Zbornik referatov 18. Goljevščkovega spominskega dne - Proceedings of 18th Goljevšček Memorial Day
© *Acta hydrotechnica* **17/26 (1999)**, 93 p., Ljubljana

UDK: 519.87:628.35
Pregledni znanstveni članek

UDC: 519.87:628.35
Scientific paper

MATEMATIČNO MODELIRANJE PRIRASTA KOSMOV V BIOKEMIJSKIH REAKTORJIH
MATHEMATICAL MODELLING OF THE GROWTH OF FLOCKS IN THE BIOCHEMICAL REACTOR

Jože PANJAN

V bazenu za poživljanje - biološkem reaktorju, v katerem v bistvu pospešeno poteka naravno samočiščenje, želimo pretvoriti raztopljene in neusedljive snovi v usedljivo obliko. To nam omogočajo mikroorganizmi, ki tvorijo razpršeno biološko maso v vodi biokemijskega reaktorja. Da pride do rasti mikroorganizmov, mora odpadna voda zadostiti najmanjšim pogojem za njihovo rast. Organizmi v poživljenem blatu prevzamejo organsko in delno mineralno snov iz odpadne vode in jo spreminjajo v nove organizme. Ti potem tvorijo kosme poživljenega blata v naknadnih (sekundarnih) usedalnikih. Procese prirasta kosmov lahko ocenjujemo eksperimentalno pa tudi z matematičnimi modeli. Pri matematičnem modeliranju uporabljamo enačbe difuzije glukoze (sladkorja) in kisika v kosem. Prikazali bomo matematične modele za kosme okrogle, cilindrične in diskaste oblike ter enačbo za strižne sile, ki določajo največjo velikosti kosmov v reaktorju ter enačbo za stabilnost kosmov.
Ključne besede: *matematični model, biološki reaktor, mikroorganizmi, kosmi, blato*

The purpose of the biochemical reactor, in which an accelerated natural self-purification is carried out, is to covert dissolved and insoluble substances into a settleable form. This process is enabled by microorganisms which constitute a part of the dispersed biomass in the water of the biochemical reactor. Wastewater should satisfy the minimum conditions for the growth of microorganisms. Organisms in the activated sludge take over organic and partially inorganic compounds and transform them into new organisms which form flocks of the activated sludge in the secondary treatment basin. Processes of the growth of flocks can be evaluated experimentally or by mathematical models. Diffusion equations for glucose and oxygen into flocks are applied. Mathematical models for spherical, cylindrical and disklike flocks, the equation for shearing forces which determine the maximal dimensions of flocks in the reactor and the stability equation will be demonstrated.
Key words: *mathematical model, biological reactor, microorganisms, flocks, sludge*

1. UVOD

Pri čiščenju odpadnih voda tako v naravi kot v biološkem reaktorju komunalnih čistilnih naprav sodelujejo mikroorganizmi. V biološkem reaktorju lahko spremljamo procese na makro ali mikro ravni. Nas v tem primeru zanimajo predvsem dogajanja na mikro ravni, to je na mikroorganizmih. Eni najpomembnejših temeljnih nosilcev biološkega čiščenja med mikroorganizmi so bakterije. Bakterije so zelo majhna bitja, ki merijo le nekaj tisočink milimetra oz. par deset

1. INTRODUCTION

Microorganisms participate in wastewater treatment in a natural state and also in the biochemical reactor of the sewage plant. Processes in the biochemical reactor can be studied on both the macro and micro levels. In this case we are especially interested in processes on the micro level, those taking place with microorganisms. Bacteria are one of the most important participants of the biochemical treatment among microorganisms. Bacteria are very small beings, measuring only a few thousandths of a millimetre, mikronov. Poznamo krogličaste, paličaste – valjaste (ravne ali krive), diskaste in spiralne. Poleg teh so še bakterije posebnih oblik, od katerih nekatere prehajajo, meje ne moremo natančno določiti, h glivam. Živijo posamezno ali v kolonijah. Pri bakterijah je težko ali nemogoče diferencirati genetski material od ostale vsebine celice in nimajo pravega jedra. So skoraj vedno brez klorofila (zelenih barvil). Bakterije se razmnožujejo z delitvijo celic. Poleg bakterij so pomembne tudi protozoe ali praživali. Protozoi so živalski enoceličarji, ki imajo v nasprotju z bakterijami, že pravo jedro, eno ali celo več. Glede na njihovo zgradbo poznamo razmeroma preproste oblike, npr. amebe ali pa tudi zelo diferencirane, predvsem kar zadeva oblikovanje jedra in načine za izločanje vode, premikanje, občutenje dražljajev, kot npr. parameciji. Protozoi so z vidika velikosti bistveno večji od bakterij, vendar so še vedno mikroskopsko majhni, saj večina akvatičnih oblik meri manj kot 1/10 mm.

V tem članku bomo prikazali prirast biomase mikroorganizmov (celic) različnih oblik na mikro ravni. Sama delitev - hitrost prirasta je različna. Večina tistih bakterij, ki živijo od organske snovi, se deli v času manj kot ene ure.Večina bakterij sicer živi od mrtve organske substance, ki jo razkraja v navzočnosti kisika, v okviru izmenjave energije in snovi. Razpolovna doba protozoev je pri optimalnih pogojih, pri manjših oblikah nekaj ur, pri večjih pa več dni.

2. PRIRAST BIOMASE IN MOŽNOST KOSMIČENJA V BIOLOŠKEM REAKTORJU

V bazenu za poživljanje - biološkem reaktorju, v katerem v bistvu pospešeno poteka naravno samočiščenje, želimo pretvoriti raztopljene in neusedljive snovi v usedljivo obliko. To nam omogočajo mikroorganizmi, ki tvorijo razpršeno biološko maso. Da pride do rasti mikroorganizmov, mora odpadna voda zadostiti minimalnim pogojem za njihovo rast.

respectively, few microns, which appear in spherical, sticklike – cylindrical (straight and curved), disklike and spiral forms. Besides these, some bacteria appear in special forms – some of them represent the transition to fungi without any clearly established boundary. They live solitary or in colonies. It is difficult or even impossible to distinguish genetic matter from the rest of the bacterial cells, as they have no discernible nucleus. They are nearly always without chlorophyll. Multiplication by cell division is common with them. Protozoa, in addition to bacteria, are also important. Protozoa are unicellular animals, which have one or more discernible nuclei in contradiction to bacteria. Relatively simple forms are known with respect to their structure, as , for instance, amoeba, or also very differentiated ones – especially with respect to the formation of their nucleus and methods for the excretion of water, movement and the perception of stimuli, as, for instance, paramecia. Although protozoa are much larger than bacteria, they are still of a microscopic size, as most of the aquatic forms measure less than 1/10 of a millimetre.

The biomass increase of microorganisms (cells) of different forms at the micro level will be presented in this article. The very cell division - the rate of growth is different. Most bacteria living on organic compounds multiply in less than an hour. They live on dead organic matter, which is degraded by them in the presence of oxygen in the cycle of the exchange of energy and matter. The half-life period of protozoa is a few hours for smaller forms and a few days for larger ones under optimal conditions.

2. THE GROWTH OF THE BIOMASS AND THE POSSIBILITY OF FLOCCULATION IN THE BIOCHEMICAL REACTOR

The biochemical reactor is intended to convert dissolved and insoluble substances into settleable ones by means of an accelerated natural self-purification. This is enabled by the microorganisms which form a dispersed biomass. Wastewater should satisfy some minimal conditions to achieve the growth of microorganisms. These are especially:

To pa so predvsem:

- zahteva po lastnosti hranilne raztopine (substrata)
- ustrezna temperatura
- zagotovljena količina zraka ali kisika
- stalno gibanje, da ne pride do usedanja in da imajo mikroorganizmi čim boljši stik s hrano in kisikom
- vrednost pH oz. nevtralnost je lahko v mejah od 5 do 9.

Prvemu in drugemu pogoju zadostuje večina komunalnih odpadnih voda. Tretjega in četrtega zagotovimo s turbinskimi mešali, z vpihovanjem zraka, s kisikovo atmosfero ali mešanjem s curkom, vrednost pH pa moramo zagotoviti že pri predčiščenju industrijskih odpadnih voda. Zagotovljeni pogoji v okolju omogočajo razvoj različnih združb mikroorganizmov (biocenoze), ki se glede na spremembe pogojev tudi sami spreminjajo - iz nižjih v višje razvite oblike in obratno, ali npr. en tip združbe prevlada nad drugimi. Organizmi v poživljenem blatu prevzamejo organsko in delno mineralno snov iz odpadne vode in jo spreminjajo v nove organizme (I. faza čiščenja), ki tvorijo kosme poživljenega blata v naknadnih (sekundarnih) usedalnikih.

Osnovni kosmi - flokule imajo velikost 10^{-3} do 1 µm, super koloidi od 1 do 50 µm in neraztopljene in težko usedljive (lebdeče) snovi velikost od 50 do 100 µm (Lau et al., 1984; Panjan, 1996). Mikroorganizmi in koloidi so usedljivi - izločljivi šele, ko tvorijo velike kosme. Usedanje se prične šele v naknadnih usedalnikih v mirujočem vodnem okolju po skosmičenju primarnih kosmov, ko imajo ti premer, večji od 50 do 100 µm. Premeri skosmičenih kosmov so sicer po naših meritvah in literaturi od 20 do 3500 µm, pri tem je srednja vrednost oz. mediana 30 do 200 µm.

Biološko kosmičenje postane mogoče šele, ko intenzivnost rasti bakterij in drugih mikroorganizmov prične upadati in ko se začnejo izločati naravni polimeri - koloidi, ki premostijo razdalje med mikroorganizmi (II. faza). Zaradi velike turbulentnosti v biološkem

- demand on the properties of a nutritional solution (substrate)
- suitable temperature
- supply of the necessary amount of air or oxygen
- continuous movement to avoid sedimentation and to ensure microorganisms the best possible contact with nutriments and oxygen
- pH value should be in the range between 5 and 9.

The first and second conditions are satisfied by most sewage wastewater. The third and forth conditions are assured by the use of a turbine mixer, aeration, with an oxygen atmosphere or by mixing with a jet. The pH value must be arranged prior to the purification of the industrial wastewater. The assured conditions in the medium enable the development of different communities of microorganisms (biocenoses) which change themselves as conditions change from lower to higher developed forms and vice versa, or, for instance - the domination of one living form over another. Organisms in the activated sludge take over the organic and partially inorganic compounds from the wastewater and transform them into new organisms (1^{st} phase of treatment), forming flocks of activated sludge in the secondary basin.

Basic flocks have a size of 10^{-3} to 1 µm, super colloids 1 - 50 µm and insoluble and hardly settleable (floating) matter, a size of 50 - 100 µm (Lau et al., 1984; Panjan, 1996). Microorganisms and colloids are settleable - eliminable - only when they form larger flocks. Sedimentation begins only after the flocculation of the primary flocks in a stagnant water medium of the secondary settling tank, when they reach a diameter greater than 50 - 100 µm. The diameter of the flocculated flocks is from 20 to 3500 µm with an average, respectively, median value of 30 - 200 µm, according to our measurement and literature data.

Biological flocculation becomes possible when the intensity of the growth of bacteria and other microorganisms begins to decline and when natural polymers - colloids start to be excreted and bridge the distances between the microorganisms (2^{nd} phase). Flocculation of the primary particles doesn't occur due to reaktorju torej ne prihaja do kosmičenja primarnih delcev, ampak se bistveno poveča njihovo število oz. volumska koncentracija delcev z rastjo mikroorganizmov, njihove žive in odmrle substance (približno 50 do 100-krat glede na dotok na komunalno čistilno napravo) (Degremont, 1991; Droste, 1997).

3. MATEMATIČNO MODELIRANJE PRIRASTA FLOKUL - KOSMOV

Kinetiko izmenjave snovi v kosmu (flokuli) lahko opišemo z difuzijskim, sorbcijskim in encimskim procesom. V prvem delu procesa gre za mehansko difuzijo v pomičnem mediju, ki poteka še znotraj celice, v drugem delu prihaja do adsorpcije zaradi površinskih sil ter absorpcije zaradi kemičnih zvez, v tretjem delu pa proti notranjosti celice-delčka potekajo encimske reakcije in difuzijski procesi (v smeri manjše koncentracije).

Zaradi zapletenosti vseh biokemijsko - fizikalnih dogajanj si poglejmo le model povečevanja poživljenega blata oz. nastajanja kosmičev bakterij - prirasta biomase po literaturi (Lau, 1884) v stacionarnem stanju za okrogle, cilindrične (valjaste) in diskaste oblike kosmov.

Za mineralizacijo organske snovi ali pa za rast novih celic je treba prevzeti organsko snov v notranjost celice, kjer potekajo najpomembnejši biokemijski procesi. Pri heterotrofnih (živalskih) organizmih biokemijske reakcije potekajo tako, kot prikazuje shema na sliki 1, ki prikazuje zgradbo kosmov in potek razkroja beljakovin v končne produkte (vodo, ogljikov dioksid, nitrate itd.). Najprej si izračunajmo masno bilanco (Panjan, 1994; 1997; Tyagi, 1996).

the high turbulence in the biochemical reactor. Instead, the number or volume concentration of the particles is essentially increased with the growth of microorganisms - their live and dead matter (approximately 50 to 100 times with respect to their affluence into the municipal sewage plant) (Degremont, 1991; Droste, 1997).

3. MATHEMATICAL MODELLING OF THE GROWTH OF FLOCULAE - FLOCKS

The kinetics of the exchange of matter in a flock can be described with diffusion, sorption and enzymatic processes. Mechanical diffusion in the moving medium and inside each cell takes place in the first part of the process. Adsorption due to the surface forces and adsorption due to the chemical bonding takes place in the second part of the process. The enzyme reactions and diffusion processes (in the direction of lower concentration) take place in the third part of the process.

Due to the high complexity of all these biochemical - physical processes, only the model of the growth of the activated sludge, respectively, the formation of the bacterial flocks - increase of biomass after literature (Lau, 1884) and in the stationary state for the spherical, cylindrical and disklike flock forms is considered.

It is necessary to introduce organic compounds into the interior of the cell, where the most important biochemical processes take place, to accomplish the mineralization of organic compounds. Biochemical reactions take place in heterotrophic (animal) organisms as is shown with the schematic in Figure 1, which represents the structure of flocks and the course of protein degradation into final products (water, carbon dioxide, nitrate etc). Let's first calculate the balance of the mass (Panjan, 1994; 1997; Tyagi, 1996).

3.1 MASNA BILANCA

3.1.1 Okrogli kosmi

Za ta primer lahko zapišemo:

$$4 \cdot \pi \cdot r^2 \cdot N_{r+dr} - 4 \cdot \pi \cdot r^2 \cdot N_r = 4 \cdot \pi \cdot r^2 \cdot R \cdot dr \qquad (1)$$

Pri tem je:

$$R = \mu \cdot \frac{M}{Y} \qquad (2)$$

$$\mu = \mu_{\max} \cdot \left[\frac{S_1}{(S_1 + K_1)} \right] \cdot \left[\frac{S_2}{(S_2 + K_2)} \right] \qquad (3)$$

kjer pomenijo:

- d_p premer delca [m]
- r odmik od centra delca [m]
- r_f polmer delca [m]
- *No* masni pretok hranil skozi površino [mg/(s.cm^2)]
- *R* prirast biomase po Monodu [mg/(s.cm^2)]
- μ specifična hitrost rasti biomase [dan^{-1}]
- μ_{max} maksimalna specifična rast biomase [dan^{-1}]
- *M* gostota suhega dela žive mase snovi (sušina) oz. kosmov [mg/l]
- *Y* razmerje med težo tvorjene biološke mase in porabo hraniv (koeficient proizvodnje oz. prirasta) [mg/mg]
- *K* Monodov koeficient: $\mu = \mu_{max}/2$
- *S* koncentracija hraniv (substrata) - indeks 1 ali S_b in koncentracija kisika O_2 - indeks 2 v [mg/l]

3.1 THE BALANCE OF THE MASS

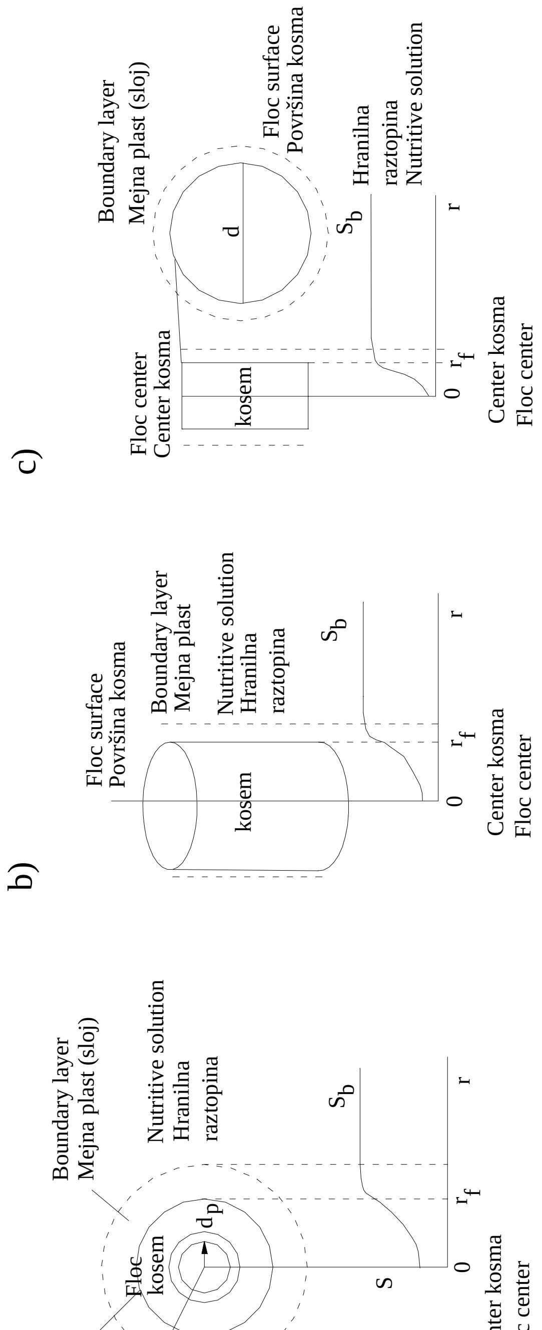
3.1.1 Spherical flocks

We can write for this case:

in which it is

where it means:

- d_p diameter of the particle [m]
- r distance from the centre of the particle [m]
- r_f radius of the particle [m]
- N_o mass flow of nutriment through the surface [mg/(s·cm^2)]
- R increment of biomass after Monod [mg/(s·cm^3)]
- μ specific rate of growth of biomass [day^{-1}]
- μ_{max} maximal specific rate of growth of biomass [day^{-1}]
- *M* density of the dry part of the live mass of matter [mg/l]
- *Y* ratio between produced biomass and consumption of nutriment (production coefficient respectively increment) [mg/mg]
- *K* Monod coefficient: $\mu = \mu_{max}/2$
- *S* concentration of nutriment (substrate) - index 1 or S_b and oxygen concentration O_2 - index 2 in [mg/l]



Slika 1. Profil koncentracije hranil (substrata) v aktivnem kosmu blata: a) kroglaste oblike, b) valjaste (cilindrične) oblike in c) diskaste oblike kosmov (Lau et al., 1984).
Figure 1. Concentration profile of nutriment (substrate) in the active sludge flock: spherical shape, b) cylindrical shape, c) disklike shape of the flock (Lau et al., 1984).

Če enačbo delimo s $4 \cdot \pi \cdot \mathrm{dr}$, dobimo:

Dividing the equation (1) with 4·π·dr we obtain

$$\frac{(r^2 \cdot N_{r+dr} - r^2 \cdot N_r)}{dr} = r^2 \cdot R \tag{4}$$

ko gre dr→0, velja:

when dr→0 it is

$$\frac{d \cdot (r^2 \cdot N_\phi)}{dr} = r^2 \cdot R \tag{5}$$

Upoštevati moramo tudi Fickov zakon za difuzijski pretok v notranjost poroznega kosma:

Fick's law for diffusion flow into the interior of a porous flock must be taken into account:

$$N_\phi = De \cdot \frac{dS}{dr} \tag{6}$$

$$De = \frac{N_\phi \cdot r}{(S_{02} - S_2)} \tag{7}$$

kjer so:

- N_Φ difuzijski pretok v notranjost kosma
- De efektivna difuzija hraniv v kosem [m^2/s]
- S_{02} nasičena koncentracija kisika v vodi [mg/l]
- S_2 povprečna koncentracija kisika v raztopini [mg/l]

where it is:

- N_Φ diffusion flow into the interior of the flock
- De effective diffusion of nutriment in the flock [m^2/s]
- S_{02} saturated concentration of oxygen in water [mg/l]
- S_2 average concentration of oxygen in solution [mg/l]

Če upoštevamo enačbo (6), dobimo:

When equation (6) is taken into account we obtain:

$$De \cdot \left(\frac{d^2 \cdot S}{dr^2} + \frac{2 \cdot dS}{r \cdot dr} \right) = R \tag{8}$$

Z uvedbo nove spremenljivke *R* pa dobimo dve diferencialni enačbi:

With introduction of the new variable *R* we get two differential equations:

za hraniva (sladkor - glukozo):

for nutriment (sugar - glucose):

$$\left(\frac{d^2 \cdot S_1}{dr^2} + \frac{2 \cdot dS_1}{(r \cdot dr)} \right) = \mu_{\max} \cdot \frac{M}{D_1 \cdot Y_1} \cdot \left(\frac{S_1}{S_1 + K_1} \right) \cdot \left(\frac{S_2}{S_2 + K_2} \right) \tag{9}$$

za kisik:

for oxygen:

$$\left(\frac{d^2 \cdot S_2}{dr^2} + \frac{2 \cdot dS_2}{(r \cdot dr)}\right) = \mu_{\max} \cdot \frac{M}{D_2 \cdot Y_2} \cdot \left(\frac{S_1}{S_1 + K_1}\right) \cdot \left(\frac{S_2}{S_2 + K_2}\right) \tag{10}$$

Upoštevati moramo naslednja robna pogoja:

1. Na površini kosma ($r = r_f$) je koncentracija hraniv enaka prostorninski koncentraciji hraniv v reaktorju:

The following boundary conditions must be considered:

1. Concentration of nutriment on the surface of the flock ($r = r_f$) is equal to the concentration of nutriment in the reactor:

$$S_{1_{r=rf}} = S_{b1}, S_{2_{r=rf}} = S_{b2} \tag{11}$$

2. Izberemo, da je v notranjosti kosma pri ($r = 0$) poraba hraniv in kisika enaka nič:

2. We assume that consumption of nutriments and oxygen in the interior ($r = 0$) of the flock is equal to zero:

$$\frac{dS_1}{dr_{r=0}} = 0, \ \frac{dS_2}{dr_{r=0}} = 0 \tag{12}$$

Da se izognemo nekaterim potrebnim podatkom o kosmih (premer, koncentracija aktivne biomase v kosmu, difuziji hrane v kosem), v zgornje enačbe uvedemo brezdimenzijske parametre:

Dimensionless parameters are introduced in the upper equations to avoid some necessary data about flocks (diameter, concentration of active biomass in the flock, diffusion of nutriment into the flock):

$$y_1 = \frac{S_1}{S_{b1}}, \ y_2 = \frac{S_2}{S_{b2}} \tag{13}$$

$$\phi_1 = \frac{K_1}{S_{b1}}, \ \phi_2 = \frac{K_2}{S_{b2}} \tag{14}$$

$$\Omega_1 = \frac{\mu_{\max} \cdot r_f^{\,2} \cdot M}{(D_1 \cdot S_{b1} \cdot Y_1)}, \ \Omega_2 = \frac{\mu_{\max} \cdot r_f^{\,2} \cdot M}{(D_2 \cdot S_{b2} \cdot Y_2)} \tag{15}$$

$$z = \frac{r}{r_f} \tag{16}$$

Tako dobimo naslednje izraze:

So the following expressions are obtained:

za hraniva (glukozo):

for nutriment (glucose):

$$\left(\frac{d^2 \cdot y_1}{dz^2} + \frac{2 \cdot dy_1}{z \cdot dz}\right) = \Omega_1 \left(\frac{y_1}{y_1 + \phi_1}\right) \cdot \left(\frac{y_2}{y_2 + \phi_2}\right) \tag{17}$$

za kisik:

for oxygen:

$$\left(\frac{d^2 \cdot y_2}{dz^2} + \frac{2 \cdot dy_2}{z \cdot dz}\right) = \Omega_2 \left(\frac{y_1}{y_1 + \phi_1}\right) \cdot \left(\frac{y_2}{y_2 + \phi_2}\right) \tag{18}$$

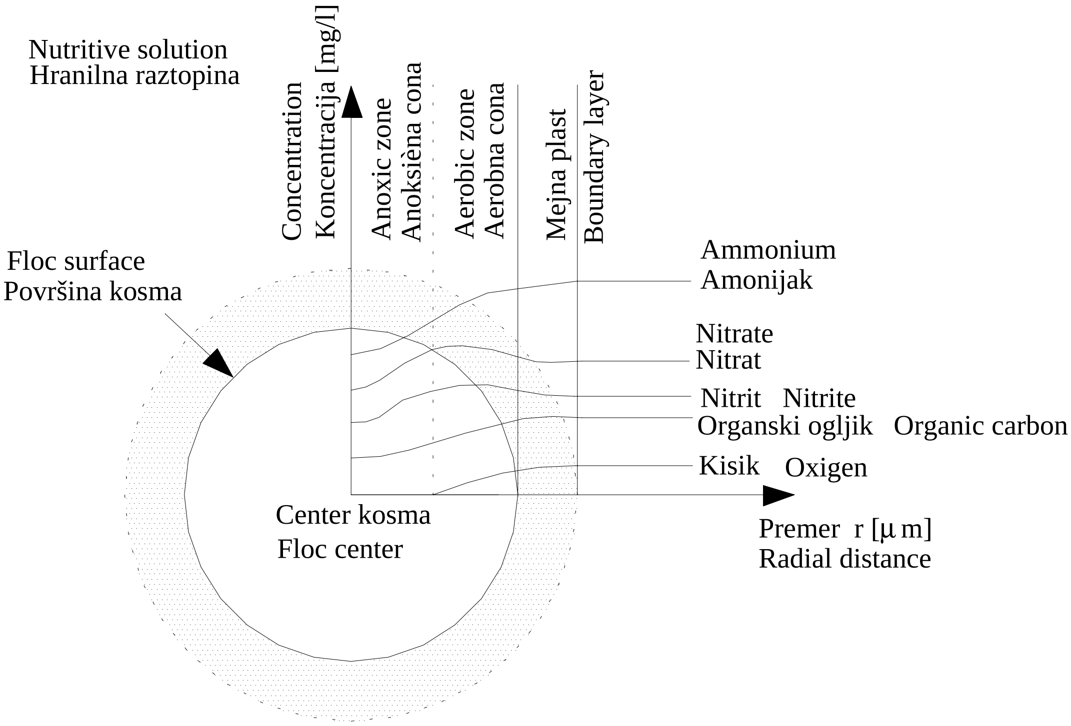
Brezdimenzijski robni pogoji pa se zdaj zapišejo v naslednji obliki:

Dimensionless boundary conditions are written now in the following form:

$$y_1\big|_{z=1} = 1\,,\; y_2\big|_{z=1} = 1\,,\; \frac{dy_1}{dz}\bigg|_{z=0} = 0\,,\; \frac{dy_2}{dz}\bigg|_{z=0} = 0 \tag{19}$$

To sta nelinearni diferencialni enačbi drugega reda, ki nimata analitične rešitve. Rešujemo ju z ortogonalno numerično metodo, skozi več (sedem) točk. Za reševanje moramo poznati enajst koeficientov.

These are non-linear differential equations of the second order, which have no analytical solution. They are solved by the orthogonal numeric method through more (seven) points. Eleven coefficients must be known for the solution.



Slika 2. Shema simultanih dogajanj nitrifikacije in denitrifikacije v okroglem kosmu (Bakti & Dick, 1992).
Figure 2. Schematic of the simultaneous process of nitrification and denitrification in the spherical flock (Bakti & Dick, 1992).

3.1.2 Valjasta oblika kosmov

Za cilindrično obliko kosmov, višine h in premera r_f, če je $h >> r_f$, je prirast masne bilance enak:

za hraniva (glukozo):

3.1.2 Cylindrical shape of flocks

The increment of the mass balance for the cylindrical shape of flocks with height h and diameter r_f, if $h >> r_f$, is equal:

for nutriment (glucose):

$$\left(\frac{d^2 \cdot y_1}{dz^2}\right) + \left(\frac{dy_1}{z \cdot dz}\right) = \Omega_1 \cdot \left(\frac{y_1}{y_1 + \phi_1}\right) \cdot \left(\frac{y_2}{y_2 + \phi_2}\right) \tag{20}$$

za kisik:

for oxygen

$$\left(\frac{d^2 \cdot y_2}{dz^2}\right)+\left(\frac{dy_2}{z \cdot dz}\right)=\Omega_2 \cdot \left(\frac{y_1}{y_1+\phi_1}\right)\cdot\left(\frac{y_2}{y_2+\phi_2}\right) \quad (21)$$

3.1.3 Diskasta oblika kosmov

Za diskasto obliko kosmov premera *d* in debeline $2r_f$, če je premer $d>>r_f$ velja:

za hraniva (glukozo):

3.1.3. Disklike shape of flocks

For disklike shape of flocks with a diameter d and thickness $2r_f$, if $d>>r_f$, it is

for nutriment (glucose):

$$\frac{d^2 \cdot y_1}{dz^2}=\Omega_1 \cdot \left(\frac{y_1}{y_1+\phi_1}\right)\cdot\left(\frac{y_2}{y_2+\phi_2}\right) \quad (22)$$

za kisik:

for oxygen:

$$\frac{d^2 \cdot y_2}{dz^2}=\Omega_2 \cdot \left(\frac{y_1}{y_1+\phi_1}\right)\cdot\left(\frac{y_2}{y_2+\phi_2}\right) \quad (23)$$

4. MAKSIMALNA VELIKOST PRIMARNIH KOSMOV IN NJIHOVA STABILNOST

Za ugotovitev stabilnosti maksimalne velikosti delcev - kosmov moramo poznati sile, ki delujejo na kosme. Po literaturi (Matsun & Unno, 1981) imamo naslednje sile, ki delujejo na kosem:

4. MAXIMAL DIMENSIONS OF PRIMARY FLOCKS AND THEIR STABILITY

Forces acting on flocks must be known to establish the stability of the maximal particles – flocks size. The following particles are taken into account after literature (Matsun & Unno, 1981):

$$\rho_s \cdot V \cdot \frac{dv_s}{dt}=k_f \cdot \rho \cdot S \cdot v^2+\rho \cdot V \cdot \frac{du}{dt}+\frac{1}{2}\cdot \rho \cdot V \cdot \left(\frac{du}{dt}-\frac{dv_s}{dt}\right)+Fe \quad (24)$$

Tu pomenijo:

- *Fe* zunanja sila (težnost) [N]
- k_f koeficient turbulentnosti [-]
- ρ_s gostota delca [kg/m^3]
- ρ gostota vode [kg/m^3]
- *V* volumen kosma - delca [m^3]
- *S* prerez kosma [m^2]
- v_s hitrost kosma - delca [m/s]
- *u* hitrost tekočine v neposredni bližini kosma [m/s]
- *v* relativna hitrost med tekočino in delcem [m/s]

Here it means:

- *Fe* exterior force (gravity) [N]
- k_f coefficient of turbulence [-]
- ρ_s density of the particle [kg/m^3]
- ρ density of water [kg/m^3]
- V volume of the flock [m^3]
- *S* intersection of the flock [m^2]
- v_s velocity of the flock [m/s]
- *u* velocity of the liquid in the immediate vicinity of the flock [m/s]
- *v* relative velocity between the liquid and the particle [m/s]

Rešitev zgornje enačbe za relativno hitrost je podana v (Matsun & Unno, 1981; Panjan, 1997) v obliki:

Solution of the upper equation is given in (Matsun & Unno, 1981; Panjan, 1997) in the following form:

$$v = \left((\rho_s - \rho) \cdot V\right)^{1/2} \cdot \frac{u}{\left((\rho_s + \frac{\rho}{2}) \cdot V + k_f \cdot \rho \cdot S \cdot \lambda\right)^{1/2}} \qquad (25)$$

Strižne napetosti, ki delujejo na delce pa so enake:

Shearing tensions acting on particles are equal to:

$$\tau = \frac{1}{2} \cdot \lambda_D \cdot \rho \cdot v^2 \qquad (26)$$

Tu so parametri enačbe podani za $\lambda = \lambda_D$ pri maksimalnem premeru kosma (Re - Reynoldsovo število, v_s - hitrost usedanja):

Here the parameters of the equation are given as follows (Re - Reynold's number, v_s - sedimentation velocity):

$$\lambda_D = \frac{K}{(\mathrm{Re})^n}; \qquad (27)$$

$$\mathrm{Re} > 10^3 : K = 0,\ n = 0; \qquad (27a)$$

$$10^3 < \mathrm{Re} < 1 : K = 12,\ n = 1/2; \qquad (27b)$$

$$\mathrm{Re} < 1 : K = 24,\ n = 1 \qquad (27c)$$

$$\mathrm{Re} = \frac{d \cdot v_s}{\nu} \qquad (28)$$

$$v_s = \frac{g}{18} \cdot \frac{d^2}{\nu} \cdot \left(\frac{\rho_s}{\rho} - 1\right) \qquad (29)$$

Maksimalni premer delca - kosma pa je določen z enačbo:

Maximal diameter of the particle - flock is determined by the following equation:

$$d_s = \frac{C}{G^n} \qquad (30)$$

kjer pomenijo:

- *C* konstanto, ki je odvisna od številnih parametrov kosmov in hranilne raztopine [-]
- *G* hitrost mešanja oz. turbulentnosti, izražena z gradientom [s^{-1}]
- ν kinematični koeficient viskoznosti [m^2/s]

where it means:

- *C* constant, which depends on numerous parameters of the flock and the nutritive solution [-]
- *G* velocity of mixing or turbulence expressed with gradient [s^{-1}]
- ν kinematic coefficient of viscosity [m^2/s]

Preglednica 1. Premeri kosmov poživljenega blata, njihov delež in hitrost usedanja - laboratorijske analize (Degremont, 1991; Sedlak, 1991; Panjan, 1994).
Table 1. Diameters of the flocks of activated sludge, their fraction and the rate of sedimentation – laboratory tests (Degremont, 1991; Sedlak, 1991; Panjan, 1994).

PREMER KOSMOV *DIAMETER OF THE FLOCKS* (μm)	DELEŽ *FRACTION* (%)	HITROST USEDANJA *RATE OF SEDIMENTATION* (m/h)
50 - 60	10	3,6
80 - 90	11	5,4
100	35	10,8
120	28	16,2
150	16	25,2

5. ZAKLJUČEK

Pri čiščenju odpadnih voda je faza biološke obdelave vode ena najpomembnejših. Biološki procesi potekajo kot difuzijski, sorbcijski in encimski. Da bi te procese čim bolje razumeli, se čedalje bolj uporabljajo matematični modeli. Modeliranje je mogoče na makro ravni (celotni volumen reaktorja) ali na mikro ravni (mikroorganizmih). Mi smo modelirali procese na mikro ravni oz. v mikroorganizmih majhne velikosti. Pri teh procesih je nekatere parametre, ki so potrebni za izračune, zelo težko določiti. Tej težavi se delno izognemo z uvedbo brezdimenzijskih števil. V našem prispevku smo prikazali matematično modeliranje vnosa hranil in kisika okroglih, valjastih in diskastih oblik mikroorganizmov - kosmov. S temi modeli poizkušamo čimbolj razumeti vnos hranil in porabo kisika ter simultani potek nitrifikacije in denitrifikacije v bioloških reaktorjih. Z njimi ugotavljamo mejo med aerobno in anoksično cono. Izpeljali smo tudi enačbo za oceno maksimalne velikosti kosmov oz. velikost primarnih kosmov v odvisnosti od turbulence oz. intenzivnosti mešanja vodne mase, v kateri se kosmi nahajajo.

5. CONCLUSION

The phase of biochemical wastewater treatment is one of the most important in the purification process. Biochemical processes take place as diffusion, sorption and enzymatic ones. Mathematical models are more and more often employed for a better understanding of these processes. Modelling is possible on the macro level (whole volume of the reactor) or on the micro level (in microorganisms). We have modelled these processes on the micro level, respectively in tiny microorganisms. It is very difficult to determine some parameters which are necessary for the calculation in these processes. This problem is partially avoided by the introduction of dimensionless numbers. In our article we have demonstrated a mathematical modelling of the introduction of nutriments and oxygen into the spherical, cylindrical and disklike microorganisms – flocks. With these models we try to understand as well as possible the introduction of the nutritive, oxygen demand and the simultaneous course of nitrification and denitrification in biochemical reactors. The boundary between the aerobic and anoxic zones is also established by their means. We have also carried out an equation for the estimation of the maximal dimension of the flocks, respectively, dimension of the primary flocks in dependence of the turbulence, respectively, intensity of the mixing of the water medium in which flocks are present.

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