

Symmetries of the Woolly Hat graphs*

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Abstract

A graph is *edge-transitive* if the natural action of its automorphism group on its edge set is transitive. An automorphism of a graph is *semiregular* if all of the orbits of the subgroup generated by this automorphism have the same length. While the tetravalent edge-transitive graphs admitting a semiregular automorphism with only one orbit are easy to determine, those that admit a semiregular automorphism with two orbits took a considerable effort

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and were finally classified in 2012. Of the several possible different “types” of potential tetravalent edge-transitive graphs admitting a semiregular automorphism with three orbits, only one “type” has thus far received no attention. In this paper we focus on this class of graphs, which we call the *Woolly Hat* graphs. We prove that there are in fact no edge-transitive Woolly Hat graphs and classify the vertex-transitive ones.

Keywords: Edge-transitive, vertex-transitive, tricirculant, Woolly Hat graph.

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Simetrije grafov volnenih kap*

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Povzetek

Graf je *povezavno tranzitiven*, če je naravno delovanje njegove grupe avtomorfizmov na njegovi množici povezav tranzitivno. Avtomorfizem grafa je *semiregularen*, če imajo vse orbite podgrupe, generirane s tem avtomorfizmom, isto dolžino. Medtem ko je tetravalentne povezavno tranzitivne grafe, ki dopuščajo semiregularni avtomorfizem z eno samo orbito,

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enostavno določiti, so tisti, ki dopuščajo semiregularni avtomorfizem z dvema orbitama, zahtevali precej dela, končno so bili klasificirani leta 2012. Od več možnih različnih “tipov” potencialnih tetravalentnih povezavno tranzitivnih grafov, ki dopuščajo semiregularen avtomorfizem s tremi orbitami, samo en “tip” do sedaj ni bil deležen nobene pozornosti. V tem članku se osredotočamo na ta razred grafov, ki ga imenujemo grafi *volnenih kap*. Dokažemo, da v resnici ni nobenih povezavno tranzitivnih grafov volnenih kap in klasificiramo točkovno tranzitivne.

Ključne besede: Povezavno tranzitiven, točkovno tranzitiven, tricirkulant, grafi volnenih kap.

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