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Renaissance Music between Science and Art: The Case of Gioseffo Zarlino

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ABSTRACT

This paper deals with the role of ancient music theory in Gioseffo Zarlino's *Istitutioni harmoniche* and, within its framework, in particular with mathematical and physical considerations and their relevance to audible music. An outline of the treatise is followed by a presentation of Zarlino's justification of music as *scienza* and *arte*. Finally, two case studies are presented on joining ancient theory with contemporary musical practice: the division of the interval and the system of the *senario*.

Keywords: Gioseffo Zarlino, music theory, Renaissance, Antiquity, senario

IZVLEČEK

Prispevek obravnava vlogo antične glasbene teorije v traktatu *Istitutioni harmoniche* Gioseffa Zarlina. V tem okviru so posebej izpostavljeni Zarlinovi matematični in fizikalni premisleki in njihov pomen za razpravo o dejanski, zveneči glasbi. Po orisu zasnove *Istitutioni* in predstavitvi Zarlinovega utemeljevanja glasbe kot *scienze* in *arte* sta podana konkretna primera povezovanja antične teorije s sodobno glasbeno prakso: delitev intervala in sistem *senario*.

Ključne besede: Gioseffo Zarlino, glasbena teorija, renesansa, antika, senario

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Renaissance writers on music based their treatises largely on the ideas of ancient music theorists and philosophers.[1] Like other scholars of the Renaissance, they took antiquity as their model: in ancient writings, they could read about what music was like in antiquity and what effects it had, and they sought ways to achieve this perfection in contemporary compositions, as well.

As is well known, in antiquity and the Middle Ages, music was regarded as one of the mathematical sciences, a *quadrivial* discipline, that is, one that studies relationships between quantities. The positioning of music among the quadrivial disciplines was perhaps most clearly defined by Boethius:

> *Now of these types, arithmetic considers that multitude which exists of itself as an integral whole; the measures of musical modulation understand that multitude which exists in relation to some other; geometry offers the notion of stable magnitude; the skill of astronomical discipline explains the science of moveable magnitude. If a searcher is lacking knowledge of these four sciences, he is not able to find the true; without this kind of thought, nothing of truth is rightly known. […] This, therefore, is the quadrivium [= four roads] by which we bring a superior mind from knowledge offered by the senses to the more certain things of the intellect.*[2]

Many ancient and medieval treatises on music therefore focused primarily on the construction of the tonal system and the study of its acoustic properties, as well as on the treatment of individual intervals expressed by mathematical ratios.

In view of their desire to revive ancient music, Renaissance music theorists seem to focus, in one way or another, precisely on an attempt to combine ancient music theory with the musical practice of the time. The fusion of ancient theory and contemporary practice is also one of the main postulates of the famous treatise *Istitutioni harmoniche*, published in 1558 by Gioseffo Zarlino (ca. 1517–1590), who is considered by many to be the central and most influential Italian music theorist of the sixteenth century. The aim of the present paper is to determine the role of ancient music theory in the *Istitutioni* and, within its framework, in particular mathematical and physical considerations and their relevance to audible music: What are mathematics and physics in the *Istitutioni harmoniche*? How and for what purpose does Zarlino use them?[3]

1 The adoption of ancient music theory by Renaissance theorists has been discussed by many authors, but for a general overview of the topic one should still consult Palisca's fundamental book: Claude V. Palisca, *Humanism in Italian Renaissance Musical Thought* (New Haven: Yale University Press, 1985).

2 Michael Masi, *Boethian Number Theory: A Translation of the* De Institutione Arithmetica (Amsterdam: Rodopi, 2006), 72–73. Underlined emphasis added by Nejc Sukljan.

3 Although in rather different contexts and with other aims, this topic has already been discussed (at least partially) by several other authors, among whom the following should be mentioned: Daniel Pickering Walter, *Studies in Musical Science in the Late Renaissance* (London: Warburg Institute London, 1978), Benito V. Rivera, "Theory Ruled by Practice: Zarlino's Reversal of the Classical System of Proportions," *Indiana Theory Review* 16 (1995): 145–170, and Robert W. Wienpahl, "Zarlino, the Senario, and Tonality," *Journal of the American Musicological Society* 12, no. 1 (1959): 27–41.

1 *Istitutioni harmoniche:* Sources, motivations, outline

The life story of Zarlino is quite interesting and still topical. If we are to believe Bernardino Baldi, who in 1595 wrote Zarlino's first biography, Zarlino's parents fled to Chioggia (a small town in the Venetian lagoon where the theorist was born) from Alexandria, near Milan.[4] Zarlino was therefore a child of newcomers, war refugees, yet later managed to obtain one of the most prestigious musical positions in Europe: in mid 1565, he was appointed *maestro di cappella* at St Mark's Basilica in Venice. The fact that he lived and worked in the very centre of the famous *Serenissima* – he moved to the city in 1541 – was decisive for him. It was here that he was able to master his compositional skills with Adrian Willaert; it was here that, as a member of the *Accademia Veneziana*, he could discuss many scientific and other topics with Venetian and other scholars; and – perhaps most importantly – it was here that he had at his disposal the vast corpus of ancient writings on music that the famous Cardinal Bessarion had previously donated to the city together with other manuscripts from his library. Besides Boethius' *Fundamentals of Music*, which was already widely known in the Middle Ages and was his main source, Zarlino was thus able to read the works of Claudius Ptolemy, Quintilian, Aristoxenus, Plutarch, Euclid and many others; he was also well acquainted with the philosophies of Plato and Aristotle. Thus, as a humanist scholar, Zarlino remained true to the Renaissance paradigm of adopting the ancient canons, and he discussed them extensively in the *Istitutioni*.

Zarlino explains why he decided to write the *Istitutioni* in its dedication and preface. Setting out from a broadly conceived human struggle for universal knowledge, in the humanistic spirit he immediately establishes a connection with the ancient philosophers: the ancient scholars agreed that the causes of things are important.[5] Therefore, man explores the origins of all being in the hope of understanding the ultimate mysteries of nature; by acquiring a knowledge of things, he strides towards perfection. Zarlino began writing the *Istitutioni* precisely because, in his opinion, music – in contrast to other sciences and arts (*le scienze e le arti*) – had not yet been fully represented: he wanted to see if he, who had studied music since his youth, could succeed in bringing both music theory (*teorica o contemplativa*) and practice (*prattica*) to perfection. To

4 Bernardino Baldi, *Le vite de' matematici Italiani* (Milano: Franco Angeli, 1998), 543.

5 Gioseffo Zarlino, *Istituzioni armoniche*, ed. Silvia Urbani (Treviso: Diastema, 2011), 3–4. Here, Zarlino obviously relies on Aristotle's concept of the principle, as presented by the philosopher at the beginning of the Book 1 of Physics: "[…] we think we know a thing only when we have grasped its first causes and principles and have traced it back to its elements." (Aristotle, *Physics I*, trans. Robin Waterfield (Oxford: Oxford University Press, 2008), 184a, 9.) In fact, Aristotle's natural-scientific or physical considerations are one of the cornerstones on which the *Istitutioni* is built; the need to understand their causes in order to fully understand things and their nature is a kind of fundamental idea, a postulate to which Zarlino often returns.

achieve this, it is necessary to know the causes of music, which in the *Istitutioni* are closely linked to the firm mathematical and physical postulates on which the tonal system, from which all music comes to be, is built.

Zarlino's discussion in the *Istitutioni* closely follows the structure of the treatise, which is in turn modelled on various considerations in Aristotle's *Physics*. Here, in Chapter 7 of Book 1,[6] the philosopher discusses the three principles of coming to be: the underlying thing (matter), privation and form, the last two being opposite (first, there is the underlying thing (matter), which takes shape when a thing comes to be). It is precisely to these principles that Zarlino refers when he explains that the *Istitutioni* is divided roughly into two parts, theoretical (contemplative) and practical.[7] However, since all things, whether natural or artificial, are made of matter and form, each of the two parts is treated appropriately in both respects and, consequently, divided into a further two parts, so that there are four in all.[8] Since matter can only be known by its form, the first part will initially present numbers and ratios that form consonances. In the second part, the tones that are their matter are discussed.[9] The structure of the second part of the treatise is based on the same principles, albeit somewhat differently justified and reversed. Zarlino states that musical practice is nothing other than the realisation of music and its purpose through compositions that are artificial, since they are composed through the art called counterpoint or composition. Compositions, too, have both matter and form: when a composer wants to write something, he first chooses the matter and then creates a suitable form for it. This is why the third part discusses the consonances and intervals that are the matter of compositions, while the fourth part deals with their form: the way music should accompany words is explained.[10]

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6 Aristotle, *Physics I*, 189b–191a, 24–28.

7 Zarlino, *Istituzioni armoniche*, 10. Among other things, Zarlino's understanding of music theory and practice is evident from the presented division. For him, music theory refers exclusively to speculative considerations on music, while the theory of counterpoint (which is now generally understood as a theoretical discipline) is seen as musical practice. Such a definition is derived from the final results of both: the theory of counterpoint, in its implementation, leads to a certain product, to an actual composition, which is why it is defined as musical practice. On the other hand, there is no tangible product in speculation: such considerations only define and describe music and, above all, examine the tonal space in which it evolves.

8 Zarlino, *Istituzioni armoniche*, 11–12.

9 That numbers and ratios are forms of consonances means that consonances exist (appear) as numbers and the ratios between them. Furthermore, consonances are a natural thing, and in the case of the latter, according to Aristotle, matter (or the underlying thing) can only be known by its form: through that which already exists, which has already come to be as a result of the process of coming to be. The matter of consonances, which we can approach only through form, are thus tones.

10 Actually, the third and fourth parts of the treatise are much more broadly conceived. At the beginning of the third part, individual intervals are indeed presented, but by far the largest part of the discussion is then devoted to the presentation of the theory of counterpoint. However, if we borrow Aristotle's categories presented above, this is neither the matter nor the form of compositions.

2 Music as *scienza* and *arte*

It is clear from the outline of the *Istitutioni* that the aim of the first half of the treatise (the first two parts) is a speculative representation of the tonal system in which the actual practical music evolves. The latter is then discussed in the second half of the treatise (the last two parts). Such an arrangement of content reflects Zarlino's fundamental postulate that it is necessary to combine the theoretical (speculative) principles of musical science with practical musical activity:

> *And even if speculation in itself does not seem to require practical implementation, a speculative scholar would not be able to effectuate any of the new things he has discovered without the help of an artist or an instrument. Therefore, even if such speculation were justified, it would not bear fruit, since it would not achieve its ultimate goal, which can only be achieved through the use of natural or artificial instruments. On the other hand, without the help of reason the artist would never achieve a perfect performance. Consequently, in music (if we imagine it in its perfection) these two fields are so closely connected that, for the reasons mentioned above, they cannot be separated.*[11]

The fact that speculative music without its practical component is of little value and imperfect can also be observed in many theorists who were not familiar with musical practice. Zarlino notes: they said a lot of nonsense and made many mistakes. Similarly, those who only deal with practical music and refuse to learn about any causes (*ragione*) have written many stupidities in their compositions.[12]

In Zarlino's opinion (which is fundamental to understanding the entire *Istitutioni* and is a kind of a cornerstone on which the treatise is built), music therefore consists of two components, scientific (speculative or contemplative music) and artistic (practical music); it is *scienza* and *arte*. However, although these two components are inseparable and cannot function properly without each other, it seems that Zarlino puts science first, which is also ontologically

Rather, its realisation is the actual process of coming to be, the moment when given matter (intervals and chords) acquires its form (tones and text arranged in a composition). Similarly, the discussion of the setting of text to music is only a small part of the fourth book of the *Istitutioni*, the main part of which is devoted to the treatment of ancient and modern modes. However, these are – if we judge them according to Zarlino's criteria – more matter (which the composer chooses before starting to compose) than form.

11 "E quantunque la speculazione da per sé non abbia dibisogno dell'opera, tuttavia non può lo speculativo produr cosa alcuna in atto ch'abbia ritrovato nuovamente, senza l'aiuto dell'artefice overo dell'istrumento; percioché tale speculazione, se ben ella non fusse vana, parrebbe nondimeno senza frutto, quando non si riducesse all'ultimo suo fine che consiste nell'essercizio de' naturali e arteficiali istrumenti, col mezo dei quali ella viene a conseguirlo, come ancora l'artefice senza l'aiuto della ragione mai potrebbe condurre l'opera sua a perfezione alcuna. E perciò nella musica (considerandola nella sua perfezione) queste due parti sono tanto insieme congiunte che per l'assegnate ragioni non si possono separare l'una dall'altra." Zarlino, *Istituzioni armoniche*, 56.

12 Zarlino, *Istituzioni armoniche*, 139.

justified: without their implementation in actual, audible music, speculative considerations may not achieve their final goal and are an end in themselves, but they can still exist, since speculation in itself does not require any practical activity. On the other hand, it is clear that the existence of practical music is not possible without a firm theoretical basis (as a part of which the tonal system is primarily contemplated). Consequently, any musical activity necessarily results from speculative considerations, which in Zarlino's musical thought are based precisely on the ancient theoretical tradition. In the *Istitutioni*, music is primarily defined as an exact scientific discipline (*scienza*) that can be accurately described and its causes determined through scientific observation. Nevertheless, it is only completed through the art (*arte*) of counterpoint, which is its inseparable and necessary component.

According to Zarlino, music as *scienza* emerged at its earliest beginnings.[13] Even before the biblical flood, it was invented by Jubal, a descendant of Cain, using the sound of hammers. It was then lost in the flood, but was reinvented by Mercury, who was the first to observe the paths of the stars, the harmony of song, and the numerical ratios. The ratios in music were then studied by Pythagoras, and his successors developed a perfect and exact science (*perfetta e certa scienza*). Like Aristotle in natural science (*filosofia naturale*), they established fixed rules, with each successive generation correcting the mistakes of the previous one. In this way, musical science became so exact that it was affirmed as mathematical science and thus gained access to the highest level of truth (*primo grado di verità*). A thought is then borrowed from Boethius' *Fundamentals of Arithmetic*: mathematics is nothing other than the ability to recognise the truth (*capacita di verità*) about being, which by its very nature is unchangeable. Its accuracy is so great that mathematicians can use numbers to describe celestial movements, different positions of the planets, lunar and solar eclipses, and many other things unambiguously and without the slightest disagreement between them.[14]

Music is particularly close to the other quadrivial disciplines: arithmetic, astronomy and geometry. Indeed, the subject of music is sounding number (*numero sonoro*),[15] which can be experienced (heard) with the help of sounding bodies (*corpi sonori*), which is why musical science is subordinate to both arithmetic and geometry.[16] According to the adopted views of Aristotle, mu-

13 Zarlino, *Istituzioni armoniche*, 14–18.

14 Ibid., 17–18.

15 The concept of sounding number is further discussed and explained below.

16 Zarlino explains that there are two kinds of sciences: (1) basic (*principali*) and superior (*subalternanti*) and (2) secondary (*non pricipali*) and subordinate (*subalternate*). The first are those whose principles (*principii*) are approached through both reason (*lume naturale*) and sensory perception; these include arithmetic and geometry, to which sensory perception is also important, but only to a certain extent. Secondary and subordinate sciences are those which, in addition to their own principles, are characterised by several principles of the superior sciences: the subordinate sciences

sic is also subordinate to the natural sciences (*scienza naturale*) in addition to mathematics; not in terms of numbers, but in terms of sound, which originates in nature and from which every song (*modulatione*), every consonance (*consonanza*), every harmony and melody is born.[17] Musical science is therefore partly natural-scientific and partly mathematical; it lies between the two fields: through natural science, the matter of consonances (tones) is approached, while their form (ratios) is examined by mathematics. However, since form is more excellent (*più nobile*) than matter, music is considered more mathematical than natural science, Zarlino concludes.[18]

Another important postulate of Zarlino is connected with the subordination of music to natural science; namely, the principle of *imitation of nature*, to which the theorist often refers. In accordance with the contemporary humanistic views derived from the considerations of the ancient philosophers, Zarlino is convinced that the perfect order of nature must be the model for everything. Music, too, must follow its example, both in the construction of the tonal system and the assessment of consonances, as well as in composing and considering the aesthetic value of compositions.[19]

Closely linked to Zarlino's positioning of music as a scientific and artistic discipline is his view of the relationship between the senses and reason.[20] In general, Zarlino places reason before the senses, which cannot be fully trusted.

take their subject from the superior sciences but add an accident (*accidente*) to it, for otherwise there would be no difference between them. Zarlino cites the example of perspective, which, in addition to its own principles, refers to several other principles specific to geometry: perspective takes a line in itself as its subject and adds visual representation (*visualità*) to it as an accident; this way, its subject becomes a visually represented line. Similarly, music, which shares number with arithmetic and body with geometry, adds sound to both as an accident. Thus, music is subordinate to both, since it not only has its own principles, but also takes them from arithmetic and geometry; it is only through the latter that we can truly know it (Zarlino, *Istituzioni armoniche*, 76–78). In this case, the term *accident* denotes a certain nonessential quality of a being. In the example used by Zarlino, line is the being, while its visual representation is the property that makes it the subject of perspective. However, the line itself would still exist even if it were not represented visually. This means that visual representation is not its essential property on which its existence depends.

17 The term *natural science* primarily refers to the physical examination of nature. Aristotle hints at this connection with music in Chapter 2 of the second book of Physics, from which Zarlino apparently borrows: "One can conceive of odd and even, and straight and curved, in isolation from change, and similarly number and line and shape; but this is impossible in the case of flesh and bone and man, which are defined like a snub nose rather than a curved thing. Further clarification comes from the branches of mathematics which are closest to natural science (such as optics, harmonics, and astronomy), since they are in a sense the converse of geometry: where geometry studies naturally occurring lines, but not as they occur in nature, optics studies mathematical lines, but as they occur in nature rather than as purely mathematical entities." For more details, see Aristotle, *Physics II*, 194a, 36–37.

18 Zarlino, *Istituzioni armoniche*, 78.

19 See, for example, Zarlino, *Istituzioni armoniche*, 416.

20 Here, Zarlino almost literally quotes the content of Chapter 9 of Book 1 of Boethius' treatise (Anicius Manlius Severinus Boethius, *Fundamentals of Music*, trans. Calvin M. Bower, ed. Claude V. Palisca (New Haven: Yale University Press, 1989), 16–17).

The use of the senses alone makes it impossible to achieve the precision required by scientific study; only reason can make definite and infallible judgments in this regard. That this is true can easily be verified: if we want to divide a random thing (e.g., a line) into two equal parts only with the help of the senses, we will never be able to do it perfectly; even if we succeed by chance, there is no way that the senses could confirm this. Similarly, if we took 50 grains from a large pile, our eyes would not be able to see the change, because the amount taken away would be too small and therefore hardly perceptible to the senses. The same thing happens with sound: although the ear cannot err in judging consonances and dissonances, it cannot judge how far an interval is from another or how much one interval exceeds another. On the other hand, reason does not rest until it has carefully examined all of the differences. Although musical science begins from the senses, Zarlino sums up, since through them we examine all things, we cannot rely entirely on their judgment; we must therefore combine them with reason, for only in this way will our judgment be exact and precise.[21]

With the definition and conceptualisation of music as musical science, Zarlino largely joins the vast majority of ancient theorists led by Ptolemy and Boethius,[22] while also adopting many of the ideas of the ancient phi-

21 Zarlino, *Istituzioni armoniche*, 722–723. Despite such clearly expressed distrust of the senses, the reality in the *Istitutioni* is somewhat different. In many places, especially when it comes to contrapuntal rules, Zarlino defends and substantiates the represented views precisely by aesthetic criteria based on auditory perception: things are set as they are just because they sound good and are so pleasing to our ear.

22 Ptolemy presents his grounds for the discussion on music or, as he puts it, harmonics (*harmonic science*) mostly in Chapters 1 and 2 of Book 1 of *Harmonics*. He first explains that harmonics deal with "the distinctions related to high and low pitch in sounds", which is then physically defined as "modification of air that has been struck". Hearing and reason are then named as criteria for judging harmony (*harmonia*), but not in the same way: "Rather, hearing is concerned with the matter and the modification, reason with the form and the cause, since it is in general characteristic of the senses to discover what is approximate and to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate, and to discover what is accurate. […] The apprehensions of senses are determined and bounded by those of reason." However, even if the senses need the help of reason to be able to know things as they truly are, they are also important: in sound perception, the correct method is the one in which the ears will not only witness, but will also agree with the result. The aim of harmonics, then, "must be to preserve in all respects the rational postulates of the *kanon* [audible music], as never in any way conflicting with the perceptions that correspond to most people's estimation". In this sense, Ptolemy combines both Pythagorean and Aristoxenian traditions in his theoretical considerations, being critical of both: "The Pythagoreans did not follow the impressions of hearing even in those things where it is necessary for everyone to do so" whereas "the Aristoxenians […] gave most weight to things grasped by perception, and misused reason as if it were incidental to the route, contrary both to reason itself and to the perceptual evidence". These are thus two different concepts, two approaches to music theoretical principles, which Ptolemy tries to combine as much as possible in his music theoretical thinking. At the centre, however, is reason, which alone enables the discovery of the true truth of things. (For a detailed account see Andrew Barker, *Greek Musical Writings II: Harmonic and Acoustic Theory* (Cambridge: Cambridge University Press, 1989): 276–279).

losophers (especially Aristotelian and Pythagorean-Platonist ideas). At the same time, however, Zarlino clearly distances himself from the tradition of ancient musical thought: he acknowledges the fact that theoretical, speculative considerations are first and foremost a necessary basis for actual music, for musical practice, which is also their ultimate goal: we speculate about music in order to achieve a better end product, which is actual, audible music, a product of artistic intention and action. Zarlino's discourse on music in the *Istitutioni* is to be understood in light of what has just been said: it is a guided path that leads from theoretical, speculative considerations to the study of actual, audible music.

With the establishment of music as *scienza* and *arte* in the *Istitutioni*, the division of musicians and, finally, the definition of the perfect musician (*musico perfetto*) are closely connected. Speculative and practical music are inseparable, claims Zarlino, but if we had to separate them, one who deals with speculation would be called a *musician* (*musico*), and one who deals with practice would be called something else: one who composes is called a composer and one who sings is a singer (*cantor*), while a player (*sonatore*) plays an instrument. This way of naming practitioners is even clearer for those who make music with their hands, as they are named after their instrument: organist by organ, citternist by cittern, lyricist by lyre, etc.[23]

A musician is a person who is an expert in music (*nella musica è perito*) and can judge it not only in terms of sound but also rationally. A practitioner (composer, singer or player), on the other hand, is one who masters the musician's findings through many years of practice and puts them into practice with his voice or an artificial instrument. In fact, no composer has learned to compose through reason or science, but through long practice (*uso*). The same is true of instrumentalists: the speed of the hands or tongue, or other movements, should only be attributed to long practice, not to science.[24]

Man was born for much more excellent things than singing and playing the lyre or other instruments with which only his hearing is satisfied, Zarlino continues to believe.[25] In this way, man neglects his nature, as he does not make

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Boethius presents his starting points for the study of music mainly at the beginning of Book 1 (Chapters 1 and 9) and at the beginning of Book 2 (Chapters 2 and 3) of *Fundamentals of Music*. Here, music is firmly positioned as one of the mathematical sciences of the quadrivium, which is approached by both hearing and reason, but as in the case of Ptolemy, the latter is undoubtedly of greater importance. This is then confirmed in more detail in Chapter 2 of Book 5, where Ptolemy's definitions of harmony are adopted almost literally ("harmonics is the faculty that weighs differences between high and low sounds"), and "sense and reason are, as it were, particular instruments for the faculty of harmonics". For a detailed account see Boethius, *Fundamentals of Music*, 163.

23 Zarlino, *Istituzioni armoniche*, 56–57.

24 Ibid., 57.

25 Ibid., 32–34.

much effort to properly nourish his intellect, which always wants to know and learn new things. Therefore, one should not learn the art of music alone – that would be madness. Even more: if man were to be active only in practice, it would be a sin, for it would lead him to drowsiness and laziness and make his spirit weak and effeminate. However, most contemporary musicians only engage in practical music, which has a bad effect on their character and makes them coarse and rude. They can only become virtuous (*virtuosi*) by studying speculative music.

It is clear, according to Zarlino, that without a solid theoretical system on which it is based and from which it is born, practical music cannot exist. This system can only be approached rationally, but it is really accessible only to those who know both the speculative and the practical parts of music, since without its audible realisation, the system is meaningless. Hence, both theorist and practical musician must go hand in hand, and it is best if they are united in one person – then this is the perfect musician (*musico perfetto*).[26]

3 From ancient music theory to contemporary musical practice: The cases of the division of the interval and the system of the *senario*

Perhaps the clearest example of Zarlino's attempt to join the postulates of ancient music theory, based on exact mathematical principles, with contemporary musical reality in the *Istitutioni* is his construction of the tonal system. Let us therefore consider two case studies: the division of the interval and the system of the *senario*.

26 What a perfect musician needs to know is explained in more detail at the end of the *Istitutioni* (Zarlino, *Istituzioni armoniche*, 718–721): if you want to achieve perfection in music, you must know many things, says Zarlino; if only one thing is missing, perfection cannot be achieved. Since the science of music is subordinate to arithmetic, the perfect musician must first be well versed in this discipline, especially in the use of numbers and ratios: he must know at least as much about numbers as merchants do (*numeri mercanteschi*). Furthermore, the ratios between tones cannot be represented in any other way than through sounding bodies, so a perfect musician must be well trained in geometry: he must at least master the division of a line with a compass (*compasso*) and understand what a point, a line (curved and straight), a surface, a body and other similar things are. In addition, a perfect musician must master at least the average playing of the monochord and harpsichord: the harpsichord is the most perfect and tuning stable of all instruments, and with the help of a monochord, he will be able to prove and validate his research on sounding numbers and implement things he finds new every day. Knowledge of both instruments also requires that one can tune them perfectly and that one has perfect hearing (*l'udito perfetto*): only in this way will a perfect musician be able to make perfect judgments without making a mistake when researching the differences between the intervals. Furthermore, a perfect musician must be well trained in the art of singing (*arte del cantare*) and counterpoint (*arte del contrapunto*) or composition: this way, he will be able to realise everything musical because the creation of musical things is the way to their ultimate goal and perfection. It will also be extremely useful for a perfect musician to master some other disciplines, especially grammar (because with the help of grammar, one can learn of long and short syllables and understand writers who write about music) and dialectics (because with the help of dialectics, one can argue); rhetoric (because with it, one can express thoughts in an orderly way) and natural science are also important.

3.1 The division of the interval

The *musical interval* is the smallest unit of the tonal system as defined by Zarlino; in the *Istitutioni*, an interval is defined as the distance between two different tones: lower and higher.[27] Mathematically speaking, these distances are expressed by numerical ratios,[28] but not by any ratios, only by *unequal ratios of comparison of the larger with the smaller, which are closer to simplicity*, explains Zarlino.[29] Since intervals are expressed by numerical ratios, they are inevitably subject to all of their laws, including the arithmetic operations between them.[30] The latter are used in the *Istitutioni* mainly in the discussion about the division of the interval. Indeed, Zarlino believes that smaller intervals come from larger ones,[31] so it is important to show how to divide them.

Since intervals are divided the same way as ratios, their division can generally be irrational (a musical scholar does not deal with this) or rational, and the latter can be arithmetic, geometric or harmonic.[32] Actually, the division of a given interval determines the arithmetic, harmonic or geometric mean of the ratio by which the divided interval is expressed; all three means are summarised in the table below.

Table 1: Arithmetic, geometric and harmonic means

division (mean)	**differences between terms**	**ratios between terms**	**mathematical formula of the mean**	**example**
arithmetic	equal	different	$m=\frac{x+y}{2}$	4:3:2
geometric	different	equal	$m=\sqrt{xy}$	4:2:1
harmonic	different	different	$m=\frac{2xy}{x+y}$	15:12:10

The determined mean of the ratio of a given musical interval is in fact a new tone placed between the two existing ones, thus dividing the initial interval

27 Zarlino, *Istituzioni armoniche*, 185.

28 In considering and defining the interval, Zarlino is somewhat inconsistent: if an interval is a distance (and as such necessarily a quantity), it cannot be expressed by a ratio (which is necessarily a comparison of two quantities – Zarlino, too, defines it as such). Strictly speaking, the musical interval, which in modern music theory is usually also defined as the distance between two tones, is mathematically and physically actually a comparison (ratio) of their frequencies.

29 Zarlino, *Istituzioni armoniche*, 81–83, 300–305.

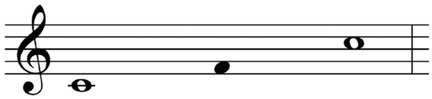
30 The basic arithmetic operations with ratios are addition, subtraction, multiplication, division, finding the lowest terms, and determining the means. For a detailed presentation of these operations, see Zarlino, *Istituzioni armoniche*, 96–102, 104–108, 110–112 in 120–121.

31 In the *Istitutioni*, Zarlino argues that intervals originate from the division of the octave into smaller parts. Zarlino, *Istituzioni armoniche*, 295–297 and 313–316.

32 Zarlino, *Istituzioni armoniche*, 207.

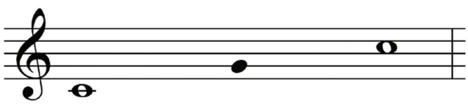
into two smaller ones. Divisions with all three means are shown in the examples below:

(1) Let us divide the octave with the arithmetic mean. Its ratio of 2:1 must first be doubled (= 4:2),[33] and then the middle term 3 is determined so that the initial ratio of the octave will be divided arithmetically: 4:3:2. We get the interval of the fourth (4:3) between the determined middle and the lower tones, while between the middle and upper tones, we get the interval of the fifth (3:2).



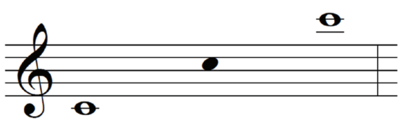
Example 1: Arithmetic division of the octave.

(2) Let us divide the octave with the harmonic mean. We take the ratio of 6:3 (= 2:1)[34] and determine the middle term 4 so that the initial ratio of the octave will be divided harmonically: 6:4:3. In this case, between the lower two tones we get the interval of the fifth (6:4 = 3:2) and between the higher two tones the interval of the fourth (4:3).



Example 2: Harmonic division of the octave.

(3) Let us divide the double octave[35] with the geometric mean. We take the ratio of 4:1 and determine the middle term 2, so that the initial ratio of the double octave will be divided geometrically: 4:2:1. Both obtained intervals are of the same ratio, differing only in terms of pitch.



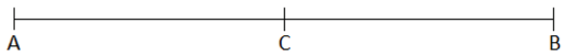
Example 3: Geometric division of the double octave.

33 The arithmetic mean of terms 2 and 1 (= $\frac{3}{2}$) cannot be written with an integer, which causes Zarlino to double the ratio of the octave.

34 The harmonic mean of terms 2 and 1 (= $\frac{4}{3}$) cannot be written with an integer, which causes Zarlino to triple the ratio of the octave.

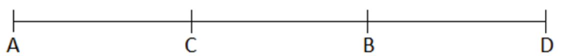
35 The geometric mean of the ratio of the octave (2:1) is $\sqrt{2}$, which is not a rational number. Zarlino therefore takes the ratio of the double octave (4:1) for his example of division by geometric mean.

However, the arithmetic procedures described for dividing an interval with the arithmetic, harmonic and geometric mean prove to be somewhat inadequate in Zarlino's further discussion, since these means do not allow each interval to be divided into two equal parts (e.g., the interval of the octave in the ratio 2:1 or the interval of the major second in the ratio 9:8).[36] Where it is not possible to divide a given interval into two equal parts following the arithmetic procedures because the result would be irrational, a geometric procedure must be applied: just as we can find a third line segment to the two given ones, which will be their mean, we can also find a third tone to the two given ones.[37] So let us divide the ratio of the octave (2:1) in half. We take the line segment AB halved by point C so that AB:CB will be in the chosen ratio of 2:1.



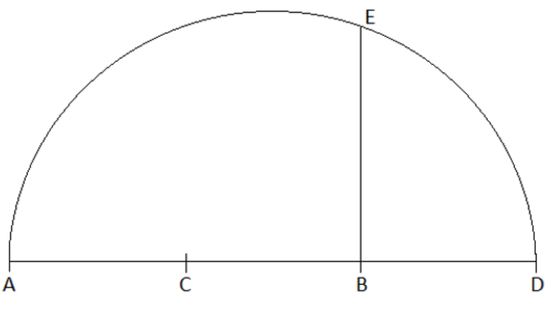
Example 4: Division of the line segment AB in half.

We then extend the line segment AB from point B to point D so that BD = CB.



Example 5: Extension of the line segment AB to point D; BD = CB.

Next, we draw a semicircle to the line segment AD and then a perpendicular line from point B to point E. The resulting line segment BE is the length we are looking for.

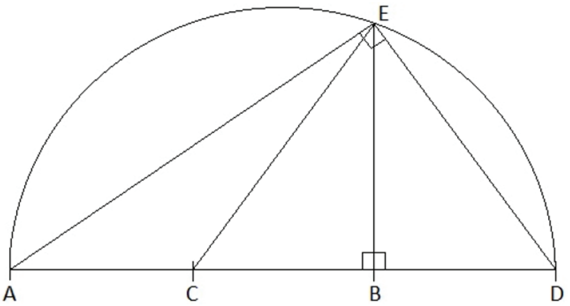


Example 6: Semicircle to the line segment AD and the result – the line segment BE.

36 A given interval can be divided into two equal intervals only by geometric mean, which divides any ratio into two equal ratios. However, the geometric mean is not always a rational number, and irrational numbers are not acceptable to music scholars.

37 Zarlino summarises the following procedure after Book 6 of Euclid's *Elements*, especially Propositions 8 and 13 (see Euclid, Euclid's Elements, trans. David E. Joyce, 1998, accessed 2 August 2020, https://mathcs.clarku.edu/~djoyce/java/elements/). In this case, the line segments must be taken as taut strings of a determined length.

Zarlino's procedure can be verified through geometric rules of similar triangles.[38] First, we have to connect the points A and E, and E and D to obtain the triangle AED. At the same time, we get the triangles ABE and EBD. Then, we connect the points C and E to get the triangle BCE. All four triangles are right-angled[39] and similar.[40]



Example 7: Similar triangles AED, ABE, EBD and BCD.

We must show that AB:BE = BE:CB is true. The triangles AED and ABE are similar, as they coincide in two angles: they have one common angle (∢EAB = ∢EAD), and the angles ∢ABE and ∢AED are right angles. Consequently, the angles ∢ADE and ∢AEB are equal. The triangles AED and EBD are also similar since they also coincide in two angles: they have one common angle (∢BDE = ∢ADE), and the angles ∢EBD and ∢AED are right angles. Consequently, the angles ∢BED and ∢EAD are equal. Furthermore, we can find that the triangles CBE and EBD are also similar, as they coincide in two sides (CB = BD, side BE is common) and the angle between them (∢CBE = ∢BED). Since all four triangles are similar, the triangles ABE and CBE are also similar. Therefore, according to the rule that the ratios of the corresponding sides (sides at the same angle) are the same, it is true that AB:BE = BE:CB.

Although Zarlino, at the beginning of the discussion on the division of the larger intervals into smaller ones, claims that a musician only deals with rational divisions, the manner of dividing an arbitrary ratio (interval) into two

38 Zarlino does not offer a proof in the *Istitutioni*, merely stating that the procedure is in accordance with Euclid's rules (Zarlino, *Istituzioni armoniche*, 209).

39 The triangle AED is right-angled according to the Thales theorem, which states that the central angle (in this case ∢ABD) is twice as large as the inscribed angle (∢AEB). Since the angle ∢ABD is 180°, ∢AED must be 90°. Euclid discusses this problem in Proposition 31 of Book 3 of *Elements* (see Euclid, *Elements*).

40 Two triangles are similar when they match: (1) in two angles (and consequently in the third); (2) in two ratios of the corresponding sides (which are by the same angle in both triangles); (3) in the ratio of two sides and the angle against the longer, and (4) in the ratio of two sides and the angle between them.

equal parts as depicted is actually nothing other than a geometric representation of irrational geometric division. In this way, the tone in the ratio 9:8 could also be divided into two equal parts; thus, in the *Istitutioni*, the door to equal temperament based on auditory perception opens widely.

3.2 Senario

After he explains how the intervals are to be divided arithmetically (with the help of means) and how any interval can be divided into two equal parts by geometric procedure, Zarlino continues with the presentation of the features of the intervals, with particular emphasis on consonances and dissonances.

The question of consonances and dissonances was one of the central questions discussed by Renaissance music theorists in their treatises. In discussing it, they departed significantly from their ancient predecessors, approaching contemporary musical practice. As is well known, according to the ancient Pythagorean system, only the intervals whose ratios could be expressed by the first four numbers were consonant: double octave (4:1), octave with fifth (3:1), octave (2:1), fifth (3:2) and fourth (4:3). Since all other intervals were considered dissonant, when reading the ancient writings on music, Renaissance theorists faced a difficult dilemma: on the one hand, there were mathematically clear, strict and rigid "Pythagorean" definitions, and on the other hand, there was the musical reality of their time, musical practice in which ear-pleasing thirds and sixths had also been in use as consonances for quite some time. These were impossible to describe with the Pythagorean system of the first four numbers, since they occur in the known ratios of 5:4 (major third), 6:5 (minor third), 5:3 (major sixth) and 8:5 (minor sixth). Their justification thus became a significant problem that many music theorists of the time addressed.

Zarlino also engaged in the difficult justification of thirds and sixths as consonances. In the *Istitutioni*, he first defined consonance and dissonance:[41] consonance is defined as the arrangement of low and high tones, reduced to a single ratio, which reaches the ear sweet and homogenous and can affect the senses (*ha posanza di mutare il senso*); such intervals are the unison, the third, the fourth, the fifth, the sixth and the octave, as well as their conjunctions with the octave. To their opposite is dissonance, which reaches the ear sharp, rough and without any grace; it arises because its tones, given the disproportion between them, do not want to unite but strive to remain separate. Dissonant intervals are the second and the seventh, as well as their conjunctions with the octave.

Zarlino then continues with a discussion of consonances, which he divides in different ways. Like intervals in general, consonances can be simple or composed,[42]

41 Zarlino, *Istituzioni armoniche*, 179–180, 185.

42 Ibid., 69–70.

with the latter further divided into three types: (1) composed of parts of the octave[43] but smaller than the octave, (2) composed of an octave and one of its parts, and (3) composed of two or more octaves. The first group includes the major sixth in the ratio 5:3 (composed of a fourth and a major third and smaller than the octave) and the minor sixth in the ratio 8:5 (composed of a fourth and a minor third and smaller than the octave). The second group includes the octave with the fifth (3:1), and the third group a double octave (4:1). Actually, only the consonances of the second and third groups are composed, Zarlino adds, whereas the consonances of the first group are incompletely composed: since they are smaller than the octave, they are almost simple and basic. In fact, in the sequence of musical consonances, almost the same order can be observed as in the sequence of numbers up to 10. Once we get to this number, we no longer add any new ones, but only duplicate those already used. Just as the number 10 is followed by the number 11, then 12, etc., the consonances following the octave and the fifth are posed almost to infinity: first there is a fourth, then major and minor thirds, then a fourth again, and so on. If necessary, the sequence could continue indefinitely this way, but music does not reach infinity, as infinity cannot be known.[44]

In the *Istitutioni*, consonances are further divided into perfect and imperfect.[45] This division is closely connected with contemporary musical practice, especially with the rules of constructing chords and voice leading in counterpoint. Zarlino classifies the unison, the fourth, the fifth and the octave as perfect consonances, while the third and the sixth are imperfect. Relying on the authority of the ancient Pythagoreans, he further explains that perfect consonances were most likely named as such because the terms of their ratios are contained in parts of the number four (*numero quaternario*) in multiple and superparticular genera.[46] Subsequently, however, he also justifies them with auditory-aesthetic criteria: these consonances please the ear in such a way that it does not wish for anything else.

On the other hand, imperfect consonances are named this way because the

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43 This refers to the intervals smaller than the octave into which the latter is divided according to Zarlino's belief that all intervals originate from the octave.

44 Zarlino speaks about the sequence of numbers between the ratios of simple consonances: 1:2 (octave) :3 (fifth) :4 (fourth) :5 (major third) :6 (minor third) :8 (3:4, fourth) :10 (4:5, major third), etc. In fact, the sequence is not quite the same as in numbers up to 10 since the numbers 7 and 9 are missing. If we arrange these ratios in a series of intervals, however, they relate to each other so that the upper tone of the previous interval is the lower tone of the following one: C – c – g – c^1 – e^1 – g^1 – c^2 – e^3 – g^3 etc.

45 Zarlino, *Istitutioni armoniche*, 322–325.

46 Two numbers (whether larger to smaller or vice versa) can be compared in five different ways, Zarlino maintains. Consequently, he speaks of five different genera of ratios, which are divided into simple (multiple, superparticular, superpartient) and compound (multiple superparticular and multiple superpartient). See Zarlino, *Istituzioni armoniche*, 81–82, 87–90. The definitions of genera of ratios are most likely taken from Boethius, who discusses them in Chapter 4 of Book 1 and Chapter 4 of Book 2 (see Boethius, *Fundamentals of Music*, 12–14 and 54–55).

terms of their ratios occur after the number four (6, 5, 4). In the first place, these are major and minor thirds, which combined with the fourth form major and minor sixths, which are in the superpartient ratio, and this, according to the Pythagoreans, cannot produce consonances. Unlike perfect consonances, imperfect ones cannot completely satisfy our hearing on their own. This only occurs when they are combined with other intervals in such a way that the highest and lowest tones of the chord form either a perfect consonance or one of the composed imperfect consonances (*imperfette replicate,* i.e., an imperfect consonance with an octave).

Finally, consonances are also divided into complete (*piene*) and pleasant (*vaghe*).[47] It is explained that contemporary musicians use both terms with the adjectives (*particella*) *more* or *less*. More complete are consonances that have a greater ability to fill the ear with different sounds. In this sense, the fifth is more complete than the octave, for its tones fill the ear more with different sound than the tones of the octave, which are of the same sound (*equisonanti*) and are therefore similar. If we leave the octave aside, however, the rule is as follows: consonances that are closer to their origin (in a greater ratio) and are more perfect than the others are also more complete. On the other hand, consonances in smaller ratios are more pleasing. This is especially true when consonances are placed in the right places. Those whose ratios are closer to double are attracted to lower pitches (*amano la parte grave*), whereas those that are more pleasant and whose ratios are more distant from the double are attracted to higher ones.[48]

Consonances are also considered to be more pleasant if they are further away from simplicity (*semplicità*). The latter does not please the ear the most, as the senses prefer composed rather than simple things. In this sense, hearing has a similar attitude to tones as sight has to colours. The latter originate in white and black, but sight is more enthusiastic about those colours that are distant from white and black. Just as sight rejoices more in the composition of colours, hearing rejoices more in the composition of tones.[49]

In addition to consonant intervals, the musician also deals with dissonant ones, the sound of which does not please the ear, causing the tune (*cantilena*) to be sharp and without any grace. In singing, however, it is not possible to proceed from one consonance to another without the help of dissonances, thus making their use necessary, albeit subject to certain rules. Of these intervals, only those serving diatonic tonal motions,[50] that is, those smaller than a third

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47 Zarlino, *Istituzioni armoniche*, 325–326.

48 Thus, the chord would be best constructed if the larger intervals (e.g., fifth) are placed in the lower voices of the composition and the smaller intervals (e.g., third) in the higher ones.

49 This is clearly in contrast to Zarlino's initial definition of perfect and imperfect consonances, where he said that the former pleased the ear in such a way that it did not wish for anything else.

50 In this case, the consecutive intervals in the melody are meant: e.g., from the (consonant) fourth

and larger than a minor semitone, are necessary; there are three such dissonances: major tone, minor tone and major semitone. In instrumental music, in addition to these, a minor semitone and a comma are sometimes used.[51]

Although Zarlino proposes auditory-aesthetic criteria and practical experience in his definitions and divisions of consonance and dissonance, he is not satisfied with this. In this case, too, he remains true to his initial belief that the judgment of the senses cannot be fully trusted. The consonance of thirds and sixths must therefore also be confirmed rationally, which means that it must be based on solid mathematical foundations. The *Istitutioni* thus brings forth a new mathematical system of consonances, different from the Pythagorean system of the first four numbers and based on the number six (*numero senario*).

Before Zarlino rejects the Pythagorean system, he presents it and tries to explain why the Pythagoreans did not consider thirds and sixths as consonances.[52] He is convinced that the main reason was the fact that Pythagoras, who studied the mysteries of nature and the causes of being, was an advocate of simple and pure things. He rejected imperfect consonances precisely because he believed that they could not be fully understood (*di esse non si potesse aver ferma ragione*). On the other hand, he allowed those consonances that were based on simple numbers and whose nature was purest. Consequently, he approved only of consonances whose ratios can be found among the parts of the number four (*numero quaternario*) and that are in the multiple or superparticular genera. From these – unlike major and minor thirds, from which intervals in the superpartient genus can be composed – no non-consonant intervals can originate.[53]

Zarlino subsequently tries to guess why it was precisely the number four that was so significant to the Pythagoreans. Three possible explanations are given: (1) the Pythagoreans may have valued the number four, which they considered perfect, because it reflects the perfection of the soul (it is obvious that Pythagoras did not realise the existence of harmony beyond this number); (2) some think that Pythagoras' prohibition of exceeding the number four in compositions (*cantilene*) actually refers to the vocal range, which would mean that the composition should not exceed a double octave (4:1), for every good voice can only encompass such a range; and (3) the Pythagoreans did not consider the imperfect consonances as consonant because they did not know them as they truly are, as their tonal system would not allow them to do so, resulting in their hearing imperfect consonances as dissonant.[54]

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to the (consonant) fifth one cannot proceed otherwise than through the (dissonant) major tone.

51 Zarlino, *Istituzioni armoniche*, 340–342.

52 Ibid., 130–133.

53 This statement of Zarlino is not strictly true: from the octave and the fourth, an eleventh can be formed in a ratio of 8:3, which is multiple superpartient. Unlike Zarlino, the Pythagoreans did not consider this interval as consonant since it is not in the multiple or superparticular ratio.

54 Zarlino does not explain the third reason in the *Istitutioni*, only mentioning that it exists, that in

The Pythagoreans were therefore mistaken: the *quaternario* system they established is clearly flawed, as it does not presuppose imperfect consonances. Instead of the number four, it is therefore necessary to consider the number six, which is the first *perfect number* (*numero perfetto*).[55] Its importance is reflected in many things, argues Zarlino.[56] Although God never needed time in his deeds, the prophet Moses chose the number six in describing the creation of the world to illustrate the majesty of the Creator's work. Indeed, many things, both natural and artificial, consist of the number six. It manifests itself in the order of the heavens, among other things in the twelve zodiac signs, six of which are always above our hemisphere, while the other six are below. The number six is also reflected in the order on Earth. There are six substantial qualities of the elements: sharpness (*acuità*) and dullness (*ottusità*), rarity (*rarità*) and density (*densità*), motion (*moto*) and stillness (*quiete*). There are six natural qualities without which nothing exists: size (*grandezza*), colour (*colore*), shape (*forma*), distance (*intervallo*), state (*stato*) and motion (*moto*). There are six types of motion: generation (*generatione*), corruption (*corrutione*), increase (*accrescimento*), decrease (*diminutione*), alteration (*alteratione*) and movement (*mutatione di luogo*). There are six different positions (*differenze dei siti over positioni*): up, down, front, behind, right, left. The number six outlines the tetrahedron and the cube,[57] and there are six stages of human life: infancy (*infantia*), boyhood (*pueritia*), adolescence (*adolescentia*), youth (*giovenezza*), old age (*vecchiezza*) and decrepitude (*decrepità*). Actually, the number six is not only perfect, but also reflects virtue.

Finally, the number six is also important in music: there are six relations between the tones, in which every musical harmony is contained.[58] There are six consonances: five simple and basic consonances (octave, fifth, fourth, major third, minor third) and the unison.[59] There are also six species of ancient

his opinion it is the most probable, and that he discussed it in *Dimostrationi harmoniche*; the topic is actually discussed at the beginning of this treatise (Gioseffo Zarlino, *Le Dimostrationi harmoniche* (Venezia, 1589), 4). A deficient tonal system, which did not allow the Pythagoreans to truly know the imperfect consonances, refers to an actual acoustic or tuning system in which the music of their time was supposed to have evolved. In it, thirds and sixths were clearly in such ratios that (in Zarlino's opinion) they were dissonant.

55 In the *Istitutioni*, Zarlino defines perfect numbers as those numbers that are the sum of their parts (*parti*). (Zarlino, *Istituzioni armoniche*, 61.) By this he means the sum of all divisors of such a number, but without the number itself (1 + 2 + 3 = 6). Besides being the sum of its parts, according to Zarlino, number six is also a *circular number* (*numero circolare*), since we can also obtain it if its parts (divisors) are multiplied (1 · 2 · 3 = 6). (Zarlino, *Istitutioni armoniche*, 65.)

56 Zarlino, *Istituzioni armoniche*, 62–65.

57 A tetrahedron has six edges, and a cube has six faces.

58 The division of intervals into unison, equison, consonant, melodic, dissonant and nonmelodic, as presented by Boethius in the *Fundamentals of Music* (and summarised after Ptolemy), is meant. See Boethius, *Fundamentals of Music*, 170–171.

59 This statement by Zarlino is not in accordance with his discussion of consonances and dissonances: there are no sixths among the listed intervals, which he otherwise places among consonances.

harmonies (Dorian, Phrygian, Lydian, Mixolydian or Locrian, Aeolian and Iastian or Ionian) and of authentic and plagal modern modes.

Since the number six is so important, it is not surprising that it is also the most important harmonic or sounding number.[60] Whichever of its parts we take (1, 2, 3, 4, 5, 6), their ratio will form one of the consonances, either simple (octave 2:1, fifth 3:2, fourth 4:3, major and minor thirds 5:4 and 6:5, respectively) or composed (major sixth 5:3, composed of a fourth and a major third).[61] Furthermore, the ratios of the largest consonances (fifth and octave) can be divided in such a way that the ratios of the other intervals within the *senario* are obtained, and the terms by which these intervals are divided form the ratio of the major sixth.[62]

Zarlino thus placed all consonances among the parts of the number six except for the minor sixth in the ratio 8:5, because the number 8 is obviously not found among the first six numbers. He therefore justified the placement of the minor sixth among (composed) consonances in three ways:[63] (1) in addition to the number six, the number eight is also important since it is the first cube; (2) the minor sixth is composed of two intervals contained in the *senario*;[64] and (3) the harmonic mean (*mezano termine harmonico*), which divides the minor sixth into the intervals of which it is composed, is exactly the number six.[65] In this way, says Zarlino, all of the simple consonances can be found between the parts of the *senario*, and all of the composed consonances can be derived from them. Both simple and composed consonances are the origin of every good and perfect harmony: if we stretched strings in the presented ratios and strummed all of them at the same time, we would not hear any discordance in the produced tones, but such a harmony would arise that the ear would accept it with the greatest pleasure.

Finally, Zarlino explains, the ratios of dissonances can potentially be formed from the parts of the number six. If we multiply these numbers each with each other and by themselves, we can also obtain the ratios of major and minor

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On the other hand, the list includes the unison, which he otherwise says is not even an interval.

60 Zarlino, *Istituzioni armoniche*, 66–68.

61 $\frac{3}{4} \cdot \frac{5}{4} = \frac{20}{12} = \frac{5}{3} = \underline{5:3}$

62 The following is meant. The ratio of the octave 2:1 is first doubled to the ratio 4:2. Between these terms, 3 can be placed, thus forming the consonances of the fifth (3:2) and the fourth (4:3). Similarly, the fifth in the ratio 3:2 can be divided; again, its ratio is first doubled to 6:4. If we insert 5 between these terms, we get the consonances of the major third (5:4) and the minor third (6:5). The terms placed in the middle of the two original ratios (5 and 3), however, form the ratio of the major sixth (5:3).

63 Zarlino, *Istituzioni armoniche*, 70.

64 It is composed of the fourth and the minor third: $\frac{4}{3} \cdot \frac{6}{5} = \frac{24}{15} = \frac{8}{5} = \underline{8:5}$.

65 If the ratio 8:5 is divided by the number 6 (8:6:5), the ratios 8:6 (= 4:3) and 6:5 are formed. However, the number 6 is not the exact harmonic mean of the numbers 8 and 5. If we tried to determine the harmonic mean of 8 and 5 (following the examples presented above), the result would be approximately 6.15, and we would get the ratio of 8:6.15:5.

tones and major and minor semitones.[66] The terms of all ratios obtained this way are the *sounding numbers* (*numeri sonori*),[67] as illustrated below.

1
octave (2:1)
2
fifth (3:2)
3
fourth (4:3)
4
major third (5:4)
5
minor third (6:5)
6
fourth (8:6 = 4:3)
8
major tone (9:8)
9
minor tone (10:9)
10
minor third (12:10 = 6:5)
12

12
major third (15:12 = 5:4)
15
major semitone (16:15)
16
major tone (18:16 = 9:8)
18
minor tone (20:18 = 10:9)
20
minor third (24:20 = 6:5)
24
minor semitone (25:24)
25
minor third (30:25 = 6:5)
30
minor third (36:30 = 6:5)
36

Example 8: Zarlino's sounding numbers.[68]

The justification of consonances by the system of the first six numbers completes Zarlino's belief that theory and practice must necessarily be connected: if he were satisfied only with auditory-aesthetic criteria, he would not remain true to the fundamental concepts on which the whole content of the *Istitutioni* is based. With regard to the ancient theoretical tradition, one could say that Zarlino in this case combined Pythagorean-Platonist rational considerations with Aristoxenous' views based on auditory experience. In so doing, however, reason seems to be of somewhat greater importance: thirds and sixths are essentially consonant only because of the firm mathematical grounds on which

66 $(3^2):(2 \cdot 4) = 9:8$; $(2 \cdot 5):(3^2) = 10:9$; $(4^2):(3 \cdot 5) = 16:15$; $(5^2):(6 \cdot 4) = 25:24$. In addition, the ratios of minor (9:5) and major (15:8) sevenths are also to be found among the numbers obtained.

67 The concept of *sounding number* is another fundamental idea presented by Zarlino in the *Istitutioni*. It represents one of the most direct connections between Zarlino's theoretical and practical considerations, as the voices in counterpoint can only move in intervals (both consonant and dissonant) whose ratios can be formed by the sounding numbers. This means that dissonances such as minor and major semitones and minor and major sevenths can also be used in composition (although with limitations). However, the augmented fourth or *tritone* (45:32) and the diminished fifth or *semidiapente* (64:45) are strictly forbidden, as the terms of their ratios are not found among the sounding numbers. See Zarlino, *Istitutioni armoniche*, 418.

68 In addition to the listed intervals, it is also possible to derive the minor sixth (8:5), the major sixth (5:3), the minor seventh (9:5) and the major seventh (15:8) from the given series of numbers.

their consonance is based, and it is precisely because their ratios have certain properties (they can be found within the perfect number six) that they sound agreeable. On the other hand, Zarlino tries to justify them by an exact mathematical system precisely because they sound agreeable: he tries to construct a system that would reflect contemporary musical practice. If these intervals were not used in practice, and if they did not please the ear, there would be no need to seek a system to justify their consonance.

4 Conclusion

In view of everything presented above, it is clear that the ancient music theory tradition and, within it, the mathematical and physical considerations in the *Istitutioni* can be seen primarily as a model and means to achieve a particular goal; namely, the construction of a tonal system within which contemporary music evolves. According to Zarlino, the art of counterpoint (that is, practical music) is not only possible within this system, it depends on it. Since the main aim of the *Istitutioni* is the recognition and affirmation of contemporary music, Zarlino adopts the ancient theorems selectively and primarily deals with those that are, at least to some extent, important for the discussion of contemporary compositional practice. As a result, the connections between ancient ideas and the music of Zarlino's time are sometimes loose and in some cases do not even stand up to critical examination.

The question, then, is: Why does Zarlino substantiate contemporary musical practice in this way at all? Why does he find it necessary to look for a system within which thirds and sixths function as consonances, if we simply hear them as consonant and use them in this way because they sound good? It seems that, for Zarlino, their justification merely with auditory-aesthetic criteria would be too much of a turn from the humanistic paradigm. This is perhaps most clearly evident in his thought that even the music of ignorant people who do not know music theory can be delightful, but it cannot be good. Only music that makes a person better and morally virtuous can be good, and only reason-based music is capable of this. In these concepts, Zarlino remained true to ancient music theorists and philosophers, and even if he adapted their theorems to contemporary music, the following is very clear in the *Istitutioni*: behind all music – and especially good music – there is a solid, rational and mathematically based system; it just has to be found.

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POVZETEK

Renesančna glasba med znanostjo in umetnostjo: primer Gioseffa Zarlina

Eden temeljnih postulatov v znamenitem traktatu *Istitutioni harmoniche*, ki ga je leta 1558 izdal Gioseffo Zarlino (ok. 1517–1590), po mnenju mnogih osrednji in najvplivnejši italijanski glasbeni teoretik 16. stoletja, je povezovanje antične glasbene teorije z glasbeno prakso časa. Kot je dobro znano, so v antiki in v srednjem veku glasbo obravnavali kot matematično znanost, eno od disciplin *kvadrivija*, in sicer tisto, ki se ukvarja z razmerji med količinami. Namen prispevka je zato ugotoviti, kakšno vlogo imajo v *Istitutioni* antična glasbena teorija in v njenem okviru predvsem matematični in fizikalni premisleki in njihove povezave z zvenečo glasbo.

Zarlino se na antične teoretike in filozofe naveže že pri zasnovi svojega traktata, ki je oblikovan v skladu z nekaterimi premisleki iz Aristotelove *Fizike*, še bolj pa jih prevzema v opredelitvi glasbe, ki jo utemeljuje kot eksaktno matematično disciplino. A spekulativne premisleke o glasbi je po njegovem mnenju vendarle nujno združevati s praktičnim glasbenim ustvarjanjem, saj spekulativna glasba brez praktične uresničitve ne doseže svojega končnega cilja in je zato le malo vredna in nepopolna. Glasba je po Zarlinovem prepričanju potemtakem sestavljena iz dveh komponent, znanstvene (spekulativna oz. kontemplativna glasba) in umetniške (praktična glasba), je *scienza* in *arte*. Čeprav sta neločljivi in druga brez druge ne

moreta, se vendarle zdi, da Zarlino na prvo mesto postavlja *scienzo*. Z opredeljevanjem glasbe kot znanstvene in umetniške discipline je tesno povezan tudi Zarlinov pogled na razmerje med čuti in razumom: v splošnem je razum postavljen pred čute, na katere se ni mogoče dokončno zanesti; dokončne in nezmotljive sodbe more podati le razum.

Morda najjasnejši primer Zarlinovega povezovanja na eksaktnih matematičnih temeljih postavljenih antičnih glasbenoteoretskih postulatov s sodobno glasbeno danostjo je njegova razprava o tonskem sistemu. Zato sta v prispevku predstavljena dva konkretna primera iz utemeljevanja tonskega sistema v *Istitutioni*, pri katerih je Zarlino izhajajoč iz antičnih premislekov iskal način, kako bi jih povezal z glasbo svojega časa: delitev intervala in sistem *senario*.

Antična glasbenoteoretska tradicija in znotraj nje obravnava matematičnih in fizikalnih premislekov se v *Istitutioni* torej kažejo predvsem kot model in sredstvo za doseganje ozkega cilja, namreč izgradnjo tonskega sistema, znotraj katerega poteka glasba časa in le znotraj katerega je po Zarlinovem prepričanju možen in od njega tudi odvisen nauk o kontrapunktu (praktična glasba). Ker je v ospredju *Istitutioni* utemeljevanje sodobne glasbe, Zarlino antične glasbene teoreme prevzema selektivno in obravnava predvsem tiste, ki so in kolikor so pomembni za kasnejšo razpravo o kompozicijski praksi časa. Kljub temu je Zarlino antičnim glasbenim teoretikom in filozofom ostajal zvest in tudi če je njihove zamisli prilagodil sodobni glasbi, je v *Istitutioni* zelo jasno naslednje: v ozadju vsake, posebno pa dobre glasbe je trden, racionalen in na matematičnih temeljih utemeljen sistem, le najti ga je potrebno.

ABOUT THE AUTHOR

NEJC SUKLJAN (nejc.sukljan@ff.uni-lj.si) studied musicology and history at the Faculty of Arts, University of Ljubljana. As an Erasmus student, he also studied in Regensburg, Germany. He graduated with honours in September 2009, and for his thesis in musicology, *Vincenzo Galilei's Musical-Theoretical and Musical-Aesthetical Thought,* he received the faculty's Prešeren Prize. His research focuses on early music history and music theory. In April 2017, he completed his PhD studies with the thesis *Gioseffo Zarlino's Istitutioni Harmoniche and Ancient Music Theory*. Since February 2010, he has been a teaching assistant and since June 2019, an assistant professor of musicology at the Department of Musicology, Faculty of Arts, University of Ljubljana. From 2008 to 2012, he was the secretary of the Slovenian Musicological Society, and he was re-elected to this position in March 2017. From 2011 to 2015, he conducted the Koper Wind Orchestra.

O AVTORJU

NEJC SUKLJAN (nejc.sukljan@ff.uni-lj.si) je po maturi na Gimnaziji Koper študiral muzikologijo in zgodovino na Filozofski fakulteti Univerze v Ljubljani. Del študijskih obveznosti je v okviru izmenjave Erasmus opravil v Regensburgu v Nemčiji. Študij je z odliko zaključil septembra 2009 in za muzikološko diplomsko nalogo *Glasbeno-teoretska in glasbeno-estetska misel Vincenza Galileija* prejel študentsko Prešernovo nagrado. Raziskovalno se ukvarja z zgodovino starejše glasbe in teorije glasbe; aprila 2017 je doktoriral s temo *Istitutioni Harmoniche Gioseffa Zarlina in antična glasbena teorija*. Od februarja 2010 je kot asistent zaposlen na Oddelku za muzikologijo Filozofske fakultete Univerze v Ljubljani, kjer je bil junija 2019 izvoljen v docenta za muzikologijo. V letih 2008–2012 je bil tajnik Slovenskega muzikološkega društva in to funkcijo ponovno opravlja od marca 2017.