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Optimization of Trapezoidal Cross Section of the Truck Crane Boom by Lagrange's Multipliers and by Differential Evolution Algorithm (DE)

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The cross sections of truck crane booms are complex box-like cross sections, which should provide continuous stress allocation. It is difficult to analytically determine optimal relations among geometric parameters of such cross sections. The paper deals with the method for determining relations among geometric parameters in order to achieve the optimal shape of the cross section. The method is based on Lagrange's multipliers used for determination of extreme values. The optimisation of geometric parameters has also been done with the method of differential evolution (DE). The optimisation of the cross section is based on the strength criterion. The results of the applied methods have been verified by means of numerical example for an existing solution. The comparative analysis of the results of both methods has also been done.

Keywords: trapezoidal cross section, truck crane, boom, Lagrange's multipliers, differential evolution algorithm

0 INTRODUCTION

The world's truck crane manufacturers have been giving significance to the design of the truck crane booms having box-like cross sections, which increase bending and torsion stiffness, and decrease the weight. Since the technology of box-like girders has been enhanced, the classic rectangular cross sections have been replaced with more complex polygonal ones [1] to [8].

The box-like girders are made of sheet metal of various thicknesses because of optimisation and material saving. Papers [1] and [9], which deal with the telescopic truck crane booms, have proved that there are some local peaks of stress at the areas where the members are in contact, so the cross sections must be made of thicker sheets at contact areas. These stress peaks are rather noticeable when the boom segments are on maximum reach [9]. Both local stresses and the stresses at the polygonal cross sections are smaller at the areas where external load is transmitted from one segment to another [1] and [9].

The research of the optimal parameters of trapezoidal cross section (Fig. 1) has been done by two methods. The results of the comparative analysis are also shown.

The first method for cross section optimisation is based on Lagrange's multipliers [8] to [10]. This method provides optimal values of geometric parameters of the cross section in the explicit form and also their functional relations. The obtained relations of geometric parameters determine the minimal cross sectional area. The method is also suitable for forming the algorithms of the cross sectional area optimisation.

The second method for cross section optimisation is based on the algorithm of differential evolution (DE). DE algorithm is efficient for solving optimisation problems where the objective function does not need to be continual in an area and where the values of design parameters do not need to be close to the initial values.

Price and Storn [11] successfully applied DE algorithm during optimisation of certain well-known non-linear, non-differentiable and non-convex functions. The papers [12] to [16] give a detailed description of DE algorithm as well as its application to various optimisation problems.

This paper proves that DE algorithm can be successfully applied to optimise the cross-sectional areas of the elements of supporting structures.

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1 MATHEMATICAL FORMULATION OF OPTIMISATION PROBLEM

The aim of the research is to define the geometric parameters of the cross section, as well as their relations, which will provide the minimum cross sectional area of the boom for defined load. The weight minimisation corresponds to the volume minimisation, i.e. to the cross sectional area minimisation, and it is determined from the condition that the stress at the appropriate cross section is less than or equal to the allowable stress. The allowable stress criterion only has been taken as boundary function AS it is typical for boom cranes [8] to [10]. The cross sectional area depends on the section height and width, sheet metal thickness, and relations among the parameters. If there are many optimisation parameters and if the optimisation of all parameters cannot be done, the dominant parameters need to be chosen.

This is a general mathematical formulation of the above-defined optimisation problem:

$$\text{minimize } f(X), \tag{1}$$

$$\text{subject to: } g_j(X) \leq 0, \quad j = 1, \ldots, m, \tag{2}$$

where: $f(X)$ is the objective function, $g_j(X) \leq 0$ represents the constraints defined by the search space, m is the total number of constraints. $X = \{x_1, \ldots, x_D\}^T$ is a design vector consisting of D design variables. The design variables are the values, which should be determined during the optimisation procedure. Each design variable is defined by its lower and upper boundaries.

2 OPTIMISATION DONE BY LAGRANGE'S MULTIPLIERS

Generally, the truck crane boom is loaded by the longitudinal force N, the bending moments M_x, M_y, and the torsion moment T [1] to [9].

In order to determine the optimal values of geometric parameters by Lagrange's multipliers we started with the expression for cross-sectional area, which is selected for the objective function $f(X)$.

Lagrange's function is defined as:

$$\Phi(X) = f(X) + \lambda g(X), \tag{3}$$

where λ is Lagrange's multiplier.

The following conditions need to be satisfied so the objective function has its minimum or maximum:

$$\frac{\partial \Phi(X)}{\partial x_i} = 0, \text{ where } i = 1, \ldots, D. \tag{4}$$

Eq. (4) can be also written as:

$$\frac{\partial f(X)}{\partial x_i} + \lambda \frac{\partial g(X)}{\partial x_i} = 0 \quad i = 1, \ldots, D. \tag{5}$$

When the multiplier λ is eliminated the equations that define the optimal values of parameters are obtained.

2.1 Objective Function and Boundary Function

The optimisation of three parameters (H, B, b) is done for trapezoidal cross-section (Fig. 1). Other geometric parameters such as wall thicknesses t_1, t_2 and t_3 are not treated in this method as optimisation parameters. Their values are taken according to manufacturers' recommendations and references [2], [5], [7] and [9].

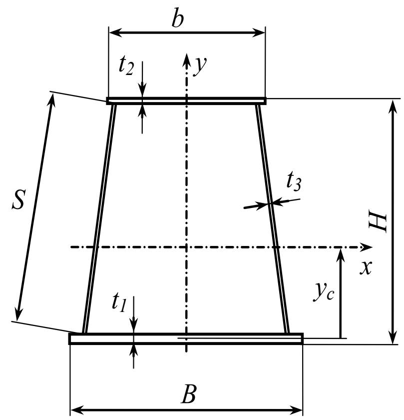

Fig. 1. *Trapezoidal cross section*

Below are the defined parameters:

N axial force acting on the centre of the cross section,
M_x and M_y bending moments for x and y axes,
T torsion moment,
σ_0 allowable stress of the basic boom material.

The geometric parameters of the truck crane boom are:

W_x and W_y resisting bending moments for x and y axes,

W_t polar moment of resistance.

The design parameters can be expressed in the form of design vector:

$$X = (k, d, H)^T$$

where:

$$k = \frac{b}{B}, \quad d = \frac{H}{B}. \tag{6}$$

The boundary function corresponds to the sum of normal and tangential stresses at the cross section [2]:

$$g_1(k,d,H) = \left(\frac{N}{A} + \frac{M_x}{W_x} + \frac{M_y}{W_y}\right)^2 + \\ + 4 \cdot \left(\frac{T}{W_t}\right)^2 - \sigma_0^2 = 0. \tag{7}$$

Since the analysis of this problem includes the ratios between the sheet metal thicknesses and height [1], [5], [7] and [9]:

$$\delta_1 = \frac{t_1}{H},\ \delta_2 = \frac{t_2}{H},\ \delta_3 = \frac{t_3}{H}, \tag{8}$$

the objective function, which stands for cross-section area, is:

$$f(k,d,H) = A = \\ = H^2\left[\frac{k}{d}\delta_2 + \frac{\delta_1}{d} + \delta_3\sqrt{4 + \left(\frac{1-k}{d}\right)^2}\right]. \tag{9}$$

Tangential stresses are much smaller in comparison to normal stresses, so the member $4(T/W_t)^2$ of the boundary function, Eq. (7) can be ignored [3] to [10]:

$$g_1(k,d,H) = \left(\frac{N}{A} + \frac{M_x}{W_x} + \frac{M_y}{W_y}\right)^2 - \sigma_0^2 = \\ = \sigma_e^2 - \sigma_0^2 = 0. \tag{10}$$

The parameter values (8) of the trapezoidal cross section (Fig. 1) are within the following limits [5], [7] and [9]:

$$\delta_1 = 0.02 \div 0.03, \\ \delta_2 = 0.02 \div 0.027, \\ \delta_3 = 0.015 \div 0.02. \tag{11}$$

Conformably to the reference [17] the accepted values of these parameters are:

$$\delta_1 = 0.0273,\ \delta_2 = 0.0221,\ \delta_3 = 0.0175, \tag{12}$$

so the values of their ratio are:

$$\frac{\delta_1}{\delta_2} = 1.56, \quad \frac{\delta_2}{\delta_3} = 1.26\ . \tag{13}$$

The accepted boundaries of defined parameters do not decrease the generality of the optimisation of the parameters H, B, b.

The relation between the bending moments is defined in practice and references [2] to [9] as:

$$M_y = \psi M_x, \tag{14}$$

where the value of ψ coefficient is within the limits [5], [7] and [9]: $\psi = 0.4$ to 0.75. The relation (14) can be expressed as:

$$M_y = \frac{M_x}{M}, \tag{15}$$

so the value of M coefficient is within the following boundaries: 1.3 to 2.5. Since the allowable stress depends on the used material, an arbitrary value of $\sigma_0 = 100$ MPa has been adopted, which does not affect the problem generality.

2.2 Optimisation of Geometric Parameters

The objective function Eq. (9) is transformed into the following form:

$$f(k,d,H) = A = B^2\left[d^2\delta_3 + \left(\frac{1-k}{2}\right)^2\delta_3 + \\ + \delta_1 + \delta_2 k^2\right] = B^2[R], \tag{16}$$

where:

$$[R] = \left[d^2\delta_3 + \left(\frac{1-k}{2}\right)^2\delta_3 + \delta_1 + \delta_2 k^2\right]. \tag{17}$$

The position of the centre of trapezoidal cross sectional area (Fig. 1) is expressed as:

$$y_c = \frac{H\left[\frac{k\delta_2}{d} + \frac{\delta_3}{2}\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right]}{\left[\frac{k\delta_2}{d} + \frac{\delta_1}{d} + \delta_3\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right]}. \quad (18)$$

The values of the moments of resistance for appropriate axes are:

$$W_x(k,d,H) = \frac{H^3}{d\left(1-\frac{y_c}{H}\right)}\left\{k\delta_2\left(1-\frac{y_c}{H}\right)^2 + \delta_2\left(\frac{y_c}{H}\right)\right\} + \frac{H^3}{\left(1-\frac{y_c}{H}\right)}\left\{\frac{\delta_3}{12}\sqrt{4+\left(\frac{1-k}{d}\right)^2} + \delta_3\left(0.5-\frac{y_c}{H}\right)^2\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right\}$$

$$W_y(k,d,H) = H^3\left\{\left[\frac{k^3\delta_2}{6d^2} + \frac{\delta_1}{6d^2}\right] + \left[\frac{\delta_3}{24d}(1-k)^2\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right] + \left[\frac{\delta_3}{8d}(1+k)^2\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right]\right\}. \quad (19)$$

The expressions for moments of resistance (19) are not suitable for the application of Lagrange's multipliers. Thus, their approximation has been done providing that their accuracy is not reduced (the error at this approximation does not exceed the value of 5%). The simplified expression for the moment of resistance for x axis is:

$$W'_x(k,d,H) = Ah\alpha. \quad (20)$$

The graphic interpretation of the approximation can be seen in Fig. 2. Approximation coefficient value of $\alpha = 0.44$ has been calculated from condition that the deviation between the values of the moments of resistance, stated in the Eqs. (19) and (20), does not exceed 5%.

The expression for the moment of resistance for y axis can be also transformed into:

$$W'_y(k,d,H) = \beta B^3\left(\frac{1+k}{4}\right)\left[\left(\frac{d}{2\alpha}\right)^2\delta_3 + \left(\frac{1-k}{2}\right)^2\delta_3 + \delta_1 + \delta_2 k^2\right] = \beta B^3\left(\frac{1+k}{4}\right)[S_1], \quad (21)$$

where:

$$[S_1] = \left[\left(\frac{d}{2\alpha}\right)^2\delta_3 + \left(\frac{1-k}{2}\right)^2\delta_3 + \delta_1 + \delta_2 k^2\right], \quad (22)$$

$$\beta = 2\alpha = 0.88.$$

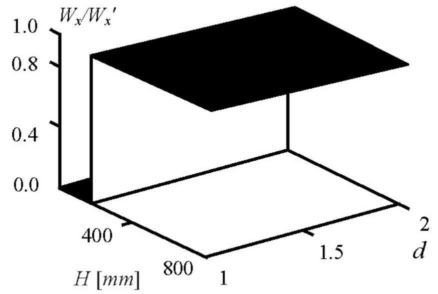

Fig. 2. *Relation between the moments of resistance for x axis*

Considering the existing solutions of truck cranes, it has been noticed that the boom first segment height is not less than 300 mm.

According to the above approximation, the boundary function (10) is:

$$g_1 = \frac{N}{B^2[R]} + \frac{M_x}{\alpha B^3 d[R]} + \frac{2M_y}{\alpha B^3(1+k)[S_1]} - \sigma_0 = 0. \quad (23)$$

In order to apply Lagrange's condition, it is necessary to differentiate the boundary function (23) and the objective function (16) with respect to the stated parameters, and then to find the following ratios:

$$\frac{\frac{\partial g_1}{\partial B}}{\frac{\partial f}{\partial B}} = -\frac{N}{B^4[R]^2} - \frac{3M_x}{2\alpha B^5 d[R]^2} - \frac{3M_y}{\alpha B^5(1+k)[S_1][R]},$$

$$\frac{\dfrac{\partial g_1}{\partial d}}{\dfrac{\partial f}{\partial d}} = -\frac{N}{B^4\left[R\right]^2} - \frac{M_x\left(\left[R\right]+2d^2\delta_3\right)}{2\alpha B^5 d^3 \delta_3 \left[R\right]^2} - \frac{M_y}{2\alpha^3 B^5 (1+k)\left[S_1\right]^2}, \tag{24}$$

$$\frac{\dfrac{\partial g_1}{\partial k}}{\dfrac{\partial f}{\partial k}} = -\frac{N}{B^4\left[R\right]^2} - \frac{M_x}{\alpha B^5 d\left[R\right]^2} - \frac{2M_y\left[2\left[S_1\right]+(1+k)\left[4k\delta_2-\delta_3(1-k)\right]\right]}{\alpha^2 B^5 (1+k)^2\left[S_1\right]^2\left[4k\delta_2-\delta_3(1-k)\right]}.$$

Setting the first and second Eq. (24) equal to each other gives the following:

$$d^3 = M(1+k)\left[\left(\frac{1-k}{2}\right)^2 + \frac{\delta_1}{\delta_3} + \frac{\delta_2}{\delta_3}k^2\right]. \tag{25}$$

The following transformation can be done for defined parameter limits (11) and for defined optimization area (Fig. 3):

$$F_1 = \left[\left(\frac{1-k}{2}\right)^2 + \frac{\delta_1}{\delta_3} + \frac{\delta_2}{\delta_3}\cdot k^2\right], \tag{26a}$$

$$F_2 = \frac{4}{3}\left(1+k\right). \tag{26b}$$

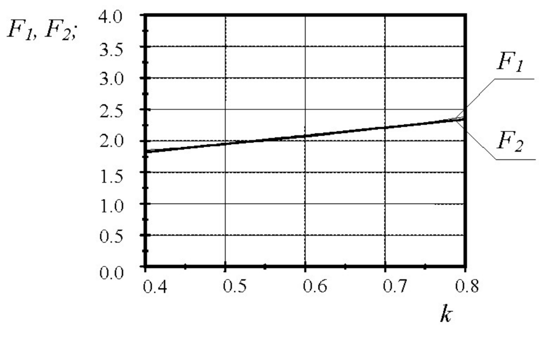

Fig. 3. *Comparative values of F_1 and F_2*

Fig. 3 shows that functions (26a) and (26b) have the same values within defined boundaries. Replacing Eqs. (26a) and (26b) into Eq. (25) and considering $F_1 = F_2$, the relation between parameters k and d is obtained:

$$d = \sqrt[3]{\frac{4M}{3}(1+k)^2}. \tag{27}$$

If we equalize the first and third Eq. (24), by means of certain transformations, the following Eq. is obtained:

$$\frac{\left[R\right]}{\left[4\delta_2 k + (k-1)\delta_3\right]} = \frac{(1+k)\left[R\right]}{8\alpha\left[S_1\right]} + \frac{\alpha\left[S_1\right]}{8\delta_3}. \tag{28}$$

Having (27) and ignoring the members of a very small value, Eq. (28) can be solved numerically. The solution, which is within the defining range of parameter k, is shown in Fig. 4.

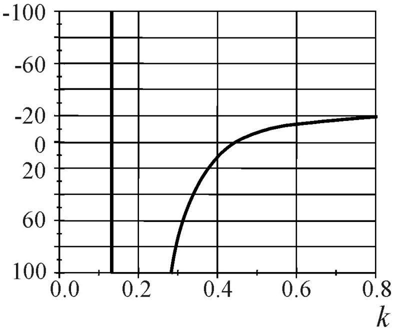

Fig. 4. *Solution to Eq. (28) within the defining range*

Fig. 4 shows solutions of Eq. (28) only for the real world values of parameter k. Other solutions ($k \to 0$, $k > 0.8$) are not considered because they are not of practical significance. If $k \to 1$, the trapezoidal cross section becomes rectangular. If $k < 0.4$, the solution cannot be realized in practice.

The trapeze height can be obtained from the boundary function (23) and relation (6), if the members of a very small value are ignored:

$$H = \sqrt[3]{\frac{M_x d^2\left\{2d\left[R\right]+M(1+k)\left[S_1\right]\right\}}{\alpha M(1+k)\left[S_1\right]\left[R\right]\sigma_0}}. \tag{29}$$

3 OPTIMISATION BY DE METHOD

3.1 Brief Description of DE Algorithm

The DE algorithm is briefly described here, and the control parameters of the algorithm are also dealt with. A detailed description of DE algorithm can be seen in references [11] to [16].

DE is a simple, but still strong evolutionary algorithm used for realization of the global minimum in numerous real world optimisation problems. The DE algorithm has the following control parameters: the population size *NP*, the crossover constant *CR* and the mutation constant *F*. Coding of chromosomes with real numbers, i.e. presentation of chromosomes as vectors of real values, is used in numerical application of DE in optimisation processes.

Generation of the initial population is performed stochastically. The population size *NP* is commonly ten times bigger than the number of design variables. At the beginning, each design variable is a random value which is found within the defined upper and lower boundaries. While defining the boundaries, attention should be paid to ensure that the values of design variables are not out of range which is really acceptable.

The mutation constant in DE is a real parameter, which controls the increase of difference between two individuals in the search space. The difference between two randomly chosen vectors defines the magnitude and direction of mutation. When the difference is added to a randomly chosen vector, it becomes a mutant vector. The basic idea of DE is that mutation is self-adaptive in the search space and the current population. At the beginning of the optimisation process, the magnitude of mutation is large because the vectors in the population are far away from the search space. When the process starts to converge, the magnitude of mutation starts to decrease. The self-adaptive mutation in DE leads the solution of the optimisation process toward the global minimum [16].

There are some basic rules, which are defined in [14], for taking the best values for *CR*. High values are effective for all problems, but they are not always the fastest. The problems with heavy interaction between design variables generally require a high *CR*. However, if interaction between design variables is lower, a lower *CR* can be used, which results in obtaining a satisfactory solution with a smaller number of iterations (faster solution). According to reference [14], the values of control parameters are presented in Table 2.

3.2 Optimisation Done by DE Algorithm

Design parameters can be expressed in the form of design vector $X = (k, d, \delta_1, \delta_2, \delta_3, H)$.

In addition to the variables k, d, H, which have been optimised by Lagrange's multipliers, the variables δ_1, δ_2 and δ_3, are also optimised.

The objective function is:

$$f(k,d,\delta_1,\delta_2,\delta_3,H) = A = = H^2\left[\frac{k}{d}\delta_2 + \frac{\delta_1}{d} + \delta_3\sqrt{4+\left(\frac{1-k}{d}\right)^2}\right], \tag{30}$$

with the following boundaries:

$$\begin{aligned} g_1(k,d,\delta_1,\delta_2,\delta_3,H) &= \left(\frac{N}{A}+\frac{M_x}{W_x}+\frac{M_y}{W_y}\right)-100 \le 0, \\ h_1(k,d,\delta_1,\delta_2,\delta_3,H) &= \delta_1 - 1.56\delta_3 = 0, \\ h_2(k,d,\delta_1,\delta_2,\delta_3,H) &= \delta_2 - 1.26\delta_3 = 0. \end{aligned} \tag{31}$$

The limit $g_1(k, d, \delta_1, \delta_2, \delta_3, H)$ results from (10) while the limits $h_1(k, d, \delta_1, \delta_2, \delta_3, H)$ and $h_2(k, d, \delta_1, \delta_2, \delta_3, H)$ result from (13).

The boundaries of design variables δ_1, δ_2, and δ_3 are defined by Eq. (11) and the boundaries of k, d, H are accepted according to references [5], [7] and [9]. Their upper and lower boundaries are shown in Table 1. Reference [14] proposes the constraints directly in the DE algorithm, which allow the values of design variables to remain within the mentioned boundaries during the whole optimisation process.

The parameters related to DE algorithm are shown in Table 2.

In Table 3 there are some final design variables for various accepted values M and M_x. On the basis of the final design variables, the values B, b, σ_e have been calculated as well as the numerical value of the objective function, i.e. the minimal trapezoidal cross sectional area of the truck crane boom has been obtained.

Table 1. *Initial values of design variables*

Boundary	k	d	δ_1	δ_2	δ_3	H [cm]
Lower	0.4	1.2	0.02	0.02	0.015	60
Upper	0.8	2.2	0.03	0.027	0.02	90

Table 2. *Parameters of DE algorithm*

NP (initial population)	*D* (number of design variables)	*CR* (crossover constant)	*F* (mutation constant)	*itermax* (maximum number of iterations)
60	6	0.5	0.5	1000

Table 3. *Accepted, final and optimised values of design variables*

		Accepted values					
		M = 1.33	M = 1.7	M = 2.5	M = 2.5	M = 1.7	M = 2.5
		M_x = 35000 [kNcm]	M_x = 40000 [kNcm]	M_x = 45000 [kNcm]	M_x = 35000 [kNcm]	M_x = 45000 [kNcm]	M_x = 40000 [kNcm]
Final values of design variables	k	0.415	0.407	0.501	0.495	0.591	0.606
	d	1.722	1.453	2.158	1.708	1.580	1.667
	δ_1	0.028	0.029	0.029	0.026	0.026	0.027
	δ_2	0.023	0.022	0.023	0.022	0.022	0.022
	δ_3	0.017	0.018	0.018	0.017	0.017	0.018
	H [cm]	86.9	82.8	86.9	76.4	86.0	76.8
Number of iterations		27	22	13	34	26	39
Calculated values	B [cm]	50.5	57.0	40.3	44.8	54.4	46.1
	b [cm]	21.0	23.2	20.2	22.2	32.2	28.0
	A_{min} [cm^2]	428.3	425.2	412.9	352.1	441.1	356.0
	σ_e [kN/cm^2]	9.70	9.53	9.86	9.77	9.08	9.59

4 COMPARISON OF OBTAINED RESULTS

Using Eqs. (9) and (29) the relation between the area and height of trapezoidal cross section as the function of external load and parameter M is obtained. The load values correspond to the load values of the truck cranes TD-16 made by [17]: IMK 14. oktobar – Krusevac and ADH-16 made by ILR – Belgrade.

Using the data from Table 3, and the objective function (9), a comparative analysis of the obtained results of both methods (Fig. 5) can be made. However, the exact comparison of the methods is not possible because the number of considered parameters is different for the first and the second method.

Analysing the obtained results (Fig. 5), it can be concluded that the results from both methods are in agreement, except for the case with values k = 0.6; $M \in$ 2 to 2.5 (Fig. 5c). In that case, the results obtained by DE method offer better solutions in terms of material saving.

5 CONCLUSION

Both optimisation methods can be successfully applied to determine the relation between the geometric parameters of trapezoidal cross sections of the truck crane booms.

The method of Lagrange's multipliers has advantages in defining the analytical form of the objective function, which is suitable for practical application. The achieved relations can be very useful for engineers, especially in the first phase of designing when facing the problems of defining the initial dimensions of the structure cross-sections, which should be close to the optimal ones.

DE method does not provide dependence of the objective function in analytical form, but it does provide the use of a larger number of boundaries, a wider range of initial design variables and a larger number of solutions, which meet the boundaries defined.

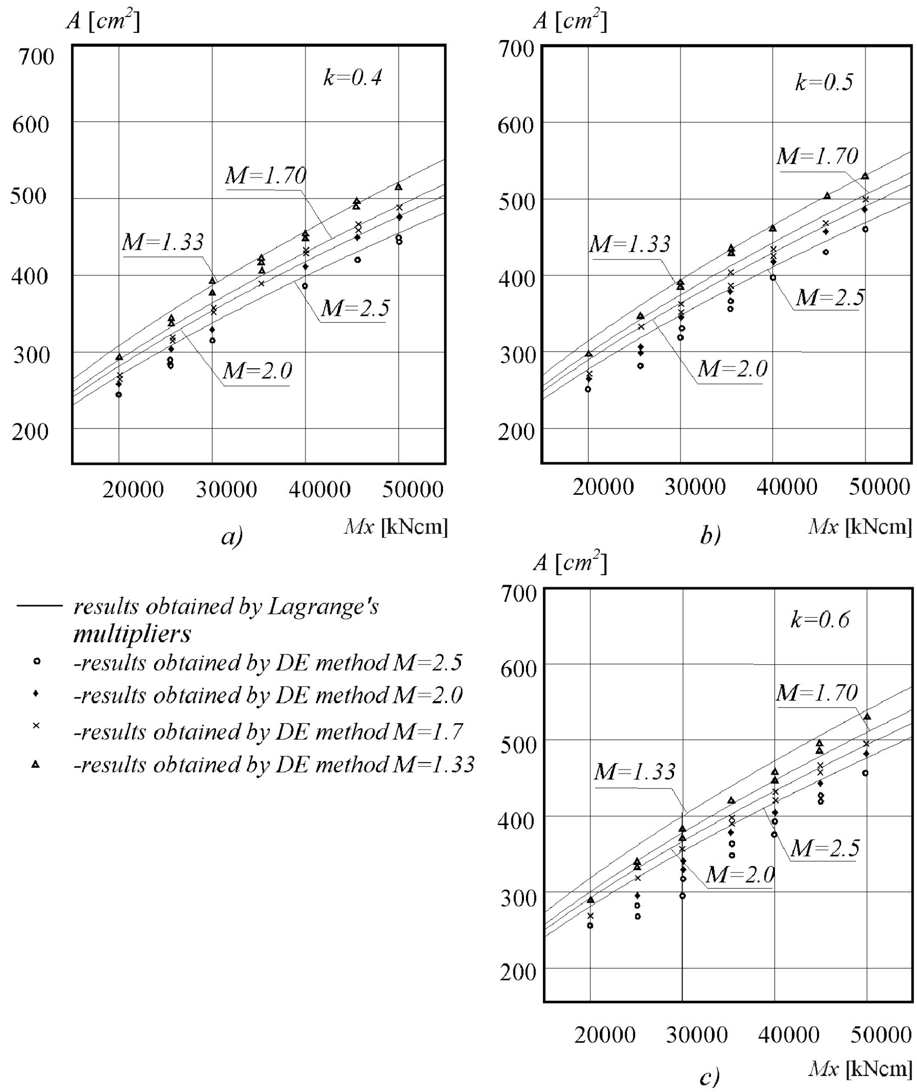

Fig. 5. *Comparative illustration of the results of both methods a) parameter value k=0.4 b) parameter value k=0.5 c) parameter value k=0.6*

DE method gives discrete values of the relation between some parameters as well as the minimal value of the objective function for defined load values. Apart from that, DE method has been applied in order to be compared to the first method.

After comparing the results obtained by means of two considered methods, we can state that there is a significant agreement between them. However, better solutions can be obtained by DE method (Fig. 5), because six parameters have been considered, unlike the first method of Lagrange's multipliers by which three parameters have been considered within optimisation.

If DE method is applied, solutions that are more accurate are reached for defined parameters $k = 0.6$, $M \in$ (2 to 2.5) (Fig. 5c).

For defined values $M \in$ (1.33 to 2), $k \in$ (0.4 to 0.5) the agreement between the results is very good (Figs. 5a and b). Due to the fact that these ranges of values match with the real world ones, the method of Lagrange's multipliers can be successfully applied to the structure optimisation, especially in the first phase of design.

However, cross-sections obtained should be further verified using other design criteria such as deflection of the boom structure and local stability of the metal sheets forming the trapezoidal cross-section.

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