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Vulnerability bounds on the number of spanning tree leaves

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Abstract

Hamiltonicity and vulnerability of graphs are in a strong connection. A basic necessary condition states that a graph containing a 2-leaf spanning tree (that is, a Hamiltonian path) cannot be split into more than $k + 1$ components by deleting k of its vertices. In this paper we consider a more general approach and investigate the connection between the number of spanning tree leaves and two vulnerability parameters, namely scattering number sc(G) [10] and cut-asymmetry ca(G) [16]. We prove that any spanning tree of a graph G has at least sc(G)+1 leaves. We also show that if $X \subset V$ is a maximum cardinality independent set of $G = (V,E)$, such that the elements of X are all leaves of a particular spanning tree then $|X| =$ ca(G)+1 $= |V| -$ cvc(G), where cvc(G) is the size of a minimum connected vertex cover of G. As a consequence we obtain a new proof for the following results: any spanning tree with independent leaves provides a 2-approximation for both the *maximum internal spanning tree* [16] and the *minimum connected vertex cover* [17] problems. We also consider the opposite point of view by fixing the number of leaves to q and looking for a q-leaf subtree of G that spans a maximum number of vertices. Bermond [2] proved that a 2-connected graph on n vertices always contains a path (a 2-leaf subtree) of length min $\{n, \delta_2\}$, where δ_2 is the minimum degree-sum of a 2-element independent set. We generalize this result to obtain a sufficient condition for the existence of a large q-leaf subtree.

Keywords: Spanning tree leaves, vulnerability, Hamiltonian path.

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Ranljivostne meje števila listov vpetega drevesa

Povzetek

Hamiltonskost in ranljivost grafov sta tesno povezani. Osnovni potrebni pogoj pravi, da se grafa, ki ima 2-listno vpeto drevo (torej Hamiltonovo pot), z izbrisom k njegovih vozlišč ne da razcepiti na več kot k+1 komponent. V članku uberemo splošnejši pristop in raziskujemo zvezo med številom listov vpetega drevesa in dvema ranljivostnima parametroma, *razpršenostjo* sc(G) [10] in *rezno-asimetrijo* ca(G) [16]. Tako dokažemo, da ima vsako vpeto drevo grafa G najmanj sc(G)+1 listov. Pokažemo tudi, da če je $X \subset V$ najmočnejša neodvisna množica v $G = (V,E)$ z lastnostjo, da so vsi njeni elementi listi nekega vpetega drevesa, potem je $|X|$ = ca(G)+1 = $|V|$ – cvc(G), kjer je cvc(G) velikost najmanjšega povezanega vozliščnega krova nad G. Od tod dobimo nov dokaz naslednjih rezultatov: vsako vpeto drevo z neodvisnimi listi nam da 2-aproksimacijo tako za *maksimalno notranje vpeto drevo* [16] kot tudi za probleme v zvezi z *minimalnim povezanim vozliščnim krovom* [17]. Problem obravnavamo tudi z nasprotnega zornega kota, tako da pri fiksnem številu listov q iščemo q-listno poddrevo grafa G, ki vsebuje maksimalno število vozlišč. Bermond [2] je dokazal, da 2-povezan graf na n vozliščih vedno vsebuje pot (2-listno poddrevo) dolžine $\min\{n, \delta_2\}$, kjer je δ_2 minimalna vsota stopenj 2-elementne neodvisne množice. S posplošitvijo tega rezultata dobimo zadosten pogoj za obstoj velikega q-listnega poddrevesa.

Ključne besede: Listi vpetega drevesa, ranljivost, Hamiltonova pot.