THE DRY FRACTION OF UNSATURATED SOILS

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Abstract

An equation to account for the shear strength of unsaturated soils is proposed in this paper. This equation is defined as the equivalent stress, and is an extension of Murray's equation. This approach applies to the general case of bi-modal structured soils showing a macrostructure and microstructure. The theoretical development considers the existence of a dry fraction in addition to the saturated and unsaturated fractions of the soil. These different fractions are included in a porous model, which allows an evaluation of the parameters of the equivalent stress equation. Finally, the paper includes a comparison between theoretical and experimental results. The comparison shows that the proposed equation can be used to estimate the shear strength of unsaturated soils.

1 INTRODUCTION

The volumetric behavior and shear strength of saturated soils can be clearly explained using the principle of effective stress, as established by Terzaghi [1]. For the case of unsaturated soils, a general effective stress equation has not been established yet. Bishop's equation [2] has been used as an effective stress equation for unsaturated soils. However, there is still no consensus on how to evaluate or determine the parameter χ, which varies between 0 and 1, for the dry and the saturated conditions, respectively. Other effective stress equations have been proposed by different researchers, for example: Croney et al. [3], Skempton [4], Aitchison [5, 6], Jennings [7], and Richards [8]. These equations hold to the idea of effective stress and include empirical factors similar to Bishop's parameter χ.

For this reason, the behavior of unsaturated soils has been studied on the basis of the independent stress variables proposed by Burland [9]. In general, the net stress and the suction have been used as the two independent stress-state variables to simulate the behavior of these materials.

Recently, Bishop's equation has regained importance, since new developments in the modeling of porous structures make it possible to determine the parameter χ. Also, the hydro-mechanical coupling observed in unsaturated materials can be easily introduced in effective stress formulations. To that purpose, the influence of the volumetric deformation on parameter χ should be included. Various equations have been proposed to determine Bishop's parameter χ, for example: Öberg and Sällfours [10], Vanapalli *et al.* [11], Khalili and Khabbaz [12] and Rojas [13]. These equations include the degree of saturation and/or the suction as variables. Rojas [13] proposes the following equation.

$$\chi = f_s + (1 - f_s) S_w^u \qquad (1)$$

where f_s and S_w^u represent the saturated fraction and the degree of saturation of the unsaturated fraction, respectively.

One way to relate the suction and the degree of saturation is by means of the soil-water characteristics curves (SWCCs). There are several proposals for a general equation for the SWCCs, most of them containing empirical fitting parameters. Some of these equations have been proposed by Gardner [14], Brooks and Corey [15], Farrell and Larson [16], Van Genuchten [17], McKee and Bumb [18], Seber and Will [19] and Fredlund and Xing [20]. There are also relations oriented to determine the SWCCs from the granulometric distributions: Arya, L.M. and Paris, J. F. [21], Huang, M. [22]. Other researchers use the properties of mass and volume, such as Fredlund, M. D. [23, 24]. Sillers [25] has applied statistical methods, while Johari et al. [26] have used numerical techniques such as genetic algorithms to predict the SWCCs.

Another way to obtain both the effective stress as well as the SWCC is by means of a porous-solid model. This type of model attempts to reproduce the porous structure of soils and is able to simulate the distribution of water in the pores of the soil depending on the value suction. This property makes it possible to determine Bishop's parameter χ as a function of the saturated and unsaturated fractions of the soil (f_s, f_u); for example, the the case of equation (1), in which $f_s + f_u = 1$. The saturated fraction represents all the zones where solids are completely surrounded by saturated pores, while the unsaturated fraction is formed by solids surrounded by a combination of saturated and unsaturated pores. However, in this paper it is demonstrated that a dry fraction may eventually appear at high suctions, affecting the value of the parameter χ. The dry fraction is formed by solids completely surrounded by dry pores. In this paper, an equation of equivalent stress that includes the dry fraction is proposed.

2 EQUIVALENT STRESS EQUATION WITH THE DRY FRACTION

An unsaturated soil is formed by solid particles where the voids are occupied by two other phases: air and water. The structure of the soils depends on the shape, size and stresses applied to the solid particles. The mechanical behavior of these materials is influenced by the presence of water and its interaction with air. The water contained in most soils is present in different states: as a solid in the inner layers of the so-called adsorption layer. When the molecules of water move away from the surface of the fine particles of soil, it becomes less viscous. Farther away, the water molecules show a normal viscosity where the capillary and gravitational forces are predominant. The capillary phenomenon is produced by the meniscus of water formed mainly at the contact between the solid particles. This capillary effect produces additional contact stresses between the solid particles. These additional contact stresses need to be evaluated in order to propose an effective stress equation for unsaturated materials. One way to obtain these additional contact stresses was proposed by Murray [27]. Murray applied the concept of enthalpy to determine the coupling stress between the solid particles p'_c as a function of the total stress p, the air pressure u_a, the value of the suction s, the void ratio e and the degree of saturation S_w, in the form

$$p'_c = p - u_a + s\left(\frac{1 + eS_w}{1 + e}\right) \qquad (2)$$

It is clear that equation (2) keeps the structure of equation (3) proposed by Bishop, in which $\chi = (1 + eS_w)/(1 + e)$.

$$\sigma' = \sigma - u_a + s\chi \qquad (3)$$

where σ' is the effective stress, $\sigma - u_a$ is the net stress, s is the suction and χ is Bishop's parameter. Following Murray's proposal, the strength of the material can be represented with the following relationship

$$\frac{q}{s} = M_t\left[\frac{p'_c}{s} - 1\right] + \Lambda \qquad (4)$$

where M_t represents the slope of the line for the coordinated system $p'_c/s - q/s$, and Λ is the coordinate to the origin of such a straight line.

Notice that equation (4) becomes undetermined for the saturated case (i.e., the suction is zero). In that sense Murray's model is unable to consider the effect of the saturated fraction of the soil.

These equations were obtained by considering a mono-modal structure of soils. However, according to Sridharan *et al.* [28], most soils show a bimodal structure: consisting of a macrostructure and a microstructure. The first one is related to larger particles and inter-particle pores, also called macropores. The second one refers to smaller particles (for example, packets of clay) and intra-particle pores or micropores. By following Murray's procedure, but considering a bi-modal structured soil, Rojas [13] obtained an alternative expression for the equivalent stress as represented by Equation (5). This equation also follows the structure of Bishop's equation (3), where χ is given by equation (1).

$$p_c' = p - u_a + s\left[f_s + S_w^u\left(1 - f_s\right)\right] \qquad (5)$$

where the term $(1 - f_s)$ represents the unsaturated fraction of the soil. These unsaturated and saturated fractions naturally appear in a soil sample during a drying or wetting process. For example, at the beginning of the drying process for an initially saturated soil, all the pores are saturated and thus only the saturated fraction exists. As the suction increases, some of the largest pores start to dry and therefore some solids are surrounded by a combination of saturated and dry pores and an unsaturated fraction emerges. However, when a soil is subjected to high suctions, most of the macrostructure dries, and therefore, large particles may be completely surrounded by dry pores, and in that sense a dry fraction appears. This paper proposes an extension of Rojas's equation [13] to include the dry fraction of the soil following the procedure established by Murray to obtain a coupling stress.

Fig. 1 represents the elemental volume of a bi-modal structured unsaturated soil subjected to a total stress p. The enthalpy H_t of the sample is the product pV, where V represents the volume of the elemental volume.

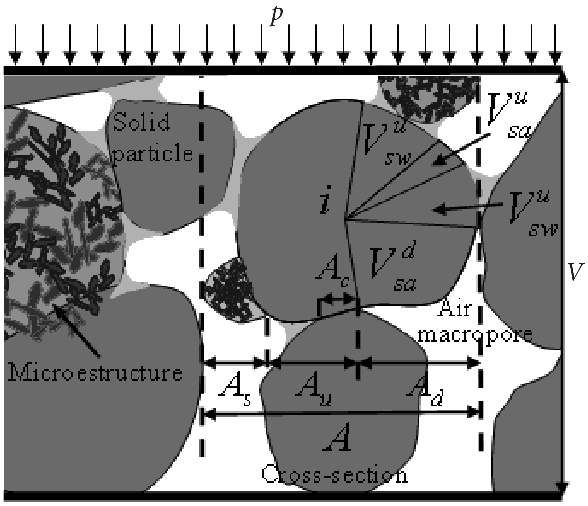

Figure 1. Soil fractions under pressure.

The total enthalpy of the sample is represented by the sum of the enthalpies of each one of its phases.

$$H_t = H_w + H_a + H_s \qquad (6)$$

where H_t is the total enthalpy, H_w is the enthalpy of the water phase, H_a is the enthalpy of the air phase, and H_s is the enthalpy of the solid phase. Following the proposal of Murray [27], and keeping the products $pV = H_t$, $H_w = u_wV_w$, $H_a = u_aV_a$, $H_s = u_sV_s + p'_cV$, it is possible to establish the following expression:

$$pV = u_wV_w + u_aV_a + u_sV_s + p'_cV \qquad (7)$$

where p is the total stress, V is the total volume, u_w is the water pressure, V_w is the volume of water, u_a is the air pressure, V_a is the volume of air, u_s is the stress in the solid particles, V_s is the volume of solids, and p'_c is the coupled stress. The coupling stress p'_c can be seen as a component of the enthalpy by unit of total volume. This stress links the independent stress variables and allows defining in a clearer way the behavior of unsaturated soils.

The volume of the sample can be divided into three parts: the saturated fraction V^s, the unsaturated fraction V^u and the dry fraction V^d in the form:

$$V = V^s + V^u + V^d \qquad (8)$$

Dividing equation (8) by V the following relationship is obtained:

$$1 = f_s + f_u + f_d \qquad (9)$$

where f_s, f_u and f_d represent the saturated, the unsaturated and the dry fractions, respectively.

Considering the unsaturated and saturated fractions according to figure 1, and using particle i as reference, equation (7) can be rewritten as follows:

$$p = \frac{u_wV_w}{V} + \frac{u_wV_{sw}^s}{V} + \frac{u_w^uV_{sw}^u}{V} + \frac{u_aV_{sa}^u}{V} + \frac{u_aV_a}{V} + u_{sc}\frac{V_{sc}}{V} + p'_c \qquad (10)$$

where $V_s = V_{sw}^s + V_{sw}^u + V_{sa}^u + V_{sc}$ is the solids' volume, V_{sw}^s is the solids' volume of the saturated fraction in contact with the water, V_{sw}^u is the solids' volume of the unsaturated fraction in contact with the water, V_{sa}^u is the solids' volume of unsaturated fraction in contact with the air, V_{sc} is the contact volume among the solid particles, $V_w = V_w^s + V_w^u$, where V_w^s is the volume of water in the pores of the saturated fraction, and V_w^u is the volume of water in the pores of the unsaturated fraction, and $V_{sc} / V \approx 0$, according to Skempton [29]. This consideration was explained by Skempton using the area-ratio concept defined as $a = A_{sc} / A$, where A_{sc} represents the area of contact between the particles and A is the total area of the cross-section, figure 1. The total area A results from the addition of saturated A_s, unsaturated A_u and dry A_d fractions, that is to say $A = A_s + A_u + A_d$. In this area, a total normal stress p, is applied. Parameter a is so small that it can be neglected for a unit thickness V_{sc} / V, meaning that the contact area between the solid particles is being neglected.

By grouping the terms with u_a and u_w in equation (10) the following is obtained

$$p_c' = p - \frac{u_w}{V}\left(V_w^s + V_w^u + V_{sw}^s + V_{sw}^u\right) - \frac{u_a}{V}\left(V_{sa}^u + V_a^u\right) \qquad (11)$$

where $V = V_w^s + V_w^u + V_{sw}^s + V_{sw}^u + V_{sa}^u + V_a^u$,
then the value $V_{sa}^u + V_a^u$ can be replaced by

$V-\left(V_w^s+V_w^u+V_{sw}^s+V_{sw}^u\right)$ resulting in

$$p'_c = p-u_a+\left(u_a-u_w\right)\left(\frac{V_w^s+V_w^u+V_{sw}^s+V_{sw}^u}{V}\right) \qquad (12)$$

The volume of the saturated fraction is $V^s=V_w^s+V_{sw}^s$, which when substituted into equation (12) and defining $f^s=\frac{V^s}{V}$ results in

$$p'_c = p-u_a+s\left(f_s+\frac{V_w^u+V_{sw}^u}{V}\right) \qquad (13)$$

Reducing the second term in the parentheses of equation (13) yields the following expression.

$$\frac{V_w^u+V_{sw}^u}{V}=\frac{V_w^u+r_{sw}^u V_s^u}{V}=\frac{S_w^u V_v^u+\frac{V_w^u}{V_v^u}V_s^u}{V} \qquad (14)$$

where $S_w^u=\frac{V_w^u}{V_v^u}$, $r_{sw}^u=\frac{V_{sw}^u}{V_s^u}\approx\frac{V_w^u}{V_v^u}$ and

$$\frac{V_w^u+V_{sw}^u}{V}=S_w^u\frac{V_v^u}{V}+S_w^u\frac{V_v^u}{V}=S_w^u\frac{V_v^u}{V}+S_w^u\frac{V_s^u}{V}=S_w^u\frac{V_v^u}{V}+S_w^u\frac{V^u-V_v^u}{V}$$
$$=S_w^u\frac{V_v^u}{V}+S_w^u f_u-S_w^u\frac{V_v^u}{V}=S_w^u f_u=S_w^u\left(1-f_s-f_d\right) \qquad (15)$$

Finally, the equation for the equivalent stress that includes the dry fraction is

$$p'_c = p-u_a+s\left[f_s+S_w^u\left(1-f_s-f_d\right)\right] \qquad (16)$$

In equation (16) the value of Bishop's parameter can be easily identified as $\chi=f_s+S_w^u\left(1-f_s-f_d\right)$. For the case of a fully saturated soil, s is substituted by u_a-u_w, $f_s=1$ and $f_d=0$ or $\chi=1$, and then equation (14) becomes Terzaghi's equation

$$p'_c = p-u_w \qquad (17)$$

For the case of a dry soil, $\chi=0$ and equation (16) transforms into

$$p'_c = p-u_a \qquad (18)$$

3 EXPERIMENTAL PROGRAM

In order to experimentally verify the proposed equation of equivalent stresses, a laboratory testing program was developed. It included the following steps: a) a mixture of 79% sand, 21% silt was made, b) the material index properties were determined, c) its grain size distribution was determined, d) a methodology to prepare compacted soil samples was established, e) using the filter paper technique, water-retention curves for wetting and drying were determined f) the porosimetry of the sample was obtained by fitting the numerical and experimental retention curves in drying and wetting, and g) triaxial tests were conducted in a controlled suction triaxial cell on samples following drying and wetting paths. Each one of these steps is explained below.

a) The sand and silt were obtained from an extraction bank located in the city of Valle de Santiago, Guanajuato, Mexico. The sand was passed through the sieve number 10, and washed to remove clay particles.
b) This material has neither plasticity nor linear shrinkage. The relative density of the solids is 2.43 and the soil was classified as SM (silty sand).
c) All the specimens were fabricated in a metallic mould by static compaction in five layers at a water content of 19.5% and a dry density of 14.889 kN/m^3. Under these conditions, the void ratio of the samples was 0.54 and their degree of saturation 87%. In order to fabricate the sample, each layer was weighed on a balance with 0.01-g precision and placed inside the mould. In order to avoid planes of contact, the surface of the previous layer was carefully scarified before placing the material for the next layer.
d) The granulometry of the material was determined using the dry and wet methods.
e) The size distribution of the macropores, sites and bonds was obtained by adjusting the water-retention curves during drying and wetting, taking into account the following: "*it is well known that the drying curve is mainly dependent on the size of bonds as they unsaturate at higher suctions than sites. On the contrary, the wetting curve depends mainly on the size of sites and macropores as they saturate at smaller suctions than bonds. Therefore, it is proposed to use of the boundary SWCC at drying in order to define the size distribution of bonds. When the porosimetry of the material is not available at all, both retention curves have to be used to define the size distribution of bonds sites and macropores. These distributions are obtained from an iterative procedure where an initially proposed size distribution for each element is consecutively modified, until the porous model reproduces with sufficient accuracy, both the wetting and the drying soil–water characteristic curves*", (Rojas *et al.* [13], p. 198).
f) The only method to obtain directly the retention curve is with the membrane apparatus. However, its suction range only reaches 10 MPa. On the other hand, the filter-paper method covers the entire range of suctions and can be easily applied for wetting and drying paths. However, mainly due to the heterogeneity of the papers during the fabrication process, it is recommended to verify the calibration every time a new batch of paper is used. That is the reason why the Sleicher and Schuell No. 589 paper batches were first calibrated with potassium chloride solutions at various concentrations. The whole process was

conducted in a controlled-temperature room where the filter paper was exposed to a saline atmosphere for a period of two weeks. Once the filter paper was calibrated, cubic samples with a side of 5 cm were made. The drying curve was obtained by introducing the compacted samples into the oven for time ranging from 1 min to 24 hours, then introduced in a sealed container along with two pieces of filter paper to measure both the total and matrix suctions. The samples were left for two weeks in a controlled--temperature room to reach equilibrium. In the case of the wetting curve the samples were initially oven dried for 24 hours and subsequently moistened using a fine spray to reach a certain degree of saturation. Once the sample reached the required humidity, the homogenization process was carried out for two weeks in an identical manner as for the drying path.

g) Triaxial tests were performed following the wetting and drying paths. The suction was controlled using the vapor-circulation technique using a peristaltic pump with an 11-ml/min flow rate, as recommended by Cunningham [30]. The soil samples were placed in the triaxial chamber, and their initial moisture content was slightly higher or lower than the moisture of the circulating vapor, so as to follow a drying or wetting path, respectively. In order to define the equilibrium time for specimens, some tests were performed previously, where the samples were mounted in the triaxial chamber and weighed at regular intervals until equilibrium was achieved. In general, it was found that three or four days were sufficient to reach equilibrium. In order to ensure constant suction tests, all the tests were performed at a strain rate of 0.001 mm/min. The confining stress for all the samples was 150 kPa. Once the triaxial tests were completed, samples were cut in sections to verify the value of the suction. Triaxial tests were performed on soil samples with water contents ranging from saturated to practically dry. For the case of the saturated soil samples, confining stresses of 50, 100 and 150 kPa were applied.

4 COMPUTATIONAL MODEL

The solid-porous model used herein is similar to that proposed by Rojas *et al.* [31], in which four types of elements were considered: the macropores, the sites, the bonds and the solids. This model is built on a regular network and includes the following elements: cavities, bonds, and solids.

Cavities contain most of the void volume of the soils. These cavities are subdivided into macropores and mesopores. The macropores are the largest pores in the soil and are responsible for most of the volumetric behavior of the soil (Simms and Yanful, [32]). The mesopores or sites are pores that do not change their size during loading. All the cavities are interconnected by the bonds or throats. Finally, the solids are considered incompressible. The model is built on a regular network, where the cavities are placed at the nodes of the network, and the connectors are the bonds. These elements are represented in Fig. 2.

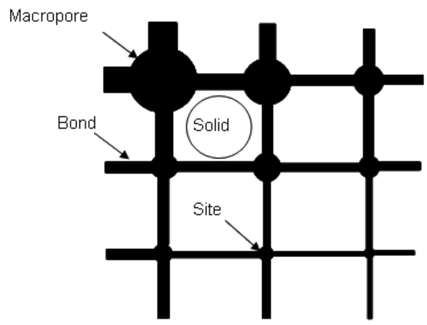

Figure 2. Network elements: macropores, sites, bonds and solids (from Rojas *et al.* [31], p. 196).

The size of the network determines the number of nodes and connectors. This defines the number of cavities, bonds and solids. The number of elements of each size is defined by the porosimetry and the grain size distribution of the material. Once the number of elements of each size is defined, sites and bonds are randomly placed on the network. In order to avoid the superposition of bonds converging on one node, a constructive principle needs to be respected. This constructive principle establishes that two bonds converging to a site at 90º should comply with expression (19).

$$\sqrt{r_{b1}^2 + r_{b2}^2} \le r_{st} \qquad (19)$$

where r_{b1} and r_{b2} are the radius of the converging bonds and r_{st} is the radius of the cavity.

To totally guarantee this principle within the network, this procedure must be followed: "*at each node, the number of violations to the principle is determined. If the number of violations at certain site is different from zero, then a substitution with another site (selected at random) is simulated. If the number of violations to the principle reduces, then the substitution is granted. If not, another site is selected at random. The same procedure is followed for bonds, and it continues until no violation to the construction principle subsists within the network*", (Rojas *et al.* [31], p. 197).

Once the sites and bonds are located, macropores are placed by substituting the required number of sites. Finally, solids are also placed at random in the spaces between the pores.

The process of drying and wetting the pores is regulated by the Young-Laplace equation (20), Defay and Prigogine [33]. This equation is a nonlinear partial differential equation that describes the capillary pressure difference sustained across the interface between two static fluids, such as water and air, due to the phenomenon of surface tension or wall tension, although usage on the latter is only applicable if we assume that the wall is very thin. The Young–Laplace equation for a spherical meniscus can be written as:

$$u_a - u_w = \frac{2T\cos\alpha}{R_c} \qquad (20)$$

where α represents the contact angle between the water and the solid particles, R_c is the critical radius and corresponds to the maximum pore size that remains saturated at a certain suction.

In addition, to comply with equation (20) a pore must also comply with the continuity principle in order to saturate or dry. This continuity principle establishes that a pore is able to dry or saturate only if it is connected to a boundary of the network where the bulk of gas or water is present, respectively, following a continuous path.

Therefore, during a drying process, pores will dry when the size of the pore is larger or equal to R_c and one of its connected bonds is already dry and connected to the bulk of gas. The drying process starts when all the pores are saturated and the suction is zero. An increase in the suction results in the drainage of the largest pores located at the boundaries of the network. The process continues only when the bonds connected to these pores dry too. This means that the drying SWCC is dependent on the size distribution of the bonds.

The saturation of pores will happen only if the pore size is smaller or equal to R_c and one of the elements connected to this pore is already saturated. During the wetting process it is considered that initially all the pores are dry and the suction is very high. Then the suction decreases in steps and the smallest bonds at the boundaries of the network start to saturate. The wetting process only continues when the sites connected to those bonds saturate. This means that the wetting process is controlled by the size distribution of the sites.

A computer program was developed to reproduce the porous structure of the soils by fitting the numerical with the experimental SWCC. The program also determines the saturated, unsaturated and dry fractions of the soil during wetting-drying cycles. Finally, the parameter χ is determined and the experimental and numerical results are compared. In general, logarithmic normal distributions are used to define the pore size distributions of soils. Logarithmic normal distributions are defined with only two parameters: the mean value and the standard deviation. Therefore, these two parameters are required for each of the elements in the network: sites, macropores, bonds and solids. Double logarithmic normal distributions are required for the case of double structured soils.

5 DISCUSSION OF THE THEORETICAL AND EXPERIMENTAL RESULTS.

Table 1 indicates the parameters required by the solid-porous model: the mean radius ($\overline{R}$) and the standard deviation (δ) for sites (SP_1 and SP_2), macropores (MP) and bonds (B_1 and B_2). Finally, the void ratio (e) of the sample is also required.

Table 1. Parameters of the solid-porous model.

Parameter	SP_1	SP_2	MP	B_1	B_2	e
$\overline{R}$	0.05	0.4	3.0	0.0004	2.0	0.54
δ	2	2	3	6	4	

The fitting of the numerical with the experimental SWCC is shown in Fig. 3. The values included in table 1 define the logarithmic normal distribution for each case. These parameters are initially proposed and then adjusted until the best fit with the experimental points is achieved. The experimental points of the SWCCs were obtained using the filter-paper method.

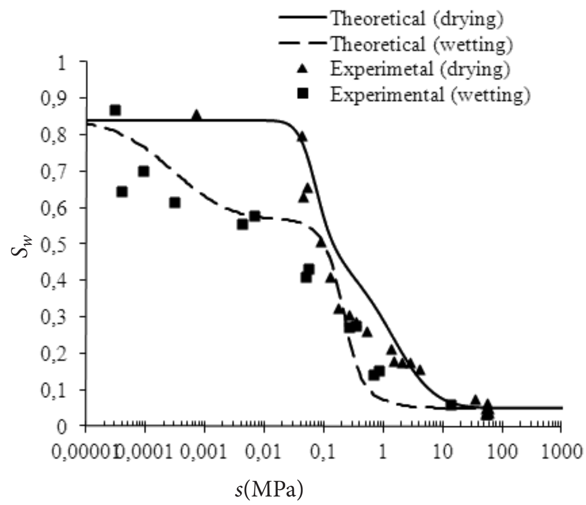

Figure 3. Numerical and experimental SWCC in drying and wetting paths (data from Leal *et al.* [34]).

The numerical pore size distribution obtained after the fitting process is shown in Fig. 4. The existence of a bimodal pore structure can be observed.

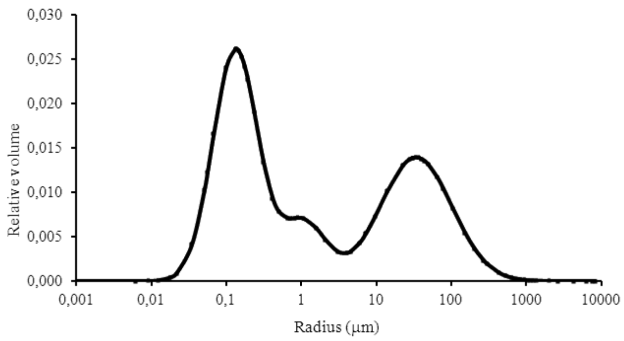

Figure 4. Theoretical pore size distribution of the soil.

In the same way, in order to obtain the number of solids of each size, the experimental grain size distribution has to be fitted with the numerical model using a single or double logarithmic normal distribution. These results are shown in Fig. 5. It is clear that the fitted curve is quite close to the experimental results.

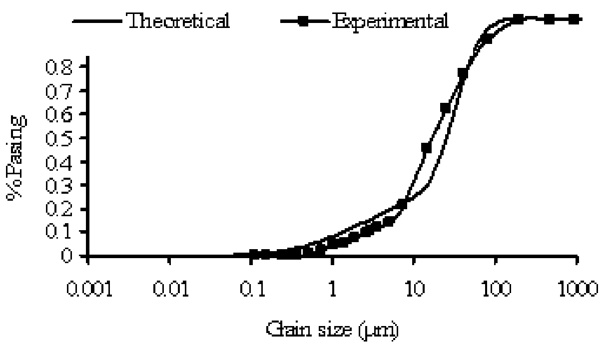

Figure 5. Theoretical and experimental grain size distribution.

With the model, the parameters f_s, f_u, f_d, and S_w^u for different degrees of saturation (S_w) are defined. These parameters are shown in Fig. 6 and Fig. 7. The volume of the saturated fraction $(f_s = 1 - f_u - f_d)$ is obtained by adding the volume of solids completely surrounded by water to the volume of voids surrounding these solids.

Fig. 6 shows the values of the dry fraction (f_d), the unsaturated fraction (f_u), and the saturated fraction (f_s) with the degree of saturation (S_w). These values were obtained for the drying and wetting paths.

It can be verified that the values shown in Fig. 6 comply with Equation (9) for any value of the degree of saturation for both the drying and wetting paths. Additionally, Fig. 7 shows the variation of the degree of saturation for the unsaturated fraction with respect to the degree of saturation for both paths.

Bishop's parameter can now be determined using the expression $\chi = f_s + S_w^u\left(1 - f_s - f_d\right)$ included in Equation (16). To obtain the experimental values of χ, the $p' - q$ diagram is plotted with the triaxial test results of saturated samples, carried out at confining pressures of 50, 100 and 150 kPa. According to the critical state theory it is possible to draw a failure line departing from the origin of the plane $p' - q$. If in this diagram the pairs of values (σ_{net}, q) of the unsaturated samples are represented, then the difference between their abscissas and the failure line represents the suction stress $s\chi$. And because the suction is known, it is possible to obtain the value of χ.

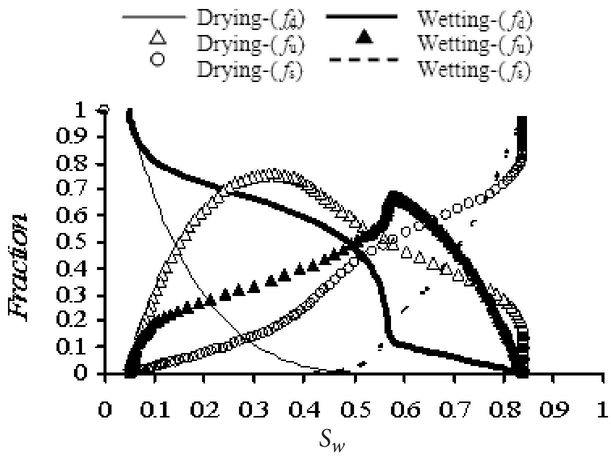

Figure 6. Theoretical values of the dry, unsaturated and saturated fractions for the drying and wetting paths.

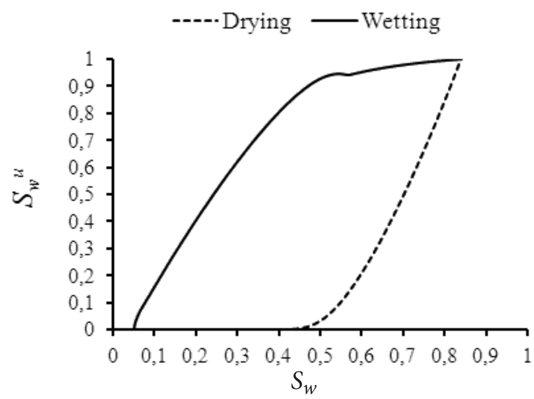

Figure 7. Theoretical values for the degree of saturation of the unsaturated fraction in drying and wetting (from Leal *et al.* [34]).

The experimental and numerical values for χ are shown in Fig. 8, for both the drying and wetting paths. In this figure, it can be observed that both the numerical and experimental values follow a general tendency, except for one experimental point obtained during wetting. "*The best results are presented in low degrees of saturation. This is because at low degrees of saturation volume changes are minor and the degree of saturation is influenced by changes in volume. One can expect some differences in the final*

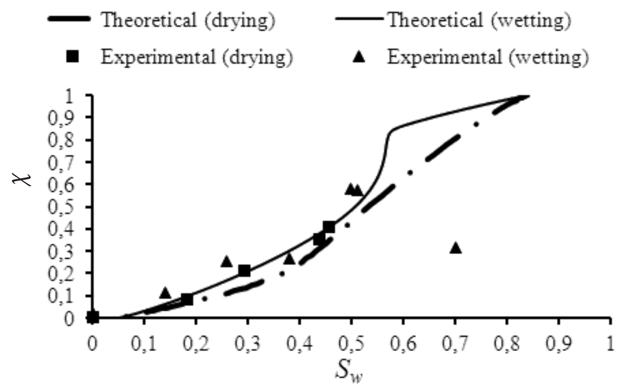

Figure 8. Variation of χ with respect to the degree of saturation (data extended, from Leal et al. [34]).

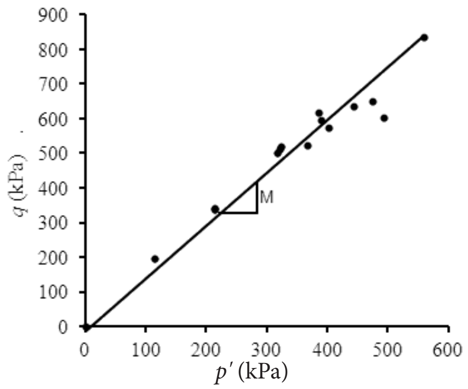

Figure 9. Numerical results of the shear strength using the solid-porous model.

results of f_s and therefore also of the resistance because the processes of wetting and drying of the solid-porous model are developed by invasion. That is, to move water or gas, continuity in the phases is required. This means that a site or a bond cannot be invaded if at least one adjacent element has not been invaded", (Leal *et al.* [34], p.401).

When the value of χ is multiplied by its corresponding suction, the cohesive stress is obtained. Once the values of the parameter χ have been determined, it is possible to estimate the equivalent stress values using the following expressions:

$$\sigma_1 = \sigma_3 + q \qquad (21)$$

$$\sigma_{i\ neto} = \sigma_i - u_a \qquad (22)$$

$$\sigma_i' = \sigma_{i\ neto} + \chi s \qquad (23)$$

In these equations, i varies from 1 to 3, σ_1 is the total vertical stress, σ_3 is the confining stress, u_a is the air pressure in the pores, q is the deviator stress applied in the triaxial tests, σ_i' is the equivalent stress and s is the suction.

All the specimens were tested up to failure according to the critical state theory. The relationship between the mean and the deviatoric stress is represented by the slope M. If the results are plotted in this plane, their alignment with the slope M can be observed, and the mean equivalent stress can be obtained using the following expression:

$$p_c' = (\sigma_1' + \sigma_2' + \sigma_3')/3 \qquad (24)$$

Following the above equations, it is possible to calculate the theoretical equivalent stress p_c', and the results are shown in Fig. 9.

Fig. 9 shows that the solid-porous model can be used to obtain Bishop's parameter χ and simulate the strength of the unsaturated soils tested at different values of suction. Finally, the inclusion of the dry fraction for the determination of Bishop's χ parameter results in a more precise description of the behavior of unsaturated soils during the wetting-drying cycles.

6 CONCLUSIONS

A general analytical equation has been established to obtain Bishop's parameter χ, that includes the saturated, the unsaturated and the dry fractions of the soil. The comparisons of the numerical and experimental results show that the proposed equation is adequate for simulating the strength of unsaturated soils. Although more comparisons are still required, the solid-porous model proposed herein to obtain the parameter χ seems to be sufficiently accurate.

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