JET Volume 6 (2013), p.p. 13 - 18

Issue 1, February 2013

http://www.fe.um.si/en/jet/e-jet.html

DESIGN OF ELECTRICAL MACHINES BY USING CONFORMAL MAPPING

KONSTRUIRANJE ELEKTRIČNIH STROJEV Z UPORABO KONFORMNIH PRESLIKAV

Jan Šlamberger[ℜ], Peter Virtič

Keywords: Electrical machine, Conformal mapping, Slot opening, Schwarz-Christoffel transformation

Abstract

The design of electrical machines requires good working knowledge of magnetic fields in air gaps, which is very difficult or analytically unsolvable in most cases. With the development of computers, numerical methods came to the fore, enabling very good approximations of real values to be calculated. One of the major disadvantages of numerical methods is the duration of the calculations, particularly with the construction of prototypes, in which the structure changes, thus requiring more calculations. With the goal of bringing about quicker calculations, analytical methods were used, and combinations of analytical methods with numerical methods were re-started. This article will present an analytical calculation of magnetic fields using conformal mappings.

Povzetek

Načrtovanje električnih strojev med drugim zahteva dobro poznavanje megnetnega polja v zračni reži, kar pa je v večini primerov zelo zahtevno oziroma analitično nerešiljivo. Z razvojem računalnikov so prišle v ospredje numerične metode, s katerimi lahko izračunamo zelo dobre približke realnim vrednostim. Ena večjih slabosti numeričnih metod so dolgotrajni izračuni, še posebaj pri konstruiranju prototipov, kjer se konstrukcija spreminja in s tem potrebujemo več izračunov. Z željo, po čim hitrejših izračunih so se ponovno začele uporabljati analitične metode

[ℜ] Corresponding author: Asst. Jan Šlamberger B.Sc.E.E., Mailing address: University of Maribor, Faculty of Energy Technology, Hočevarjev trg 1, SI-8270 Krško Tel.: +386 3 777 0407, Fax: +386 3 777 0413, E-mail address: jan.slamberger@uni-mb.si

----

ter kombinacije analitičnih metod z numeričnimi metodami. V tem članku bo predstavljen analitičen izračun magnetnega polja s pomočjo konformnih preslikav.

1 INTRODUCTION

One of the most important necessities in the design of electrical machines is knowledge of magnetic fields in air gaps. There are several types of approaches to solving the issues of magnetic fields in air gaps; one such approach is analytical methods, a combination of analytical and numerical methods. In this paper, analytical conformal mapping is presented, which connects the symmetrical slotted air gap to the slot-less air gap. The connection between the two gaps was given by Zarko, Ban and Lipo, [1], and Gibbs, [2]. The slot opening of a symmetrical slotted air gap in the Z and W planes was given by Markovic, Jufer and Perriardin, [3], and Zhu and Howe, [4].

2 SLOT OPENING

There are four conformal transformations necessary to transform the slotted air gap into a slot-less air gap. A single slot of the original geometry is shown in Fig.1. This geometric shape needs to be transformed into its linear model in the *Z* plane, shown in Fig. 2, using a logarithmic conformal transformation defined as

$$z = \ln(s)\,, \tag{2.1}$$

where $s = m + jn = re^{j\theta}$, $z = x + jy$. The link between the coordinates in the *S* and *Z* plane is

$$x = \theta \tag{2.2}$$

and

$$y = \ln\left(\frac{r}{R_r}\right) \tag{2.3}$$

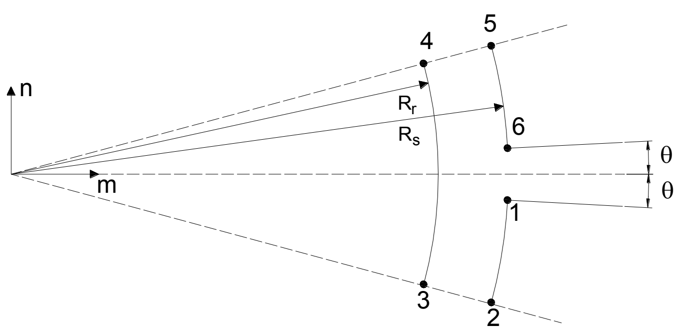

Figure 1: *Slot opening in the S plane*

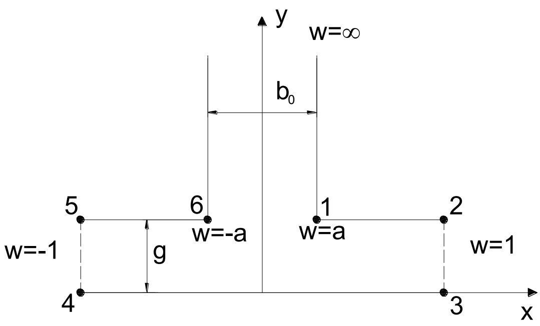

Figure 2: *Slot opening in the Z plane*

The coefficients b_0 and g are defined as

$$b_0 = 2 \cdot \theta \qquad g = \ln\left(\frac{R_s}{R_r}\right). \tag{2.4}$$

The second transformation is to transform the geometric structure in the *Z* plane into the upper half of the *W* plane, using a Schwarz-Christoffel (SC) transformation, shown in Fig. 3. In the symmetrical slotted air gap, the SC transformation will have the form

$$z = \frac{b}{j\pi}\int \frac{(w-a)^{\frac{1}{2}}(w+a)^{\frac{1}{2}}}{(w-1)(w+1)}dw. \tag{2.5}$$

The unknown coefficient *a*, which represents the values of *w* at the corner points, is defined as [4]

$$a = \sqrt{1+\left(\frac{2g}{b_0}\right)^2}. \tag{2.6}$$

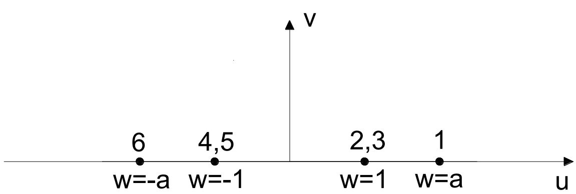

Figure 3: *Slot opening in the W plane*

The next transformation is required from the *T* plane where the field is regular to the *W* plane. The slot opening in the *T* plane represents two parallel plates extending an infinite distance in both directions, as shown in Fig. 4. The transformation from the *T* plane into the W plane is given by

----

$$t = \frac{b}{j\pi}\int \frac{1}{(w-1)(w+1)}dw \quad . \tag{2.7}$$

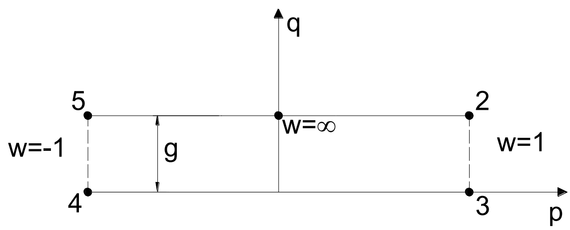

Figure 4: *Slot opening in the T plane*

The transformation of linear geometry in the *T* plane into curved geometry in the *K* plane (Fig. 5) requires an exponential function in the form

$$k = e^t \, . \tag{2.8}$$

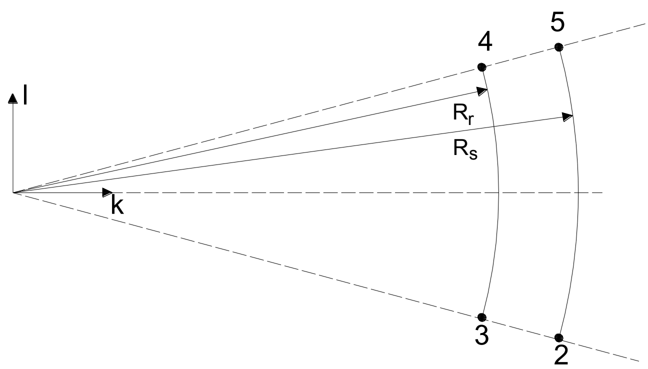

Figure 5: *Slot opening in the K plane*

3 FIELD SOLUTION IN THE SLOTTED AIR GAP

The field solution in the *K* plane, which represents a slot-less air gap, can now be mapped back to the *S* plane. The connection between magnetic field in the *K* and the *S* plane is, [1],

$$B_s = B_k \left(\frac{\partial k}{\partial s}\right)^* , \tag{3.1}$$

where $(\partial k/\partial s)^*$ conjugate value of $(\partial k/\partial s)$ is. The partial derivate $(\partial k/\partial s)$ can be expressed as

$$\frac{\partial k}{\partial s} = \frac{\partial k}{\partial t}\frac{\partial t}{\partial w}\frac{\partial w}{\partial z}\frac{\partial z}{\partial s} . \tag{3.2}$$

The partial derivatives in (3.2) are defined by conformal transformations between the corresponding complex planes

$$\begin{aligned} \frac{\partial k}{\partial t} &= e^{t} = e^{\ln k} = k \\ \frac{\partial t}{\partial w} &= \frac{b}{j\pi}\frac{1}{w^2-1} \\ \frac{\partial w}{\partial z} &= \frac{j\pi}{b}\frac{w^2-1}{\sqrt{w^2-a^2}} \\ \frac{\partial z}{\partial s} &= \frac{1}{s} \end{aligned} \tag{3.3}$$

Considering (3.2) and (3.3) in (3.1) yields

$$B_s = B_k\left(\frac{k}{s}\frac{1}{\sqrt{w^2-a^2}}\right)^{*} \tag{3.4}$$

The variables *k* and *s* can both be expressed as a function of *w*. Combining (2.1) and (2.5) yields

$$s = e^{\frac{b_0}{\pi}\left(\sin^{-1}\left(\frac{w}{a}\right)+\frac{\sqrt{a^2-1}}{2}\ln\left|\frac{\sqrt{a^2-w^2}+w\sqrt{a^2-1}}{\sqrt{a^2-w^2}-w\sqrt{a^2-1}}\right|\right)}. \tag{3.5}$$

Combining (2.7) and (2.8) yields

$$k = e^{\frac{jb}{2\pi}\ln\left|\frac{w+1}{w-1}\right|}. \tag{3.6}$$

4 CONCLUSION

This paper presents the analytical conformal mapping that connects a slotted air gap with a slot-less air gap. With this mapping, the magnetic field in slotted air gap can be transformed into a slot-less air gap, be solved and then be mapped back to a slotted air gap. This method is very useful in the early design stages of electrical machines, because it is much faster than numerical methods.

References

[1] **D. Žarko, D. Ban, T.A. Lipo:** *Analytical Calculation on Magnetic Field Distribution in the Slotted Air Gap of a Surface Permanent-Magnet Motor Using Complex Relative Air-Gap Permeance*, IEEE Trans. Magn., vol. 42, no. 7, pp. 1828–1837, 2006

[2] **W.J. Gibbs:** *Conformal Transformations in Electrical Engineering*, Chapman & Hall, 1958

[3] **M. Marković, M. Jufer, Y. Perriard:** *Determination of Tooth Cogging Force in a Hard-Disk Brushless DC Motor*, IEEE Trans. Magn., vol. 41, no. 12, pp. 4421–4426, 2005

----

[4] **Z. Q. Zhu, D. Howe:** *Instantaneous Magnetic Field Distribution in Brushless Permanent Magnet dc Motors, Part III: Effect of Stator Slotting*, IEEE Trans. Magn., vol. 29, no.1, 1993