DESIGN OPTIMIZATION FOR SYMMETRICAL GRAVITY RETAINING WALLS

EROL SADOĞLU

about the author

Erol Sadoğlu
Karadeniz Technical University,
Faculty of Engineering
Trabzon, Turkey
E-mail: erolsadoglu@hotmail.com

Abstract

The optimization for symmetrical gravity retaining walls of different heights is examined in this study. For this purpose, an optimization problem of continuous functions is developed. The continuous functions are the objective function defined as the cross-sectional area of the wall and the constraint functions derived from external stability and internal stability verifications. The verifications are listed as the overturning, the forward sliding, the bearing capacity, the shears in the stem and the bendings in the stem. The heights of the walls are selected as 2.0, 3.0, and 4.0 m in order to investigate the outline of the optimum cross-section and the effect of the wall height on the outline. Additionally, the physical and mechanical properties of the soil are kept constant in order to compare only the effect of the height on the geometry. The upper and lower bounds of the solution space are specified to be as wide as possible and the minimum dimensions suggested for the gravity retaining walls are not taken into account. A common feature of the optimum cross-sections of walls with different heights is to have a very wide lower part like a wall foundation and a slender stem. However, other than the forward sliding constraint, the bending constraints are active at the optimum values of the variables.

Keywords

gravity retaining wall, nonlinear optimization, continuous variables, interior point method

1 INTRODUCTION

A difference in the ground elevation over a random horizontal distance is usually confronted during the use of land for civil engineering purposes. The first solution to this situation is brought into being by organizing a slope. However, in some cases the proposed slope cannot support itself, and so another kind of solution called a retaining structure is required. Thus, earth-retaining structures are normally used to support soils and structures in order to maintain a difference in the elevation of the ground surface and are normally grouped into gravity walls, embedded walls, and reinforced earth walls [1].

The weight of the gravity wall provides the required stability against the effects of the retained soil and the ground water. This type of wall is generally constructed of plain concrete and masonry. In some cases, the provision of sand, gravel and cement are easier and cheaper than masonry, so it is preferable to use concrete as a construction material rather than masonry. Various cross-sections of gravity retaining walls are given in Fig. 1.

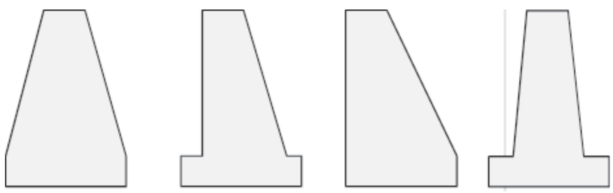

Figure 1. Cross-sections of gravity retaining walls.

The retained soil provides pressure, known as earth pressure, on the back face of the retaining wall, since the horizontal deformation of the soil is restricted by the retaining wall. One of the most crucial stages when designing a retaining wall is the determination of the earth pressure. There are several theories (Coulomb's theory [2], Rankine's theory [3], etc.) and approaches (Terzaghi-Peck Charts [4] and finite-element analysis [5,6,7,8,9]) to determine the earth pressures.

There have been a lot of studies to calculate the passive or active earth pressure since Coulomb's theory [2,3,10,11,12]. However, because of its simplicity, Rankine's theory [3] is still widely used for the determination of earth pressures acting on retaining walls. The theory is based on the assumptions that the soil is in a state of plastic equilibrium and that the Mohr-Coulomb failure criterion is valid [2]. Thus, Rankine obtained the formula below to calculate the lateral pressures on vertical planes within a mass of homogeneous, isotropic, and cohesionless soil behind a smooth wall:

$$\sigma_i = K_i \sigma_v = K_i \gamma_n z \qquad (i{=}a,\, p) \qquad (1)$$

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (2)$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) \qquad (3)$$

where:

K_a = Coefficient of earth pressure in the active state
K_p = Coefficient of earth pressure in the passive state.
σ_a = Lateral soil pressure in the active state
σ_p = Lateral soil pressure in the passive state
σ_v = Vertical pressure
ϕ = Internal friction angle

The proper design of a gravity retaining wall satisfies both the external and internal stability. The external stability is related to the interaction of the wall with the surrounding soil. The stability is ensured by verifying some failures, called sliding (on the ground), overturning, bearing capacity, and overall failures. In this way it is shown that the wall remains fixed in the desired place. The external stability verifications are carried out by defining a factor of safety, which is the ratio of the stabilizing forces (or moments) over the destabilizing forces (or moments).

The internal stability requirements are satisfied by developing a structural design with sufficient structural integrity to resist the applied loads safely [13]. The design of the wall may be carried according to the *Building Code Requirements for Structural Concrete* (ACI 318-99) [14]. The ACI code uses the ultimate strength design. Therefore, the computed loads are multiplied by the ACI load factors, which are equal to 1.6 for the earth pressure loads and 1.2 for the dead loads. However, this method

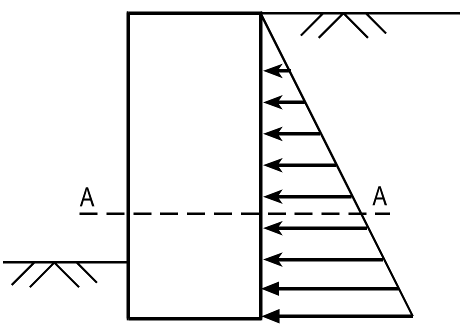

Figure 2. Shear and bending verifications.

uses a resistance factor (0.85) to the ultimate capacity for a strength-limit analysis. As a result, an evaluation between the factored forces and the nominal capacity is made. More obviously, the internal stability is checked in terms of the bending and shear verifications for some sections of the wall (Fig. 2).

Optimization is the process of obtaining the 'best', if it is possible to measure and change what is 'good' or 'bad'. Optimization practice, on the other hand, is the collection of techniques, methods, procedures, and algorithms that can be used to find the optima. Optimization problems are abundant in various fields of engineering, like electrical, mechanical, civil, chemical, and structural engineering. In recent decades, optimization methods have been widely applied to the problems of geotechnical engineering [15,16,17,18,19].

There are several general approaches to optimization, including analytical methods, graphical methods, experimental methods, and numerical methods. Analytical methods are based on the classical techniques of differential calculus and cannot be applied to highly nonlinear problems and problems involving more than two or three independent variables. Graphical methods require a plot of the function to be maximized and minimized. However, the number of independent variables does not exceed two. Experimental methods use a setup and change variables while the performance criterion is measured directly in each case (e.g., the Standard Proctor Test). Numerical methods can be used to solve highly complex optimization problems of the type that cannot be solved analytically. The discipline encompassing the theory and practice of numerical optimization methods has come to be known as mathematical programming. The branches of mathematical programming are linear programming, integer programming, quadratic programming, nonlinear programming, and dynamic programming.

The most general class of optimization problems that has both nonlinear objective functions and constraint functions is nonlinear programming. These problems can be solved using a variety of methods, such as penalty- and barrier-function methods, gradient projection methods, sequential quadratic-programming (SQP) methods, interior point methods, etc.

Rhomberg and Street [20] presented a method of proportioning for cantilever walls. The design variables were selected as the base size, the proportion of the stem to the base, the thickness and the reinforcing of the stem. Many combinations of the selected design parameters were evaluated to satisfy the basic design requirements for a minimum factor of safety, a maximum toe pressure and a minimum cost. Thus, the trial proportions were provided.

Alshawi et al. [21] proposed a new type of retaining wall called a tied-back retaining wall, for which the stem is tied to the base with inclined ties. The design bending moments were calculated for various dimensions of the retaining wall and various positions of the tie system. They found the position for the tie system where the design bending moment is a minimum. For this position of the tie system, the bending moments are greatly reduced with respect to that of the cantilever retaining wall. The design tables of the optimum cases were presented.

Other researchers also developed constrained nonlinear programming problems dealing with cantilever retaining walls [22,23]. The design variables were generally chosen as the total base width, the toe projection, the stem thickness at the bottom, the thickness of the base slab, the vertical steel area of the stem per unit length of the wall, the horizontal steel area of the toe per unit length of the wall, the horizontal steel area of the heel per unit length of the wall, etc. The total cost of the reinforced concrete wall was considered as the objective function for the analysis. The constraints were derived not only from structural safety and stability considerations but also some code provisions and practice. Sensitivity analyses were carried out to study the effect of the variation of problem parameters on the objective function.

In this work, a constrained nonlinear programming problem is defined and solved in order to find the general outline of a plain concrete gravity retaining wall in a cohesionless soil. The constraints are derived from external and internal stability criteria and the objective function is defined as the cross-sectional area of the wall. The nonlinear programming problem is solved using the interior point method.

2 OPTIMIZATION PROCEDURE

The engineering aspects that govern the design of a retaining wall are safety, stability, and cost. Safety and stability are interconnected concepts, i.e., the results of stability verifications provide information about safety, since safety is defined mostly on the results of stability verifications. Cost requires building with enough safety and usability for the least cost. Therefore, the cost of the wall should be minimized while some verifications are fulfilled. Consequently, the design of a gravity retaining wall can be thought of as an optimization problem. Clearly, a cost-related function emerges as an objective function and the constraints are derived from the stability verifications. The design variables of this optimization problem are chosen as the thicknesses of the gravity retaining wall at regular distances from the base (Fig. 3). The vertical distances between the thicknesses are chosen as one-sixth of the wall's height. Thus, the number of design parameters is prevented from becoming too large and sufficient points are determined in order to obtain the optimum outline of the concrete gravity retaining wall.

2.1 OBJECTIVE FUNCTION

An objective function is a mathematical expression that should be maximized or minimized in certain conditions and chosen as the volume, cost, weight, etc. in structural engineering [24]. The aim of this optimization problem is to determine the cross-section of the wall that minimizes the cost. Therefore, the objective function is chosen as the cross-sectional area, because the cost of the formwork and the scaffolding is mainly dependent on the wall's height. Thus, the wall with

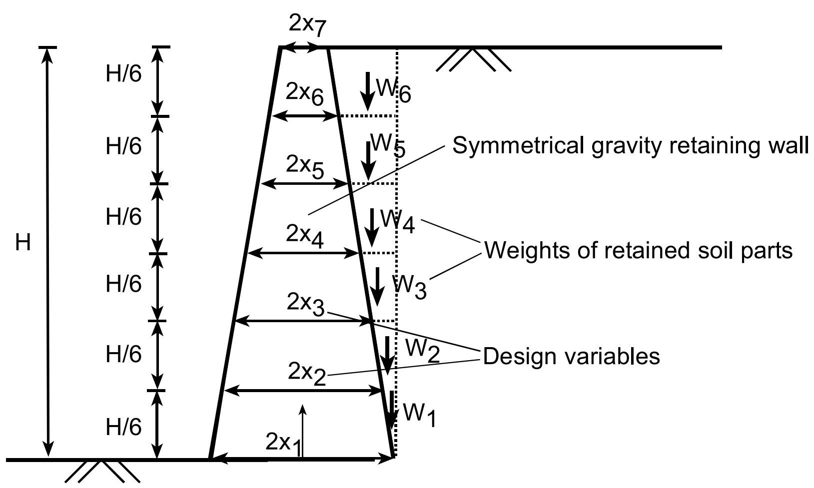

Figure 3. Design variables of gravity retaining wall.

minimum cross-sectional area can be considered to have the lowest cost.

$$f(x) = \frac{H}{6}(x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + x_7) \quad (4)$$

2.2 CONSTRAINTS

The design of a gravity retaining wall must ensure that the wall has enough external and internal stability. External stability is satisfied if the wall remains fixed in its desired position, except for small movements causing active and passive states in the nearby soil. The forces causing instability are the resultant of earth pressures in this case. The earth pressures are calculated according to Rankine's Theory [3]. Duncan et al. [25] proposed the following external stability criteria for granular backfill and foundation soils:

a) *N* within the middle third of the base (*N* = the sum of vertical forces acting on the wall),
b) $q_{allowable} \geq q_{max}$, ($q_{allowable}$ = allowable bearing capacity, q_{max} = maximum base pressure)
c) Safe against sliding,
d) Settlement within tolerable limits

Overturning about the toe criterion is often met in the design stage in addition to the foregoing criteria. The settlement criterion is mainly related to the layers beneath the wall, and therefore it is neglected in this study.

Internal stability guarantees that the wall carries the loads acting on it without rupturing. In other words, a gravity retaining wall must be capable of resisting the internal shear forces and bending moments caused by earth pressures and for other reasons. Apparently, the distribution and magnitude of lateral earth pressures should be known to find the internal forces and moments to evaluate the internal stability criteria. The lateral earth pressure is calculated according to Rankine's theory, because it is widely used and easily adapted to this optimization study [3].

2.2.1 overturning constraint

The factor of safety against overturning is determined by dividing the resisting moments by the driving moments. The resisting moments are caused by the weight of the wall and the weight of the retained soil above the base of the wall (Fig 3.). The driving moment is the overturning effect of the active earth pressure. However, the factor of safety depends on the point around which we compute the moments. The overturning stability is evaluated only around the toe point in practice [13]. The constraint from overturning about the toe is derived as below.

$$\begin{aligned}
g_1(x) = {} & FS_o \cdot \frac{\gamma_n}{6} \cdot H^3 \cdot K_a - \frac{H}{6}\gamma_c(x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 + 2x_6 + x_7)x_1 \\
& - \frac{H}{12}\gamma_n[(x_1 - x_2)\left(2x_1 - \frac{(x_1 - x_2)}{3}\right) \\
& + (2x_1 - x_2 - x_3)\left(2x_1 - \frac{(x_1-x_3)^2 + (x_1-x_3)(x_1-x_2) + (x_1-x_2)^2}{3(2x_1 - x_3 - x_2)}\right) \\
& + (2x_1 - x_3 - x_4)\left(2x_1 - \frac{(x_1-x_4)^2 + (x_1-x_4)(x_1-x_3) + (x_1-x_3)^2}{3(2x_1 - x_4 - x_3)}\right) \\
& + (2x_1 - x_4 - x_5)\left(2x_1 - \frac{(x_1-x_5)^2 + (x_1-x_4)(x_1-x_5) + (x_1-x_4)^2}{3(2x_1 - x_5 - x_4)}\right) \\
& + (2x_1 - x_5 - x_6)\left(2x_1 - \frac{(x_1-x_6)^2 + (x_1-x_5)(x_1-x_6) + (x_1-x_5)^2}{3(2x_1 - x_6 - x_5)}\right) \\
& + (2x_1 - x_6 - x_7)\left(2x_1 - \frac{(x_1-x_7)^2 + (x_1-x_6)(x_1-x_7) + (x_1-x_6)^2}{3(2x_1 - x_6 - x_7)}\right)] \leq 0
\end{aligned} \quad (5)$$

where:

γ_n = Unit weight of the soil
γ_c = Unit weight of the concrete.
FS_o= Factor of safety against overturning

2.2.2 sliding constraint

The sliding verification is carried out by comparing the forces causing the sliding with the forces resisting it. The earth pressure acting on the back of the wall is the only sliding force and the friction along the bottom of the base is the only resisting force in this case. The resultant of the friction is calculated by multiplying the weight of the wall and the soil acting as part of the wall by the coefficient of the base friction. Thus, the sliding constraint is defined as follows:

$$\begin{aligned}
g_2(x) = {} & FS_s\left(\frac{1}{2}\gamma_n H^2 K_a\right) - [\frac{H}{6}\gamma_c(x_1 + 2x_2 + 2x_3 + \\
& 2x_4 + 2x_5 + 2x_6 + x_7) + \\
& \frac{H}{12}\gamma_n((x_1 + x_2) + (2x_1 - x_2 - x_3) + (2x_1 - x_3 - x_4) \\
& + (2x_1 - x_4 - x_5) \\
& + (2x_1 - x_5 - x_6) + (2x_1 - x_6 - x_7))]\tan\delta \leq 0
\end{aligned} \quad (6)$$

where:

$\tan\delta$ = Friction coefficient between the wall base and the foundation soil
FS_s = Factor of safety against sliding

2.2.3 Total vertical forces within the middle third of the base constraint

Duncan et al. [25] suggested that eccentricity should be within the middle third of the base. Thus, the separation between the wall base and the soil does not occur. This constraint is inspired by the idea that tensile stresses do not develop in the base-pressure distribution. The constraint is given as below:

$$
\begin{aligned}
g_3(x) = \{ & \frac{1}{6}\gamma_n H^3 K_a - \frac{H}{12}\gamma_n[(x_1-x_2)\left(x_1-\frac{(x_1-x_2)}{3}\right)+\\
&(2x_1-x_2-x_3)\left(x_1-\frac{(x_1-x_3)^2+(x_1-x_3)(x_1-x_2)+(x_1-x_2)^2}{3(2x_1-x_2-x_3)}\right)+\\
&(2x_1-x_3-x_4)\left(x_1-\frac{(x_1-x_3)^2+(x_1-x_3)(x_1-x_4)+(x_1-x_4)^2}{3(2x_1-x_3-x_4)}\right)+\\
&(2x_1-x_4-x_5)\left(x_1-\frac{(x_1-x_4)^2+(x_1-x_4)(x_1-x_5)+(x_1-x_5)^2}{3(2x_1-x_4-x_5)}\right)+\\
&(2x_1-x_4-x_5)\left(x_1-\frac{(x_1-x_4)^2+(x_1-x_4)(x_1-x_5)+(x_1-x_5)^2}{3(2x_1-x_4-x_5)}\right)+\\
&(2x_1-x_5-x_6)\left(x_1-\frac{(x_1-x_5)^2+(x_1-x_5)(x_1-x_6)+(x_1-x_6)^2}{3(2x_1-x_5-x_6)}\right)+\\
&(2x_1-x_6-x_7)\left(x_1-\frac{(x_1-x_6)^2+(x_1-x_6)(x_1-x_7)+(x_1-x_7)^2}{3(2x_1-x_6-x_7)}\right)]\}/\left(4x_1^2/6\right)-\\
&\{\frac{H}{6}\gamma_c(x_1+2x_2+2x_3+2x_4+2x_5+2x_6+x_7)+\frac{H}{12}\gamma_n((x_1-x_2)+(2x_1-x_2-x_3)+\\
&(2x_1-x_3-x_4)+(2x_1-x_4-x_5)+(2x_1-x_5-x_6)+(2x_1-x_6-x_7))\}/(2x_1)\le 0
\end{aligned}
\quad (7)
$$

2.2.4 Bearing capacity constraint

Bearing-capacity failure happens in soil when the contact pressure between the footing and the soil causes shear failure in the foundation soil. In this optimization problem the Terzaghi bearing-capacity theory [26] is used to calculate the safe bearing capacity and compared with the maximum base pressure. The constraint from the bearing-capacity failure is given as below:

$$
\begin{aligned}
g_4(x) = \{ & \frac{1}{6}\gamma_n H^3 K_a - \frac{H}{12}\gamma_n[(x_1-x_2)\left(x_1-\frac{(x_1-x_2)}{3}\right)+\\
&(2x_1-x_2-x_3)\left(x_1-\frac{(x_1-x_3)^2+(x_1-x_3)(x_1-x_2)+(x_1-x_2)^2}{3(2x_1-x_2-x_3)}\right)+\\
&(2x_1-x_3-x_4)\left(x_1-\frac{(x_1-x_3)^2+(x_1-x_3)(x_1-x_4)+(x_1-x_4)^2}{3(2x_1-x_3-x_4)}\right)+\\
&(2x_1-x_4-x_5)\left(x_1-\frac{(x_1-x_4)^2+(x_1-x_4)(x_1-x_5)+(x_1-x_5)^2}{3(2x_1-x_4-x_5)}\right)+\\
&(2x_1-x_5-x_6)\left(x_1-\frac{(x_1-x_5)^2+(x_1-x_5)(x_1-x_6)+(x_1-x_6)^2}{3(2x_1-x_5-x_6)}\right)+\\
&(2x_1-x_6-x_7)\left(x_1-\frac{(x_1-x_6)^2+(x_1-x_6)(x_1-x_7)+(x_1-x_7)^2}{3(2x_1-x_6-x_7)}\right)]\}\\
&/\left(4x_1^2/6\right)+\{\frac{H}{6}\gamma_c(x_1+2x_2+2x_3+2x_4+2x_5+2x_6+x_7)\\
&+\frac{H}{12}\gamma_n((x_1-x_2)+(2x_1-x_2-x_3)+\\
&(2x_1-x_3-x_4)+(2x_1-x_4-x_5)+(2x_1-x_5-x_6)+\\
&(2x_1-x_6-x_7))\}/(2x_1)-\frac{\gamma_n x_1 N_\gamma}{FS_b}\le 0
\end{aligned}
\quad (8)
$$

where:

N_γ = Bearing capacity factor
FS_b = Factor of safety against bearing capacity failure

2.2.5 Shear verification constraints

Shear and bending verifications are carried out to satisfy and verify the internal stability of the retaining wall. Therefore, the shear capacity of the different stem sections of a concrete gravity wall is calculated according to the *Building Code Requirements for Structural Concrete* [14]. As there are no stirrups, the concrete of the stem must have adequate capacity to resist the shear force. The nominal shear capacity is:

$$V_n/b = \frac{1}{6} b_w\, d\sqrt{f'_c} \quad (9)$$

where:

V_n/b = Nominal shear capacity per unit length of the wall
b_w = Width of shear surface (unit length for the wall)
d = Effective depth (thickness of the stem)
f'_c = 28-day compressive strength of the concrete

ACI 318 uses the ultimate strength design so the computed lateral earth pressures must be multiplied by the load factors. The factors for earth pressure and dead load are 1.6 and 1.2, respectively.

The stem must have adequate thickness so that the following condition is fulfilled.

$$V_u/b \le 0.5\varphi V_n\ /b \quad (10)$$

where:

V_u/b = Factorized shear force per unit length of the wall.
φ = Resistance factor = 0.85

As a result, the following constraints are obtained for five different sections, from the bottom to the top.

$$
\begin{aligned}
g_5(x) &= 1.6\cdot\frac{1}{2}\cdot\gamma_z\cdot\left(\frac{H}{6}\right)^2\cdot K_a-\frac{0.85}{6}\cdot x_6\cdot\sqrt{f'_c}\le 0\\
g_6(x) &= 1.6\cdot\frac{1}{2}\cdot\gamma_z\cdot\left(\frac{H}{}\right)^2\cdot K_a-\frac{0.85}{}\cdot x_5\cdot\sqrt{f'_c}\le 0\\
g_7(x) &= 1.6\cdot\frac{1}{2}\cdot\gamma_z\cdot\left(\frac{H}{}\right)^2\cdot K_a-\frac{0.85}{}\cdot x_4\cdot\sqrt{f'_c}\le 0\\
g_8(x) &= 1.6\cdot\frac{1}{2}\cdot\gamma_z\cdot\left(\frac{2H}{3}\right)^2\cdot K_a-\frac{0.85}{6}\cdot x_3\cdot\sqrt{f'_c}\le 0\\
g_9(x) &= 1.6\cdot\frac{1}{2}\cdot\gamma_z\cdot\left(\frac{5H}{6}\right)^2\cdot K_a-\frac{0.85}{6}\cdot x_2\cdot\sqrt{f'_c}\le 0
\end{aligned}
\quad (11)
$$

2.2.6 Bending verification constraints

The thicknesses of the stem must have enough strength to resist the bending moments. The tensile strength of the concrete can be used in the design, because the stem of the wall can tolerate random cracks without detrimentally affecting their structural integrity, and ductility is not an essential feature of the design. The constraints from the bending verification can be given as follows:

$$g_{10}(x)=1.6\times\frac{\gamma_n\left(\frac{H}{6}\right)^3 K_a}{4x_6^{\,2}}-1.2\times\frac{\gamma_c\frac{H}{6}(x_6+x_7)}{2x_6}-f_{ctd}\le 0$$

$$g_{11}(x)=1.6\times\frac{\gamma_n\left(\frac{H}{3}\right)^3 K_a}{4x_5^{\,2}}-1.2\times\frac{\gamma_c\frac{H}{6}(x_5+2x_6+x_7)}{2x_5}-f_{ctd}\le 0$$

$$g_{12}(x)=1.6\times\frac{\gamma_n\left(\frac{H}{2}\right)^3 K_a}{4x_4^{\,2}}-1.2\times\frac{\gamma_c\frac{H}{6}(x_4+2x_5+2x_6+x_7)}{2x_4}-f_{ctd}\le 0$$

$$g_{13}(x)=1.6\times\frac{\gamma_n\left(\frac{2H}{3}\right)^3 K_a}{4x_3^{\,2}}-1.2\times\frac{\gamma_c\frac{H}{6}(x_3+2x_4+2x_5+2x_6+x_7)}{2x_3}-f_{ctd}\le 0 \quad (12)$$

$$g_{14}(x)=1.6\times\frac{\gamma_n\left(\frac{5H}{6}\right)^3 K_a}{4x_2^{\,2}}-1.2\times\frac{\gamma_c\frac{H}{6}(x_2+2x_3+2x_4+2x_5+2x_6+x_7)}{2x_2}-f_{ctd}\le 0$$

where:

f_{ctd} = Concrete design strength in tension

2.2.7 Design constraints

The minimum thickness for the concrete gravity wall at the top is recommended to be 0.3 m [27]. However, this is in contradiction with the aim of this study, i.e., to see the outline of the optimum cross-section this aspect is not taken into account. However, the x_i dimensions must be greater than zero. Therefore, the following constraints can be defined.

$$\begin{bmatrix}-1&0&0&0&0&0&0\\0&-1&0&0&0&0&0\\0&0&-1&0&0&0&0\\0&0&0&-1&0&0&0\\0&0&0&0&-1&0&0\\0&0&0&0&0&-1&0\\0&0&0&0&0&0&-1\end{bmatrix}\times\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\end{bmatrix}\le\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\end{bmatrix} \quad (13)$$

2.3 OPTIMIZATION METHOD

The constrained nonlinear optimization problem is defined as below:

$$\text{minimize } f(x) \qquad (14)$$

$$\text{subject to: } g_i(x) \le 0 \text{ for } i = 1, 2, \ldots, n \qquad (15)$$

where $f(x)$ and $g_i(x)$ are continuous and have continuous second partial derivatives, and the feasible region described by Eq. (15) is non-empty. A computer program was developed to solve the problem using the interior point method. The details and algorithms of this method can be found in books about nonlinear optimization [28].

3 OPTIMIZATION EXAMPLES

Several physical and mechanical properties of the walls and soils are assigned to the functions of the optimization problem in order to obtain the optimum cross-section of these situations. The assigned parameters corresponding to the optimization problem are given in

Table 1. Input parameters for design examples.

Input Parameters	Unit	Symbol	Value
Height of the wall	m	H	2.0
Internal friction angle of the retained soil	degree	ϕ	35
Internal friction angle of the base soil	degree	ϕ_b	35
Friction angle between the wall base and the soil	degree	δ	25.35
Unit weight of the retained soil	kN/m^3	γ_n	16.0
Unit weight of the base soil	kN/m^3	γ_{nb}	16.0
Concrete design strength in tension	Mpa	f_{ctd}	0.9
Concrete characteristic strength in compression	Mpa	f'_c	16.0
Unit weight of the concrete	kN/m^3	γ_c	25.0
Factor of safety for the overturning stability	-	FS_o	2.0
Factor of safety against sliding	-	FS_s	1.5
Factor of safety for the bearing capacity	-	FS_b	3.0

Table 1. The heights of the walls are chosen to be 2.0, 3.0 and 4.0 m, because gravity retaining walls are generally used for heights of less than 5 m. The other parameters are selected as typical values for concrete and cohesionless soil. The coefficient of base friction between medium sand and cast in place concrete can be taken as between 0.45 and 0.55, according to the NAVFAC Design Manual [29].

The first example is the wall with a height of 2 m, and the x_i values of the wall acquired from the optimization study are x_1=0.6578 m, x_2= 0.0934 m, x_3=0.0670 m, x_4=0.0436 m, x_5=0.0238 m, x_6=0.0084 m, and x_7=0.0000 m. The cross-sectional area of the wall is 0.3767 m^2, which is the result of the objective function too. The first striking feature of the cross-section is that a wide part occurs at the bottom of the wall like a footing (Fig. 4). It is seen from the results that active constraints determining the optimum values are the sliding and bending in the stem (Table 2). The constraints derived from the overturning, the total vertical force in the middle third, the bearing capacity, and the shear in the stem verifications are inactive.

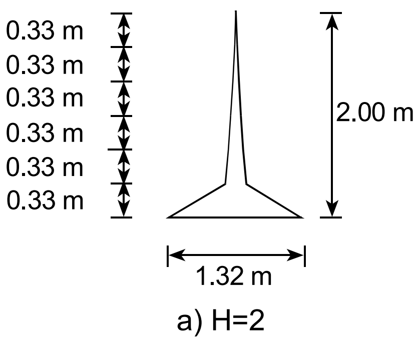

Figure 4. Optimum cross-section of a gravity retaining wall with a height of 2 m.

The second example is a retaining wall of 3.0 m in height. The optimum xi values are x_1=0.9633 m, x_2=0.1707 m, x_3=0.1225 m, x_4=0.0798 m, x_5=0.0436 m, x_6=0.0155 m, and x_7=0.0000, and the objective function takes the value of 0.9137 (Fig. 5). The factors of safety for the overturning, the sliding, and the bearing capacity are 4.1068, 1.5000, and 7.7099, respectively. The sliding and bending in the stem verifications are the constraints that are equal to zero. In other words, these constraints are active (Table 2). The cross-section of the wall has a very narrow upper part, almost zero at the top and a wide lower part like a footing. The wide lower part contributes the stability of the wall with the retained soil over the base of the wall, especially for the overturning and sliding checks.

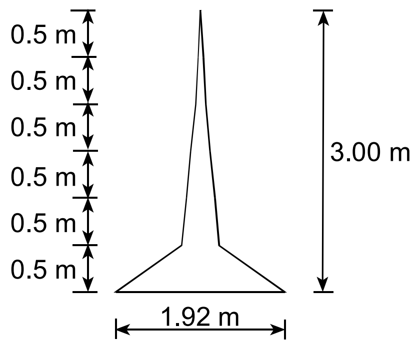

Figure 5. Optimum cross-section of a gravity retaining wall with a height of 3 m.

The retaining wall with a height of 4.0 m is considered as the third example. The factors of safety for the overturning, the sliding and the bearing capacity are 3.9975, 1.5000, and 7.0530, respectively. The optimum values of the variables are x_1=1.2584 m, x_2=0.2613 m, x_3=0.1878 m, x_4=0.1225 m, x_5=0.0670 m x_6=0.0238 m and x_7=0.0000 m and area of the cross-section is equal to 1.7220 m^2 (Fig. 6). The $g_2(x)$, $g_{10}(x)$, $g_{11}(x)$, $g_{12}(x)$, $g_{13}(x)$ and $g_{14}(x)$ constraints derived from the sliding and bending in the stem verifications, respectively, are zero at the optimum x_i values. The cross-section of the wall is very similar to the walls with heights of 2 and 3 m.

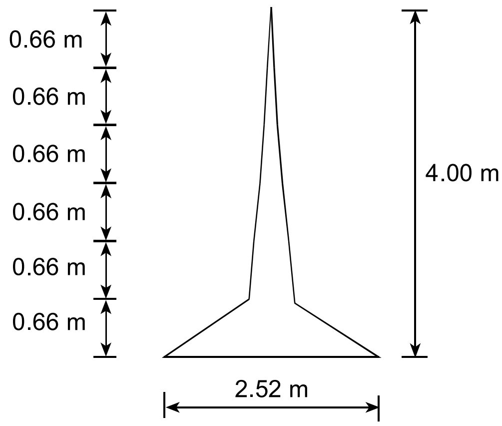

Figure 6. Optimum cross section of a gravity retaining wall with a height of 3 m.

The constraint functions are evaluated at the optimum solution points and the results are given in Table 2. The g2(x) constraint is equal to zero for all the gravity walls mentioned above. Thus, it can be said that the sliding constraint is active for all the walls. Additionally, the constraints derived from the bending in the stem verifications are also equal to zero for the walls. Therefore, bending in the stem checks becomes the other active constraint.

Table 2. Results of the constraint functions for the optimum x_i values.

Constraints	H=2 m	H=3 m	H=4 m
$g_1(x)$	-12.9428	-41.1071	-92,3876
$g_2(x)$	0.0000	0.0000	0,0000
$g_3(x)$	-23.1743	-33.8651	-44,0500
$g_4(x)$	-140.7466	-203.0383	-261,5659
$g_5(x)$	-4.3915	-7.8965	-11,9324
$g_6(x)$	-11.9432	-21.2465	-31,7960
$g_7(x)$	-21.2523	-37.4571	-55,5754
$g_8(x)$	-31.8122	-55.5870	-81,8056
$g_9(x)$	-43.3270	-75.0844	-109,6252
$g_{10}(x)$	0.0000	0.0000	0,0000
$g_{11}(x)$	0.0000	0.0000	0,0000
$g_{12}(x)$	0.0000	0.0000	0,0000
$g_{13}(x)$	0.0000	0.0000	0,0000
$g_{14}(x)$	0.0000	0.0000	0,0000

A common feature of the optimum cross-sections of the walls is to have a wide lower part like a strip footing. In addition, the bending in the stem verifications determines the thicknesses of the stems at a regular interval from the base. The general outlines of the optimum cross-section are similar to each other (Fig. 4, 5, and 6). The normalized thicknesses obtained by dividing the thicknesses by the height of the wall are given in Fig. 7. The normalized thicknesses are almost equal for the walls and the normalized thicknesses increase slightly with increasing height, except for the thickness at the base (corresponding to x_1).

The normalized dimensions of the condition that has an internal friction angle of 25° for the retained soil and the

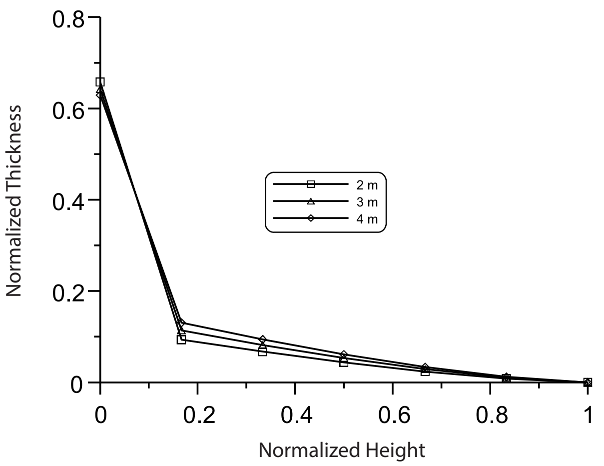

Figure 7. Normalized dimensions for ϕ=35°.

foundation soil are given in Fig. 8. The other parameters are assigned to the constrained nonlinear programming problem, as in Table 1. Here, a 10° decrease in internal friction angle causes an approximately 52% increase in the normalized thickness corresponding to x_1, and a nearly 22% increase occurs in the normalized thicknesses corresponding to x_2, x_3, x_4, x_5, x_6. The active constraints are sliding and bending in the stem verifications in this case too. Such an increase in the width corresponding to x_1 is caused by a high lateral earth pressure due to a low internal friction angle. The base key can be utilized in order to obtain a viable bottom width.

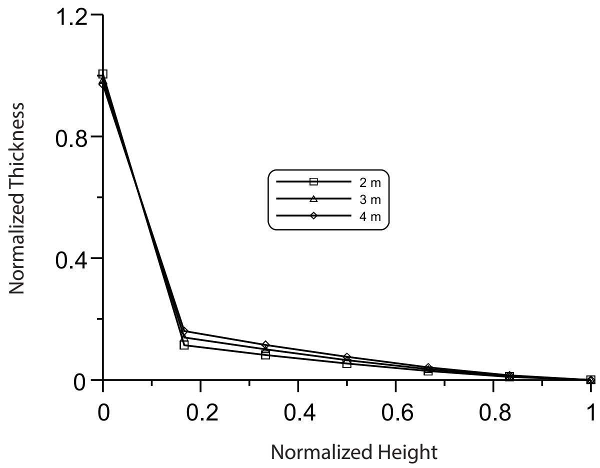

Figure 8. Normalized dimensions for ϕ=25°.

The normalized dimensions given in Fig. 7 and Fig. 8 can be used for proportioning and the normalized thicknesses of the internal friction angles between 25° and 35° can be found by linear interpolation. The sliding constraint is the active constraint for all cases, so some measures, for example, adding a base key beneath the footing, installing anchors, etc., may be used to design more economic cross-sections for gravity retaining walls.

4 CONCLUSIONS

This study aimed to determine the optimum cross-section outline of a symmetrical gravity retaining wall on granular soil. The cross-sectional area of the plain concrete wall is assumed to be a direct indicator of the cost. Therefore, the objective function is defined as the cross-sectional area. Additionally, the constraints of the optimization problem are derived from the verifications that a concrete gravity retaining wall should satisfy. Thus, the constraint nonlinear optimization problem, defined by the objective function and constraints, is obtained. The problem is solved by developing a computer-program-based interior point method. The check of the sliding is an active constraint for external stability verifications and the bending in the stem verifications is an active constraint of the internal stability checks. The cross-sections of the walls with different heights have similar outlines. The outlines of the optimum cross-sections have wide lower parts, like wall footings, and slender stems that have minimum thicknesses satisfying the bending verifications. The areas of the optimum cross-sections are less than those of conventional plain concrete gravity retaining walls. The use of these optimum cross-sections will substantially reduce the costs.

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