $\left\{\begin{array}{l} (\alpha; \alpha \models \mathfrak{I}_\alpha) \models \mathfrak{I}_\alpha; \\ (\mathfrak{I}_\alpha; \mathfrak{I}_\alpha \models \mathfrak{C}_\alpha) \models \mathfrak{C}_\alpha; \\ (\mathfrak{C}_\alpha; \mathfrak{C}_\alpha \models \mathfrak{c}_\alpha) \models \mathfrak{c}_\alpha; \\ (\mathfrak{c}_\alpha; \mathfrak{c}_\alpha \models \mathfrak{E}_\alpha) \models \mathfrak{E}_\alpha; \\ (\mathfrak{E}_\alpha; \mathfrak{E}_\alpha \models \mathfrak{e}_\alpha) \models \mathfrak{e}_\alpha; \\ (\mathfrak{e}_\alpha; \mathfrak{e}_\alpha \models \alpha) \models \alpha \end{array}\right.$

GMV. $\begin{pmatrix} (\sigma_\alpha(\beta) \subset (\beta \models \alpha)); \\ ((\sigma_\alpha(\beta) \models (\alpha \models \alpha)) \models \\ \beta_{\text{behavior}}(\alpha, \beta)) \end{pmatrix} \models \beta_{\text{behavior}}(\alpha, \beta);$

GMPo. $((\alpha, \beta, \pi_\Diamond); ((\alpha, \beta \models_\Diamond \alpha) \models \pi_\Diamond)) \models (\pi_1, \pi_2, \cdots, \pi_n \subset \pi_\Diamond);$

GMN. $((\alpha, \beta, \nu_\Box); ((\alpha, \beta \models_\Box \alpha) \models \nu_\Box)) \models (\nu_1, \nu_2, \cdots, \nu_n \subset \nu_\Box)$

This list of inference-axiom formulas includes the modi 1–9, discussed merely in a superficial (conceptually possible) way in Subsection 4.3. The main operators between the premise and consequence of each rule can be particularized by the use of the so-called detachment operator which, from case to case, can be specifically decomposed, according to the transition between the premise system (operand) and the consequence system (operand). Any inference transition from the premise to the conclusion can be understood to be a real informational discourse which underlies the concept of regular informational arising (phenomenalism) in the scope of principles of circularity, spontaneity, serialism, parallelism, etc.

6 Axioms and Theorems of Decomposition within Informational Calculus

The philosophy of informational decomposition of informational entities brings to the surface a substantial matter of deconstruction of entities which is not only semantic but causes semantic changes also through syntactic possibilities of formula-representing entities. Informational decomposition is a term which replaces the traditional concepts of induction, deduction, abduction and the role of inference. Decomposition means that we are usually concerned with general entity concepts which have to be decomposed into greater and greater detail. Decomposition also has the meaning of interpretation when formulas are added to the already informationally determined formula systems. Decomposition follows the informational possibilities of analysis with synthesis of formulas, by modeling, describing and realizing actively a certain informational system.

6.1 Axioms of the Informational Particularism and Universalism

Particularizing and universalizing the existing informational formulas means to decompose (determine, concretize, supplement) them into greater (more adequate, realistic) details. For example, formula $\alpha \models \beta$ can be decomposed in several ways, that is, in respect to operand α, operator $\models$, and operand β. Such decompositions can be simple and also very complex. We have learned about two possibilities of operator decomposition using operator frames in Subsection 4.3, discussing the informational modus vivendi.

Particularization and universalization as a decompositional procedure performs as a local transformation in the sense of analyzing and synthesizing informational operands and operators by bringing to the surface essential and possible informational details and building them into existing formula structures. Both operands and operators can be universalized (generalized) and particularized (individualized), directly or interpretively in parallel, by operand decomposition and operator composition, that is in several possible ways.

6.2 Axioms of Informational Serialism

Informational serialism springs from the question how could an informational entity be decomposed in a serial form, to determine the serial structure (components) concerning the entity. The basic axiom on this way is

$$\frac{\alpha}{\alpha \models \beta};\ \beta \subset \alpha$$

which is recursive regarding β. Such a transitive recursiveness transits into a system of the form

$$\frac{\alpha}{\alpha \models \alpha_1};\ \frac{\alpha_1}{\alpha_1 \models \alpha_2};\ \cdots;\ \frac{\alpha_{n-1}}{\alpha_{n-1} \models \alpha_n};$$
$$\alpha_1, \alpha_2, \cdots, \alpha_{n-1}, \alpha_n \subset \alpha$$

By the strict substitution from the beginning to the end (from the left to the right), one of the resulting axioms of informational serialism becomes

$$\frac{\alpha}{\alpha \models (\alpha_1 \models (\alpha_2 \cdots \models (\alpha_{n-1} \models \alpha_n) \cdots))};$$
$$\alpha_1, \alpha_2, \cdots, \alpha_{n-1}, \alpha_n \subset \alpha$$

In fact, instead of a single decomposition formula, there can exist various decomposition formulas of length $\ell = n$, where operands α_i $(i = 1, 2, \cdots, n)$ and to them belonging operators preserve their places and only the parentheses pairs are displaced in all possible manners.

On the other side, an informing entity α can form a serial composition with exterior components α_i, where $\alpha_i \not\subset \alpha$.

6.3 Axioms of Informational Parallelism

Informational parallelism emerges from the question how could an informational entity be decomposed in a parallel form, to determine the parallel structure (components) concerning the entity. The basic axiom in this direction is

$$\frac{\alpha}{\alpha; \alpha_i}; \frac{\alpha_i}{\alpha_i; \alpha_i \models \alpha_j}; \alpha_i, \alpha_j \subset \alpha$$

which is recursive regarding α_i and α_j. Such a transitive recursiveness transits, among other possibilities, into a parallel inference system of the form

$$\frac{\alpha}{\alpha; \alpha \models \alpha_1}; \frac{\alpha_1}{\alpha_1; \alpha_1 \models \alpha_2}; \cdots; \frac{\alpha_{n-1}}{\alpha_{n-1}, \alpha_{n-1} \models \alpha_n}; \\ \alpha_1, \alpha_2, \cdots, \alpha_{n-1}, \alpha_n \subset \alpha$$

By such and such application of the listed inferential rules, one of the formulas for the parallel decomposition becomes

$$\frac{\alpha}{\begin{pmatrix} \alpha; \alpha_i; \alpha_i \models \alpha_j; \\ i, j = 1, 2, \cdots, n \end{pmatrix}}$$

In fact, instead of a single decomposition formula, there can exist various formulas for parallel decomposition of different lengths regarding the participating components α_i and transitions $\alpha_i \models \alpha_j$, which can be mixed in all possible manners. If components α_i are interior to α, there is $\alpha_i \subset \alpha$. This condition [8] brings various possibilities for the decomposition-rule construction.

On the other side, an informing entity α can form a parallel composition with exterior components α_i, where $\alpha_i \not\subset \alpha$.

6.4 Axioms of Informational Circularity

The axiomatic view of informational circularity can stay within the framework of informational serialism. The only difference is that the first and the last operand in a serial formula are the same.

The most important case of informational circularity seems to be the metaphysicalistic one in which the participating components belong to the main circular entity. This does not mean that circularity cannot perform in an exterior way when information informed by an information source returns to it, adequately transformed by the exterior informational entities. Thus, evidently,

$$\frac{\alpha}{\alpha \models (\alpha_1 \models (\alpha_2 \cdots \models (\alpha_{n-1} \models (\alpha_n \models \alpha)) \cdots))}; \\ \alpha_1, \alpha_2, \cdots, \alpha_{n-1}, \alpha_n \subset \alpha$$

could be one of other possible rules for serial circular decomposition. In a parallel case, the canonical circular decomposition

$$\frac{\alpha}{\begin{pmatrix} \alpha \models \alpha_1; \alpha_i \models \alpha_{i+1}; \alpha_n \models \alpha \\ i = 1, 2, \cdots, n \end{pmatrix}}$$

could be one of the parallel characteristic cases.

A particular and the most interesting case arises with the question of the circular causal informing. In this case, several other rules can be constructed which even the participation of the components (entities, operands, formulas) within a causal loop. Causal problems of informing are studied fundamentally and will be treated on some other places[2].

6.5 Decomposition and Composition Theorems of Informational Calculus

According to deduction theorems of propositional and predicate calculus (Subsection 3.4 and Subsection 3.7, respectively), it is possible to list some fundamental axioms and theorems of the informational calculus (see also Subsubsection 4.2.7). But, decomposition theorems of informational calculus can also become inferential, and new modi informationis can be derived according to informational axioms also beyond the discussed inferential rules in Subsection 4.3. In distinction from inference rules in traditional logic, all of which are axiomatically determined, the general informational theory enables a decompositional rise of inference rules, which become an essential source of the possibilities of informational arising of entities (informational operands and operators, the last ones by a procedure of decomposition).

Decomposition is a much broader term than are, for instance, deduction, induction and abduction for it not only unites them but adds new possible principles of the spontaneously arising in the sense of modi informationis.

Decomposition and Composition Theorem. *If formula β is informationally derivable from formulas α_1, α_2, $\cdots$, α_n, then*

[2]Causal informing of serially and in parallel decomposed informational entities is studied by the author in the paper entitled "Causality of the Informational" which is in preparation and will appear soon.

$$\alpha_1 \models (\alpha_2 \models (\cdots(\alpha_n \models \beta)\cdots))$$

is an informationally regular serial decomposition or composition formula and

$$\begin{array}{l} \alpha_1 \models \alpha_2; \\ \alpha_2 \models \alpha_3; \\ \cdots \\ \alpha_{n-1} \models \alpha_n; \\ \alpha_n \models \beta \end{array}$$

is an informationally regular parallel decomposition or composition formula system. □

By induction it is possible to show the following: if

$$\alpha_1, \alpha_2, \cdots, \alpha_{n-1}, \alpha_n \twoheadrightarrow \beta$$

then

$$\alpha_1, \alpha_2, \cdots, \alpha_{n-1} \twoheadrightarrow (\alpha_n \models \beta)$$

where symbol $\twoheadrightarrow$ is used for marking a derived formula within the informational calculus.

A Decomposition Theorem Concerning Informational Gestalt of a Serial Formula. *Let $\alpha \models (\alpha_1 \models (\alpha_2 \models (\cdots(\alpha_{n-1} \models \alpha_n)\cdots)))$ be a serial formula, where $\alpha_1, \alpha_2, \cdots, \alpha_n \subset \alpha$. It is to stress that the serial formula of length $\ell = n$ can also have arbitrarily distributed parentheses pairs and we mark it by $\sigma^{\rightarrow}(\alpha, \alpha_1, \alpha_2, \cdots, \alpha_n)$. Let mark by $\Gamma^n_{\rightarrow}(\alpha)$ the so-called informational gestalt assigned to a serial formula of length*[3] *n, informing from the left to the right, and let this gestalt be a parallel informational system of all possible formulas of length n. Then,*

$$\frac{\sigma^{\rightarrow}(\alpha, \alpha_1, \alpha_2, \cdots, \alpha_n)}{\Gamma^n_{\rightarrow}(\alpha)}$$

where system $\Gamma^n_{\rightarrow}(\alpha)$ includes

$$\frac{1}{n+1}\binom{2n}{n}$$

formulas of length n. □

This theorem will be proved on some other place[4].

7 A Preliminary Catalogue of Informational Rules of Decomposition

Rules which can be used in a decomposition (composition) process can be listed by a short catalogue. From these rules more complex and particularized rules can be constructed being adapted to a concrete (semantic, interpretative, arising) situation (possible deconstruction). The presented rule catalogue has to be considered as a preliminary one.

7.1 Basic Phenomenalistic Rules

An informational entity α can be decomposed in four basic forms, which are externalistic, internalistic, metaphysicalistic and phenomenalistic, respectively, that is,

$$\frac{\alpha}{\alpha \models}; \quad \frac{\alpha}{\models \alpha}; \quad \frac{\alpha}{\alpha \models \alpha}; \quad \frac{\alpha}{\alpha \models; \models \alpha}$$

This initial system of transformation (decomposition) rules offers the necessary alternatives by which the informational scope (interpretation) of an informational entity is extended.

An essential comment to the basic phenomenalistic rules concerns operand α which represents any informational entity in the following sense: if α appears in a rule several times, then each appearance could be comprehended as another entity, say, β, γ, etc. Such cases can become informationally legal in several decomposition processes of α's decomposition. Thus, also,

$$\frac{\alpha}{\beta \models}; \quad \frac{\alpha}{\models \beta}; \quad \frac{\alpha}{\alpha \models \beta}; \quad \frac{\alpha}{\beta \models \alpha}; \quad \frac{\alpha}{\beta \models \gamma};$$
$$\frac{\alpha}{\alpha \models; \models \beta}; \quad \frac{\alpha}{\beta \models; \models \beta}; \quad \frac{\alpha}{\beta \models; \models \gamma}$$

etc., derived from basic phenomenalistic rules, are legal and senseful. Another problem is the so-called mutual independence of axioms which may not concern the catalogued informational rules at all.

[3] The length ℓ of an informational formula is always determined by the number of the occurring binary informational operators of the type $\models$. In the debated case, the number of occurring operands in the serial formula is $\ell + 1$. Thus, the length of the formula is n, where there are n binary operators and $n+1$ operands.

[4] Gestalts as informational entities are examined exhaustively by the author within the study entitled "Informational Frames and Gestalts" which is in preparation and will be issued soon.

7.2 A Formal Extension of the Basic Phenomenalistic Rules

The basic transformation rules in the preceding subsection can be systematically permuted regarding to the externalistic, internalistic, metaphysicalistic and phenomenalistic case, respectively. There is

$$\frac{\alpha \models}{\alpha}; \frac{\alpha \models}{\models \alpha}; \frac{\alpha \models}{\alpha \models \alpha}; \frac{\alpha \models}{\alpha \models; \models \alpha};$$

$$\frac{\models \alpha}{\alpha}; \frac{\models \alpha}{\alpha \models}; \frac{\models \alpha}{\alpha \models \alpha}; \frac{\models \alpha}{\alpha \models; \models \alpha};$$

$$\frac{\alpha \models \alpha}{\alpha}; \frac{\alpha \models \alpha}{\alpha \models}; \frac{\alpha \models \alpha}{\models \alpha}; \frac{\alpha \models \alpha}{\alpha \models; \models \alpha};$$

$$\frac{\alpha \models; \models \alpha}{\alpha}; \frac{\alpha \models; \models \alpha}{\alpha \models}; \frac{\alpha \models; \models \alpha}{\models \alpha}; \frac{\alpha \models; \models \alpha}{\alpha \models \alpha}$$

By these rules, premises can be replaced by the corresponding conclusions of the rules.

As one may observe, the rules

$$\frac{\alpha \models}{\alpha}; \frac{\models \alpha}{\alpha}; \frac{\alpha \models \alpha}{\alpha}; \frac{\alpha \models; \models \alpha}{\alpha}$$

act in an reductionistic way (lowering the degree of a formula complexity).

7.3 Rules of Informational Parallelism

Rules of parallel decomposition of an informational entity root in the axioms of conjunction as particular cases of informational parallelism. It is to stress that the listed rules are in no way informationally independent because the aim of the rule catalogue is to show the reader various possibilities of informational decomposition.

Parallel decomposition covers the view of the possibilities of parallel interpretation of an informational case. The rules guarantee an introduction of parallel cases which explain the respective case into more and more details and in additional manners. In this way, the informational system representing an entity grows and becomes more and more complex, that is, informationally interweaved by its parallel structure.

7.3.1 Rules of Interior Parallel Decomposition

Although the basic rule of an interior parallel decomposition of an entity α is only one, that is,

$$\frac{\alpha}{\alpha; \alpha}$$

this rule can be "multiplied" by considering the rules from Subsection 7.2.

7.3.2 Rules of Exterior Parallel Decomposition

Which formulas can be understood as a parallel performing of their components? An evident informational rule resulting from the basic axiomatic philosophy is, for instance,

$$\frac{\alpha \models \beta}{\begin{pmatrix} \alpha; \\ \beta \end{pmatrix}}$$

This mean that the rule of interior parallel decomposition of entity α, which is $\frac{\alpha}{\alpha; \alpha}$, originates from the initial metaphysicalistic nature of the entity, that is,

$$\frac{\alpha \models \alpha}{\begin{pmatrix} \alpha; \\ \alpha \end{pmatrix}}$$

Thus, rule $\frac{\alpha}{\alpha; \alpha}$ is a consequence of implication

$$\left(\frac{\alpha}{\alpha \models \alpha}; \frac{\alpha \models \alpha}{\alpha; \alpha}\right) \Longrightarrow \frac{\alpha}{\alpha; \alpha}$$

7.4 Rules of Serial Decomposition

If decomposition of an entity α means an interior deconstruction (interpretation, analysis, synthesis) of α's informational structure, the appearing entities in a decomposition formula are informational components of entity α. In the rules of the so-called canonical[5] serial decomposition of α,

[5] As mentioned in a footnote before, the canonical and non-canonical decomposition of informational entities (operands) will be discussed on some other place, that is, in *Informational Frames and Gestalts* of the author, where concepts of canonical and non-canonical formulas will be presented in detail.

$$\frac{\alpha}{\alpha \models (\alpha_1 \models (\alpha_2 \models (\cdots(\alpha_{n-1} \models \alpha_n)\cdots)))};$$

$$\frac{\alpha}{(\alpha \models \alpha_1) \models (\alpha_2 \models (\cdots(\alpha_{n-1} \models \alpha_n)\cdots))};$$

$$\vdots$$

$$\frac{\alpha}{(\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models (\alpha_{n-1} \models \alpha_n)};$$

$$\frac{\alpha}{((\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models \alpha_{n-1}) \models \alpha_n}$$

where

$$\alpha_1, \alpha_2, \cdots, \alpha_n \subset \alpha$$

This means: as soon as an entity α_i, $i = 1, 2, \cdots, n$ is introduced (comes to the surface, arises, appears) in the serial structure of α's decomposition, it is internalized through the transition formula $\alpha_i \subset \alpha$ [8]. Other forms of serial decomposition are the so-called non-canonical formulas of length n, for instance formula

$$\frac{\alpha}{(\alpha \models (\alpha_1 \models \alpha_2)) \models (\alpha_3 \models (\cdots(\alpha_{n-1} \models \alpha_n)\cdots))}$$

in which the parenthesis pairs are non-canonically distributed.

7.5 Rules of Circular Decomposition

Rules of circular decomposition consider serial, metaphysicalistic, causal, and parallel decomposition.

7.5.1 Rules of Circular Serial Decomposition

A primitive, non-trivial rule of circular serial decomposition of entity α concerns α's informing $\mathfrak{I}_\alpha$, that is,

$$\frac{\alpha}{\alpha \models (\mathfrak{I}_\alpha \models \alpha)} \quad \text{and} \quad \frac{\alpha}{(\alpha \models \mathfrak{I}_\alpha) \models \alpha}$$

Rules for a general circular serial decomposition are characterized by conclusion formulas in which the first and the last entity are marked equally. Thus, the $n+1$ canonically-serial circular formulas are

$$\frac{\alpha}{\alpha \models (\alpha_1 \models (\alpha_2 \models (\cdots(\alpha_n \models \alpha)\cdots)))};$$

$$\frac{\alpha}{(\alpha \models \alpha_1) \models (\alpha_2 \models (\cdots(\alpha_n \models \alpha)\cdots))};$$

$$\vdots$$

$$\frac{\alpha}{(\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models (\alpha_n \models \alpha)};$$

$$\frac{\alpha}{((\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models \alpha_n) \models \alpha}$$

where

$$\alpha_1, \alpha_2, \cdots, \alpha_n \subset \alpha$$

On the other side, there are still

$$\frac{1}{n+2}\binom{2n+2}{n+1} - (n+1)$$

non-canonical serial circular formulas of length $n+1$.

7.5.2 Rules of Serial Metaphysicalistic Decomposition

Metaphysicalistic decomposition considers standard interior components of an entity α which are the following: informing $\mathfrak{I}_\alpha$, counterinforming $\mathfrak{C}_\alpha$, counterinformational entity $\mathfrak{c}_\alpha$, informational embedding $\mathfrak{E}_\alpha$, and informational embedding entity $\mathfrak{e}_\alpha$. Six canonical forward-cycle (right-loop) metaphysicalistic rules are:

$$\frac{\alpha}{\alpha \models (\mathfrak{I}_\alpha \models (\mathfrak{C}_\alpha \models (\mathfrak{c}_\alpha \models (\mathfrak{E}_\alpha \models (\mathfrak{e}_\alpha \models \alpha)))))};$$

$$\frac{\alpha}{(\alpha \models \mathfrak{I}_\alpha) \models (\mathfrak{C}_\alpha \models (\mathfrak{c}_\alpha \models (\mathfrak{E}_\alpha \models (\mathfrak{e}_\alpha \models \alpha))))};$$

$$\frac{\alpha}{((\alpha \models \mathfrak{I}_\alpha) \models \mathfrak{C}_\alpha) \models (\mathfrak{c}_\alpha \models (\mathfrak{E}_\alpha \models (\mathfrak{e}_\alpha \models \alpha)))};$$

$$\frac{\alpha}{(((\alpha \models \mathfrak{I}_\alpha) \models \mathfrak{C}_\alpha) \models \mathfrak{c}_\alpha) \models (\mathfrak{E}_\alpha \models (\mathfrak{e}_\alpha \models \alpha))};$$

$$\frac{\alpha}{((((\alpha \models \mathfrak{I}_\alpha) \models \mathfrak{C}_\alpha) \models \mathfrak{c}_\alpha) \models \mathfrak{E}_\alpha) \models (\mathfrak{e}_\alpha \models \alpha)};$$

$$\frac{\alpha}{(((((\alpha \models \mathfrak{I}_\alpha) \models \mathfrak{C}_\alpha) \models \mathfrak{c}_\alpha) \models \mathfrak{E}_\alpha) \models \mathfrak{e}_\alpha) \models \alpha}$$

To these 6 rules 126 non-canonical forward-cycle metaphysicalistic rules can be added.

Six canonical backward-cycle (left-loop) metaphysicalistic rules are:

$$\frac{\alpha}{\alpha \models (\mathfrak{e}_\alpha \models (\mathfrak{E}_\alpha \models (\mathfrak{c}_\alpha \models (\mathfrak{C}_\alpha \models (\mathfrak{I}_\alpha \models \alpha)))))};$$

$$\frac{\alpha}{(\alpha \models \mathfrak{e}_\alpha) \models (\mathfrak{E}_\alpha \models (\mathfrak{c}_\alpha \models (\mathfrak{C}_\alpha \models (\mathfrak{I}_\alpha \models \alpha))))};$$

$$\frac{\alpha}{((\alpha \models \mathfrak{e}_\alpha) \models \mathfrak{E}_\alpha) \models (\mathfrak{c}_\alpha \models (\mathfrak{C}_\alpha \models (\mathfrak{I}_\alpha \models \alpha)))};$$

$$\frac{\alpha}{(((\alpha \models \mathfrak{e}_\alpha) \models \mathfrak{E}_\alpha) \models \mathfrak{c}_\alpha) \models (\mathfrak{C}_\alpha \models (\mathfrak{I}_\alpha \models \alpha))};$$

$$\frac{\alpha}{((((\alpha \models \mathfrak{e}_\alpha) \models \mathfrak{E}_\alpha) \models \mathfrak{c}_\alpha) \models \mathfrak{C}_\alpha) \models (\mathfrak{I}_\alpha \models \alpha)};$$

$$\frac{\alpha}{(((((\alpha \models \mathfrak{e}_\alpha) \models \mathfrak{E}_\alpha) \models \mathfrak{c}_\alpha) \models \mathfrak{C}_\alpha) \models \mathfrak{I}_\alpha) \models \alpha}$$

To these 6 rules 126 non-canonical backward-cycle metaphysicalistic rules can be added.

7.5.3 Rules of Circular Serial Causal Decomposition

Causal decomposition of an entity (operand) can be serial and serially circular. In the first case, the difference between the ordinary serial and serially causal is only in the nature of informational operators. In the causal case, they must perform in a causal manner, possessing the property of causality.

In a circular serial case of causal decomposition of an entity, the circularly involved entities (components of the decomposed entity) become equally entitled in respect to the cycle. This can cause the arising of components parallel loops and their gestalts. The consequence can be an enormous rise of the number of parallel cycles[6].

7.5.4 Rules of Circular Parallel Decomposition

An operand α can be circularly decomposed in a parallel way by the following rule:

$$\frac{\alpha}{\begin{pmatrix} \alpha \models \alpha_1; \\ \alpha_1 \models \alpha_2; \\ \vdots \\ \alpha_{n-1} \models \alpha_n; \\ \alpha_n \models \alpha \end{pmatrix}}$$

[6]The case of causal circularity is studied exhaustively in *Causality of the Informational*, prepared by the author.

Such a rule describes a circular graph as shown in Fig. 4.

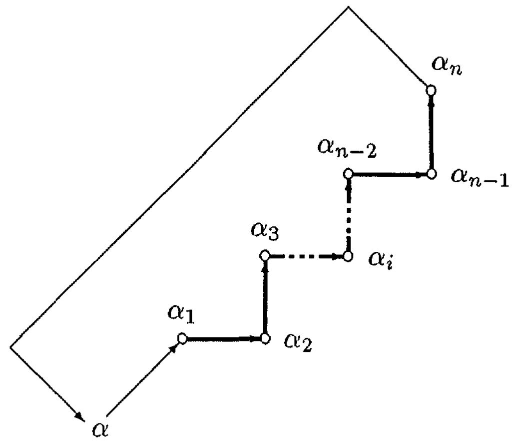

Figure 4: *A circular graph determined by the parallel system of transitions* ($\alpha \models \alpha_1; \alpha_1 \models \alpha_2; \cdots; \alpha_{n-1} \models \alpha_n; \alpha_n \models \alpha$). *The zig-zag course of connections (informational operators) between the informing components α_i of α points to a spontaneously discursive kind of informing.*

As we see, the graph in Fig. 4 can be the source of different circular interpretations, stretching from parallel circularity to serial causal circularity. It can simply mean that a parallel circularity implies all sorts of circular serial and circular causal informing with all possible circular gestalts. Within this comprehension, parallel circularity is the source of all possible rules of serial circularity within the informational calculus. For instance,

$$\frac{\begin{pmatrix} \alpha \models \alpha_1; \\ \alpha_1 \models \alpha_2; \\ \vdots \\ \alpha_{n-1} \models \alpha_n; \\ \alpha_n \models \alpha \end{pmatrix}}{\begin{pmatrix} \alpha \models (\alpha_1 \models (\alpha_2 \models (\cdots(\alpha_n \models \alpha)\cdots))); \\ (\alpha \models \alpha_1) \models (\alpha_2 \models (\cdots(\alpha_n \models \alpha)\cdots)); \\ \vdots \\ (\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models (\alpha_n \models \alpha); \\ ((\cdots((\alpha \models \alpha_1) \models \alpha_2)\cdots) \models \alpha_n) \models \alpha \end{pmatrix}}$$

is an example of the rule by which from a circular parallel system of $n+1$ simple transitional formulas an adequate canonical circular serial system (gestalt) of $n+1$ formulas is obtained. Certainly, such a parallel system can cause also the arising of a non-canonical gestalt of circular serial formulas.

The most general rule for the replacement of a circular system of parallel transitions, marked by $\pi^{\|}(\alpha, \alpha_1, \cdots, \alpha_n)$ into a parallel system of circular formulas, that is, gestalts, becomes, for instance,

$$\frac{\pi^{\|}(\alpha, \alpha_1, \cdots, \alpha_n)}{\begin{pmatrix} \Gamma_{\circlearrowright}^{i+1}(\alpha); \Gamma_{\circlearrowleft}^{i+1}(\alpha); \\ \Gamma_{\circlearrowright}^{i+1}(\alpha_j); \Gamma_{\circlearrowleft}^{i+1}(\alpha_j); \\ i, j = 1, 2, \cdots, n \end{pmatrix}}$$

where $\Gamma_{\circlearrowright}^{i+1}(\alpha)$ is the gestalt of all forward-cycle (subscript $\circlearrowright$) formulas of length $i+1$ ($i = 1, 2, \cdots, n$). Similarly, $\Gamma_{\circlearrowleft}^{i+1}(\alpha)$ is the gestalt of all backward-cycle (subscript $\circlearrowleft$) formulas of length $i+1$ ($i = 1, 2, \cdots, n$). In the case of the so-called causal circularity, components $\alpha_1, \alpha_2, \cdots, \alpha_n \subset \alpha$ have the possibility to become informers and observers in informational forward and backward loops of lengths $i+1$, etc. So, $\Gamma_{\circlearrowright}^{i+1}(\alpha_j)$ and $\Gamma_{\circlearrowleft}^{i+1}(\alpha_j)$ are the corresponding gestalts.

7.6 Rules of Alternative Decomposition

Rules of alternative decomposition embrace all discussed rules in this section if the general informational operator (joker) $\models$ is replaced by the general alternative operator (alternative joker) $=\!\mid$. By this replacement, formulas' informing(ness) operator becomes the informedness operator, that is, the operation of the Inform(s) is replaced by the operation of the Is/Are_Informed. The deduction of alternative informing originates from two different sources. The first one comes from regular informing, where the basic rule of informational transition could be

$$\frac{\alpha \models \beta}{\beta =\!\mid \alpha}$$

This rule determines nothing more than that formula $\alpha \models \beta$ can be read alternatively as β is informed by α, that is, $\beta =\!\mid \alpha$.

On the other hand, the alternative informing originates in the informational phenomenalism (Subsection 7.1), where

$$\frac{\alpha}{=\!\mid \alpha}; \frac{\alpha}{\alpha =\!\mid}; \frac{\alpha}{\alpha =\!\mid \alpha}; \frac{\alpha}{=\!\mid \alpha; \alpha =\!\mid}$$

are the principled alternative rules.

The basic alternative transformation rules can be systematically inverted regarding to the externalistic, internalistic, metaphysicalistic and phenomenalistic case, respectively. There is

$$\frac{=\!\mid \alpha}{\alpha}; \frac{=\!\mid \alpha}{\alpha =\!\mid}; \frac{=\!\mid \alpha}{\alpha =\!\mid \alpha}; \frac{=\!\mid \alpha}{=\!\mid \alpha; \alpha =\!\mid};$$

$$\frac{\alpha =\!\mid}{\alpha}; \frac{\alpha =\!\mid}{=\!\mid \alpha}; \frac{\alpha =\!\mid}{\alpha =\!\mid \alpha}; \frac{\alpha =\!\mid}{=\!\mid \alpha; \alpha =\!\mid};$$

$$\frac{\alpha =\!\mid \alpha}{\alpha}; \frac{\alpha =\!\mid \alpha}{=\!\mid \alpha}; \frac{\alpha =\!\mid \alpha}{\alpha =\!\mid}; \frac{\alpha =\!\mid \alpha}{=\!\mid \alpha; \alpha =\!\mid};$$

$$\frac{=\!\mid \alpha; \alpha =\!\mid}{\alpha}; \frac{=\!\mid \alpha; \alpha =\!\mid}{=\!\mid \alpha}; \frac{=\!\mid \alpha; \alpha =\!\mid}{\alpha =\!\mid}; \frac{=\!\mid \alpha; \alpha =\!\mid}{\alpha =\!\mid \alpha}$$

By these alternative rules, premises can be replaced by the corresponding conclusions of the rules.

As an example, the informational alternativeness enables the following implication:

$$(\alpha \models \beta) \Longrightarrow \begin{pmatrix} \beta =\!\mid \alpha; \\ \alpha =\!\mid_{\text{observe}} \beta; \\ \beta \models_{\text{observe}} \alpha; \\ \alpha \models_{\text{observingly}} \beta; \\ \beta =\!\mid_{\text{impact}} \alpha \\ \vdots \end{pmatrix}$$

The meanings of parallel formulas on the right side of $\Longrightarrow$ are not only informationally significant (logically instructive) but also various in a semantic manner and are the following:

- β is informed by α;
- α is observed by β;
- β observes α;
- α informs observingly β;
- β is informationally impacted by α;
- etc.

8 Conclusion

One of the aims of this paper is to show that the construction of an informational calculus is possible in a sufficiently (technically) rigorous way by a similar axiomatic approach known in the propositional and predicate calculus. The difference is certainly obvious: informational calculus covers much wider realm of informational possibilities which may lie beyond the propositional and predicate philosophy[7]. What could such assumption mean at all?

The author is aware that the presented construction of the informational calculus is preliminary and that still many theoretical concepts must be developed prior to a satisfactory final result. But it becomes also evident that a sufficiently developed theoretical approach builds up the fundament for something called the informational machine [11]. The main problem as seen by the author on this way is the so-called problem of informational arising which is the basic property of any informing entity. This basic property must be supported by the machine hardware and its operating system, that is automatically, when an entity begins to inform as an informational entity of the machine. And, as we have experienced during the reading of the paper, a strong support to the informational arising of any kind lies in the decomposition phenomenality of entities informing serially, in parallel, circularly, metaphysicalistically, and causally.

A great deal of the problems pointed out already in this paper will be unfolded in the context of the ongoing studies concerning:

1. informational frames and gestalts,
2. informational transition of the form $\alpha \models \beta$,
3. causality of the informational,
4. understanding and interpretation, and
5. informational memory.

By these studies, various cases of informational philosophy and formal theory will be presented and thematically rounded up with the aim to enable the concept and implementation of the informational machine.

[7] A good example of axiomatic informational possibilities is a derived axiom which considers the axioms of type I.3 in the mixed informing-implicative form, that is,

$$(\alpha \models \beta) \Longrightarrow \left((\beta \models \gamma) \Longrightarrow \left(\begin{matrix} \alpha \models (\beta \models \gamma); \\ (\alpha \models \beta) \models \gamma) \end{matrix}\right)\right)$$

This axiom is very useful and causes the so-called gestalt and causal development of formulas.

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