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What Comes Beyond the Standard Models

Bled, July 20–28, 2014

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- ▷ *Hadrons as Solitons* (July 6–17, 1999)
- ▷ *Few-Quark Problems* (July 8–15, 2000), Vol. **1** (2000) No. 1
- ▷ *Selected Few-Body Problems in Hadronic and Atomic Physics* (July 7–14, 2001), Vol. **2** (2001) No. 1
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Preface

The series of workshops on "What Comes Beyond the Standard Models?" started in 1998 with the idea of Norma and Holger for organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. It is the seventeenth workshop which took place this year in the picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks and mountaineering.
In our very open minded, friendly, cooperative, long, tough and demanding discussions several physicists and even some mathematicians have contributed. Most of topics presented and discussed in our Bled workshops concern the proposals how to explain physics beyond the so far accepted and experimentally confirmed both standard models - in elementary particle physics and cosmology. Although most of participants are theoretical physicists, many of them with their own suggestions how to make the next step beyond the accepted models and theories, experts from experimental laboratories were very appreciated, helping a lot to understand what do measurements really tell and which kinds of predictions can best be tested.
The (long) presentations (with breaks and continuations over several days), followed by very detailed discussions, have been extremely useful, at least for the organizers. We hope and believe, however, that this is the case also for most of participants, including students. Many a time, namely, talks turned into very pedagogical presentations in order to clarify the assumptions and the detailed steps, analysing the ideas, statements, proofs of statements and possible predictions, confronting participants' proposals with the proposals in the literature or with proposals of the other participants, so that all possible weak points of the proposals showed up very clearly. The ideas therefore seem to develop in these years considerably faster than they would without our workshops.
In the seventeen years of our workshops the organizers, together with the participants, are trying to answer several open questions of the elementary particle physics and cosmology. Experiments have made large steps in this time. Among the most notable and might be also among the most important ones was two years ago the LHC confirmation that the scalar field, the Higgs, is like other fermionic and bosonic fields - just a field. And yet it is a very unusual field: A boson with the fractional weak and hyper charges. Do we have the explanation for that? Can we explain the origin of families and Yukawa couplings? Can we understand the origin of the matter-antimatter asymmetry? Can we explain all the assumptions of the *standard Model*?

The evolution of the universe and the dynamics of it on all levels, from the elementary particles to the matter, can be understood only, if we have the theory behind, which explains the observations and predicts new phenomena. Should we design theories and models in steps, each one more or less adapted for explaining a new experimental observation? Or can we suggest the theory which answers several (all?) open questions at the same time? Do the laws of Nature emerge at the low energy regime in the way we observe them? Or are the laws of Nature simple and elegant on all scales, while the observation of systems of many degrees of freedom do not show up the laws behind in a transparent way?
Can it happen that at the LHC no new fields, scalars, vectors or fermions will be observed, so that there will be no sign which will help to make a trustable step beyond the standard model?
If trusting the *spin-charge-family* theory, predicting the fourth family, coupled to the observed three families, and several scalar fields, this is not possible. This theory offers the explanation for all the assumptions of the *standard model*. It answers the question why does the Higgs's scalar (in this theory a superposition of several scalar fields) carry the weak charge and the hyper charge equal to $(\pm\frac{1}{2}, \mp\frac{1}{2})$, respectively. Since these scalar fields carry also quantum numbers of the family groups, the theory offers as well the explanation for the origin of the Yukawa couplings. Predicting the second group of four families, the lowest of these four families might explain the origin of the dark matter.
There are colour triplet scalars in this theory, transforming antileptons and antiquarks into quarks and back, transforming antimatter into matter and back. The scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families breaks the matter-antimatter symmetry. This could, hopefully, explain the observed matter-antimatter asymmetry. What is beautiful in this theory is, that a simple and elegant starting action has (only) all the needed degrees of freedom to explain in the low energy regime all the properties of quarks and leptons and of the vector and scalar gauge fields, predicting even how will more accurate measurements of the mixing matrices among the observed families of fermions change the values. The experiments might observe, due to this theory, that the space time is more than 3+1 dimensional.
There are inventive new predictions in this proceeding, in the main part and also in the Discussion section: Like the one that the local gauge symmetries appear due to the spontaneous break of the Lorentz invariance and supersymmetry, and there are interesting trials to built the origin of all the gauge fields in a common theory within the theory of higher spins, there are trials to explain emergency of the space and the influence of the future on the past.
There are attempts to extend the *standard model* with new fields of fermionic and bosonic origin to explain mass matrices and correspondingly masses and mixing matrices of quarks and leptons. Although it is not easy to see, to which extent different theories overlap when describing the same physical phenomena, yet our discussions helped to clarify many points a lot.
There is the work of new string theory, made of non interacting equal scalar objects, which reproduces the Veneziano scattering amplitude. These objects, which do not recognize "their own identity" when they meet with another group of equal

objects and the scattering matrix in the Hilbert space of which is equal to unity, manifest all the properties of strings, with the tension included.

It is the experimental contribution presenting the dark matter observations of DAMA/LIBRA through more than a decade of years. It includes also explanation why other experiments have not confirmed the DAMA/LIBRA experiments yet.

There are works trying to explain the direct observations from different laboratories with the idea that new particles, like OHe, exist and form the dark matter.

There were works in this year workshop with very promising ideas and developments of the ideas, the authors of which have not succeeded to send in time their contributions. And there were very discerning discussions among the participants, for which there has been not enough of time to matured the discussions into the written contribution.

Since the time to prepare the proceedings is indeed very short, three months if vacations are not counted, authors did not have a time to polish their contributions carefully enough.

Bled Workshops owe their success to participants who have at Bled in the heart of Slovene Julian Alps enabled friendly and active sharing of information and ideas, yet their success was boosted by vidoeconferences. Questions and answers as well as lectures enabled by M.Yu. Khlopov via Virtual Institute of Astroparticle Physics (www.cosmovia.org) of APC have in ample discussions helped to resolve many dilemmas.

The reader can find the records of all the talks delivered by cosmovia since Bled 2009 on www.cosmovia.org in Previous - Conferences. The four talks delivered by: R.Bernabei (Dark matter particles in the galactic halo), M. Laletin and M.Yu. Khlopov (Dark Atoms and their Decaying Constituents), N.S. Mankoč Borštnik (The Spin-Charge-Family theory offers the explanation for the assumptions of the Standard model, for the Dark matter, for the Matter-antimatter asymmetry), H.B.F. Nielsen (Novel string field theory), can be accessed directly at
http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xvii.html
Most of the talks can be found on the workshop homepage
http://bsm.fmf.uni-lj.si/.

Let us conclude this preface by thanking cordially and warmly to all the participants, present personally or through the teleconferences at the Bled workshop, for their excellent presentations and in particular for really fruitful discussions and the good and friendly working atmosphere.

The workshops take place in the house gifted to the Society of Mathematicians, Physicists and Astronomers of Slovenia by the Slovenian mathematician Josip Plemelj, well known to the participants by his work in complex algebra.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(the Organizing comittee)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(the Editors)

Ljubljana, December 2014

1 Predgovor (Preface in Slovenian Language)

Serija delavnic "Kako preseči oba standardna modela, kozmološkega in elektrošibkega" ("What Comes Beyond the Standard Models?") se je začela leta 1998 z idejo Norme in Holgerja, da bi organizirali delavnice, v katerih bi udeleženci v izčrpnih diskusijah kritično soočili različne ideje in teorije. Letos smo imeli sedemnajsto delavnico v mestu Bled ob slikovitem jezeru, kjer prijetni sprehodi in pohodi na čudovite gore, ki kipijo nad mestom, ponujajo priložnosti in vzpodbudo za diskusije.
K našim zelo odprtim, prijateljskim, dolgim in zahtevnim diskusijam, polnim iskrivega sodelovanja, je prispevalo veliko fizikov in celo nekaj matematikov. Večina predlogov teorij in modelov, predstavljenih in diskutiranih na naših blejskih delavnicah, išče odgovore na vprašanja, ki jih v fizikalni skupnosti sprejeta in s številnimi poskusi potrjena standardni model osnovnih fermionskih in bozonskih polj ter kozmološki standardni model puščata odprta. Čeprav je večina udeležencev teoretičnih fizikov, mnogi z lastnimi idejami kako narediti naslednji korak onkraj sprejetih modelov in teorij, so še posebej dobrodošli predstavniki eksperimentalnih laboratorijev, ki nam pomagajo v odprtih diskusijah razjasniti resnično sporočilo meritev in katere napovedi lahko poskusi najzanesljiveje preverijo.
Organizatorji moramo priznati, da smo se na blejskih delavnicah v (dolgih) predstavitvah (z odmori in nadaljevanji čez več dni), ki so jim sledile zelo podrobne diskusije, naučili veliko, morda več kot večina udeležencev. Upamo in verjamemo, da so veliko odnesli tudi študentje in večina udeležencev. Velikokrat so se predavanja spremenila v zelo pedagoške predstavitve, ki so pojasnile predpostavke in podrobne korake, soočile predstavljene predloge s predlogi v literaturi ali s predlogi ostalih udeležencev ter jasno pokazale, kje utegnejo tičati šibke točke predlogov. Zdi se, da so se ideje v teh letih razvijale bistveno hitreje, zahvaljujoč prav tem delavnicam.
V teh sedemnajstih letih delavnic smo organizatorji skupaj z udeleženci poskusili odgovoriti na marsikatero odprto vprašanje v fiziki osnovnih delcev in kozmologiji. Na vsakoletnem napovedniku naše delavnice objavimo zbirko odprtih vprašanj, na katera bi udeleženci utegnili predlagati rešitve. V sedemnajstih letih so eksperimenti napravili velike korake. Med najpomembnejšimi dosežki je potrditev LHC, da je skalarno pole, Higgsov delec, prav tako polje kot ostala fermionska in bozonska polja. In vendar je to skalarno polje zelo nenavadno polje: Je bozon s polovičnim šibkim in hiper nabojem. Ali to razumemo? Ali lahko pojasnimo izvor družin in Yukawinih sklopitev? Znamo pojasniti nesimetrijo med snovjo in antisnovjo v vesolju? Znamo razložiti privzetke *standardnega modela*?

Dinamiko vesolja na vseh nivojih, od osnovnih delcev do snovi, lahko razumemo samo, če ponudimo teorijo, ki opaženja razloži in napove nova spoznanja. Je prava pot pri postavljanju teorij ta, da prilagodimo teorijo eksperimentalnim spoznanjem po korakih? Ali lahko ponudimo teorijo, ki odgovori na mnoga (morda vsa) doslej odprta vprašanja?
Ali se naravni zakoni manifestirajo le pri nizkih energijah? Ali pa so preprosti in elegantni na vseh nivojih, le da jih pri nizkih energijah opazujemo v sistemih z velikim številom sodelujočih, kjer jih je težko razspoznati?
Kaj pa, če na LHC ne bodo izmerili nobenega novega polja, ne skalarja, ne vektorja, ne fermiona in ne bo ponudil eksperiment nobenega napotka, kako izbrati naslednji korak od *standardnega modela?*
Če ima teorija *spina, naboja in družin* prav, potem bodo na LHC izmerili četrto družino, ki je z že opaženimi sklopljena ter verjetno še kakšnega od dveh tripletov in treh singletov, ki jih teorija napoveduje. Teorija razloži vse predpostavke *standardnega modela*, pojasni izvor družin kvarkov in leptonov, vektorskih umeritvenih polj in izvor Higgsovega skalarja ter Yukawinih sklopitev. Razloži tudi, zakaj ima Higgsov skalar polštevilčen šibki in hiper naboj. To lastnost imata oba tripleta (ki nosita tudi družinska kvantna števila) in trije singleti skalarnih polj, ki poskrbijo za masne matrike kvarkov in leptonov ter razložijo Yukawine sklopitve. Teorija napove še eno skupino štirih družin kvarkov in leptonov. Družina z najnižjo energijo razloži nastanek temne snovi.
Teorija napove skupino skalarnih polj, ki so barvni tripleti. Povzročijo rojstvo kvarkov iz antileptonov in antikvarkov, s tem pa tudi rojstvo nukleonov. Reakcije tečejo tudi v obratni smeri. Kondenzat iz dveh desnoročnih nevtrinov z družinskimi kvantnimi števili zgornjih štirih družin zlomi simetrijo med obem reakcijama in ponudi odgovor na vprašanje, kaj je vzrok presežku snovi nad antisnovjo v opazljivem delu vesolja.
Teorija je lepa in elegantna, ker ponudi preprosta začetna akcija vsa fermionska (družine kvarkov in leptonov z opaženimi lastnostmi) in bozonska (vektorska umeritvena polja in skalarna polja) polja, ki so že opažena, direktno ali indirektno, napove nove fermione, nova skalarna polja in napove, kako se bodo spreminjali matrični elementi mešalne matrike pri bolj natančnih meritvah. Novi eksperimenti bodo prej ali slej dokazali, če ima teorija „prav”, da je dimenzija prostora-časa več kot le $(3 + 1)$.
V zborniku so nove zelo preroške teorije: Lokalne umeritvene simetrije se pojavijo kot odziv sistema na spontano zlomitev Lorentzove invariance, pa tudi supersimetrije. Zanimiva je prav gotovo tudi teorija, ki gradi na spinih večjih kot 2, da razloži vsa umeritvena polja. Zbornik vsebuje tudi prispevek, ki razširi *standardni model* z novimi fermionskimi in bozonskimi polji, da bi pojasnil masne matrike kvarkov in leptonov, ter s tem njihove mase in mešalne matrike.
Je tudi delo, nova teorija strune, ki jo sestavljajo enaki skalarni objekti, ki med seboj sploh ne interagirajo. In vendar reproducira teorija Venezianovo sipalno amplitudo. Ti objekti, ki „pozabijo”, ko srečajo drugo struno iz enakih objektov, kateri struni so pripadali in katerih sipalna matrika v njihovem Hilbertovem prostoru je identiteta, manifestirajo vse lastnosti teorije bozonskih strun, vključno z napetostjo strune. Fermionska inačica nove strune je v delu.

Vselej smo veseli eksperimentalnih prispevkov, ki tokrat predstavi več kot desetletje meritev temne snovi na DAMA/LIBRA. Pojasni tudi, zakaj jih ostali eksperimenti še niso potrdili.
Predstavljeni sta tudi deli, ki poskušata razložiti obstoječe neskladje med različnimi laboratotiji, ki merijo direktno interakcijo z delci temne snovi. Avtorji študirajo interakcijo novih delcev, kot je O*He*, z jedri v merilnih aparaturah.
Je tudi nekaj del, ki so bila predstavljena na delavnici, ki pa jih avtorji v tako kratkem času niso uspeli pripraviti za zbornik. Je tudi kar precejšnje število zelo obetavnih diskusij, ki jih avtorji prav tako niso utegnili pravočasno pripraviti.
Četudi so k uspehu „Blejskih delavnic" največ prispevali udeleženci, ki so na Bledu omogočili prijateljsko in aktivno izmenjavo mnenj v osrčju slovenskih Julijcev, so k uspehu prispevale tudi videokonference, ki so povezale delavnice z laboratoriji po svetu. Vprašanja in odgovori ter tudi predavanja, ki jih je v zadnjih letih omogočil M.Yu. Khlopov preko Virtual Institute of Astroparticle Physics (www.cosmovia.org, APC, Pariz), so v izčrpnih diskusijah pomagali razčistiti marsikatero dilemo.
Bralec najde zapise vseh predavanj, objavljenih preko "cosmovia" od leta 2009, na www.cosmovia.org v povezavi Previous - Conferences. Štiri letošnja predavanja: R. Bernabei (Dark matter particles in the galactic halo), M. Laletin in M.Yu. Khlopov (Dark Atoms and their Decaying Constituents), N.S. Mankoč Borštnik (The Spin-Charge-Family theory offers the explanation for the assumptions of the Standard model, for the Dark matter, for the Matter-antimatter asymmetry) in H.B.F. Nielsen (Novel string field theory), so dostopna na
http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xvii.html
Večino predavanj najde bralec na spletni strani delavnice na
http://bsm.fmf.uni-lj.si/.
Naj zaključimo ta predgovor s prisrčno in toplo zahvalo vsem udeležencem, prisotnim na Bledu osebno ali preko videokonferenc, za njihova predavanja in še posebno za zelo plodne diskusije in odlično vzdušje.
Delavnica poteka v hiši, ki jo je Društvu matematikov, fizikov in astronomov Slovenije zapustil v last slovenski matematik Josip Plemelj, udeležencem delavnic, ki prihajajo iz različnih koncev sveta, dobro poznan po svojem delu v kompleksni algebri.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(Organizacijski odbor)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(uredniki)

Ljubljana, grudna (decembra) 2014

Talk Section

All talk contributions are arranged alphabetically with respect to the authors' names.

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1 Dark Atoms and Their Decaying Constituents

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Abstract. The nonbaryonic dark matter of the Universe might consist of new stable charged species, bound by ordinary Coulomb interactions in various forms of heavy neutral "dark atoms". The existing models offer natural implementations for the dominant and subdominant forms of dark atom components. In the framework of Walking Technicolor the charge asymmetric excess of both stable negatively doubly charged technilepton ζ^{--} and metastable but longliving positively doubly charged technibaryon UU^{++} can be generated in the early Universe together with the observed baryon asymmetry. If the excess of ζ exceeds by several orders of magnitude the excess of UU, dark matter might consist dominantly by $He\zeta$ dark atoms of nuclear interacting O-helium (OHe) bound state of ζ with primordial helium. This dominant dark matter component causes negligible nuclear recoil in underground experiments, but can explain positive results of DAMA/NaI and DAMA/LIBRA experiments by annual modulations of few keV energy release in radiative capture of OHe by sodium. However, a sufficiently small subdominant component of WIMP-like objects $UU\zeta$ can also form. Making up a small fraction of dark matter, it can also evade the severe constraints on WIMPs from underground detectors. Although sparse, this subdominant component can lead to observable effects, since leptonic decays of technibaryons UU give rise to two positively charged leptons contrary to the pairs of opposite charge leptons created in decays of neutral particles. We show that decays of $UU^{++} \to e^+e^+, \mu^+\mu^+, \tau^+\tau^+$ of the subdominant $UU\zeta$ component of dark matter, can explain the observed high energy positron excess in the cosmic rays if the fraction of $UU\zeta$ is $\sim 10^{-6}$ of the total dark matter density, the mass of UU^{++} about 1 TeV and the lifetime about 10^{20} s. Optimizing fit of recent AMS-02 data by model parameters, the predicted mass range of such long-living double charge particle is challenging for its search at the LHC.

Povzetek. Avtorji predpostavijo, da sestavlja nebarionsko temno snov v vesolju nova, stabilna, nabita vrsta snovi, katere delce v obliki težkih nevtralnih "temnih atomov" veže običajna Coulombska interakcija. V obstoječih modelih, denimo v "Walking Technicolor", najdejo potrditev za tako domnevo: V zgodnjem vesolju lahko nastane presežek stabilnih tehnileptonov ζ^{--} z nabojem -2 in metastabilnih vendar dovolj dolgoživih tehnibarionov UU^{++} z nabojem $+2$, ki pojasnijo barionsko-antibarionsko asimetrijo. Če je presežek delcev ζ za več velikostnih redov večji od presežka delcev UU, sestavljajo temno snov pretežno temni atomi $He\zeta$ O-helija (OHe), ki je vezano stanje delca ζ in jedra običajnega helija. Če je OHe prevladujoči del temne snovi, je njegov prispevek k odrivu na jedrih merilnih aparatur, ki so postavljene pod zemljo, zanemarljiv, vendar lahko pojasni poskuse na DAMA/NaI in DAMA/LIBRA, ki izmerita periodično letno modulacijo, v območju nekaj keV energije.

Opazljiv pa je, pravijo avtorji, tudi majhen delež $UU\zeta$, ki tudi sestavlja temno snov, ker nastaneta pri leptonskem razpadu tehnibarionov UU dva pozitivno nabita leptona, pri razpadu nevtralnih delcev pa nastaneta dva nasprotno nabita leptona. Avtorji pokažejo, da lahko razpadi $UU^{++} \to e^+e^+, \mu^+\mu^+, \tau^+\tau^+$ $UU\zeta$ pojasnijo izmerjeni presežek pozitronov v kozmičnih žarkih, če je masa UU^{++} približno 1 TeV, njihov prispevek h gostoti temne snovi $\sim 10^{-6}$ in življenska doba okrog 10^{20} s. Avtorji napovedujejo, da bodo te delce izmerili tudi na LHC.

1.1 Introduction

Dark atoms offer an interesting possibility to solve the puzzles of dark matter searches. It turns out that even stable electrically charged particles can exist hidden in such atoms, bound by ordinary Coulomb interactions (see [1–3] and references therein). Stable particles with charge -1 are excluded due to overproduction of anomalous isotopes. However, there doesn't appear such an evident contradiction for negatively doubly charged particles.

There exist several types of particle models where heavy stable -2 charged species, O^{--}, are predicted:

(a) AC-leptons, predicted as an extension of the Standard Model, based on the approach of almost-commutative geometry [4–7].
(b) Technileptons and anti-technibaryons in the framework of Walking Technicolor (WTC) [8–14].
(c) stable "heavy quark clusters" $\bar{U}\bar{U}\bar{U}$ formed by anti-U quark of 4th generation [4,15–19]
(d) and, finally, stable charged clusters $\bar{u}_5\bar{u}_5\bar{u}_5$ of (anti)quarks $\bar{u}_5$ of 5th family can follow from the approach, unifying spins and charges[20].

All these models also predict corresponding +2 charge particles. If these positively charged particles remain free in the early Universe, they can recombine with ordinary electrons in anomalous helium, which is strongly constrained in terrestrial matter. Therefore a cosmological scenario should provide a mechanism which suppresses anomalous helium. There are two possible mechanisms than can provide a suppression:

(i) The abundance of anomalous helium in the Galaxy may be significant, but in terrestrial matter a recombination mechanism could suppress this abundance below experimental upper limits [4,6]. The existence of a new U(1) gauge symmetry, causing new Coulomb-like long range interactions between charged dark matter particles, is crucial for this mechanism. This leads inevitably to the existence of dark radiation in the form of hidden photons.
(ii) Free positively charged particles are already suppressed in the early Universe and the abundance of anomalous helium in the Galaxy is negligible [1,16].

These two possibilities correspond to two different cosmological scenarios of dark atoms. The first one is realized in the scenario with AC leptons, forming neutral AC atoms [6]. The second assumes a charge asymmetry of the O^{--} which forms the atom-like states with primordial helium [1,16].

If new stable species belong to non-trivial representations of the SU(2) electroweak group, sphaleron transitions at high temperatures can provide the relation between baryon asymmetry and excess of -2 charge stable species, as it was demonstrated in the case of WTC [8,21–23].

After it is formed in the Standard Big Bang Nucleosynthesis (BBN), ^4He screens the O^{--} charged particles in composite $(^4He^{++}O^{--})$ OHe "atoms" [16]. In all the models of OHe, O^{--} behaves either as a lepton or as a specific "heavy quark cluster" with strongly suppressed hadronic interactions. The cosmological scenario of the OHe Universe involves only one parameter of new physics − the mass of O^{--}. Such a scenario is insensitive to the properties of O^{--} (except for its mass), since the main features of the OHe dark atoms are determined by their nuclear interacting helium shell. In terrestrial matter such dark matter species are slowed down and cannot cause significant nuclear recoil in the underground detectors, making them elusive in direct WIMP search experiments (where detection is based on nuclear recoil) such as CDMS, XENON100 and LUX. The positive results of DAMA and possibly CRESST and CoGeNT experiments (see [24] for review and references) can find in this scenario a nontrivial explanation due to a low energy radiative capture of OHe by intermediate mass nuclei [1–3]. This explains the negative results of the XENON100 and LUX experiments. The rate of this capture is proportional to the temperature: this leads to a suppression of this effect in cryogenic detectors, such as CDMS. OHe collisions in the central part of the Galaxy lead to OHe excitations, and de-excitations with pair production in E0 transitions can explain the excess of the positron-annihilation line, observed by INTEGRAL in the galactic bulge [2,3,21,25].

One should note that the nuclear physics of OHe is in the course of development and its basic element for a successful and self-consistent OHe dark matter scenario is related to the existence of a dipole Coulomb barrier, arising in the process of OHe-nucleus interaction and providing the dominance of elastic collisions of OHe with nuclei. This problem is the main open question of composite dark matter, which implies correct quantum mechanical solution [26]. The lack of such a barrier and essential contribution of inelastic OHe-nucleus processes seem to lead to inevitable overproduction of anomalous isotopes [27].

It has been shown [8,21–23,28] that a two-component dark atom scenario is also possible and can be naturally realized in the framework of a WTC model, in which both stable double charged technilepton ζ^{--}, playing the role of O^{--}, and positively double charged technibaryon UU are predicted. Along with the dominant ζ^{--} abundance, a much smaller excess of positively doubly charged techniparticles UU can be created. These positively charged particles are hidden in WIMP-like atoms, being bound to ζ^{--}. In the framework of WTC such positively charged techniparticles can be metastable, with a dominant decay channel to a pair of positively charged leptons. We have shown in [28] that even a 10^{-6} fraction of such positively charged techniparticles with a mass of 1 TeV or less and a lifetime of 10^{20} s, decaying to e^+e^+, $\mu^+\mu^+$, and $\tau^+\tau^+$ can explain the observed excess of cosmic ray positrons, being compatible with the observed gamma-ray background.

The anomalous excess of high-energy positrons in cosmic rays was first observed by PAMELA [29] and was later confirmed by AMS-02 [30]. These results generated widespread interest, since the corresponding effect cannot be explained by positrons of only secondary origin and requires primary positron sources, e.g. annihilations or decays of dark matter particles. Recently AMS-02 collaboration has reported new results on positron and electron fluxes in cosmic rays [31] and positron fraction [32]. These measurements cover the energy ranges 0.5 to 700 GeV for electrons and 0.5 to 500 GeV for positrons and provides important information on the origins of primary positrons in the cosmic rays. In particular, new results show, for the first time, that above ∼ 200 GeV the positron fraction no longer exhibits an increase with energy. The possibility to explain the cosmic positron excess by the decays of UU particles, comprising the tiny WIMP-component of dark matter in the considered scenario, was discussed in detail in [28]. Here we estimate the optimal values of model parameters by achieving the best agreement with AMS-02 data on cosmic positron flux and FERMI-LAT data on diffuse gamma-ray flux [33].

1.2 Cosmic positron excess and fit to the latest AMS-02 data

In the considering scenario the metastable UU particles, which together with ζ forms the subdominant component of dark matter, decays as

$$UU \to e^+e^+, \mu^+\mu^+, \tau^+\tau^+$$

in principle with different branching ratios. All decay modes give directly or through cascades positrons and gamma photons. The latter are hereafter referred to as final state radiation (FSR). The positron flux at the top of the Earth's atmosphere can be estimated as

$$F(E) = \frac{c}{4\pi}\frac{n_{loc}}{\tau}\frac{1}{\beta E^2}\int_E^{m/2}\frac{dN}{dE_0}Q(\lambda(E_0, E))dE_0, \tag{1.1}$$

where $n_{loc} = \xi \cdot (0.3\,\mathrm{GeV/cm^3})m_{UU}^{-1}$ is the local number density of UU particles with $\xi = 10^{-6}$, dN/dE_0 is the number of positrons produced in a single decay (obtained using Pythia 6.4 [34]), $\beta \sim 10^{-16}\ \mathrm{s^{-1}GeV^{-1}}$ and

$$Q = 1 - \frac{(\lambda-h)^2(2\lambda+4)}{2\lambda^3}\eta(\lambda-h) - \frac{2h(\lambda^2-r^2)}{3\lambda^3}\eta(\lambda-R), \tag{1.2}$$

(see [28] for details).

Below ∼ 10 GeV behavior of positrons is affected by solar modulation. This effect can be in principle taken into account, using the force field model with two different parameters ϕ both for electrons and positrons, which can be easily adjusted in order to fit the data points at low energies. However, the effects of solar modulations are insignificant at the energies above ∼ 20 GeV and thus for our analysis we consider the positron spectrum from 20 to 500 GeV. The positron background component was taken from [35].

Any scenario that provides positron excess is also constrained by other observational data, mainly from the data on cosmic antiprotons and gamma-radiation from our halo (diffuse gamma-background) and other galaxies and clusters. If dark matter does not produce antiprotons, then the diffuse gamma-ray background gives the most stringent and model-independent constraints. For the FSR photons produced by UU decays in our Galaxy, the flux arriving at the Earth is given by

$$F_{FSR}^{(G)}(E) = \frac{n_{loc}}{\tau}\frac{1}{4\pi\Delta\Omega_{obs}}\int_{\Delta\Omega_{obs}}\frac{n(r)}{n_{loc}}dl d\Omega\cdot\frac{dN_\gamma}{dE}, \qquad (1.3)$$

where we use an isothermal number density distribution $\frac{n(r)}{n_{loc}} = \frac{(5\,\mathrm{kpc})^2+(8.5\,\mathrm{kpc})^2}{(5\,\mathrm{kpc})^2+r^2}$, r and l are the distances from the Galactic center and the Earth respectively. We obtain the averaged flux over the solid angle $\Delta\Omega_{obs}$ corresponding to $|b| > 10°$, $0 < l < 360°$.

Out of our Galaxy, decays of UU homogeneously distributed over the Universe should also contribute to the observed gamma-ray flux. For FSR photons this contribution can be estimated as

$$F_{FSR}^{(U)}(E) = \frac{c}{4\pi}\frac{\langle n_{mod}\rangle}{\tau}\int\frac{dN}{dE}dt = \frac{c\langle n_{mod}\rangle}{4\pi\tau}\times$$
$$\times\int_0^{\min(1100,\frac{m}{2E}+1)}\frac{dN}{dE_0}(E_0 = E(z+1))\frac{H_{mod}^{-1}dz}{\sqrt{\Omega_\Lambda+\Omega_m(z+1)^3}}, \qquad (1.4)$$

where $z = 1100$ corresponds to the recombination epoch, $\langle n_{mod}\rangle$ is the current cosmological number density of UU, $H_{mod}^{-1} = \frac{3}{2}t_{mod}\sqrt{\Omega_\Lambda}\ln^{-1}\left(\frac{1+\sqrt{\Omega_\Lambda}}{\sqrt{\Omega_m}}\right)$ is the inverse value of the Hubble parameter with t_{mod} being the age of the Universe. Ω_Λ and $\Omega_m = 1-\Omega_\Lambda$ are respectively the current vacuum and matter relative densities.

Contribution into gamma-ray flux induced by scattering off background electromagnetic radiations of electrons and positrons from decays is small at high energy tail of spectrum, where observation data put the strongest constraint [28], and is not taken into account here.

The best-fit parameter values were obtained by fitting the curve, given by Eq.(1.1), to the AMS-02 data points in the least squares sense. Since the same parameters define the predicted gamma-ray flux in order not to contradict the FERMI-LAT data we extend the fitting procedure, involving several lowest points in the diffuse gamma-background spectrum above ~ 100 GeV to be fit. For each choice of m_{UU} from 700 to 1400 GeV we have evaluated the best-fit values of the lifetime τ and three branching ratios Br_e, Br_μ and $Br_\tau = 1 - Br_e - Br_\mu$. To choose the scenario, which is most consistent with the experimental data, a χ^2 statistical test was used. The best fit ($\chi^2/n.d.f. = 0.57$) corresponds to the following parameter values: $M = 900\,\mathrm{GeV}$, $\tau = 4.59\ 10^{20}\,\mathrm{s}$, $Br_e = 0.195$, $Br_\mu = 0.129$, $Br_\tau = 0.676$. Positron flux, positron fraction and gamma-ray flux in the best-fit case are shown in Figs. 1.1,1.2 and 1.3 respectively.

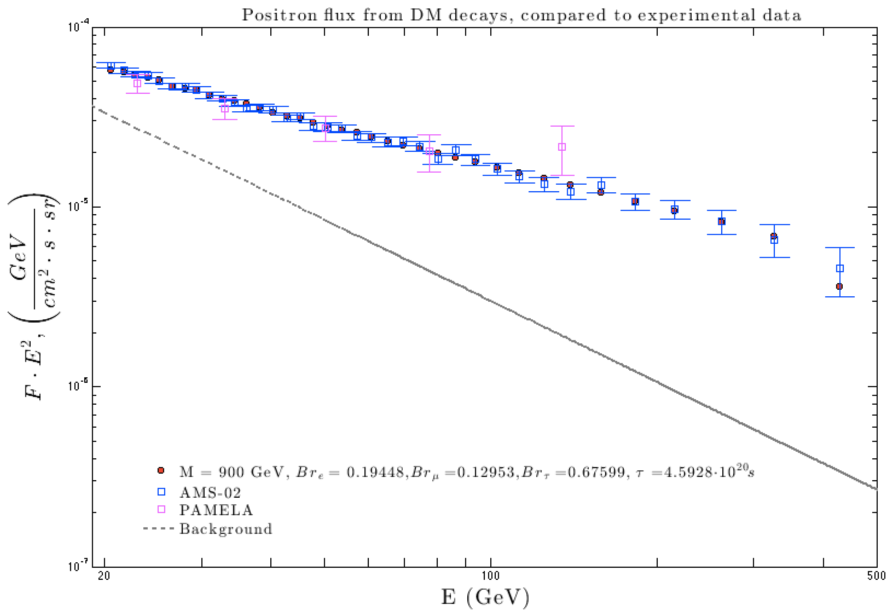

Fig. 1.1. Positron flux from UU decays compared to PAMELA and AMS-02 data

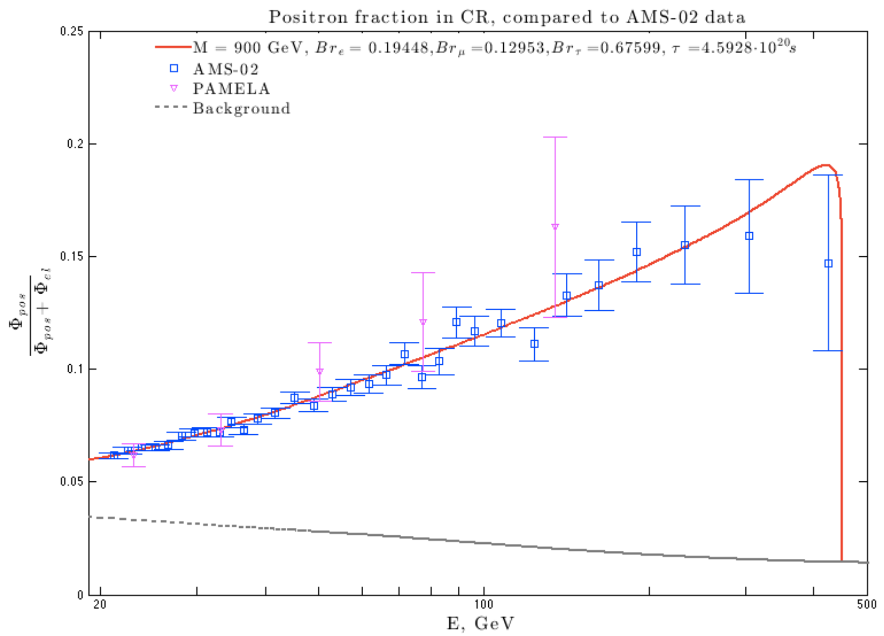

Fig. 1.2. Positron excess due to UU decays compared to PAMELA and AMS-02 data

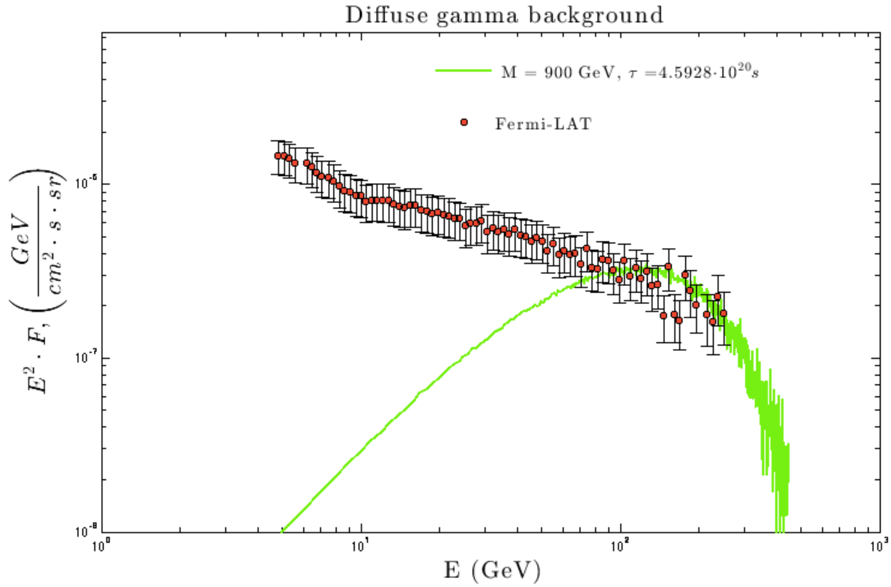

Fig. 1.3. Gamma-ray flux from UU decays in the Galaxy ($|b| \geq 10^0$) compared to the Fermi/LAT data on diffuse gamma-background

1.3 Conclusions

Being the reflection of fundamental particle symmetry beyond the Standard model, the set of stable particles – dark matter candidates – can hardly be reduced to one single species [2]. It makes natural to consider multi-component dark matter and one can hardly expect that various components put equal or comparable contribution into the total density. The situation with dominance of one component coexisting with some other subdominant components doesn't seem too exotic in this case.

Dark matter solution for the puzzles of dark matter searches can involve the form of neutral OHe dark atoms made of stable heavy doubly charged particles and primordial He nuclei bound by ordinary Coulomb interactions. This scenario can be realized in the framework of Minimal Walking Technicolor, in which an exact relation between the dark matter density and baryon asymmetry can be naturally obtained. Strict conservation of technilepton charge together with approximate conservation of technibaryon charge results in the prediction of two types of doubly charged species with strongly unequal excess – dominant negatively charged technileptons ζ^{--} and a strongly subdominant component of technibaryons UU^{++}, bound with ζ^{--} in a sparse component of WIMP-like dark atoms ($\zeta^{--}UU^{++}$). Direct searches for WIMPs put severe constraints on the presence of this component. However we have demonstrated in [28] that the existence of a metastable positively doubly charged techniparticle, forming this tiny subdominant WIMP-like dark atom component and satisfying the direct WIMP searches constraints, can play an important role in the indirect effects of dark matter. We found that decays of such positively charged constituents of WIMP-like dark atoms to the leptons $e^+e^+, \mu^+\mu^+, \tau^+\tau^+$ can explain the observed excess of high energy cosmic ray positrons, while being compatible with the observed gamma-ray background. These decays are naturally facilitated by GUT scale interactions. The best fit of the data takes place for a mass of this doubly charged particle of 1 TeV or below making it accessible in the next run of LHC. Our refined analysis of the best fit description of the recent data of the AMS-02 experiment, presented here, can provide a crucial test for the decaying dark atom hypothesis in the experimental searches for stable doubly charged lepton-like particles at the LHC.

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2 Dark Matter Particles in the Galactic Halo

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Abstract. The DAMA/LIBRA–phase1 and the former DAMA/NaI data (cumulative exposure 1.33 ton $\times$ yr, corresponding to 14 annual cycles) give evidence at 9.3 σ C.L. for the presence of Dark Matter (DM) particles in the galactic halo, on the basis of the exploited model independent DM annual modulation signature by using highly radio-pure NaI(Tl) target. Results and comparisons will be shortly addressed as well as perspectives of the presently running DAMA/LIBRA-phase2. Finally, some arguments arisen in the discussion section of this workshop are mentioned in the Appendix.

Povzetek. Avtorica, ki je postavila in vodi laboratorij DAMA/NaI in DAMA/LIBRA za merjenje delcev, ki tvorijo temno snov, poroča o rezultatih poskusov, ki tečejo že štirinajsto leto. Poroča o tem, kako in do kolikšne mere so uspeli izločiti nečistoče in poznane izvore signalov na merilni aparaturi. Poroča o odvisnosti števila izmerjenih delcev temne snovi v odvisnosti od relativne hitrosti Zemlje okoli Sonca glede na hitrost Sonca okoli centra galaksije. Prikaže napredek v zanesljivosti meritev v odvisnosti od let ter izrazitost letne modulacije meritev. Komentira dosedanje rezultate drugih laboratorijev in vzroke, zaradi katerih drugi njihovih meritev še niso potrdili. Napove drugi ciklus meritev ter odgovori na vprašanja, ki so jih postavili udeleženci delavnice.

2.1 Introduction

About 80 years of experimental observations and theoretical arguments have pointed out that a large fraction of the Universe is composed by Dark Matter particles [1].

The presently running DAMA/LIBRA ($\simeq$ 250 kg of full sensitive target-mass) [1–9] experiment, as well as the former DAMA/NaI ($\simeq$ 100 kg of full sensitive

[1] For completeness, it is worth recalling that some efforts to find alternative explanations to Dark Matter have been proposed such as MOdified Gravity Theory (MOG) and MOdified Newtonian Dynamics (MOND); they hypothesize that the theory of gravity is incomplete and that a new gravitational theory might explain the experimental observations. MOND modifies the law of motion for very small accelerations, while MOG modifies the Einstein's theory of gravitation to account for an hypothetical fifth fundamental force in addition to the gravitational, electromagnetic, strong and weak ones. However, e.g.: i) there is no general underlying principle; ii) they are generally unable to account for all small and large scale observations; iii) they fail to reproduce accurately the Bullet Cluster; iv) generally they require some amount of DM particles as seeds for the structure formation.

target-mass) [10–16], has the main aim to investigate the presence of DM particles in the galactic halo by exploiting the model independent DM annual modulation signature (originally suggested in Ref. [17]).

As a consequence of the Earth's revolution around the Sun, which is moving in the Galaxy with respect to the Local Standard of Rest towards the star Vega near the constellation of Hercules, the Earth should be crossed by a larger flux of DM particles around $\simeq$ 2 June and by a smaller one around $\simeq$ 2 December. In the former case the Earth orbital velocity is summed to the one of the solar system with respect to the Galaxy, while in the latter the two velocities are subtracted[2]. This DM annual modulation signature is very distinctive since the effect induced by DM particles must simultaneously satisfy all the following requirements: the rate must contain a component modulated according to a cosine function (1) with one year period (2) and a phase that peaks roughly $\simeq$ 2 June (3); this modulation must only be found in a well-defined low energy range, where DM particle induced events can be present (4); it must apply only to those events in which just one detector of many (9 in DAMA/NaI and 25 in DAMA/LIBRA) actually "fires" (*single-hit* events), since the DM particle multi-interaction probability is negligible (5); the modulation amplitude in the region of maximal sensitivity must be $\simeq$ 7% for usually adopted halo distributions (6), but it can be larger (even up to $\simeq$ 30%) in case of some possible scenarios such as e.g. those in Ref. [18,19]. Thus, this signature is model independent and very effective; moreover, the developed highly radio-pure NaI(Tl) target-detectors [1] and the adopted procedures assure sensitivity to a wide range of DM candidates (both inducing nuclear recoils and/or electromagnetic radiation), interaction types and astrophysical scenarios.

In particular, the experimental observable in DAMA experiments is the modulated component of the signal in NaI(Tl) target and not the constant part of it as in other approaches as those by CDMS, Xenon, etc., where in addition e.g.: i) different target materials are used; ii) the sensitivity is mainly restricted to candidates inducing just nuclear recoils; iii) many (by the fact largely uncertain) selections/subtractions of detectors and of data and (highly uncertain) extrapolations of detectors' features are applied.

The DM annual modulation signature might be mimicked only by systematic effects or side reactions able to account for the whole observed modulation amplitude and to simultaneously satisfy all the requirements given above. No one is available or suggested by anyone over more than a decade [1,2,5,6,8,13,9].

It is also worth noting that the DM annual modulation signature acts itself as a strong background reduction as pointed out since the early paper by Ref. [17], and especially when all the above peculiarities can be experimentally verified in suitable dedicated set-ups as it is the case of the DAMA experiments.

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[2] Thus, the DM annual modulation signature has a different origin and peculiarities than the seasons on the Earth and than effects correlated with seasons (consider the expected value of the phase as well as other requirements listed below).

2.2 The DAMA results

The total exposure of DAMA/LIBRA–phase1 is: 1.04 ton $\times$ yr in seven annual cycles; when including also that of the first generation DAMA/NaI experiment it is 1.33 ton $\times$ yr, corresponding to 14 annual cycles. The variance of the cosine during the DAMA/LIBRA–phase1 data taking is 0.518, showing that the set-up has been operational evenly throughout the years [2,6].

Many independent data analyses have been carried out [2,6] and all of them confirm the presence of a peculiar annual modulation in the *single-hit* scintillation events in the 2-6 keV energy interval, which – in agreement with the requirements of the DM signature – is absent in other parts of the energy spectrum and in the *multiple-hit* scintillation events in the same 2-6 keV energy interval (this latter condition correspond to have "switched off the beam" of DM particles). All the analyses and details can be found in the literature given above. In particular, Fig. 2.1 shows the time behaviour of the experimental residual rates of the *single-hit* scintillation events for DAMA/NaI [13] and DAMA/LIBRA–phase1 [2,6] cumulatively in the (2–6) keV energy interval. The data points present the experimental errors as vertical bars and the associated time bin width as horizontal bars. The superimposed curve is the cosinusoidal function $A\cos\omega(t-t_0)$ with a period $T=\frac{2\pi}{\omega}=1$ yr, a phase $t_0=152.5$ day (June 2^{nd}) and modulation amplitude, A, equal to the central value obtained by best fit on the data points. The dashed vertical lines correspond to the maximum expected for the DM signal, while the dotted vertical lines correspond to the expected minimum. The major upgrades are also pointed out.

In order to continuously monitor the running conditions, several pieces of information are acquired with the production data and quantitatively analysed. In particular, all the time behaviours of the running parameters, acquired with the production data, have been investigated: the modulation amplitudes obtained for each annual cycle when fitting the time behaviours of the parameters including a cosine modulation with the same phase and period as for DM particles are well compatible with zero. In particular, no modulation has been found in any possible source of systematics or side reactions; thus, cautious upper limits (90% C.L.) on possible contributions to the DAMA/LIBRA measured modulation amplitude have been derived (see e.g. [2]). It is worth noting that they do not quantitatively account for the measured modulation amplitudes, and are not able to simultaneously satisfy all the many requirements of the signature. Similar analyses have also been carried out for the DAMA/NaI data[13].

No other experimental result has been verified over so long time so accurately and with various significant upgrades of the set-ups.

For completeness I mention that sometimes naive statements were put forwards as the fact that in nature several phenomena may show some kind of periodicity. The point is whether they could mimic the annual modulation signature in DAMA/LIBRA (and former DAMA/NaI), i.e. whether they could quantitatively account for the observed modulation amplitude and also simultaneously satisfy all the requirements of the DM annual modulation signature. The same is also for side reactions. This has already been deeply investigated in Ref. [1,2]

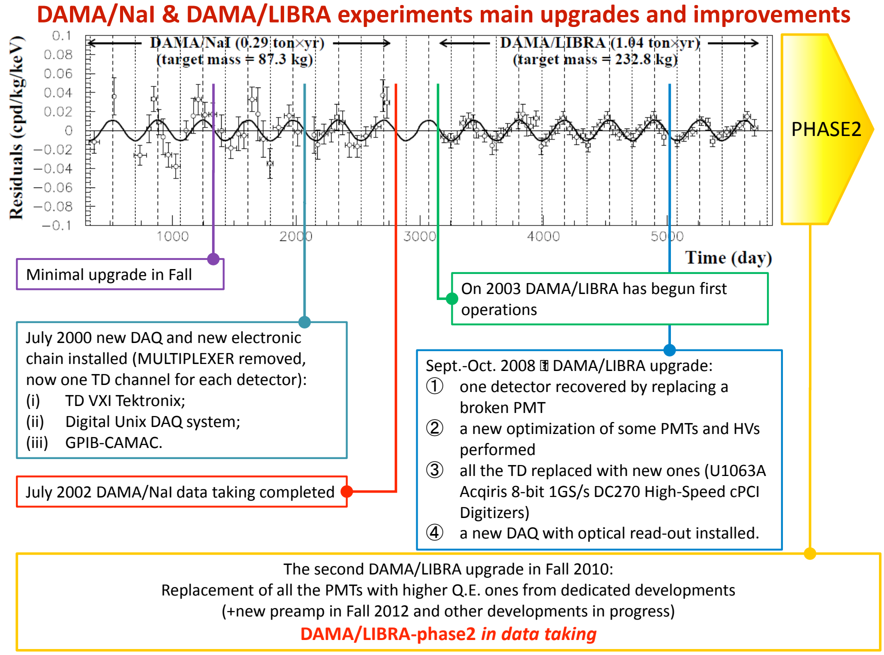

Fig. 2.1. Experimental residual rate of the *single-hit* scintillation events measured by DAMA/NaI and DAMA/LIBRA–phase1 in the (2–6) keV energy interval as a function of the time. The data points present the experimental errors as vertical bars and the associated time bin width as horizontal bars; see text. As always in DAMA results, the given rate is already corrected for the overall efficiency. The major upgrades of the experiment are also pointed out.

and references therein; the arguments and the quantitative conclusions, presented there, also apply to the entire DAMA/LIBRA–phase1 data. Additional arguments can be found in Ref. [5,6,8,9]. In particular, Ref. [9] further outlines in a simple and intuitive way why neutrons (of whatever origin), muons and solar neutrinos cannot give any significant contribution to the DAMA annual modulation results and – in addition – can never mimic the DM annual modulation signature since some of its specific requirements fail. Table 2.1 summarizes the safety upper limits on the contributions (if any) to the observed modulation amplitude due to the total neutron flux at LNGS, either from (α,n) reactions, from fissions and from muons' and solar-neutrinos' interactions in the rocks and in the lead around the experimental set-up; the direct contributions of muons and solar neutrinos are also reported there. As seen in Table 2.1, they are all negligible and they cannot give any significant contribution to the observed modulation amplitude; in addition, neutrons, muons and solar neutrinos are not a competing background when the DM annual modulation signature is investigated since they cannot mimic this signature. For details see Ref. [9] and references therein.

Source		$\Phi_{0,k}^{(n)}$ (neutrons cm^{-2} s^{-1})	η_k	t_k	$R_{0,k}$ (cpd/kg/keV)	$A_k = R_{0,k}\eta_k$ (cpd/kg/keV)	A_k/S_m^{exp}
SLOW neutrons	thermal n ($10^{-2} - 10^{-1}$ eV)	1.08×10^{-6}	$\simeq 0$ however $\ll 0.1$	–	$< 8 \times 10^{-6}$	$\ll 8 \times 10^{-7}$	$\ll 7 \times 10^{-5}$
	epithermal n (eV-keV)	2×10^{-6}	$\simeq 0$ however $\ll 0.1$	–	$< 3 \times 10^{-3}$	$\ll 3 \times 10^{-4}$	$\ll 0.03$
FAST neutrons	fission, $(\alpha, n) \to n$ (1-10 MeV)	$\simeq 0.9 \times 10^{-7}$	$\simeq 0$ however $\ll 0.1$	–	$< 6 \times 10^{-4}$	$\ll 6 \times 10^{-5}$	$\ll 5 \times 10^{-3}$
	$\mu \to n$ from rock ($>$ 10 MeV)	$\simeq 3 \times 10^{-9}$	0.0129	end of June	$\ll 7 \times 10^{-4}$	$\ll 9 \times 10^{-6}$	$\ll 8 \times 10^{-4}$
	$\mu \to n$ from Pb shield ($>$ 10 MeV)	$\simeq 6 \times 10^{-9}$	0.0129	end of June	$\ll 1.4 \times 10^{-3}$	$\ll 2 \times 10^{-5}$	$\ll 1.6 \times 10^{-3}$
	$\nu \to n$ (few MeV)	$\simeq 3 \times 10^{-10}$	0.03342*	Jan. 4th*	$\ll 7 \times 10^{-5}$	$\ll 2 \times 10^{-6}$	$\ll 2 \times 10^{-4}$
	direct μ	$\Phi_0^{(\mu)} \simeq 20\ \mu\ m^{-2} d^{-1}$	0.0129	end of June	$\simeq 10^{-7}$	$\simeq 10^{-9}$	$\simeq 10^{-7}$
	direct ν	$\Phi_0^{(\nu)} \simeq 6 \times 10^{10}\ \nu\ cm^{-2} s^{-1}$	0.03342*	Jan. 4th*	$\simeq 10^{-5}$	3×10^{-7}	3×10^{-5}

Table 2.1. Summary of the contributions to the total neutron flux at LNGS; the value, the relative modulation amplitude, and the phase of each component is reported. It is also reported the counting rate in DAMA/LIBRA for *single-hit* events, in the $(2-6)$ keV energy region induced by neutrons, muons and solar neutrinos, detailed for each component. The modulation amplitudes, A_k, are reported as well, while the last column shows the relative contribution to the annual modulation amplitude observed by DAMA, $S_m^{exp} \simeq 0.0112$ cpd/kg/keV [2]. As can be seen, they are all negligible and they cannot give any significant contribution to the observed modulation amplitude. In addition, neutrons, muons and solar neutrinos are not a competing background when the DM annual modulation signature is investigated since in no case they can mimic this signature. For details see Ref. [9] and references therein.

* The annual modulation of solar neutrino is due to the different Sun-Earth distance along the year; so the relative modulation amplitude is twice the eccentricity of the Earth orbit and the phase is given by the perihelion.

In conclusion, DAMA gives a model-independent evidence – at 9.3σ C.L. over 14 independent annual cycles – for the presence of DM particles in the galactic halo.

2.2.1 On comparisons

No direct model independent comparison is possible in the field when different target materials and/or approaches are used; the same is for the strongly model dependent indirect searches[3].

[3] It should be noted that the rising behaviour of the positron flux reported in Ref. [20,21] does not give any intrinsic evidence for production due to DM annihilation; this may arise only when a particular model of the competing background is assumed as e.g. the GALPROP code. But other more complete models exist which do not support any significant excess evidence. Moreover, an interpretation in terms of DM particle annihilation would require the assumption of: i) a very large boost factor (~ 400) of the density; ii) to boost the annihilation cross section through an assumed new interaction type; iii) to

In order to perform corollary investigations on the nature of the DM particles, model-dependent analyses are necessary[4]. Thus, many theoretical and experimental parameters and models are possible (see e.g. in [2,6,22,23]) and many hypotheses must also be exploited, while specific experimental and theoretical assumptions are generally adopted in the field assuming a single arbitrary scenario without accounting neither for existing uncertainties nor for alternative possible scenarios, interaction types, etc.

The obtained DAMA 9.3 σ C.L. model independent evidence is compatible with a wide set of scenarios regarding the nature of the DM candidate and related astrophysical, nuclear and particle Physics. For examples some scenarios and parameters are discussed e.g. in Ref. [10,11,13,2,6,22,23]. Further large literature is available on the topics (see for example in the bibliography of Ref. [6]). By the fact, both the negative results and all the possible positive hints are largely compatible with the DAMA model-independent DM annual modulation results in various scenarios considering also the existing experimental and theoretical uncertainties; the same holds for the strongly model dependent indirect approaches.

It is also worthwhile to further recall that these DAMA experiments are not only sensitive to DM particles with spin-independent coupling inducing just nuclear recoils, but also to other couplings and to other DM candidates as those giving rise to part or all the signal in electromagnetic form. Finally, scenarios exist in which other kind of targets/approaches are disfavoured or even blind.

2.3 DAMA/LIBRA–phase2 and perspectives

An important upgrade has started at end of 2010 replacing all the PMTs with new ones having higher Quantum Efficiency; details on the developments and on the reached performances in the operative conditions are reported in Ref. [4]. They have allowed us to lower the software energy threshold of the experiment to 1 keV and to improve also other features as e.g. the energy resolution [4].

Since the fulfillment of this upgrade and after some optimization periods, DAMA/LIBRA–phase2 is continuously running in order e.g.: (1) to increase the experimental sensitivity thanks to the lower software energy threshold; (2) to improve the corollary investigation on the nature of the DM particle and related astrophysical, nuclear and particle physics arguments; (3) to investigate other

adjust the propagation parameters; iv) to consider extra-source (subhalos, IMBHs); v) to consider only a leptophilic candidate to justify the absence of any excess in the antiproton spectrum. Finally, other well known sources can account for a similar positron fraction: pulsars, supernova explosions near the Earth, SNR.

[4] For completeness, it is worth recalling that it does not exist any approach to investigate the nature of the candidate in the direct and indirect DM searches, which can offer this information independently on assumed astrophysical, nuclear and particle Physics scenarios. On the other hand, searches for new particles beyond the Standard Model of particle Physics at accelerators cannot credit by themselves that a certain particle is in the halo as a solution or the only solution for DM particles, and – in addition – DM candidates and scenarios (even for the neutralino) exist which cannot be investigated there.

signal features and second order effects. This requires long and dedicated work for reliable collection and analysis of very large exposures.

In the future DAMA/LIBRA will also continue its study on several other rare processes as also the former DAMA/NaI apparatus did.

Finally, further future improvements of the DAMA/LIBRA set-up to increase the sensitivity (possible DAMA/LIBRA-phase3) and the developments towards the possible DAMA/1ton (1 ton full sensitive mass on the contrary of other kind of detectors), we proposed in 1996, are considered at some extent. For the first case developments of new further radiopurer PMTs with high quantum efficiency are starting, while in the second case it would be necessary to overcome the present problems regarding: i) the supplying, selection and purifications of a large number of high quality NaI and, mainly, TlI powders; ii) the availability of equipments and competence for reliable measurements of small trace contaminants in ppt or lower region; iii) the creation of updated protocols for growing, handling and maintaining the crystals; iv) the availability of large Kyropoulos equipments with suitable platinum crucibles; v) etc.. At present, due to the change of rules for provisions of strategical materials, the large costs and the lost of some equipments and competence also at industry level, a satisfactory development appears quite difficult.

2.4 Conclusions

The data of DAMA/LIBRA–phase1 have further confirmed the presence of a peculiar annual modulation of the *single-hit* events in the (2–6) keV energy region satisfying all the many requirements of the DM annual modulation signature; the cumulative exposure by the former DAMA/NaI and DAMA/LIBRA–phase1 is 1.33 ton $\times$ yr (orders of magnitude larger than those typically released in the field).

As required by the DM annual modulation signature: 1) the *single-hit* events show a clear cosine-like modulation as expected for the DM signal; 2) the measured period is equal to (0.998 ± 0.002) yr well compatible with the 1 yr period as expected for the DM signal; 3) the measured phase (144 ± 7) days is compatible with $\simeq$ 152.5 days as expected for the DM signal; 4) the modulation is present only in the low energy (2–6) keV interval and not in other higher energy regions, consistently with expectation for the DM signal; 5) the modulation is present only in the *single-hit* events, while it is absent in the *multiple-hit* ones as expected for the DM signal; 6) the measured modulation amplitude in NaI(Tl) of the *single-hit* events in the (2–6) keV energy interval is: (0.0112 ± 0.0012) cpd/kg/keV (9.3 σ C.L.). No systematic or side processes able to simultaneously satisfy all the many peculiarities of the signature and to account for the whole measured modulation amplitude is available.

DAMA/LIBRA–phase2 is continuously running in its new configuration with a lower software energy threshold aiming to improve the knowledge on corollary aspects regarding the signal and on second order effects as discussed e.g. in Ref. [6,8].

Few comments on model–dependent comparisons have also been addressed here.

Acknowledgments

It is a pleasure to thank all my DAMA collaborators who effectively dedicated their efforts to this experimental activity and the colleagues in this Workshop for the interesting topics we have discussed, for the question section, and for the pleasant scientific environment.

Appendix: Questions & Answers

This section shortly summarizes some of the topics extensively discussed at the Workshop, where the time dedicated to discussions and the interest in deeply understanding the topics were rather large.

Question 1: may you comment about the ratio of the measured dark matter particles modulation amplitude to the total signal: the S_m/S_0 ratio?

Answer 1: the measured counting rate in the cumulative energy spectrum is about 1 cpd/kg/keV in the lowest energy bins; this is the sum of the background contribution and of the constant part of the signal S_0. As discussed e.g. in TAUP2011 [24], the background in the 2-4 keV energy region is estimated to be not lower than about 0.75 cpd/kg/keV; this gives an upper limit on S_0 of about 0.25 cpd/kg/keV. Thus, the S_m/S_0 ratio is equal or larger than about $0.01/0.25 \simeq 4$ %.

Question 2: may you comment on the quenching factors, on their dependence on the type of the particles, and on some typical examples of extreme properties?

Answer 2: The quenching factor values play a role only when corollary model-dependent analyses for DM candidates inducing just nuclear recoils are carried out, in order to derive the energy scale in terms of nuclear recoil energy.

As is widely known, the quenching factor is a specific property of the employed detector and not a general quantity universal for a given material. For example, in liquid noble-gas detectors, it depends – among others – on the level of trace contaminants which can vary in time and from one liquefaction process to another, on the cryogenic microscopic conditions, etc.. In bolometers it depends for instance on specific properties, trace contaminants, cryogenic conditions, etc. of each specific detector, while generally it is assumed exactly equal to unity (the maximum possible value). The quenching factors in scintillators depend, for example, on the dopant concentration, on the growing method/procedures, on residual trace contaminants, etc., and are expected to be energy dependent. Thus, all these aspects are already by themselves relevant sources of uncertainties when interpreting whatever result in terms of DM candidates inducing just nuclear recoils. Similar arguments have been addressed e.g. in Ref. [2,3,13,15,25].

Question 3: May you comment under which extreme conditions your experiment is successful and comment what can at most the experiment which does not fulfil one of the conditions or more than one of them at most can "see"?

Answer 3: The full description and potentiality of the DAMA/LIBRA set-up have been discussed in details in Refs. [1,2,4] and references therein. Obviously all the set-up specific features and adopted procedures contribute to the possibility to point out the signal through the model independent DM annual modulation signature. The absence/difference of one of them would limit whatever else result.

Question 4: May you comment about muons?

Answer 4: An extensive discussion on this topics can be found in the dedicated Ref. [5,9], where its has been quantitatively demonstrated (see also Table 2.1 in this paper) that – for many reasons (and just one would suffice) - muons cannot play (directly or indirectly) any role in the DAMA annual modulation effect.

Question 5: May you comment about neutrinos?

Answer 5: The contribution from solar, atmospheric, .. neutrinos is obviously negligible; a quantitative discussion can be found in Ref. [9] (see also Table 2.1 in this paper).

Question 6: May you comment about the operating temperature of your measuring apparatus?

Answer 6: The DAMA set-ups operate at environmental temperature maintained stable by suitable and redundant air-conditioning system (2 independent devices for redundancy); moreover, the Cu housings of the detectors are in direct contact with the multi-ton metallic shield, thus there is a huge heat capacity ($\sim 10^6$ cal/^{0}C). In addition, the operating temperature of the detectors is continuously monitored and analysed as the production data. A discussion on temperature in operating condition can be found e.g. in Ref. [2,6].

Question 7: May you comment about the Snowmass plots and its meaning?

Answer 7: The recent plot from Snowmass and that in Ref. [26] about the "status of the Dark Matter search" do not point out at all the real status of Dark Matter searches since e.g.: i) Dark Matter has wider possibilities than WIMPs inducing just nuclear recoil with spin-independent interaction under single (largely arbitrary) set of assumptions; ii) neither the uncertainties for existing experimental and theoretical aspects nor alternative possible assumptions are accounted for; iii) they do not include possible systematic errors affecting the data (such as e.g. "extrapolations" of energy threshold, of energy resolution and of efficiencies, quenching factors values, convolution with poor energy resolution, correction for non-uniformity of the detector, multiple subtractions/selection of detectors and/or data, assumptions on quantities related to halo model, form factors, scaling laws, etc.); iv) the DAMA implications – even adopting the many arbitrary assumptions considered there – appear incorrect, for example the S_0 prior is not accounted for, etc., etc.. The perspectives as well appear incorrect/too optimistic.

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3 New Experimental Data for the Quarks Mixing Matrix are in Better Agreement with the *Spin-charge-family* Theory Predictions

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Abstract. The *spin-charge-family* theory [1–14] predicts before the electroweak break four - rather than the coupled and observed three - massless families of quarks and leptons. Mass matrices of all the family members demonstrate in this proposal the same symmetry, determined by the scalar fields: There are two $SU(2)$ triplets, the gauge fields of the family groups, and the three singlets, the gauge fields of the three charges (Q, Q' and Y') distinguishing among family members - all with the quantum numbers of the *standard model* scalar Higgs with respect to the weak and the hyper charge [13]: $\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively. Respecting by the *spin-charge-family* theory proposed symmetry of mass matrices and simplifying the study by assuming that mass matrices are hermitian and real and mixing matrices real, we fit the six free parameters of each family member mass matrix to the experimental data of twice three measured masses of quarks and to the measured quarks mixing matrix elements, within the experimental accuracy. Since any 3×3 sub matrix of the 4×4 matrix (either unitary or orthogonal) determine the whole 4×4 matrix uniquely, we are able to predict the properties of the fourth family members provided that the experimental data for the 3×3 sub matrix are enough accurate, which is not yet the case. However, new experimental data [15] fit better to the required symmetry of mass matrices than the old data [16]. The obtained mass matrices are very closed to the democratic ones.

Povzetek. Teorija *spinov-nabojev-družin* [1–14] napoveduje štiri in ne le tri opažene družine kvarkov in leptonov. Simetrija masnih matrik je v tej teoriji enaka za vse člane družine. Določajo jo skalarna polja: Dve tripletni upodobitvi grupe $SU(2)$, ki določata družinska kvantna števila in tri singletne upodobitve grup nabojev (Q, Q' in Y'). Vsa skalarna polja, to je obe tripletni in vsa tri singletna polja, nosijo tudi šibki in hiper naboj kot ju za Higgsov skalarni delec privzame *standardni model* [13]: ali ($\frac{1}{2}$ in $-\frac{1}{2}$), ali pa ($-\frac{1}{2}$ in $\frac{1}{2}$). Prva vrednost v oklepaju velja za šibki in druga za hiper naboj. Avtorja v prispevku upoštevata simetrijo masnih matrik, ki jo za štiri družine predlaga teorija *spinov-nabojev-družin*. Parametre masnih matrik določita iz izmerjenih podatkov. Izračun masnih matrik in mešalnih matrik poenostavita s privzetkom, da so masne metrike hermitske in realne. Mešalne matrike so tedaj ortogonalne. Vsaka matrika ima v tem primeru šest prostih parametrov. Matrični elementi vsake unitarne matrike nxn so enolično določeni z z matričnimi elementi podmatrike $(n-1)x(n-1)$, če je $n \geq 4$, pri ortogonalnih matrikah pa za vsak n. Ker pa eksperimentalni podatki nosijo napako, je mešala matrika 4x4 lahko samo približno določena. Tedaj lahko iz eksperimentalnih podatkov za 2 krat po 3 mase in iz mešalne matrike določimo lastnosti četrte družine v okviru eksperimentalne natančnosti. Izmerjeni matrični elementi mešalne matrike nosijo preveliko napako, da bi avtorja lahko napovedala mase četrte družine bolj

natančno kot da so blizu 1 TeV. Vzpodbudno pa je, da se novi eksperimentalni podatki za mešalno matriko kvarkov bolje ujemajo z zahtevano simetrijo masnih matrik kot stari. Avtorja ugotavljata, da so masne matrike zelo blizu edinkam (demokratičnim matrikam).

3.1 Introduction

There are several attempts in the literature to reconstruct mass matrices of quarks and leptons out of the observed masses and mixing matrices and correspondingly to learn more about properties of the fermion families [17–28]. The most popular is the $n \times n$ matrix, close to the democratic one, predicting that $(n-1)$ families must be very light in comparison with the n^{th} one. Most of attempts treat neutrinos differently than the other family members, relying on the Majorana part, the Dirac part and the "sea-saw" mechanism. Most often are the number of families taken to be equal to the number of the so far observed families, while symmetries of mass matrices are chosen in several different ways [29–31]. Also possibilities with four families are discussed [32–34].

In this paper we follow the *spin-charge-family* theory [1–14], which predicts four families of quarks and leptons and the symmetries of their mass matrices, the same for all the family members.

The mass matrix of each family member is in the *spin-charge-family* theory determined by the scalar fields, which carry besides by the *standard model* required weak and hyper charges [13] ($\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively) also the additional charges: There are two $SU(2)$ triplets, the gauge fields of the family groups, and three singlets, the gauge fields of the three charges (Q, Q' and Y'), which distinguish among family members. These scalar fields cause, after getting nonzero vacuum expectation values [13], the electroweak break. Assuming that the contributions of all the scalar (and in loop corrections also of other) fields to mass matrices of fermions are real and symmetric, we are left with the following symmetry of mass matrices

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha}, \tag{3.1}$$

the same for all the family members $\alpha \in \{u, d, \nu, e\}$. In appendix 3.5.1 the evaluation of this mass matrix is presented and the symmetry commented. The symmetry of the mass matrix Eq.(3.1) is kept in all loop corrections.

A change of phases of the left handed and the right handed basis - there are $(2n-1)$ free choices - manifests in a change of phases of mass matrices.

The differences in the properties of the family members originate in the different charges of the family members and correspondingly in the different couplings to the corresponding scalar and gauge fields.

We fit (sect. 3.3.1) the mass matrix (Eq. (3.1)) with 6 free parameters of any family member to the so far observed properties of quarks and leptons within the experimental accuracy. That is: *For a pair of either quarks or leptons, we fit twice* 6 *free parameters of the two mass matrices to twice three so far measured masses and to the corresponding mixing matrix.*

Since we have the same number of free parameters (6 parameters determine in the *spin-charge-family* theory the mass matrix of any family member after the mass matrices are assumed to be real) as there are measured quantities for either quarks or leptons (two times 3 masses and 6 angles of the orthogonal mixing matrix under the simplification that the mixing matrix is real and hermitian), we should predict the fourth family masses and the missing mixing matrix elements ($V_{u_i d_4}, V_{u_4 d_i}, i \in (1,2,3)$) uniquely, provided that the measured quantities are accurate. The $n-1$ sub matrix of any unitary matrix determines the unitary matrix uniquely for $n \geq 4$. The experimental inaccuracy, in particular for leptons and also for some of the matrix elements of the mixing matrix of quarks, is too large to be able to estimate the fourth family masses better than very roughly even for quarks. Yet we found out that our fitting to the experimental data for quarks are better when using the new experimental data for the quarks mixing matrix [15] than the old ones [16], which mainly differ in the second and the third diagonal values. This might be a signal that the *spin-charge-family* theory is the right step beyond the *standard model* (if taking into account also other predictions of this theory [1–14]), although we assume in this calculations the real mass matrices (Eq. (3.1)) and the orthogonal mixing matrices.

We treat all the family members, the quarks and the leptons, equivalently, as required by the *spin-charge-family* theory. We take into account the estimations of the influence of the fourth family masses to the mesons decays of the refs. [43], making also our own estimations (pretty roughly so far, this work is not presented in this paper) [1].

We can say that the so far obtained data do not contradict the prediction of the *spin-charge-family* theory that there are four coupled families of quarks and leptons, the mass matrices of which manifest the symmetry determined by the family groups – the same for all the family members, quarks and leptons. The mass matrices are quite close to the "democratic" ones, in particular for leptons.

Since the mass matrices offer an insight into the properties of the scalar fields, which determine mass matrices (together with other fields), manifesting effectively as the observed Higgs and the Yukawa couplings, we hope to learn about the properties of these scalar fields also from the mass matrices of quarks and leptons.

In sect. 3.2 the procedure to fit free parameters of mass matrices (Eq. (3.1) to the experimental data is discussed.

We comment our studies in sect. 3.4.

[1] M.I.Vysotsky and A.Lenz comment in their papers [43] that the fourth family is excluded provided that one assumes the *standard model* with one scalar field (the scalar Higgs) while extending the number of families from three to four when, in loop corrections, evaluating the decay properties of the scalar Higgs. We have, however, several scalars: Two times three triplets with respect to the family quantum numbers and three singlets, which distinguish among the family members [13], all with the quantum numbers of the scalar Higgs with respect to the weak and hyper charge. These scalar fields determine all the masses and the mixing matrices of quarks and leptons and of the weak gauge fields, what in the *standard model* is achieved by the choice of the scalar Higgs properties and the Yukawa couplings. Our rough estimations of the decay properties of mesons show that he fourth family quarks might have masses close to 1 TeV.

In appendix 3.5 we offer a very brief introduction into the *spin-charge-family* theory, which the reader, accepting the proposed symmetry of mass matrices without knowing the origin of this symmetry, can skip. In Appendix neutrino the old results [11] for leptons are presented *What follows must be carefully checked and corrected. It must go to the discussion section.

In appendix 3.5 we offer a very brief introduction into the *spin-charge-family* theory, which the reader, accepting the proposed symmetry of mass matrices without knowing the origin of this symmetry, can skip.

3.2 Procedure used to fit free parameters of mass matrices to experimental data

This part repeats in many points the ref. [11]

Matrices, following from the *spin-charge-family* theory, might not be hermitian (appendix 3.7). We, however, simplify our study, presented in this paper, by assuming that the mass matrix for any family member, that is for quarks and 0leptons, is real and symmetric. We take the simplest phases up to signs, which depend on the choice of phases of the basic states, as discussed in appendices 3.5.1 [2].

The matrix elements of mass matrices, with the loop corrections in all orders taken into account, manifesting the symmetry of Eq. (3.1), are in this paper taken as free parameters. Due to this symmetry, required by the family quantum numbers of the scalar fields [13], there 6 parameters having $(n-1)\cdot(2-2)/2)$ complex phases. Assuming, to simplify the calculations, that mass matrices are real, there are correspondingly 6 free real parameter for the mass matrix for u and d quarks and for ν and e leptons.

Let us first briefly overview properties of mixing matrices, a more detailed explanation of which can be found in subsection 3.2.1 of this section.

Let M^{α}, α denotes the family member ($\alpha = u, d, \nu, e$), be the mass matrix in the massless basis (with all loop corrections taken into account). Let $V_{\alpha\beta} = S^{\alpha}S^{\beta\dagger}$, where α represents either the u-quark and β the d-quark, or α represents the ν-lepton and β the e-lepton, denotes a (in general unitary) mixing matrix of a particular pair: the quarks one or the leptons one.

For $n \times n$ matrix ($n = 4$ in our case) it follows:

i. If a known sub matrix $(n-1)\times(n-1)$ of an unitary matrix $n \times n$ with $n \geq 4$ is extended to the whole unitary matrix $n \times n$, the n^2 unitarity conditions determine $(2(2(n-1)+1))$ real unknowns completely. If the sub matrix $(n-1)\times(n-1)$ of an unitary matrix is made unitary by itself, then we loose the information of the last row and last column.

ii. If the mixing matrix is assumed to be orthogonal, then the $(n-1)\times(n-1)$ sub matrix contains all the information about the $n \times n$ orthogonal matrix to which it belongs and the $n(n+1)/2$ conditions determine the $2(n-1)+1$ real unknowns

[2] In the ref. [9] we made a similar assumption, except that we allow that the symmetry on the tree level of mass matrices might be changed in loop corrections. We got in that study dependence of mass matrices and correspondingly mixing matrices for quarks on masses of the fourth family.

completely for any n.
If the sub matrix of the orthogonal matrix is made orthogonal by itself, then we loose all the information of the last row and last column.

We make in this paper, to simplify the present study, several assumptions [39], as it has been already written in the introduction. In what follows we present the procedure used in our study and repeat the assumptions.

1. If the mass matrix M^{α} is hermitian, then the unitary matrices S^{α} and T^{α}, introduced in appendix 3.7 to diagonalize a non hermitian mass matrix, differ only in phase factors depending on phases of basic vectors and manifesting in two diagonal matrices, $F^{\alpha\, S}$ and $F^{\alpha\, T}$, corresponding to the left handed and the right handed basis, respectively. For hermitian mass matrices we therefore have: $T^{\alpha} = S^{\alpha}\, F^{\alpha\, S} F^{\alpha\, T\, \dagger}$. By changing phases of basic vectors we can change phases of $(2n-1)$ matrix elements.
2. We take the diagonal matrices $\mathcal{M}^{\alpha}_{d}$ and the mixing matrices $V_{\alpha\beta}$ from the available experimental data. The mass matrices M^{α} in Eq. (3.1) have, if they are hermitian and real, 6 free real parameters $(a^{\alpha}, a^{\alpha}_{1}, a^{\alpha}_{2}, b^{\alpha}, e^{\alpha}, d^{\alpha})$, $\alpha = (u, d, \nu, e)$.
3. We limit the number of free parameters of the mass matrix of each family member α by taking into account n relations among free parameters, in our case $n = 4$, determined by the invariants
$$\begin{aligned} I^{\alpha}_{1} &= -\sum_{i=1,4} m^{\alpha}_{i}, \quad I^{\alpha}_{2} = \sum_{i>j=1,4} m^{\alpha}_{i}\, m^{\alpha}_{j}, \\ I^{\alpha}_{3} &= -\sum_{i>j>k=1,4} m^{\alpha}_{i}\, m^{\alpha}_{j}\, m^{\alpha}_{k}, \quad I^{\alpha}_{4} = m^{\alpha}_{1}\, m^{\alpha}_{2}\, m^{\alpha}_{3}\, m^{\alpha}_{4}\,, \\ \alpha &= u, d, \nu, e\,, \end{aligned} \tag{3.2}$$
which are expressions appearing at powers of λ_{α}, $\lambda^{4}_{\alpha} + \lambda^{3}_{\alpha} I_1 + \lambda^{2}_{\alpha} I_2 + \lambda^{1}_{\alpha} I_3 + \lambda^{0}_{\alpha} I_4 = 0$, in the eigenvalue equation. The invariants are fixed, within the experimental accuracy of the data, by the observed masses of quarks and leptons and by the fourth family mass, if we make a choice of it. for a chosen m^{α}_{4}. Correspondingly there are $(6-4)$ free real parameters left for each mass matrix, after a choice is made for the mass of the fourth family member.
4. The diagonalizing matrices S^{α} and S^{β}, each depending on the reduced number of free parameters, are for real and symmetric mass matrices orthogonal. They follow from the procedure
$$\begin{aligned} M^{\alpha} &= S^{\alpha}\, \mathbf{M}^{\alpha}_{d}\, T^{\alpha\, \dagger}\,, \quad T^{\alpha} = S^{\alpha}\, F^{\alpha\, S} F^{\alpha\, T\, \dagger}\,, \\ \mathbf{M}^{\alpha}_{d} &= (m^{\alpha}_{1}, m^{\alpha}_{2}, m^{\alpha}_{3}, m^{\alpha}_{4})\,, \end{aligned} \tag{3.3}$$
provided that S^{α} and S^{β} fit the experimentally observed mixing matrices $V^{\dagger}_{\alpha\beta}$ within the experimental accuracy and that M^{α} and M^{β} manifest the symmetry presented in Eq. (3.1). We keep the symmetry of the mass matrices accurate. One can proceed in two ways.
$$\begin{aligned} &A.: \quad S^{\beta} = V^{\dagger}_{\alpha\beta} S^{\alpha}\,, \qquad B.: \quad S^{\alpha} = V_{\alpha\beta} S^{\beta}\,, \\ &A.: \quad V^{\dagger}_{\alpha\beta}\, S^{\alpha}\, \mathbf{M}^{\beta}_{d}\, S^{\alpha\dagger} V_{\alpha\beta} = M^{\beta}\,, \qquad B.: \quad V_{\alpha\beta}\, S^{\beta}\, \mathbf{M}^{\alpha}_{d}\, S^{\beta\dagger} V^{\dagger}_{\alpha\beta} = M^{\alpha}\,. \end{aligned} \tag{3.4}$$

In the case A. one obtains from Eq. (3.3), after requiring that the mass matrix M^{α} has the desired symmetry, the matrix S^{α} and the mass matrix M^{α} ($= S^{\alpha}\mathbf{M}^{\alpha}_{d}\, S^{\alpha\dagger}$), from where we get the mass matrix $M^{\beta} = V^{\dagger}_{\alpha\beta}\, S^{\alpha}\, \mathbf{M}^{\beta}_{d}\, S^{\alpha\dagger}V_{\alpha\beta}$. In case B. one obtains equivalently the matrix S^{β}, from where we get M^{α} ($= V_{\alpha\beta}\, S^{\beta}\, \mathbf{M}^{\alpha}_{d}\, S^{\beta\dagger}V^{\dagger}_{\alpha\beta}$). We use both ways iteratively taking into account the experimental accuracy of masses and mixing matrices.

5. Under the assumption of the present study that the mass matrices are real and symmetric, the orthogonal diagonalizing matrices S^{α} and S^{β} form the orthogonal mixing matrix $V_{\alpha\beta}$, which depends on at most 6 ($= \frac{n(n-1)}{2}$) free real parameters (appendix 3.7). Since, due to what we have explained at the beginning of this section, the experimentally measured matrix elements of the 3×3 sub matrix of the 4×4 mixing matrix (if not made orthogonal by itself) determine (within the experimental accuracy) the 4×4 mixing matrix, also the fourth family masses are determined, again within the experimental accuracy. We must not forget, however, that the assumption of the real and symmetric mass matrices, leading to orthogonal mixing matrices, might not be an acceptable simplification, since we do know that the 3×3 sub matrix of the mixing matrix has one complex phase, while the unitary 4×4 has three complex phases. (In the next step of study, with hopefully more accurate experimental data, we shall relax conditions on hermiticity of mass matrices and correspondingly on orthogonality of mixing matrices.) We expect that too large experimental inaccuracy leave the fourth family masses in the present study quite undetermined, in particular for leptons.
6. We study quarks and leptons equivalently. The difference among family members originate on the tree level in the eigenvalues of the operators (Q^{α}, Q'^{α}, Y'^{α}), which in loop corrections together with other contributors in all orders contribute to all mass matrix elements and cause the difference among family members [3].

Let us conclude. If the mass matrix of a family member obeys the symmetry required by the *spin-charge-family* theory, which in a simplified version (as it is taken in this study) is real and symmetric, the matrix elements of the mixing matrices of quarks and leptons are correspondingly real, each of them with $\frac{n(n-1)}{2}$ free parameters. These six parameters of each mixing matrix are, within the experimental inaccuracy, determined by the three times three experimentally determined sub matrix. After taking into account three so far measured masses of each family member, the six parameters of each mass matrix reduce to three. Twice three free parameters are within the experimental accuracy correspondingly determined by the 3×3 sub matrix of the mixing matrix. The fourth family masses are correspondingly determined - within the experimental accuracy.

Since neither the measured masses nor the measured mixing matrices are determined accurately enough to reproduce the 4×4 mixing matrices, we can in the best case expect that the masses and mixing matrix elements of the fourth family will be determined only within some quite large intervals.

[3] There are also Majorana like terms contributing in higher order loop corrections [7] which might strongly influence in particular the neutrino mass matrix.

3.2.1 Submatrices and their extensions to unitary and orthogonal matrices

In this part well known properties of $n \times n$ matrices, extended from $(n-1)\times(n-1)$ submatrices are discussed. We make a short overview of the properties, needed in this paper, although all which will be presented here, is the knowledge on the level of text books.

Any $n \times n$ complex matrix has $2n^2$ free parameters. The $n + 2n(n-1)/2$ unitarity requirements reduce the number of free parameters to n^2 $(= 2n^2 - (n + 2n(n-1)/2))$. Let us assume a $(n-1) \times (n-1)$ known sub matrix of the unitary matrix. The sub matrix can be extended to the unitary matrix by $(2 \times [2(n-1)+1])$ real parameters of the last column and last row. The n^2 unitarity conditions on the whole matrix reduce the number of unknowns to $(2(2n-1) - n^2)$. For $n = 4$ and higher the $(n-1) \times (n-1)$ sub matrix contains all the information about the unitary $n \times n$ matrix.
The ref. [37] proposes a possible extension of an $(n-1) \times (n-1)$ unitary matrix $V_{(n-1)(n-1)}$ into $n \times n$ unitary matrices V_{nn}.

The choice of phases of the left and the right basic states which determine the unitary matrix (like this is the case with the mixing matrices of quarks and leptons) reduces the number of free parameters for $(2n-1)$. Correspondingly is the number of free parameters of such an unitary matrix equal to $n^2-(2n-1)$, which manifests in $\frac{1}{2}n(n-1)$ real parameters and $\frac{1}{2}(n-1)(n-2)$ $(= n^2 - \frac{1}{2}n(n-1) - (2n-1))$ phases (which determine the number of complex parameters).

Any real $n \times n$ matrix has n^2 free parameters which the $\frac{1}{2}n(n+1)$ orthogonality conditions reduce to $\frac{1}{2}n(n-1)$. The $(n-1)\times(n-1)$ sub matrix of this orthogonal matrix can be extended to this $n \times n$ orthogonal matrix with $[2(n-1)+1]$ real parameters. The $\frac{1}{2}n(n+1)$ orthogonality conditions reduce these $[2(n-1)+1]$ free parameters to $(2n-1-\frac{1}{2}n(n+1))$, which means that the $(n-1) \times (n-1)$ sub matrix of an $n \times n$ orthogonal matrix determine properties of its $n \times n$ orthogonal matrix completely. Any $(n-1) \times (n-1)$ sub matrix of an orthogonal matrix contains all the information about the whole matrix for any n. Making the sub matrix of the orthogonal matrix orthogonal by itself one looses the information about the $n \times n$ orthogonal matrix.

3.2.2 Free parameters of mass matrices after taken into account invariants

It is useful for numerical evaluation purposes to take into account for each family member its mass matrix invariants (sect. 3.2), expressible with three within the experimental accuracy known masses, while we keep the fourth one as a free parameter. We shall make a choice of $a^\alpha = \frac{1}{4} I_1^\alpha$ (Eqs. (3.1, 3.6)) instead of the fourth family mass.

We shall skip in this section the family member index α and introduce new parameters as follows

$$a, b\,, \quad f = d + e\,, \quad g = d - e\,, \quad q = \frac{a_1 + a_2}{\sqrt{2}}\,, \quad r = \frac{a_1 - a_2}{\sqrt{2}}\,. \qquad (3.5)$$

After choosing as a free parameter $a = \frac{I_1}{4}$ (Eq. (3.6)), which is indeed the fourth family mass - summed together with the three known (from the experiment) masses in I_1 - the four invariants of Eq. (3.2) reduce the number of free parameters to 2. The four invariants therefore relate six parameters leaving three of them undetermined. There are for each pair of family members the measured mixing matrix elements, assumed in this paper to be orthogonal and correspondingly determined by six parameters, which then fixes these two times 3 parameters. The (accurately enough) measured 3×3 sub matrix of the (assumed to be orthogonal) 4×4 mixing matrix namely determines these 6 parameters within the experimental accuracy.

Using the starting relation among the invariants I_i, $i \in (1,2,3,4)$ and replacing new parameters (a, b, f, g, q, r) from Eq. (3.5) we obtain

$$\begin{aligned} a &= \frac{I_1}{4}\,, \\ I_2' &= -I_2 + 6a^2 - q^2 - r^2 - 2b^2 = f^2 + g^2\,, \\ I_3' &= -\frac{1}{2b}(I_3 - 2aI_2 + 4a^2) = f^2 - g^2\,, \\ I_4' &= I_4 - aI_3 + a^2 I_2 - 3a^4 \\ &= \frac{1}{4}(q^2 - r^2)^2 + (q^2 + r^2)b^2 + \frac{1}{2}(q^2 - r^2) \cdot (\pm) \cdot [\pm]\, 2gf \\ &\qquad + b^2(f^2 + g^2) + \frac{1}{4}(2gf)^2\,. \end{aligned} \tag{3.6}$$

We eliminate, using the first two equations, the parameters f and g, expressing them as functions of I_2' and I_3', which depend, for a particular family member, on the three known masses, the parameter a and the three parameters r, q and b. We are left with the four free parameters (a, b, q, r) and the below relation among these parameters

$$\begin{aligned} &\{-\frac{1}{2}(q^4 + r^4) + (-2b^2 + \frac{1}{2}(-I_2 + 6a^2 - 2b^2))(q^2 + r^2) \\ &+ (I_4' - \frac{1}{4}((-I_2 + 6a^2 - 2b^2)^2 + I_3'^2) + b^2(-I_2 + 6a^2 - 2b^2))\}^2 \\ &= -\frac{1}{4}(q^2 - r^2)^2((-I_2 + 6a^2 - 2b^2 - (q^2 + r^2))^2 - I_3'^2)\,, \end{aligned} \tag{3.7}$$

which reduces the number of free parameters to 3. These 3 free parameters must be determined, together with the corresponding three parameters of the partner, from the measured mixing matrix.

We eliminate one of the 4 free parameters in Eq. (3.7) by solving the cubic equation for, let us make a choice, q^2

$$\alpha q^6 + \beta q^4 + \gamma q^2 + \delta = 0\,. \tag{3.8}$$

Parameter $(\alpha, \beta, \gamma, \delta)$ depend on the 3 free remaining parameters (a, b, r) and the three, within experimental accuracy, known masses.

To reduce the number of free parameters from the starting 6 in Eq. (3.1) to the 3 left after taking into account invariants of each mass matrix, we look for the solution of Eq (3.8) for all allowed values for (a, b, r). We make a choice for a in the interval of (a_{min}, a_{max}), determined by the requirement that a, which solves the equations, is a real number. Allowing only real values for parameters f and g we end up with the equation

$$-I_2 + 6a^2 - 2b^2 - (q^2 + r^2) > |\frac{I_3 + 8a^3 - 2aI_2}{2b}|, \tag{3.9}$$

which determines the maximal positive b for $q = 0 = r$ and also the minimal positive value for b. For each value of the parameter a the interval (b_{min}, b_{max}), as well as the interval $(r_{min} = 0, r_{max})$, follow when taking into account experimental values for the three lower masses.

Trying to fit the free parameters to the experimental values of the 3×3 sub matrix to the mixing matrix we minimize the uncertainty defined in Eq. (3.10)

$$\sigma = \sqrt{\sum_{(i,j)=1}^{3} \left(\frac{V_{u_i d_j\, exp} - V_{u_i d_j\, cal}}{\sigma_{V_{u_i d_j\, exp}}} \right)^2},$$
$$\delta V_{u_i d_j} = |\frac{V_{u_i d_j\, exp} - V_{u_i d_j\, cal}}{\sigma_{V_{u_i d_j\, exp}}}|, \tag{3.10}$$

where expressions $\sigma_{V_{u_i d_j\, exp}}$ stay for the experimental uncertainties, presented in Eqs. (3.11, 3.12).

3.3 Numerical results

Taking into account the assumptions and the procedure explained in sect. 3.2 and in the ref. [39] we are looking for the 4×4 in this paper taken to be real and symmetric mass matrices for quarks and leptons, obeying the symmetry of Eq. (3.1) and manifesting observed properties - masses and mixing matrices - of the so far observed three families of quarks in agreement with the experimental limits for the appearance of the fourth family masses and mixing matrix elements to the lower three families, as presented in the refs. [16,15,43]. We also take into account our so far made rough estimations of possible contributions of the fourth family members to the decay of mesons. More detailed estimations are in progress. The results for leptons, presented in Appendix 3.6 are the old ones, taking from [11]. They are added only for the comparison.

We hope that we shall be able to learn from the mass matrices of quarks and leptons also about the properties of the scalar fields, which cause masses of quarks and leptons, manifesting effectively so far as the measured Higgs and Yukawa couplings.

We take the 3×3 measured mixing matrices for quarks and leptons and the measured masses, all with the experimental inaccuracy. We extend the measured nine mixing matrix elements for each pair to the corresponding 4×4 mass matrix, by taking into account the unitarity of the 4×4 matrix, in our case indeed the orthogonality of the 4×4 matrices. We then look for twice 4×4 mass matrices with the symmetry of Eq. (3.1), and correspondingly for the fourth family masses, for quarks and leptons.

We perform the calculations for quarks with the old [16] and new [15] experimental data for the quarks mixing matrix, to see, whether or not the more accurate values fit better into by the *spin-charge-family* theory predicted symmetry of mass matrices (Eq. (3.1)). We present in appendix 3.6 also one trial for the lepton mass matrices. Since the experimental data for the mixing matrix and masses are for leptons known so inaccurate, the results do not tell much.

To test the predicting power of our model in dependence of the experimental inaccuracy of masses and mixing matrices, we compare the calculated mass matrices for quarks, obtained when choosing different values for the fourth family masses, among themselves and with the experimental data, the old [16] ones and the new [15] ones.

3.3.1 Numerical results for the observed quarks with mass matrices obeying Eq. (3.1)

We take for the quarks masses the experimental values [16], recalculated to the Z boson mass scale. We take two kinds of the experimental data for the quark mixing matrices, the older data from [16] and the last data [15], with the experimentally declared inaccuracies for the so far measured 3×3 mixing matrix. We assume, as suggested by the *spin-charge-family* theory, that these nine matrix elements belong to the 4×4 unitary mixing matrix. We take into account the experimentally allowed values for the fourth family masses and other limitations, presented in refs. [43,32–34]. We have made also our own rough estimations for the limitations which follow from the meson decays to which the fourth family members participate. Our estimations are still in progress.

A lot of effort was put into the numerical procedure to be sure as much as one can, that we fit the parameters of mass matrices to the experimental values within the experimental inaccuracy, in the best way, that is with the smallest errors.

It is expected that the inaccuracy, mainly due to the quarks mixing matrix, masses do not influence the results so strongly, does not allow to tell much about the fourth family masses. Yet, what we have learned not only supports the predicted symmetry of the *spin-charge-family* theory, but also predicts to what values will the more accurately measured matrix elements of the 3×3 sub matrix of the 4×4 mixing matrix move.

Let us admit that from the so far obtained results we are not yet able to predict the fourth family quarks mass accurately enough, although the results show that the most trustable might be results pushing the fourth family quarks to 1 TeV or above.

The results manifest that the mass matrices are very close to the democratic ones, which is, as expected, more and more the case the higher might be the fourth family masses, and it is true for quarks and leptons.

The calculated 4×4 mixing matrix predicts, in dependence of the fourth family masses, not only the fourth family matrix elements of the mixing matrix, but also the direction in which will the matrix elements of the 3×3 sub matrix move in the future more accurate measurements - under the assumption that the *spin-charge-family* theory is offering the right next step beyond the standard model.

In this paper we do not take yet into account the complex phases of the mass matrix elements and correspondingly of the mixing matrices. Sooner or latter we ought to do that.

We present below two types of the experimental values for the quarks 3×3 mixing matrix, taken as the sub matrix of the 4×4 matrix, the older experimental data [16] and the newer experimental data [15].

We start with the older experimental data [16]

$$|V_{ud}| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 & |V_{u_1 d_4}| \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 & |V_{u_2 d_4}| \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 & |V_{u_3 d_4}| \\ |V_{u_4 d_1}| & |V_{u_4 d_2}| & |V_{u_4 d_3}| & |V_{u_4 d_4}| \end{pmatrix}, \tag{3.11}$$

and then repeat all the calculations also with the new experimental data [15]

$$|V_{ud}| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2253 \pm 0.0008 & 0.00413 \pm 0.00049 & |V_{u_1 d_4}| \\ 0.225 \pm 0.008 & 0.986 \pm 0.016 & 0.0411 \pm 0.0013 & |V_{u_2 d_4}| \\ 0.0084 \pm 0.0006 & 0.0400 \pm 0.0027 & 1.021 \pm 0.032 & |V_{u_3 d_4}| \\ |V_{u_4 d_1}| & |V_{u_4 d_2}| & |V_{u_4 d_3}| & |V_{u_4 d_4}| \end{pmatrix}. \tag{3.12}$$

The matrix elements of the 4×4 quark mixing matrix will be determine in the numerical procedure, which searches for the best fit of the two quarks mass matrices free parameters presented in Eq. (3.1) to the experimental data, taking into account the experimental inaccuracy and unitarity of the mixing matrix, ensuring as much as possible, the best fit.

Let us notice that in the new experimental data differ slightly from the old ones only in the two diagonal matrix elements, $V_{cs} = V_{u_2 d_2}$ and $V_{tb} = V_{u_3 d_3}$, appearing in new data with smaller inaccuracy. The corresponding fourth family mixing matrix elements ($|V_{u_i d_4}|$ and $|V_{u_4 d_j}|$) are accordingly in both cases determined from the unitarity condition for the 4×4 mixing matrix through the fitting procedure, as also all the other matrix elements of the mixing matrix are.

Using first the old experimental data we predict the direction in which new more accurately measured matrix elements should move and then check if this is happening with the new experimental data.

Then we use new experimental data, repeat the procedure in look at what are the new results predicting.

For the quark masses at the energy scale of M_Z we take

$$\begin{aligned} \mathbf{M}^u_d/\mathrm{MeV}/c^2 &= (1.3 + 0.50 - 0.42, 619 \pm 84, 172\,000. \pm 760., \\ & \qquad m^{u_4} = 700\,000., 1\,200\,000.)\,, \\ \mathbf{M}^d_d/\mathrm{MeV}/c^2 &= (2.90 + 1.24 - 1.19, 55 + 16 - 15, 2\,900. \pm 90., \\ & \qquad m^{d_4} = 700\,000., 1\,200\,000.)\,. \end{aligned} \tag{3.13}$$

We found that the results are not influenced much if changing the masses within the experimental uncertainties.

Experimental values for leptons as well as the obtained mass matrices are presented in appendix 3.6.

Following the procedure explained in sect. 3.2 we look for the mass matrices for the u-quarks and the d-quarks by requiring that the mass matrices reproduce experimental data while manifesting symmetry of Eq. (3.1), predicted by the *spin-charge-family* theory.

We look for several properties of the obtained mass matrices:
i. We test the influence of the experimentally declared inaccuracy of the 3×3 sub matrices of the 4×4 mixing matrices and of the twice 3 measured masses on the prediction of the fourth family masses.
ii. We look for how do the old and the new matrix elements of the measured mixing matrix influence the accuracy with which the experimental data are reproduced in the procedure which takes into account the symmetry of mass matrices.
iii. We look for how different choices for the masses of the fourth family members limit the inaccuracy of particular matrix elements of the mixing matrices or the inaccuracy of the three lower masses of family members.
iv. We test how close to the democratic mass matrix are the obtained mass matrices in dependence of the fourth family masses.
v. We look for the predictions of the 4×4 mass matrices with the symmetry presented in Eq. (3.1).

The numerical procedure, used in this contribution, is designed for quarks and leptons. We present in this paper the results for quarks. The results for leptons, presented in appendix 3.6 is only to manifest the general properties of leptons, since the experimental data for leptons are far too non accurate to lead to trustable predictions.

In the next subsection 3.3.1 the numerical results are presented for the 4×4 mass matrices of the u-quarks and the d-quarks as they follow from the by the *spin-charge-family* theory required symmetry after fitting the experimental data.

Mass matrices for quarks In order to test whether or not our results have some experimental support, we use two kinds of the experimental values for the quark mixing matrix, presented in Eqs. (3.11, 3.12), respectively, for several values of the fourth family quark masses.

Searching for mass matrices with the symmetry of Eq. (3.1) to determine the interval for the fourth family quark masses in dependence of the values of the mixing matrix elements within the experimental inaccuracy, we repeat the numerical procedure for data with several values of masses of the fourth family quarks. Here we present results for two of them: for $m_{u_4} = 700$ GeV $= m_{d_4}$ and for $m_{u_4} = 1\,200$ GeV $= m_{d_4}$.

We present below the results for the two experimental matrix elements [16,15] for the quark mixing matrix, first for the data [16] and then for the data [15], in both cases first for $m_{u_4} = 700$ GeV $= m_{d_4}$ and then for $m_{u_4} = 1\,200$ GeV $= m_{d_4}$.

Having results from the fitting procedure when used the old experimental data for the quark mixing matrix, we look for the predictions, which the calculated 3×3 matrix elements of the 4×4 mixing matrix obeying the symmetry of Eq. (3.1) offer, and then check to which extend the predictions agree with new experimental data.

Then we repeat calculations with new experimental data.

- Results for the mass matrices of the two quarks family members, fitted to the mixing matrix elements presented in the ref. [16]. The fit offers the smallest common deviation (Eq. 3.10)) of the sum of all the average values of the nine matrix elements of the 3×3 sub matrix. The masses of quarks and the mixing matrix resulting from diagonalizing the two best fitted mass matrices are also presented.
  1. Here $m_{u_4} = 700$ GeV $m_{d_4} = 700$ GeV is chosen.

$$M^u = \begin{pmatrix} 227623. & 131877. & 132239. & 217653. \\ 131877. & 222116. & 217653. & 132239. \\ 132239. & 217653. & 214195. & 131877. \\ 217653. & 132239. & 131877. & 208687. \end{pmatrix},$$
$$M^d = \begin{pmatrix} 175797. & 174263. & 174288. & 175710. \\ 174263. & 175666. & 175710. & 174288. \\ 174288. & 175710. & 175813. & 174263. \\ 175710. & 174288. & 174263. & 175682. \end{pmatrix}, \qquad (3.14)$$

$$V_{ud} = \begin{pmatrix} -0.97423 & 0.22531 & -0.003 & 0.01021 \\ 0.22526 & 0.97338 & -0.042 & 0.0016 \\ -0.00663 & -0.04197 & -0.9991 & -0.0004 \\ 0.00959 & -0.00388 & -0.0003 & 0.99995 \end{pmatrix}. \qquad (3.15)$$

The corresponding absolute values for the deviations from the average experimental values (Eq.(3.10)) are

$$\delta V_{ud} = \begin{pmatrix} 0.091 & 0.117 & 2.339 \\ 0.431 & 1.418 & 1.348 \\ 2.951 & 0.358 & 1.559 \end{pmatrix}. \qquad (3.16)$$

The corresponding total absolute average deviation Eq. (3.10) is 4.5579.

The two mass matrices correspond to the diagonal masses

$$\mathbf{M}^u_d/\text{MeV}/c^2 = (1.3, 620.0, 172\,000., 700\,000.)\,,$$
$$\mathbf{M}^d_d/\text{MeV}/c^2 = (2.88508, 55.024, 2\,899.99, 700\,000.)\,. \qquad (3.17)$$

  2. In the next case $m_{u_4} = 1\,200$ GeV and $m_{d_4} = 1\,200$ GeV are chosen, again fitting the old [16] experimental for quark mixing matrix elements.

$$M^u = \begin{pmatrix} 351916. & 256894. & 257204. & 342714. \\ 256894. & 344411. & 342714. & 257204. \\ 257204. & 342714. & 341900. & 256894. \\ 342714. & 257204. & 256894. & 334395. \end{pmatrix},$$
$$M^d = \begin{pmatrix} 300783. & 299263. & 299288. & 300709. \\ 299263. & 300623. & 300709. & 299288. \\ 299288. & 300709. & 300856. & 299263. \\ 300709. & 299288. & 299263. & 300696. \end{pmatrix}, \qquad (3.18)$$

$$V_{ud} = \begin{pmatrix} -0.97425 & 0.22536 & -0.00301 & 0.00474 \\ 0.22534 & 0.97336 & -0.04239 & 0.00212 \\ -0.00663 & -0.04198 & -0.9991 & -0.00021 \\ 0.00414 & -0.00315 & -0.00011 & 0.99999 \end{pmatrix} . \quad (3.19)$$

The corresponding values for the deviations from the average experimental value of the matrix elements of the 3×3 sub matrix are

$$\delta V_{ud} = \begin{pmatrix} 0.003 & 0.226 & 2.335 \\ 0.424 & 1.419 & 1.357 \\ 2.949 & 0.355 & 1.559 \end{pmatrix} . \quad (3.20)$$

The corresponding total average deviation Eq. (3.10) is 4.5595.

The two mass matrices correspond to the diagonal masses

$$\begin{aligned} \mathbf{M}^{u}_{d}/\mathrm{MeV}/c^2 &= (1.3, 620.0, 172\,000., 700\,000.)\,, \\ \mathbf{M}^{d}_{d}/\mathrm{MeV}/c^2 &= (2.9, 55.0, 2\,900.0, 700\,000.)\,. \end{aligned} \quad (3.21)$$

Let us notice, that while the mass matrices of the u and the d quarks change for a factor of ≈ 1.5, becoming more "democratic" (that is the matrix elements become more and more equal), when changing the fourth family masses from 700 GeV to 1 200 GeV, the mixing matrix elements of the 3×3 sub matrix do not change a lot (Eqs.(3.15, 3.19)).
Let us now see what does our calculations say. We first make comparison for the old [16] (exp_o) mixing matrix with the calculated ones when the fourth family quark masses are 700 GeV, and 1 200 GeV. Results are presented in Eq. (3.22)

$$|V_{(ud)_{old}}| = \begin{pmatrix} \begin{array}{l|ccc} exp_o & 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 \\ \hline old_1 & 0.97423 & 0.22531 & 0.003 \\ old_2 & 0.97425 & 0.22536 & 0.00301 \\ \hline exp_o & 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\ \hline old_1 & 0.22526 & 0.97338 & 0.042 \\ old_2 & 0.22534 & 0.97336 & 0.04239 \\ \hline exp_o & 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \\ \hline old_1 & 0.00663 & 0.04197 & 0.9991 \\ old_2 & 0.00663 & 0.04198 & 0.9991 \end{array} \end{pmatrix} . \quad (3.22)$$

The calculated mixing predicts:
i. The matrix element $V_{u_1 d_1}$ should almost not change, $V_{u_1 d_2}$ may slightly rise, and ($V_{u_2 d_3}$ and $V_{u_3 d_3}$) will also rise.
ii. The matrix elements ($V_{u_1 d_3}$, $V_{u_2 d_1}$, $V_{u_2 d_2}$, $V_{u_3 d_1}$, $V_{u_3 d_2}$) should lower. Checking the new experimental values one sees that the prediction was in all the cases in agreement with those new experimental data which were done with better accuracy.

- let us repeat the calculations with new experimental data [15] to see how will the new data influence the mass matrices and the mixing matrix elements.

Results for the mass matrices of the two quarks family members, fitted to the new mixing matrix elements [15], which lead to the smallest common deviation for the sum of all the average values of the nine matrix elements of the 3×3 sub matrix, are presented, together with the masses of quarks and the mixing matrix resulting from diagonalizing the two mass matrices. Again the fourth quark masses are first ($m_{u_4} = 700$ GeV, $m_{d_4} = 700$ GeV) and then ($m_{u_4} = 1\,200$ GeV, $m_{d_4} = 1\,200$ GeV)

1. Here $m_{u_4} = 700$ GeV and $m_{d_4} = 700$ GeV is chosen.

$$M^u = \begin{pmatrix} 226521. & 131887. & 132192. & 217715. \\ 131887. & 219347. & 217715. & 132192. \\ 132192. & 217715. & 216964. & 131887. \\ 217715. & 132192. & 131887. & 209790. \end{pmatrix},$$
$$M^d = \begin{pmatrix} 175776. & 174263. & 174288. & 175709. \\ 174263. & 175622. & 175709. & 174288. \\ 174288. & 175709. & 175857. & 174263. \\ 175709. & 174288. & 174263. & 175703. \end{pmatrix}, \tag{3.23}$$

$$V_{ud} = \begin{pmatrix} -0.97423 & 0.22539 & -0.00299 & 0.00776 \\ 0.22534 & 0.97335 & -0.04245 & 0.00349 \\ -0.00667 & -0.04203 & -0.99909 & -0.00038 \\ 0.00677 & -0.00517 & -0.00020 & 0.99996 \end{pmatrix}. \tag{3.24}$$

The corresponding values Eq. (3.10) for the deviations from the average experimental values are

$$\delta V_{ud} = \begin{pmatrix} 0.074 & 0.109 & 2.339 \\ 0.043 & 0.791 & 1.032 \\ 2.291 & 0.753 & 0.685 \end{pmatrix}. \tag{3.25}$$

The corresponding total absolute average deviation Eq. (3.10) is 4.07154.

The two mass matrices correspond to the diagonal masses

$$\mathbf{M}^u_d/\text{MeV}/c^2 = (1.3, 620.0, 172\,000., 700\,000.)\,,$$
$$\mathbf{M}^d_d/\text{MeV}/c^2 = (2.9, 55.0, 2\,900.0, 700\,000.)\,. \tag{3.26}$$

2. Here $m_{u_4} = 1\,200$ GeV $m_{d_4} = 1\,200$ GeV is chosen.

$$M^u = \begin{pmatrix} 354761. & 256877. & 257353. & 342539. \\ 256877. & 350107. & 342539. & 257353. \\ 257353. & 342539. & 336204. & 256877. \\ 342539. & 257353. & 256877. & 331550. \end{pmatrix},$$
$$M^d = \begin{pmatrix} 300835. & 299263. & 299288. & 300710. \\ 299263. & 300714. & 300710. & 299288. \\ 299288. & 300710. & 300765. & 299263. \\ 300710. & 299288. & 299263. & 300644. \end{pmatrix}, \tag{3.27}$$

$$V_{ud} = \begin{pmatrix} 0.97423 & 0.22538 & 0.00299 & 0.00793 \\ -0.22531 & 0.97336 & 0.04248 & -0.00002 \\ 0.00667 & -0.04206 & 0.99909 & -0.00024 \\ -0.00773 & -0.00178 & 0.00022 & 0.99997 \end{pmatrix} . \tag{3.28}$$

The corresponding values for the deviations from the average experimental value for each matrix element are

$$\delta V_{ud} = \begin{pmatrix} 0.07 & 0.097 & 2.329 \\ 0.038 & 0.79 & 1.061 \\ 2.889 & 0.762 & 0.685 \end{pmatrix} . \tag{3.29}$$

The corresponding total average deviation Eq. (3.10) is 4.0724.

The two mass matrices correspond to the diagonal masses

$$\mathbf{M}^{u}_{d}/\mathrm{MeV}/c^2 = (1.3, 620.0, 172\,000., 1\,200\,000.)\,,$$
$$\mathbf{M}^{d}_{d}/\mathrm{MeV}/c^2 = (2.88508, 55.024, 2\,899.99, 1\,200\,000.)\,. \tag{3.30}$$

Again we notice that the mass matrices of the u and the d quarks change for a factor of $\approx$ 1.5 when the masses of the fourth family members grow from 700 GeV to 1 200 GeV. The mass matrices become more "democratic". The mixing matrix elements of the 3×3 sub matrix do not change a lot (Eqs.(3.24, 3.28)) with the masses of the fourth family quarks, but they do agree better with the newer [15] than with the older [16] experimental values.

Let us now compare the old [16] (exp_o) and the new [15] (exp_n) mixing matrix elements of the 3×3 sub matrix with the calculated ones for either the old [16] or for the new [15] experimental values, fitting them to the mass matrices of Eq. (3.1), in both cases for $m_{u_4} = m_{d_4} = 700$ GeV and for $m_{u_4} = m_{d_4} = 1\,200$ GeV, to see whether we can learn something out of this comparison.

We present below the old data (exp_o), the new data (exp_n) and both calculated values, each for $m_{u_4} = m_{d_4} = 700$ GeV (old_1, new_1) and $m_{u_4} = m_{d_4} = 1\,200$

GeV (old_2, new_2), putting together all these values in the same matrix.

$$|V_{(ud)}| = \begin{pmatrix} exp_o & 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 \\ exp_n & 0.97425 \pm 0.00022 & 0.2253 \pm 0.0008 & 0.00413 \pm 0.00049 \\ \hline old_1 & 0.97423 & 0.22531 & 0.003 \\ old_2 & 0.97425 & 0.22536 & 0.00301 \\ new_1 & 0.97423 & 0.22531 & 0.00299 \\ new_2 & 0.97423 & 0.22538 & 0.00299 \\ \hline exp_o & 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\ exp_n & 0.225 \pm 0.008 & 0.986 \pm 0.016 & 0.0411 \pm 0.0013 \\ \hline old_1 & 0.22526 & 0.97338 & 0.042 \\ old_2 & 0.22534 & 0.97336 & 0.04239 \\ new_1 & 0.22534 & 0.97335 & 0.04245 \\ new_2 & 0.22531 & 0.97336 & 0.04248 \\ \hline exp_o & 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \\ exp_n & 0.0084 \pm 0.0006 & 0.0400 \pm 0.0027 & 1.021 \pm 0.032 \\ \hline old_1 & 0.00663 & 0.04197 & 0.9991 \\ old_2 & 0.00663 & 0.04198 & 0.9991 \\ new_1 & 0.00667 & 0.04203 & 0.99909 \\ new_2 & 0.00667 & 0.04206 & 0.99909 \end{pmatrix} . \tag{3.31}$$

Comparing the above results and the results for mass matrices and 4×4 mixing matrices one finds:
i. The old and new experimental data differ mainly in the diagonal matrix elements.
ii. The old and new experimental data lead in the fitting procedure to quite similar 3×3 sub matrix, while their influence on the fourth family matrix elements are stronger.
iii. The fourth family masses change the mass matrices considerably, while their influence on the 3×3 sub matrix of the 4×4 mixing matrix is much weaker.
iv. The prediction (Eq. (3.22)) of the calculated mixing matrix elements, obtained by fitting the symmetry of the mass matrices (Eq. (3.1)) to the experimental data [16], was confirmed by improved experimental data [15]. In all cases are the calculated 3×3 matrix elements closer to the new experimental values than to the old experimental values.
v. Calculations with new experimental data predict: We expect (Eq. (3.31)) that more accurate experiments will bring a slightly smaller values for ($V_{u_1 d_1}$, $V_{u_1 d_3}$, $V_{u_3 d_3}$), smaller ($V_{u_2 d_2}$, $V_{u_3 d_1}$), ($V_{u_1 d_2}$, $V_{u_2 d_1}$) will slightly grow and ($V_{u_2 d_3}$) $V_{u_3 d_2}$ will grow.
vi. The matrix elements $V_{u_i d_4}$ and $V_{u_4 d_i}$ change considerably with the mass of the fourth family members, and they differ quite a lot also when using new instead of the old experimental data for the mixing matrix.
vii. Fitting the free parameters of the mass matrices to the new experimental data [15] gives smaller parameter σ (Eq. sigma) than when fitting old experimental data [16]: 4.07154 with respect to 4.5579 for the masses 700 GeV and 4.0724 with respect to 4.5595 for the masses 1 200 GeV, while with the mass σ does not really

change. Only very accurate mixing matrix elements would allow to determine fourth family quarks masses more accurately. Since the choice of the fourth family quark masses does not appreciable influence either the fitting procedure or the obtained 3×3 mixing matrix, and also not the accuracy of the masses of the three lower families, it is difficult to predict the interval for the masses of the fourth family members. For the masses of the fourth family quarks to be close or above 1 TeV speak more other experimental data, like decays of mesons.

An estimation how trustable is the numerical procedure, used to fit free parameters of the quarks mass matrices to the experimental data, can be made by comparing the results for the mixing matrix for several choices of the fourth family masses. The fitting procedure shows up that the 3×3 mixing matrix does not change appreciable, even not for much lower masses from 300 GeV up.

We can conclude: Requiring that the experimental data respect the symmetry of the mass matrices (Eq. (3.1)) (suggested by the *spin-charge-family* theory) the prediction can be made for the change of the matrix elements of the 3×3 sub matrix in future experiments. The masses of the fourth family members are more difficult to predict, since the accuracy of the experimental data for the quark masses and in particular for the mixing matrix should be extremely high to really limit the fourth family masses. For a known fourth family masses the fourth family matrix elements of the mixing matrix are accurate. For masses of the fourth family quarks to be close or above 1 TeV speak more other experimental data, like decays of mesons.

3.4 Discussions and conclusions

One of the most important open questions in the elementary particle physics is: Where do the family originate? Explaining the origin of families would answer the question about the number of families possibly observable at the low energy regime, about the origin of the scalar field(s) and Yukawa couplings and would also explain differences in the fermions properties - the differences in masses and mixing matrices among family members – quarks and leptons.

Assuming that the prediction of the *spin-charge-family* theory that there are four rather than so far observed three coupled families, the mass matrices of which demonstrate in the massless basis the $SU(2) \times SU(2)$ (each of two $SU(2)$ is a subgroup of its own $SO(4)$) symmetry of Eq. (3.1), the same for all the family members - the quarks and the leptons - and simplifying the numerical procedure by the assumption that the mass matrices are symmetric and real and correspondingly the mixing matrices orthogonal, we fit the free parameters of the quarks mass matrices (6 for u-quarks and 6 for d-quarks to twice three masses of quarks and to the mixing matrix 4×4, extracted from the 3×3 sub matrix elements, fitted to 6 parameters of the orthogonal matrix) to the experimental data. Every unitary $n \times n$ matrix is for $n \geq 4$, through the unitary conditions, uniquely determined by the 3×3 sub matrix.

The numerical procedure, explained in this paper, to fit free parameters to the experimental data within the experimental inaccuracy of masses and in particular of the mixing matrix is very tough. The accurate mixing matrix elements and masses would completely determine the fourth family masses. The experimental inaccuracies are too large to tell the trustable mass interval, within which the fourth family masses of quarks lie.

In this paper we are not yet able to tell the mass intervals for the fourth family quarks. But since the matrix elements of the 3×3 sub matrix depend very weakly on the fourth family masses, the calculated matrix (from the experimental data under the assumption that the mass matrices manifest the symmetry of Eq. (3.1)) offer the prediction to what values will more accurate measurements move the present experimental data. We checked this prediction by performing calculations with the old matrix elements [16] and then test the prediction on the new ones [15]. The results are presented in Eq. (3.22). Repeating calculations with the new matrix elements for several masses of the fourth family quarks we predict further change of the 3×3 sub matrix elements, presented in Eq. (3.31).

We expect: More accurate experiments will bring a slightly smaller values for (V_{ud}, V_{ub}, V_{tb}), smaller (V_{cs}, V_{td}), (V_{us}, V_{cd}) will slightly grow and (V_{cb}) V_{ts} will grow.

The fourth family mixing matrix elements depend, as expected, strongly on the fourth family masses. For chosen masses of the fourth family members their matrix elements can be quite accurately predicted (Eqs. (3.24, 3.28)).

Mass matrices are quite close to the "democratic" ones not only for leptons but also for quarks. With the growing fourth family masses the "democracy" in matrix elements grow (Eqs. (3.23, 3.23)), as expected.

Although we have not study complex mass matrices, we do not expect that the presented results would change considerably after taking into account the complex phases of mass matrices and correspondingly also of the mixing matrices. We estimate the accuracy of our calculations by comparing the results of the calculated 3×3 matrix elements for the interval of the fourth family masses, from 300 GeV to 1 200 GeV. It look very trustable, offering for all these masses only slowly changing matrix elements.

We are concluding: Requiring that the experimental data respect the symmetry of the mass matrices (Eq. (3.1)) (suggested by the *spin-charge-family* theory) the prediction is made for the change of the matrix elements of the 3×3 sub matrix in future more accurate experiments. More (much more) accurate measured 3×3 sub matrix elements in future will determine, following the spin-charge-family theory, the fourth family masses and the fourth family matrix elements. However, even with the present experimental data our calculations, respecting the symmetry of the mass matrices (Eq. (3.1)) offer the prediction for the direction to which will more accurately measured matrix elements move.

Since the symmetry of the mass matrices are determined in the *spin-charge-family* theory by two triplet (with respect to the family charges) and tree singlet (with respect to the family members charges (Q, Q', Y)) scalar fields [13,14], all with the weak and the hyper charges as assumed in the *standard model* for the scalar fields, we hope to learn from the properties of the mass matrices and the corresponding mixing matrices more about these scalar fields.

3.5 APPENDIX: A brief presentation of the *spin-charge-family* theory

We present in this section a very brief introduction into the *spin-charge family* theory [1–14]. The reader can skip this appendix taking by the *spin-charge family* theory required symmetry of mass matrices of Eq. (3.1) as an input to the study of properties of the 4×4 mass matrices - with the parameters which depend on charges of the family members - and can come to this part of the paper, if and when would like to learn where do families and scalar fields possibly originate from.

Let us start by directing attention of the reader to one of the most open questions in the elementary particle physics and cosmology: Why do we have families, where do they originate and correspondingly where do scalar fields, manifesting as Higgs and Yukawa couplings, originate? The *spin-charge-family* theory is offering a possible explanation for the origin of families and scalar fields, and in addition for the so far observed charges and the corresponding gauge fields.

There are, namely, two (only two) kinds of the Clifford algebra objects: One kind, the Dirac γ^a, takes care of the spin in $d = (3+1)$, while the spin in $d \geq 4$ (rather than the total angular momentum) manifests in $d = (3+1)$ in the low energy regime as the charges. In this part the *spin-charge family* theory is like the Kaluza-Klein theory, unifying spin (in the low energy regime, otherwise the total angular momentum) and charges, and offering a possible answer to the question about the origin of the so far observed charges and correspondingly also about the so far observed gauge fields. The second kind of the Clifford algebra objects, forming the equivalent representations with respect to the Dirac kind, recognized by one of the authors (SNMB), is responsible for the appearance of families of fermions.

There are correspondingly also two kinds of gauge fields, which appear to manifest in $d = (3+1)$ as the so far observed vector gauge fields (the number of - obviously non yet observed - gauge fields grows with the dimension) and as the scalar gauge fields. The scalar fields are responsible, after gaining nonzero vacuum expectation values, for the appearance of masses of fermions and gauge bosons. They manifest as the so far observed Higgs [36] and the Yukawa couplings.

All the properties of fermions and bosons in the low energy regime originate in the *spin-charge-family* theory in a simple starting action for massless fields in $d = [1 + (d-1)]$. Fermions interact with the vielbeins $f^{\alpha}{}_{a}$ and correspondingly with the two kinds of the spin connection fields: with $\omega_{abc} = f^{\alpha}{}_{c}\, \omega_{ab\alpha}$ which are the gauge fields of $S^{ab} = \frac{i}{4}\,(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and with $\tilde{\omega}_{abc} = f^{\alpha}{}_{c}\, \tilde{\omega}_{ab\alpha}$ which are the gauge fields of $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$. $\alpha, \beta, \dots$ is the Einstein index and $a, b, \dots$ is the flat index. The starting action is the simplest one

$$S = \int d^d x\, E\, \mathcal{L}_f + \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \qquad \mathcal{L}_f = \frac{1}{2}(\bar{\psi}\, \gamma^a p_{0a} \psi) + h.c.$$

$$p_{0a} = f^{\alpha}{}_{a}\, p_{0\alpha} + \frac{1}{2E}\, \{p_{\alpha}, E f^{\alpha}{}_{a}\}_{-}\,,\ p_{0\alpha} = p_{\alpha} - \frac{1}{2} S^{ab}\, \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab}\, \tilde{\omega}_{ab\alpha}, \tag{3.32}$$

$$R = \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} \;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta})\} + h.c. \,,$$
$$\tilde{R} = \frac{1}{2} \,f^{\alpha[a} f^{\beta b]} \;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^{c}{}_{b\beta}) + h.c.\,. \tag{3.33}$$

$E = \det(e^{a}{}_{\alpha})$ and $e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$. Fermions, coupled to the vielbeins and the two kinds of the spin connection fields, *manifest* (after several breaks of the starting symmetries) *before the electroweak break four massless families of quarks and leptons*, the left handed fermions are weak charged and the right handed ones are weak chargeless. The vielbeins and the two kinds of the spin connection fields manifest effectively as the observed gauge fields and (those with the scalar indices in $d = (1+3)$) as several scalar fields. The mass matrices of the four family members (quarks and leptons) are after the electroweak break expressible on a tree level by the vacuum expectation values of the two kinds of the spin connection fields and the corresponding vielbeins with the scalar indices ([1,2,6,7,12,13]):
i. One kind originates in the scalar fields $\tilde{\omega}_{abc}$, manifesting as the two $SU(2)$ triplets – $\tilde{A}^{\tilde{N}_L\, i}_{s}, i = (1,2,3)\,, s = (7,8)$; $\tilde{A}^{\tilde{1}\, i}_{s}, i = (1,2,3)\,, s = (7,8)$; – and one singlet – $\tilde{A}^{\tilde{4}}_{s}\,, s = (7,8)$ – contributing equally to all the family members.
ii. The second kind originates in the scalar fields ω_{abc}, manifesting as three singlets – $A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}\,, s = (7,8)$ – contributing the same values to all the families and distinguishing among family members. Q and Q' are the quantum numbers from the *standard model*, Y' originates in the second $SU(2)$ (a kind of a right handed "weak") charge.

All the scalar fields manifest, transforming the right handed quarks and leptons into the corresponding left handed ones [4] and contributing also to the masses of the weak bosons, as doublets with respect to the weak charge. Loop corrections, to which all the scalar and also gauge vector fields contribute coherently, change contributions of the off-diagonal and diagonal elements appearing on the tree level, keeping the tree level symmetry of mass matrices unchanged [5].

3.5.1 Mass matrices on the tree level and beyond which manifest $SU(2) \times SU(2)$ symmetry

Let us make a choice of a massless basis ψ_i, $i = (1,2,3,4)$, for a particular family memeber α. And let us take into account the two kinds of the operators, which transform the basis vectors into one another

$$\tilde{N}^{i}_{L}\,,\; i = (1,2,3)\,, \qquad \tilde{\tau}^{i}_{L}\,,\; i = (1,2,3)\,, \tag{3.34}$$

---

[4] It is the term $\gamma^0 \gamma^s\, \phi^{Ai}_{s}$, where ϕ^{Ai}_{s}, with $s = (7,8)$ denotes any of the scalar fields, which transforms the right handed fermions into the corresponding left handed partner [1,7,2,6,12–14]. This mass term originates in $\bar{\psi}\, \gamma^{a} p_{0a} \psi$ of the action Eq.(3.32), with $a = s = (7,8)$ and $p_{0s} = f^{\sigma}_{s}\, (p_{\sigma} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\sigma} - \frac{1}{2} S^{st} \omega_{st\sigma})$.

[5] It can be seen that all the loop corrections keep the starting symmetry of the mass matrices unchanged. We have also started [7,42] with the evaluation of the loop corrections to the tree level values. This estimation has been done so far [42] only up to the first order and partly to the second order.

with the properties

$$\begin{aligned}
\tilde{N}^3_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,\psi_2,-\psi_3,\psi_4)\,,\\
\tilde{N}^+_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\psi_2,\;\;0,\psi_4,\;\;0)\,,\\
\tilde{N}^-_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (0\;\;,\psi_1,\;\;0,\psi_3)\,,\\
\tilde{\tau}^3\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,-\psi_2,\psi_3,\psi_4)\,,\\
\tilde{\tau}^+\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\psi_3,\psi_4,\;\;0,\;\;0)\,,\\
\tilde{\tau}^-\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\;\;0,\;\;0,\psi_1,\psi_2)\,.
\end{aligned} \tag{3.35}$$

This is indeed what the two $SU(2)$ operators in the *spin-charge-family* theory do. The gauge scalar fields of these operators determine, together with the corresponding coupling constants, the off diagonal and diagonal matrix elements on the tree level. In addition to these two kinds of $SU(2)$ scalars there are three $U(1)$ scalars, which distinguish among the family members, contributing on the tree level the same diagonal matrix elements for all the families. In loop corrections in all orders the symmetry of mass matrices remains unchanged, while the three $U(1)$ scalars, contributing coherently with the two kinds of $SU(2)$ scalars and all the massive fields to all the matrix elements, manifest in off diagonal elements as well. All the scalars are doublets with respect to the weak charge, contributing to the weak and the hypercharge of the fermions so that they transform the right handed members into the left handed onces.

With the above (Eq. (3.35) presented choices of phases of the left and the right handed basic states in the massless basis the mass matrices of all the family members manifest the symmetry, presented in Eq. (3.1). One easily checks that a change of the phases of the left and the right handed members, there are $(2n-1)$ possibilities, causes changes in phases of matrix elements in Eq. (3.1).

3.6 APPENDIX: Mass matrices for leptons

We evaluate 3×3 matrix elements from the data [16]

$$\begin{aligned}
&7.05\cdot 10^{-17} \le \Delta(\mathbf{m}_{21}/\mathrm{MeV}/c^2)^2 \le 8.34\cdot 10^{-17}\,,\\
&2.07\cdot 10^{-15} \le \Delta(\mathbf{m}_{(31),(32)}/\mathrm{MeV}/c^2)^2 \le 2.75\cdot 10^{-15}\,,\\
&0.25 \le \sin^2\theta_{12} \le 0.37\,,\quad 0.36 \le \sin^2\theta_{23} \le 0.67\,,\\
&\sin^2\theta_{13} < 0.035(0.056)\,,\quad \sin^2 2\theta_{13} = 0.098\pm 0.013\,,
\end{aligned} \tag{3.36}$$

which means that $\frac{\pi}{4}-\frac{\pi}{10}\le\theta_{23}\le\frac{\pi}{4}+\frac{\pi}{10}$, $\frac{\pi}{5.4}-\frac{\pi}{10}\le\theta_{12}\le\frac{\pi}{4}+\frac{\pi}{10}$, $\theta_{13}<\frac{\pi}{13}$. This reflects in the lepton mixing matrix $V_{\nu e} = S^{\nu}\, S^{e\,\dagger}$

$$|V_{\nu e}| = \begin{pmatrix} 0.8224 & 0.5200 & 0.1552 & |V_{\nu_1 e_4}|\\ 0.3249 & 0.7239 & 0.6014 & |V_{\nu_2 e_4}|\\ 0.4455 & 0.4498 & 0.7704 & |V_{\nu_3 e_4}|\\ |V_{\nu_4 e_1}| & |V_{\nu_4 e_2}| & |V_{\nu_4 e_3}| & |V_{\nu_4 e_4}| \end{pmatrix}\,, \tag{3.37}$$

determining for each assumed value for any mixing matrix element within the experimentally allowed inaccuracy the corresponding fourth family mixing matrix elements ($|V_{\nu_i e_4}|$ and $|V_{\nu_4 e_j}|$) from the unitarity condition for the 4×4 mixing matrix. The masses of leptons are taken from [15,16] while we take the fourth family masses as free parameters, checking how much does the experimental inaccuracy influence a possible prediction for the fourth family leptons masses and how does this prediction agree with experimentally allowed values [15,16,43] for the fourth family lepton masses.

$$\begin{aligned}
\mathbf{M}^{\nu}_{d}/\mathrm{MeV}/c^2 &= (1\cdot 10^{-9},\, 9\cdot 10^{-9},\, 5\cdot 10^{-8},\, m^{\nu_4} > 90\,000.)\,,\\
\mathbf{M}^{e}_{d}/\mathrm{MeV}/c^2 &= (0.486\,570\,161 \pm 0.000\,000\,042,\\
&\ 102.718\,135\,9 \pm 0.000\,009\,2,\ 1746.24 \pm 0.20, m^{e_4} > 102\,000\,)\,. \qquad (3.38)
\end{aligned}$$

3.6.1 Numerical results for leptons

We present here the old results [11] for leptons, manifesting properties of the lepton mass matrices. These results are less informative than those for quarks, since the experimental results are for leptons mixing matrix much less accurate than in the case of quarks and also masses are known less accurately.

We make a choice of the fourth family masses and take the mixing matrix elements from the old experimental data [16]

We have

•

$$\begin{aligned}
M^{\nu} &= \begin{pmatrix} 14\,021. & 14\,968. & 14\,968. & -14\,021.\\ 14\,968. & 15\,979. & 15\,979. & -14\,968.\\ 14\,968. & 15\,979. & 15\,979 & -14\,968.\\ -14\,021. & -14\,968. & -14\,968. & 14\,021. \end{pmatrix},\\
M^{e} &= \begin{pmatrix} 28\,933. & 30\,057. & 29\,762. & -27\,207.\\ 30\,057. & 32\,009. & 31\,958. & -29\,762.\\ 29\,762. & 31\,958. & 32\,009. & -30\,057.\\ -27\,207. & -29\,762. & -30\,057. & 28\,933. \end{pmatrix},
\end{aligned} \qquad (3.39)$$

which leads to the mixing matrix $V_{\nu e}$

$$V_{\nu e 1} = \begin{pmatrix} 0.82363 & 0.54671 & -0.15082 & 0.\\ -0.50263 & 0.58049 & -0.64062 & 0.\\ -0.26268 & 0.60344 & 0.75290 & 0.\\ 0. & 0. & 0. & 1. \end{pmatrix}, \qquad (3.40)$$

and the masses

$$\begin{aligned}
\mathbf{M}^{\nu}_{d}/\mathrm{MeV}/c^2 &= (5\cdot 10^{-9}\,, 1\cdot 10^{-8}\,, 4.9\cdot 10^{-8}\,, 60\,000.)\,,\\
\mathbf{M}^{e}_{d}/\mathrm{MeV}/c^2 &= (0.510999\,, 105.658\,, 1\,776.82\,, 120\,000)\,. \qquad (3.41)
\end{aligned}$$

We did not adapt lepton masses to Z_m mass scale. Zeros (0.) for the matrix elements concerning the fourth family members means that the values are less than 10^{-5} and 1. means that the difference from 1 occurs on the sixth digit at most.

We notice:
i. The required symmetry, Eq. (3.1), is kept exactly.
ii. The mass matrices of leptons are very close to the "democratic" matrix.
iii. The mixing matrix elements among the first three and the fourth family members are very small, what is due to our choice, since the matrix elements of the 3×3 sub matrix of the 4×4 unitary matrix, predicted by the *spin-charge-family* theory are very inaccurately known.

3.7 APPENDIX: Properties of non Hermitian mass matrices

This pedagogic presentation of well known properties of non Hermitian matrices can be found in many textbooks, for example [44]. We repeat this topic here only to make our discussions transparent.

Let us take a non Hermitian mass matrix M^α as it follows from the *spin-charge-family* theory, α denotes a family member (index $_\pm$ used in the main text is dropped).

We always can diagonalize a non Hermitian M^α with two unitary matrices, S^α ($S^{\alpha\,\dagger}\, S^\alpha = I$) and T^α ($T^{\alpha\,\dagger}\, T^\alpha = I$)

$$S^{\alpha\,\dagger}\, M^\alpha\, T^\alpha = \mathbf{M}^\alpha_d = (m^\alpha_1 \dots m^\alpha_i \dots m^\alpha_n). \tag{3.42}$$

The proof is added below.

Changing phases of the basic states, those of the left handed one and those of the right handed one, the new unitary matrices $S'^\alpha = S^\alpha\, F_{\alpha S}$ and $T'^\alpha = T^\alpha\, F_{\alpha T}$ change the phase of the elements of diagonalized mass matrices $\mathbf{M}^\alpha_d$

$$\begin{aligned} S'^{\alpha\,\dagger}\, M^\alpha\, T'^\alpha &= F^\dagger_{\alpha S}\, \mathbf{M}^\alpha_d\, F_{\alpha T} = \\ &\quad diag(m^\alpha_1 e^{i(\phi^{\alpha S}_1 - \phi^{\alpha T}_1)} \dots m^\alpha_i\, e^{i(\phi^{\alpha S}_i - \phi^{\alpha T}_i)}, \dots m^\alpha_n\, e^{i(\phi^{\alpha S}_n - \phi^{\alpha T}_n)}), \\ F_{\alpha S} &= diag(e^{-i\phi^{\alpha S}_1}, \dots, e^{-i\phi^{\alpha S}_i}, \dots, e^{-i\phi^{\alpha S}_n}), \\ F_{\alpha T} &= diag(e^{-i\phi^{\alpha T}_1}, \dots, e^{-i\phi^{\alpha T}_i}, \dots, e^{-i\phi^{\alpha T}_n}). \end{aligned} \tag{3.43}$$

In the case that the mass matrix is Hermitian T^α can be replaced by S^α, but only up to phases originating in the phases of the two basis, the left handed one and the right handed one, since they remain independent.

One can diagonalize the non Hermitian mass matrices in two ways, that is either one diagonalizes $M^\alpha M^{\alpha\,\dagger}$ or $M^{\alpha\dagger} M^\alpha$

$$\begin{aligned} (S^{\alpha\dagger} M^\alpha T^\alpha)(S^{\alpha\dagger} M^\alpha T^\alpha)^\dagger &= S^{\alpha\dagger} M^\alpha M^{\alpha\,\dagger} S^\alpha = \mathbf{M}^{\alpha 2}_{dS}, \\ (S^{\alpha\dagger} M^\alpha T^\alpha)^\dagger (S^{\alpha\dagger} M^\alpha T^\alpha) &= T^{\alpha\dagger} M^{\alpha\,\dagger} M^\alpha T^\alpha = \mathbf{M}^{\alpha 2}_{dT}, \\ \mathbf{M}^{\alpha\,\dagger}_{dS} = \mathbf{M}^\alpha_{dS}, &\quad \mathbf{M}^{\alpha\,\dagger}_{dT} = \mathbf{M}^\alpha_{dT}. \end{aligned} \tag{3.44}$$

One can prove that $\mathbf{M}^\alpha_{dS} = \mathbf{M}^\alpha_{dT}$. The proof proceeds as follows. Let us define two Hermitian (H^α_S, H^α_T) and two unitary matrices (U^α_S, H^α_T)

$$\begin{aligned} H^\alpha_S &= S^\alpha \mathbf{M}^\alpha_{dS} S^{\alpha\,\dagger}, \qquad H^\alpha_T = T^\alpha \mathbf{M}^{\alpha\dagger}_{dT} T^{\alpha\,\dagger}, \\ U^\alpha_S &= H^{\alpha -1}_S M^\alpha, \qquad U^\alpha_T = H^{\alpha -1}_T M^{\alpha\,\dagger}, \end{aligned} \tag{3.45}$$

It is easy to show that $H_S^{\alpha\,\dagger} = H_S^{\alpha}$, $H_T^{\alpha\,\dagger} = H_T^{\alpha}$, $U_S^{\alpha}\, U_S^{\alpha\,\dagger} = I$ and $U_T^{\alpha}\, U_T^{\alpha\,\dagger} = I$. Then it follows

$$S^{\alpha\dagger}\, H_S^{\alpha}\, S^{\alpha} = \mathbf{M}_{dS}^{\alpha} = \mathbf{M}_{dS}^{\alpha\,\dagger} = S^{\alpha\dagger}\, M^{\alpha}\, U_S^{\alpha\,-1}\, S^{\alpha} = S^{\alpha\dagger}\, M^{\alpha}\, T^{\alpha}\,,$$
$$T^{\alpha\dagger}\, H_T^{\alpha}\, T^{\alpha} = \mathbf{M}_{dT}^{\alpha} = \mathbf{M}_{dT}^{\alpha\,\dagger} = T^{\alpha\dagger}\, M^{\alpha\,\dagger}\, U_T^{\alpha\,-1}\, T^{\alpha} = T^{\alpha\dagger}\, M^{\alpha\dagger}\, S^{\alpha}\,, \quad (3.46)$$

where we recognized $U_S^{\alpha\,-1}\, S^{\alpha} = T^{\alpha}$ and $U_T^{\alpha\,-1}\, T^{\alpha} = S^{\alpha}$. Taking into account Eq. (3.43) the starting basis can be chosen so, that all diagonal masses are real and positive.

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4 Emergent SUSY Theories: QED, SM & GUT

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Abstract. It might be expected that only global symmetries are fundamental symmetries of Nature, whereas local symmetries and associated massless gauge fields could solely emerge due to spontaneous breaking of underlying spacetime symmetries involved, such as relativistic invariance and supersymmetry. This breaking, taken in the form of the nonlinear σ-model type pattern for vector fields or superfields, puts essential restrictions on geometrical degrees of freedom of a physical field system that makes it to adjust itself in such a way that its global internal symmetry G turns into the local symmetry G_{loc}. Remarkably, this emergence process may naturally be triggered by spontaneously broken supersymmetry, as is illustrated in detail by an example of a general supersymmetric QED model which is then extended to electroweak models and grand unified theories. Among others, the $U(1) \times SU(2)$ symmetrical Standard Model and flipped $SU(5)$ GUT appear preferable to emerge at high energies.

Povzetek. Avtor dokazuje, da so samo globalne simetrije temeljne simetrije narave, lokalne simetrije in njim pridružena umeritvena polja pa se pojavljajo samo zaradi spontane zlomitve simetrije prostora-časa, kot sta relativistična invarianca in supersimetrija. Avtor uporabi nelinearni model σ za obravnavo sistema z relativistično simetrijo in supersimetrijo. Ugotavlja, da zlomitev relativistične invariance ter supersimetrije bistveno zoži geometrijske prostostne stopnje vektorskih in superpolja, zato se ta odzove s spremembo globalne notranje simetrije G v lokalno simetrijo G_{loc}. Spontano zlomljena supersimetrija lahko naravno sproži ta proces porajanja. Avtor ta spontani prehod ilustrira na primeru splošnega modela supersimetrične kvantne elektordinamike. Ta model razširi tudi na elektrošibki *standardni model* ter na teorije, imenovane "velike teorije poenotenja" (GUT). Pri visokih energijah se pri takih zlomitvah simetrij pojavita prav *standardni model* in „prekucnjen" (flipped) model $SU(5)$.

4.1 Introduction

We all believe that internal gauge symmetries form the basis of modern particle physics being most successfully realized within the celebrated Standard Model (SM) of quarks and leptons and their fundamental strong, weak and electromagnetic interactions. At the same time, local gauge invariance, contrary to a global symmetry case, may look like a cumbersome geometrical input rather than a "true" physical principle, especially in the framework of an effective quantum field theory (QFT) becoming, presumably, irrelevant at very high energies. In this connection,

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one could wonder whether there is any basic dynamical reason that necessitates gauge invariance and the associated masslessness of gauge fields as some emergent phenomenon arising from a more profound level of dynamics. By analogy with a dynamical origin of massless scalar particle excitations, which is very well understood in terms of spontaneously broken global internal symmetries [1], one could think that the origin of massless gauge fields as vector Nambu-Goldstone (NG) bosons are related to the spontaneous Lorentz invariance violation (SLIV) that is the minimal spacetime global symmetry underlying particle physics. This well-known approach providing a viable alternative to quantum electrodynamics [2], gravity [3] and Yang-Mills theories [4] has a long history started over fifty years ago, though has been significantly revised in the recent years [5–8].

4.1.1 An emergence conjecture

Directly or indirectly, the approach mentioned includes several key points which in a conventional QFT framework may be formulated nowadays in the following way (see [9] and comprehensive references therein):

- Only global symmetries are fundamental symmetries of Nature. Local symmetries and associated massless gauge vector (tensor) fields could only emerge due to some phase transition producing them as appropriate Nambu-Goldstone modes,
- The underlying Lorentz invariance is proposed to be spontaneously broken since only spacetime symmetry breaking could basically provide an existence of vector (tensor) emerging modes which mediate all interactions involved,
- The theory itself is proposed to be "physically" viable in the sense that any appropriate initial value condition (IVC), which determines the subsequent dynamical evolution of a physical field system, is uniquely satisfied. This means in turn that an interacting field system can not be superfluously restricted in the number of physical degrees of freedom in order to remain physical,
- Together, they naturally lead to the **gauge symmetry emergence** (GSE) conjecture which I will follow throughout the paper: *Let there be given an interacting field system containing some vector field (or vector field multiplet)* A_μ *together with fermion (*ψ*), scalar (*ϕ*) and other matter fields in an arbitrary relativistically invariant Lagrangian* $L(A_\mu, \psi, \phi, ...)$ *which possesses only global Abelian or non-Abelian internal symmetry* G*. Suppose that an underlying relativistic invariance of this field system is spontaneously broken in terms of the "length-fixing" covariant constraint put on vector fields,*
$$A_\mu A^\mu = n^2 M^2 \tag{4.1}$$
(where M *stands for the proposed SLIV scale, while* n_μ *is a properly-oriented unit Lorentz vector,* $n^2 = n_\mu n^\mu = \pm 1$*). If this constraint is preserved under the time development given by the field equations of motion, then in order to be protected from further reduction in degrees of freedom this system will modify its global symmetry* G *into a local symmetry* G_{loc}*, that will in turn convert the vector field constraint itself into a gauge condition thus virtually resulting in gauge invariant and Lorentz invariant theory.*

To see how technically a global internal symmetry may be converted into a local one, let us consider in some detail the question of consistency of a possible constraint for a general 4-vector field A_μ with its equation of motion in an Abelian symmetry case, $G = U(1)$. In the presence of the SLIV constraint $C(A) = A_\mu A^\mu - n^2 M^2 = 0$ (4.1), it follows that the equations of motion can no longer be independent. The important point is that, in general, the time development would not preserve the constraint. So the parameters in the starting Lagrangian have to be adjusted in such a way that effectively we have one less equation of motion for the vector field A_μ not to be superfluously restricted. This means that there should be some relationship given by a functional equation $F(C = 0;\ E_A, E_\psi, ...) = 0$ between all the vector and matter field Eulerians involved[1] which are individually satisfied on the mass shell. According to Noether's second theorem [10] such a relationship gives rise to an emergence of local symmetry for the field system considered provided that the functional F satisfies the same symmetry requirements of Lorentz and translational invariance, as well as all the global internal symmetry requirements, as the general starting Lagrangian does.

In this way, the nonlinear SLIV condition (4.1), due to which true vacuum in the theory is chosen and massless gauge fields are generated, may provide a dynamical setting for all underlying internal symmetries involved through the GSE conjecture [9]. One might think that the length-fixing vector field constraint (4.1) itself first introduced by Nambu in a conventional QED framework [11] (for some extensions and generalizations, see also [12–17]) does not especially stand out in the present context. Actually, it seems that the GSE conjecture might be equally formulated for any type of covariant constraint, say for the spin-1 vector field condition, $\partial_\mu A^\mu = 0$ [18]. However, as is generally argued in [9], the SLIV constraint (4.1) appears to be the only one whose application leads to a full conversion of an internal global symmetry G into a local symmetry G_{loc} that forces a given field system to remain always physical. Other constraints could only lead to partial gauge invariance being broken by some terms in an emerging theory.

Based upon the SLIV constraint (4.1), the starting vector field A_μ may be expanded around the true vacuum configuration in the theory,

$$A_\mu = a_\mu + n_\mu \sqrt{M^2 - n^2 a^2}\ ,\ \ n_\mu a_\mu = 0\ \ (a^2 \equiv a_\mu a^\mu)\ , \tag{4.2}$$

which means that it develops the vacuum expectation value (VEV) $\langle A_\mu \rangle = n_\mu M$. Meanwhile, its a_μ components which are orthogonal to the Lorentz violating direction n_μ describe a massless vector NG boson being an eventual gauge field (photon) candidate.

4.1.2 Gauge invariance *versus* spontaneous Lorentz violation

One can see that the gauge theory framework, be it taken from the outset or emerged, makes in turn spontaneous Lorentz violation to be physically unobserv-

[1] The field Eulerians (E_A, E_ψ, ...) are determined, as usual, $(E_A)^\mu \equiv \partial L/\partial A_\mu - \partial_\nu[\partial L/\partial(\partial_\nu A_\mu)]$, and so forth.

able both in Abelian and non-Abelian symmetry case. In substance, the essential part of the SLIV pattern (4.2), due to which the vector field $A_\mu(x)$ develops the VEV M, may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = n_\mu x^\mu M$. This is what one could refer to as the generic non-observability of SLIV in gauge invariant theories. I shall call it the "inactive" SLIV in contrast to the "active" SLIV case where physical Lorentz invariance could effectively occur. From the present standpoint, the only way for an active SLIV to occur would be if emergent gauge symmetries presented above were slightly broken at small distances. This could inevitably happen, for example, in a partially gauge invariant theory which might appear if the considered field system could become "a little unphysical" at distances being presumably controlled by quantum gravity [19]. One may think that quantum gravity could in principle hinder the setting of the required IVC in the appropriate Cauchy problem (thus admitting a superfluous restriction of vector fields) due to the occurrence of some gauge-noninvariant high-order operators near the Planck scale. As a consequence, through special dispersion relations appearing for matter and gauge fields, one is led a new class of phenomena which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the Greizen-Zatsepin-Kouzmin cutoff for ultra-high energy cosmic-ray nucleons, stability of high-energy pions and *W* bosons, modification of nucleon beta decays, and some others just in the presently accessible energy area in cosmic ray physics [19] (for many phenomenological aspects, see pioneering works [20,21]).

4.1.3 SUSY profile of emergent theories

The role of Lorentz invariance may change, and its spontaneous violation may not be the only reason why massless photons and other gauge fields could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest is related to supersymmetry which has made a serious impact on particle physics in the last decades (though has not been yet discovered). Actually, as we will see, the situation is changed dramatically in the SUSY inspired emergent gauge theories. In sharp contrast to non-SUSY analogs, it appears that the spontaneous Lorentz violation caused by an arbitrary potential of vector superfield $V(x, \theta, \overline{\theta})$ never goes any further than some nonlinear gauge condition put on its vector field component $A_\mu(x)$ associated with a photon or any other gauge field. Remarkably, this condition coincides, as we shall see below, with the SLIV constraint (4.1) given above in the GSE conjecture. This allows to think that physical Lorentz invariance is somewhat protected by SUSY, thus only requiring the "condensation" of the gauge degree of freedom in the vector field A_μ. The point is, however, that even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance.

While there are many papers in the literature on Lorentz noninvariant extensions of supersymmetric models (for some interesting ideas, see [22,23] and references therein), an emergent gauge theory in a SUSY context has only recently been introduced [9,24]. Actually, the situation was shown to be seriously changed in a SUSY context which certainly disfavors some emergent models considered above. It appears that, while the constraint-based models of an inactive SLIV successfully matches supersymmetry, the composite and potential-based models of an active SLIV leading to physical Lorentz violation cannot be conceptually realized in the SUSY context. The reason is that, in contrast to an ordinary vector field theory where all kinds of polynomial terms $(A_\mu A^\mu)^n$ $(n = 1, 2, ...)$ can be included into the Lagrangian in a Lorentz invariant way, SUSY theories only admit the bilinear mass term $A_\mu A^\mu$ in the vector field potential energy. As a result, without a stabilizing high-linear (at least quartic) vector field terms, the potential-based SLIV never occurs in SUSY theories. The same could be said about composite models [2–4] as well: a fundamental Lagrangian with multi-fermi current-current interactions can not be constructed from any matter chiral superfields. So, all the models mentioned above, but the constraint-based models determined by the GSE conjecture (4.1), are ruled out in the SUSY framework and, therefore, between the two basic SLIV versions, active and inactive, SUSY unambiguously chooses the inactive SLIV case.

4.1.4 Outline of the paper

The paper is organized in the following way. In the next section 2 I consider supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that breaks gauge invariance in the SUSY invariant phase. However, the requirement of vacuum stability in such class of models makes both supersymmetry and Lorentz invariance to become spontaneously broken. As a consequence, the massless photino and photon appear as the corresponding Nambu-Goldstone zero modes in an emergent SUSY QED, and also a special gauge invariance is simultaneously generated. Due to this invariance all observable relativistically noninvariant effects appear to be completely cancelled out and physical Lorentz invariance is recovered. Further in section 3, all basic arguments developed in SUSY QED are generalized successively to the Standard Model and Grand Unified Theories (GUTs). For definiteness, I focus on the $U(1) \times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case one only has the SUSY invariant phase in the theory that makes it inappropriate for an outgrowth of an emergence process. As possible realistic realizations, the Standard Model case with the electroweak $U(1) \times SU(2)$ symmetry and flipped $SU(5)$ GUT including some immediate applications are briefly discussed. And finally in section 4, I summarize the main results and conclude.

The present talk is complimentary to my last year talk in Bled [25]. Some more detail can also be found in the recent extended paper [9].

4.2 Emergent SUSY theories: a QED primer

In contrast to attempts simply probing physical Lorentz noninvariance through some SM extensions [8,20] with hypothetical external vector (tensor) field backgrounds originated around the Planck scale, we will principally focus here on a spontaneous Lorentz violation in an ordinary Standard Model framework itself. Particularly, we will try to extend an emergent SM with electroweak bosons appearing as massless vector NG modes to their supersymmetric analogs [9,24]. Such theories seem to open a new avenue for exploring the origin of gauge symmetries. Indeed, as I discussed at the previous workshop [25], the emergent SUSY theories, in contrast to the non-SUSY ones, could naturally have some clear observational signature. Actually, we have seen above that ordinary emergent gauge theories are physically indistinguishable from the conventional ones unless gauge invariance becomes broken being caused by some high-dimension couplings. Meanwhile, their SUSY counterparts - supersymmetric QED, SM and GUT - can be experimentally verified in another way. The point is that they generically emerge only if supersymmetry is spontaneously broken in a visible sector in order to ensure stability of the underlying theory. Therefore, the verification of emergent theories is now related to an inevitable emergence of a goldstino-like photino state in the SUSY particle spectrum at low energies, while physical Lorentz invariance may be still left intact.

4.2.1 Spontaneous SUSY violation

Since gauge invariance is not generically assumed in an emergent approach, all possible gauge-noninvariant couplings could in principle occur in the theory in a pre-emergent phase. The most essential couplings, as I discussed earlier [25], appear to be the vector field self-interaction terms triggering an emergence process in non-SUSY theories. Starting from this standpoint, I consider a conventional supersymmetric QED being similarly extended by an arbitrary polynomial potential of a general vector superfield $V(x,\theta,\overline{\theta})$ which in the standard parametrization [26] has a form

$$V(x,\theta,\overline{\theta}) = C + i\theta\chi - i\overline{\theta}\overline{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^{*} \\ -\theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta}\overline{\lambda'} - i\overline{\theta\theta}\theta\lambda' + \frac{1}{2}\theta\theta\overline{\theta\theta}D', \tag{4.3}$$

where its vector field component A_μ is usually associated with a photon. Note that, apart from an ordinary photino field λ and an auxiliary D field, the superfield (4.3) contains in general some additional degrees of freedom in terms of the dynamical C and χ fields and nondynamical complex scalar field S (I have used the brief notations, $\lambda' = \lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}$ and $D' = D + \frac{1}{2}\partial^2 C$ with $\sigma^\mu = (1,\overrightarrow{\sigma})$ and $\overline{\sigma}^\mu = (1,-\overrightarrow{\sigma})$). The corresponding Lagrangian can be written as

$$L = L_{SQED} + \frac{1}{2}D^2 + \sum_{k=1} b_k V^k|_D \tag{4.4}$$

where, besides a standard SUSY QED part, new potential terms are presented in the sum by corresponding D-term expansions $V^k|_D$ of the vector superfield (4.3) into the component fields (b_k are some constants). It can readily be checked that the first term in this expansion is the known Fayet-Iliopoulos D-term, while other terms only contain bilinear, trilinear and quartic combination of the superfield components A_μ, S, λ and χ, respectively.

Actually, the higher-degree terms only appear for the scalar field component $C(x)$. Expressing them all in terms of the C field polynomial

$$P(C) = \sum_{k=1} \frac{k}{2} b_k C^{k-1}(x) \tag{4.5}$$

and its first three derivatives

$$P'_C \equiv \frac{\partial P}{\partial C}\,, \quad P''_C \equiv \frac{\partial^2 P}{\partial C^2}\,, \quad P'''_C \equiv \frac{\partial^3 P}{\partial C^3} \tag{4.6}$$

one has for the whole Lagrangian L

$$\begin{aligned} L = L_{SQED} + \frac{1}{2}D^2 + \ P\left(D + \frac{1}{2}\partial^2 C\right) \\ +P'_C\left(\frac{1}{2}SS^* - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}A_\mu A^\mu\right) \\ + \frac{1}{2}P''_C\left(\frac{i}{2}\overline{\chi\chi}S - \frac{i}{2}\chi\chi S^* - \chi\sigma^\mu\overline{\chi}A_\mu\right) + \frac{1}{8}P'''_C(\chi\chi\overline{\chi\chi})\,. \end{aligned} \tag{4.7}$$

As one can see, extra degrees of freedom related to the C and χ component fields in a general vector superfield $V(x, \theta, \overline{\theta})$ appear through the potential terms in (4.7) rather than from the properly constructed supersymmetric field strengths, as appear for the vector field A_μ and its gaugino companion λ.

Note that all terms in the sum in (4.4) except Fayet-Iliopoulos D-term explicitly break gauge invariance. However, as we will see later in this section, the special gauge invariance constrained by some gauge condition will be recovered in the Lagrangian in the broken SUSY phase. Furthermore, as is seen from (4.7), the vector field A_μ may only appear with bilinear mass term in the polynomially extended superfield Lagrangian (4.4) in sharp contrast to the non-SUSY theory case where, apart from the vector field mass term, some high-linear stabilizing terms necessarily appear in a similar polynomially extended Lagrangian. This means in turn that physical Lorentz invariance is still preserved. Actually, only supersymmetry appears to be spontaneously broken in the theory.

Indeed, varying the Lagrangian L with respect to the D field we come to

$$D = -P(C) \tag{4.8}$$

that finally gives the following potential energy for the field system considered

$$U(C) = \frac{1}{2}[P(C)]^2\,. \tag{4.9}$$

The potential (4.9) may lead to spontaneous SUSY breaking in the visible sector provided that the polynomial P (4.5) has no real roots, while its first derivative has,

$$P \neq 0\,, \ P'_C = 0. \tag{4.10}$$

This requires $P(C)$ to be an even degree polynomial with properly chosen coefficients b_k in (4.5) that will force its derivative P'_C to have at least one root, $C = C_0$, in which the potential (4.9) is minimized. Therefore, supersymmetry is spontaneously broken and the C field acquires the VEV

$$\langle C \rangle = C_0 \ , \ P'_C(C_0) = 0 \ . \tag{4.11}$$

As an immediate consequence, that one can readily see from the Lagrangian L (4.7), a massless photino λ being Goldstone fermion in the broken SUSY phase make all the other component fields in the superfield $V(x, \theta, \overline{\theta})$ including the photon to also become massless. However, the question then arises whether this masslessness of the photon will be stable against radiative corrections since gauge invariance is explicitly broken in the Lagrangian (4.7). I show below that it could be the case if the vector superfield $V(x, \theta, \overline{\theta})$ would appear properly constrained.

4.2.2 Instability of superfield polynomial potential

Let us first analyze possible vacuum configurations for the superfield components in the polynomially extended QED case taken above. In general, besides the "standard" potential energy expression (4.9) determined solely by the scalar field component $C(x)$ of the vector superfield (4.3), one also has to consider other field component contributions into the potential energy. A possible extension of the potential energy (4.9) seems to appear only due to the pure bosonic field contributions, namely due to couplings of the vector and auxiliary scalar fields, A_μ and S, in (4.7)

$$U_{tot} = \frac{1}{2}P^2 + \frac{1}{2}P'_C\,(A_\mu A^\mu - SS^*) \tag{4.12}$$

rather than due to the potential terms containing the superfield fermionic components. It can be immediately seen that these new couplings in (4.12) can make the potential unstable since the vector and scalar fields mentioned may in general develop any arbitrary VEVs. This happens, as emphasized above, due the fact that their bilinear term contributions are not properly compensated by appropriate four-linear field terms which are generically absent in a SUSY theory context.

4.2.3 Stabilization of vacuum by constraining vector superfield

The only possible way to stabilize the theory seems to seek the proper constraints on the superfield component fields (C, A_μ, S) themselves rather than on their expectation values. This will be done again through some invariant Lagrange multiplier coupling simply adding its D term to the above Lagrangian (4.4, 4.7)

$$L_{tot} = L + \frac{1}{2}\Lambda(V - C_0)^2|_D \ , \tag{4.13}$$

where $\Lambda(x, \theta, \overline{\theta})$ is some auxiliary vector superfield, while C_0 is the constant background value of the C field which minimizes the potential U (4.9). Accordingly, the potential vanishes for the supersymmetric minimum or acquires some positive value corresponding to the SUSY breaking minimum (4.10) in the visible sector. I shall consider both cases simultaneously using the same notation C_0 for either of the background values of the C field.

Writing down the Lagrange multiplier D term in (4.13) through the component fields

$$C_\Lambda,\ \chi_\Lambda,\ S_\Lambda,\ A^\mu_\Lambda,\ \lambda'_\Lambda = \lambda_\Lambda + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}_\Lambda,\ D'_\Lambda = D_\Lambda + \frac{1}{2}\partial^2 C_\Lambda \tag{4.14}$$

and varying the whole Lagrangian (4.13) with respect to these fields one finds the constraints which appear to put on the V superfield components [25]

$$C = C_0,\ \ \chi = 0,\ \ A_\mu A^\mu = SS^*. \tag{4.15}$$

They also determine the corresponding D-term (4.8), $D = -P(C_0)$, for the spontaneously broken supersymmetry. As usual, I only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (4.14). This will provide a ghost-free theory with a positive Hamiltonian.

Finally, implementing the constraints (4.15) into the total Lagrangian L_{tot} (4.13, 4.7) through the Lagrange multiplier terms for component fields, we come to the emergent SUSY QED appearing in the broken SUSY phase

$$L^{em} = L_{SQED} + P(C)D\ + \frac{D_\Lambda}{4}(C - C_0)^2 - \frac{C_\Lambda}{4}\left(A_\mu A^\mu - SS^*\right)\ . \tag{4.16}$$

The last two term with the component multiplier functions C_Λ and D_Λ of the auxiliary superfield Λ (4.14) provide the vacuum stability condition of the theory. In essence, one does not need now to postulate from the outset gauge invariance for the physical SUSY QED Lagrangian L_{SQED}. Rather, one can derive it following the GSE conjecture (section 1.1) specified for Abelian theory. Indeed, due to the constraints (4.15), the Lagrangian L_{SQED} is only allowed to have a conventional gauge invariant form

$$L_{SQED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\overline{\lambda} + \frac{1}{2}D^2 \tag{4.17}$$

Thus, for the constrained vector superfield involved

$$\widehat{V}(x,\theta,\overline{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^* - \theta\sigma^\mu\overline{\theta}A_\mu + i\theta\theta\overline{\theta\lambda} - i\overline{\theta\theta}\theta\lambda + \frac{1}{2}\theta\theta\overline{\theta\theta}D, \tag{4.18}$$

we have the almost standard SUSY QED Lagrangian with the same states - a photon, a photino and an auxiliary scalar D field - in its gauge supermultiplet, while another auxiliary complex scalar field S gets only involved in the vector field constraint in (4.15). The linear (Fayet-Iliopoulos) D-term with the effective coupling constant $P(C_0)$ in (4.16) shows that supersymmetry in the theory is spontaneously broken due to which the D field acquires the VEV, $D = -P(C_0)$. Taking the nondynamical S field in the constraint (4.15) to be some constant background field we come to the SLIV constraint (4.1) underlying the GSE conjecture. As is seen from this constraint in (4.16), one may only have the time-like SLIV in a SUSY framework but never the space-like one. There also may be a light-like SLIV, if the S field vanishes[2]. So, any possible choice for the S field corresponds to the particular gauge choice for the vector field A_μ in an otherwise gauge invariant theory. So, the massless photon appearing first as a companion of a massless photino (being a Goldstone fermion in the visible broken SUSY phase) remains massless due to this recovering gauge invariance in the emergent SUSY QED. At the same time, the "built-in" nonlinear gauge condition in (4.16) allows to treat the photon as a vector Goldstone boson induced by an inactive SLIV.

4.3 On emergent SUSY Standard Models and GUTs

4.3.1 Potential of Abelian and non-Abelian vector superfields

Now, we extend our discussion to the non-Abelian internal symmetry case given by some group G with generators t^p

$$[t^p, t^q] = if^{pqr}t^r \ , \ Tr(t^p t^q) = \delta^{pq} \ \ (p, q, r = 0, 1, ..., \Upsilon - 1) \tag{4.19}$$

where f^{pqr} stand structure constants, while Υ is a dimension of the G group. This case may correspond in general to some Grand Unified Theory which includes the Standard Model and its possible extensions. For definiteness, I will be further focused on the $U(1) \times SU(N)$ symmetrical theories, though any other non-Abelian group in place of $SU(N)$ is also admissible. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case supersymmetry does not get spontaneously broken in a visible sector that makes it inappropriate for an outgrowth of an emergence process[3]. So, the theory now contains the Abelian vector superfield V, as is given in (4.3), and non-Abelian superfield multiplet $\mathbf{V}^p$

$$\mathbf{V}^p(x, \theta, \overline{\theta}) = \mathbf{C}^p + i\theta\chi^p - i\overline{\theta}\overline{\chi}^p + \frac{i}{2}\theta\theta\mathbf{S}^p - \frac{i}{2}\overline{\theta\theta}\mathbf{S}^{*p}$$
$$-\theta\sigma^\mu\overline{\theta}\mathbf{A}^p_\mu + i\theta\theta\overline{\theta}\overline{\lambda'}^p - i\overline{\theta\theta}\theta\lambda'^p + \frac{1}{2}\theta\theta\overline{\theta\theta}\mathbf{D}'^p, \tag{4.20}$$

where its vector field components $\mathbf{A}^p_\mu$ are usually associated with an adjoint gauge field multiplet, $(\mathbf{A}_\mu)^i_j \equiv (\mathbf{A}^p_\mu t^p)^i_j$ $(i, j, k = 1, 2, ..., N\,; p, q, r = 1, 2, ..., N^2 - 1)$. Note that, apart from the conventional gaugino multiplet $\boldsymbol{\lambda}^p$ and the auxiliary fields $\mathbf{D}^p$, the superfield $\mathbf{V}^p$ contains in general the additional degrees of freedom in terms of the dynamical scalar and fermion field multiplets $\mathbf{C}^p$ and χ^p and nondynamical complex scalar field $\mathbf{S}^p$. Note that for the non-Abelian superfield components I use hereafter the bold symbols and take again the brief notations, $\boldsymbol{\lambda}'^p = \boldsymbol{\lambda}^p + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}^p$ and $\mathbf{D}'^p = \mathbf{D}^p + \frac{1}{2}\partial^2\mathbf{C}^p$.

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[2] Indeed, this case, first mentioned in [11], may also mean spontaneous Lorentz violation with a nonzero VEV $< A_\mu > = (\widetilde{M}, 0, 0, \widetilde{M})$ and Goldstone modes $A_{1,2}$ and $(A_0 + A_3)/2$ $-\widetilde{M}$. The "effective" Higgs mode $(A_0 - A_3)/2$ can be then expressed through Goldstone modes so as the light-like condition $A_\mu A^\mu = 0$ to be satisfied.

[3] In principle, SUSY may be spontaneously broken in the visible sector even in the pure non-Abelian symmetry case provided that the vector superfield potential includes some essential high-dimension couplings.

Augmenting the SUSY and $U(1)\times SU(N)$ invariant GUT by some polynomial potential of vector superfields V and $\mathbf{V}^p$ one comes to

$$\mathcal{L}=\mathcal{L}_{SGUT}+\frac{1}{2}D^2+\frac{1}{2}\mathbf{D}^p\mathbf{D}^p+[\xi V+b_1V^3/3+b_2V(\mathbf{V}\mathbf{V})+b_3(\mathbf{V}\mathbf{V}\mathbf{V})/3]_D \quad (4.21)$$

where ξ and $b_{1,2,3}$ stand for coupling constants, and the last term in (4.21) contains products of the Abelian superfield V and the adjoint $SU(N)$ superfield multiplet $\mathbf{V}^i_j\equiv(\mathbf{V}^p t^p)^i_j$. The round brackets denote hereafter traces for the superfield $\mathbf{V}^i_j$

$$(\mathbf{V}\mathbf{V}...)\equiv \mathrm{Tr}(\mathbf{V}\mathbf{V}...) \quad (4.22)$$

and its field components (see below). For simplicity, we restricted ourselves to the third degree superfield terms in the Lagrangian $\mathcal{L}$ to eventually have a theory at a renormalizible level. Furthermore, I have only taken the odd power superfield terms that provides, as we see below, an additional discrete symmetry of the potential with respect to the scalar field components in the V and $\mathbf{V}^p$ superfields

$$C\to -C,\quad \mathbf{C}^p\to -\mathbf{C}^p. \quad (4.23)$$

Finally, eliminating the auxiliary D and $\mathbf{D}^p$ fields in the Lagrangian $\mathcal{L}$ we come to the total potential for all superfield bosonic field components written in terms of traces mentioned above (4.22)

$$\begin{aligned}\mathcal{U}_{tot}=&\,\mathcal{U}(C,\mathbf{C})+\frac{1}{2}\,b_1C(A_\mu A^\mu-S_\alpha S_\alpha)+\frac{1}{2}\,b_2C[(\mathbf{A}_\mu\mathbf{A}^\mu)-(\mathbf{S}_\alpha\mathbf{S}_\alpha)]\\&+\frac{1}{2}b_2[A_\mu(\mathbf{A}^\mu\mathbf{C})-S_\alpha(\mathbf{S}_\alpha\mathbf{C})]+\frac{1}{2}b_3[(\mathbf{A}_\mu\mathbf{A}^\mu\mathbf{C})-(\mathbf{S}_\alpha\mathbf{S}_\alpha\mathbf{C})]\,. \quad (4.24)\end{aligned}$$

Note that the potential terms depending only on scalar fields C and $\mathbf{C}^i_j\equiv(\mathbf{C}^a t^a)^i_j$ are collected in

$$\mathcal{U}(C,\mathbf{C})=\frac{1}{8}[\xi+b_1C^2+b_2(\mathbf{C}\mathbf{C})]^2+\frac{1}{2}[b_2^2C^2(\mathbf{C}\mathbf{C})+b_2b_3C(\mathbf{C}\mathbf{C}\mathbf{C})+\frac{1}{4}b_3^2(\mathbf{C}\mathbf{C}\mathbf{C}\mathbf{C})] \quad (4.25)$$

and complex scalar fields S_α and $\mathbf{S}^p_\alpha$ ($\alpha=1,2$) are now taken in the real field basis like as

$$S_1=(S+S^*)/2,\quad S_2=(S-S^*)/2i\,, \quad (4.26)$$

an so on. One can see that all these terms are invariant under the discrete symmetry (4.23), whereas the vector field couplings in the total potential $\mathcal{U}_{tot}$ (4.24) break it. However, they vanish when the V and $\mathbf{V}^p$ superfields are properly constrained that we actually confirm in the next section.

Let us consider first the pure scalar field potential $\mathcal{U}$ (4.25). The corresponding extremum conditions for C and $\mathbf{C}^a$ fields are,

$$\begin{aligned}\mathcal{U}'_C&=b_1(\xi+b_1C^2)C+b_2(b_1-2b_2)C(\mathbf{C}\mathbf{C})=0,\\ \mathrm{Tr}(\mathcal{U}'_{\mathbf{C}^i_j})&=3b_2C(\mathbf{C}\mathbf{C})+b_3(\mathbf{C}\mathbf{C}\mathbf{C})=0\,, \quad (4.27)\end{aligned}$$

respectively. As shows the second partial derivative test, the simplest solution to the above equations

$$C_0=0\,,\ \ \mathbf{C}^i_j=0 \quad (4.28)$$

provides, under conditions put on the potential parameters,

$$\xi,\ b_1 > 0\,,\, b_2 \geq 0 \ \text{ or } \ \xi,\ b_1 < 0\,,\, b_2 \leq 0 \tag{4.29}$$

its global minimum

$$\mathcal{U}(C, \mathbf{C})^{as}_{\min} = \frac{1}{8}\xi^2\,. \tag{4.30}$$

This minimum corresponds to the broken SUSY phase with the unbroken internal symmetry $U(1) \times SU(N)$ that is just what one would want to trigger an emergence process. This minimum appears in fact due to the Fayet-Iliopoulos linear term in the superfield polynomial in (4.21). As can easily be confirmed, in absence of this term, namely, for $\xi = 0$ and any arbitrary values of all other parameters, there is only the SUSY symmetrical solution with unbroken internal symmetry

$$\mathcal{U}(C, \mathbf{C})^{sym}_{\min} = 0\,. \tag{4.31}$$

Interestingly, the symmetrical solution corresponding to the global minimum (4.31) may appear for the nonzero parameter ξ as well

$$C_0^{(\pm)} = \pm\sqrt{-\xi/b_1},\ \ \mathbf{C}^i_j = 0 \tag{4.32}$$

provided that

$$\xi b_1 < 0\,. \tag{4.33}$$

However, as we saw in the QED case, in the unbroken SUSY case one comes to the trivial constant superfield when all factual constraints are included into consideration [25] and, therefore, this case is in general of little interest.

4.3.2 Constrained vector supermultiplets

Let us now take the vector fields A_μ and $\mathbf{A}^p_\mu$ into consideration that immediately reveals that, in contrast to the pure scalar field part (4.25), $\mathcal{U}(C, \mathbf{C})$, the vector field couplings in the total potential (4.24) make it unstable. This happens, as was emphasized before, due the fact that bilinear term VEV contributions of the vector fields A_μ and $\mathbf{A}^p_\mu$, as well as the auxiliary scalar fields S_α and $\mathbf{S}^p_\alpha$, are not properly compensated by appropriate four-linear field terms which are generically absent in a supersymmetric theory framework.

Again, as in the supersymmetric QED case considered above, the only possible way to stabilize the ground state (4.28, 4.29, 4.30) seems to seek the proper constraints on the superfields component fields (C, $\mathbf{C}^p$; A_μ, $\mathbf{A}^p$; S_α, $\mathbf{S}^p_\alpha$) themselves rather than on their expectation values. Provided that such constraints are physically realizable, the required vacuum will be automatically stabilized. This will be done again through some invariant Lagrange multiplier couplings simply adding their D terms to the above Lagrangian (4.21)

$$\mathcal{L}_{tot} = \mathcal{L} + \frac{1}{2}\Lambda(V - C_0)^2|_D + \frac{1}{2}\Pi(\mathbf{V}\mathbf{V})|_D\,, \tag{4.34}$$

where $\Lambda(x, \theta, \overline{\theta})$ and $\Pi(x, \theta, \overline{\theta})$ are auxiliary vector superfields. Note that C_0 presented in the first multiplier coupling is just the constant background value of the

C field for which the potential part $\mathcal{U}(C, \mathbf{C})$ in (4.24) vanishes as appears for the supersymmetric minimum (4.31) or has some nonzero value corresponding to the SUSY breaking minimum (4.30) in the visible sector.

I will consider both cases simultaneously using the same notation C_0 for either of the potential minimizing values of the C field. The second multiplier coupling in (4.34) provides, as we will soon see, the vanishing background value for the non-Abelian scalar field, $\mathbf{C}^a = 0$, due to which the underlying internal symmetry $U(1) \times SU(N)$ is left intact in both unbroken and broken SUSY phase. The Lagrange multiplier terms presented in (4.34) have in fact the simplest possible form that leads to some nontrivial constrained superfields $V(x, \theta, \overline{\theta})$ and $\mathbf{V}^p(x, \theta, \overline{\theta})$. Writing down their invariant D terms through the component fields one finds the precisely the same expression as in the SUSY QED [25] case for the Abelian superfield V and the slightly modified one for the non-Abelian superfield $\mathbf{V}^a$

$$\begin{aligned}
\Pi(\mathbf{V}\mathbf{V})|_D = {} & C_\Pi \left[\mathbf{C}\mathbf{D}' + \left(\frac{1}{2}\mathbf{S}\mathbf{S}^* - \boldsymbol{\chi}\boldsymbol{\lambda}' - \overline{\boldsymbol{\chi}}\overline{\boldsymbol{\lambda}'} - \frac{1}{2}\mathbf{A}_\mu\mathbf{A}^\mu\right)\right] \\
& + \chi_\Pi \left[2\mathbf{C}\boldsymbol{\lambda}' + i(\boldsymbol{\chi}\mathbf{S}^* + i\sigma^\mu\overline{\boldsymbol{\chi}}\mathbf{A}_\mu)\right] + \overline{\chi}_\Pi\left[2\mathbf{C}\overline{\boldsymbol{\lambda}'} - i(\overline{\boldsymbol{\chi}}\mathbf{S} - i\boldsymbol{\chi}\sigma^\mu\mathbf{A}_\mu)\right] \\
& + \frac{1}{2}S_\Pi\left(\mathbf{C}\mathbf{S}^* + \frac{i}{2}\overline{\boldsymbol{\chi}}\overline{\boldsymbol{\chi}}\right) + \frac{1}{2}S^*_\Pi\left(\mathbf{C}\mathbf{S} - \frac{i}{2}\boldsymbol{\chi}\boldsymbol{\chi}\right) \\
& + 2A^\mu_\Pi(\mathbf{C}\mathbf{A}_\mu - \boldsymbol{\chi}\sigma_\mu\overline{\boldsymbol{\chi}}) + 2\lambda'_\Pi(\mathbf{C}\boldsymbol{\chi}) + 2\overline{\lambda}'_\Pi(\mathbf{C}\overline{\boldsymbol{\chi}}) + \frac{1}{2}D'_\Pi(\mathbf{C}\mathbf{C}) \qquad (4.35)
\end{aligned}$$

where the pairly grouped field bold symbols mean hereafter the $SU(N)$ scalar products of the component field multiplets (for instance, $\mathbf{C}\mathbf{D}' = \mathbf{C}^p\mathbf{D}'^p$, and so forth) and

$$C_\Pi,\ \chi_\Pi,\ S_\Pi,\ A^\mu_\Pi,\ \lambda'_\Pi = \lambda_\Pi + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}_\Pi,\ D'_\Pi = D_\Pi + \frac{1}{2}\partial^2 C_\Pi \qquad (4.36)$$

are the component fields of the Lagrange multiplier superfield $\Pi(x, \theta, \overline{\theta})$ in the standard parametrization (4.20).

Varying the total Lagrangian (4.34) with respect to the component fields of both multipliers, (4.14) and (4.36), and properly combining their equations of motion we find the constraints which appear to put on the V and $\mathbf{V}^a$ superfields components [9]

$$\begin{aligned}
& C = C_0,\ \chi = 0,\ \ A_\mu A^\mu = S_\alpha S_\alpha, \\
& \mathbf{C}^p = 0,\ \boldsymbol{\chi}^p = 0,\ \ (\mathbf{A}_\mu\mathbf{A}^\mu) = (\mathbf{S}_\alpha\mathbf{S}_\alpha)\ ,\ \alpha = 1,2\ . \qquad (4.37)
\end{aligned}$$

As before in the SUSY QED case, one may only have the time-like SLIV in a supersymmetric $U(1) \times SU(N)$ framework but never the space-like one (there also may be a light-like SLIV, if the S and **S** fields vanish). Also note that we only take the solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (4.14) and (4.36) that will provide a ghost-free theory with a positive Hamiltonian. Again, apart from the constraints (4.37), one has the equations of motion for all fields involved in the basic superfields $V(x, \theta, \overline{\theta})$ and $\mathbf{V}^p(x, \theta, \overline{\theta})$. With vanishing multiplier component fields (4.14) and (4.36), as was proposed above, these equations appear in fact as extra constraints on components of the V and $\mathbf{V}^p$ superfields. Indeed, equations of motion for the S_α, χ and C fields, on the one hand hand, and for the $\mathbf{S}^p_\alpha$, $\boldsymbol{\chi}^p$ and $\mathbf{C}^p$ fields, on the other, are obtained by the corresponding variations of the total Lagrangian $\mathcal{L}^{tot}$ (4.34) including the potential (4.24).

They are turned out to be, respectively,

$$\begin{aligned} & S_\alpha C_0 = 0\,,\ \lambda C_0 = 0\,,\ (\xi + b_1 C_0^2)C_0 = 0\,, \\ & \mathbf{S}^p_\alpha C_0 = 0\,, \boldsymbol{\lambda}^p C_0 = 0,\ b_2[A_\mu \mathbf{A}^{\mu}{}^i_j - S_\alpha \mathbf{S}_\alpha{}^i_j] + b_3[(\mathbf{A}_\mu \mathbf{A}^\mu)^i_j - (\mathbf{S}_\alpha \mathbf{S}_\alpha)^i_j] = 0 \end{aligned} \tag{4.38}$$

where the basic constraints (4.37) emerging at the potential $\mathcal{U}(C, \mathbf{C})$ extremum point (C_0, $\mathbf{C}^p_0 = 0$) have been also used for both broken and unbroken SUSY case. Note also that the equations for gauginos λ and $\boldsymbol{\lambda}^p$ in (4.38) are received by variation of the potential terms in (4.21) containing fermion field couplings

$$\begin{aligned} \mathfrak{U} = {} & b_1 C(\chi\lambda' + \overline{\chi}\overline{\lambda'}) + \ b_2 C[(\boldsymbol{\chi}\boldsymbol{\lambda}') + (\overline{\boldsymbol{\chi}}\overline{\boldsymbol{\lambda}'})] \\ & + \frac{1}{2} b_2 [\chi(\boldsymbol{\lambda}'\mathbf{C}) + \overline{\chi}(\overline{\boldsymbol{\lambda}'}\mathbf{C}) + \lambda'(\boldsymbol{\chi}\mathbf{C}) + \overline{\lambda'}(\overline{\boldsymbol{\chi}}\mathbf{C})] \\ & + b_3(\boldsymbol{\chi}\boldsymbol{\lambda}'\mathbf{C}) + (\overline{\boldsymbol{\chi}}\overline{\boldsymbol{\lambda}'}\mathbf{C})]\,. \end{aligned} \tag{4.39}$$

One can immediately see now that all equations in (4.38) but the last equation system turn to trivial identities in the broken SUSY case (4.28) in which the corresponding C field value appears to be identically vanished, $C_0 = 0$. In the unbroken SUSY case (4.32), this field value is definitely nonzero, $C_0 = \pm\sqrt{-\xi/b_1}$, and the situation is radically changed. Indeed, as follows from the equations (4.38), the auxiliary fields $S(x)$ and $\mathbf{S}^p$, as well as the gaugino fields $\lambda(x)$ and $\boldsymbol{\lambda}^p(x)$ have to be identically vanished. This causes in turn that the gauge vector fields field A_μ and $\mathbf{A}^p_\mu$ should also be vanished according to the basic constraints (4.37). So, we have to conclude, as in the SUSY QED case, that the unbroken SUSY fails to provide stability of the potential (4.12) even by constraining the superfields V and $\mathbf{V}^p$ and, therefore, only the spontaneously broken SUSY case could in principle lead to a physically meaningful emergent theory.

4.3.3 Broken SUSY phase: an emergent $U(1) \times SU(N)$ theory

With the constraints (4.37) providing vacuum stability for the total Lagrangian $\mathcal{L}_{tot}$ (4.34) we eventually come to the emergent theory with a local $U(1) \times SU(N)$ symmetry that appears in the broken SUSY phase (4.28). Actually, implementing these constraints into the Lagrangian through the Lagrange multiplier terms for component fields one has

$$\begin{aligned} \mathcal{L}^{em} = {} & \mathcal{L}_{SGUT} + \frac{1}{2}\xi D + \frac{D_\Lambda}{4}(C - C_0)^2 - \frac{C_\Lambda}{4}(A_\mu A^\mu - SS^*) \\ & + \frac{D_\Pi}{4}(\mathbf{C}\mathbf{C}) - \frac{C_\Pi}{4}(\mathbf{A}_\mu \mathbf{A}^\mu - \mathbf{S}\mathbf{S}^*) \end{aligned} \tag{4.40}$$

with the multiplier component functions C_Λ and D_Λ of the auxiliary superfield Λ (4.14) and component functions C_Π and D_Π of the auxiliary superfield Π (4.36) presented in the Lagrangian (4.34). Again, with these constraints and the GSE conjecture (section 1.1) specified for non-Abelian theories, one does not need to postulate gauge invariance for the physical SUSY GUT Lagrangian $\mathcal{L}_{SGUT}$ from the outset. Instead, one can derive it starting from an arbitrary relativistically invariant theory. Indeed, even if the Lagrangian $\mathcal{L}_{SGUT}$ is initially taken to only possess the global $U(1) \times SU(N)$ symmetry it will tend to uniquely acquire a standard gauge invariant form

$$\begin{aligned} \mathcal{L}_{SGUT} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^2 \\ & -\frac{1}{4}\mathbf{F}^{p\mu\nu}\mathbf{F}^{p}_{\mu\nu} + i\boldsymbol{\lambda}^{p}\sigma^{\mu}\mathcal{D}_{\mu}\overline{\boldsymbol{\lambda}}^{p} + \frac{1}{2}\mathbf{D}^{p}\mathbf{D}^{p} \end{aligned} \tag{4.41}$$

where the conventional gauge field strengths for both $U(1)$ and $SU(N)$ part and terms with proper covariant derivatives for gaugino fields $\boldsymbol{\lambda}^p$ necessarily appear [9]. Again as in the pure Abelian case, for the respectively constrained vector superfields V and $\mathbf{V}^p$ we come in fact to a conventional SUSY GUT Lagrangian with a standard gauge supermultiplet containing gauge bosons A_μ and $\mathbf{A}^p$, gauginos λ and $\boldsymbol{\lambda}^p$, and auxiliary scalar D and $\mathbf{D}^p$ fields, whereas other auxiliary scalar fields S_α and $\mathbf{S}^p_\alpha$ get solely involved in the Lagrange multiplier terms (4.41). Actually, the only remnant of the polynomial potential of vector superfields V and $\mathbf{V}^p$ (4.21) survived in the emergent theory (4.40) appears to be the Fayet-Iliopoulos D-term which shows that supersymmetry in the theory is indeed spontaneously broken and the D field acquires the VEV, $D = -\frac{1}{2}\xi$.

Let us show now that this theory is in essence gauge invariant and the constraints (4.37) on the field space appearing due to the Lagrange multiplier terms in (4.34) are consistent with supersymmetry. Namely, as was argued in [25] (see also [9]), though constrained vector superfield (4.18) in QED is not strictly compatible with the linear superspace version of SUSY transformations, its supermultiplet structure can be restored by appropriate supergauge transformations. Following the same argumentation, one can see that similar transformations keep invariant the constraints (4.37) put on the vector fields A_μ and $\mathbf{A}^p$. Leaving aside the $U(1)$ sector considered in [25] in significant details, I will now focus on the $SU(N)$ symmetry case with the constrained superfield $\mathbf{V}^p$ transformed as

$$\mathbf{V}^p \to \mathbf{V}^p + \frac{i}{2}(\boldsymbol{\Omega} - \boldsymbol{\Omega}^*)^p \tag{4.42}$$

The essential part of this transformation which directly acts on the vector field constraint

$$\mathbf{A}^p_\mu \mathbf{A}^{p\mu} = \mathbf{S}^p \mathbf{S}^{*p} \tag{4.43}$$

has the form

$$\mathbf{V}^p \to \mathbf{V}^p + \frac{i}{2}\theta\theta\mathbf{F}^p - \frac{i}{2}\overline{\theta\theta}\mathbf{F}^{*p} - \theta\sigma^\mu\overline{\theta}\partial_\mu\boldsymbol{\varphi}^p \tag{4.44}$$

where the real and complex scalar field components, $\boldsymbol{\varphi}^p$ and F^p, in a chiral superfield parameter $\boldsymbol{\Omega}^p$ are properly activated. As a result, the corresponding vector and scalar component fields, $\mathbf{A}^p_\mu$ and $\mathbf{S}^p_\alpha$, in the constrained supermultiplet $\mathbf{V}^p$ transform as

$$\mathbf{A}^p_\mu \to \mathbf{a}^p_\mu = \mathbf{A}^p_\mu - \partial_\mu\boldsymbol{\varphi}^p, \quad \mathbf{S}^p \to \mathbf{s}^p = \mathbf{S}^p + \mathbf{F}^p\ . \tag{4.45}$$

One can readily see that our basic Lagrangian $\mathcal{L}^{em}$ (4.40) being gauge invariant and containing no the auxiliary scalar fields $\mathbf{S}^p$ is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (4.43), though it is also turned out to be invariant under supergauge transformations (4.45), but only if they act jointly. Indeed, for any choice of the scalar $\boldsymbol{\varphi}^p$ in (4.45) there can always be found such a scalar $\mathbf{F}^a$ (and vice versa) that the constraint remains invariant. In other words, the vector field constraint is invariant under supergauge transformations (4.45) but not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the scalar field bilinear $\mathbf{S}^p\mathbf{S}^{*p}$ to some constant background value and find a final equation for the gauge function $\boldsymbol{\varphi}^p(x)$. It is convenient to come to real field basis (4.26) for scalar fields $\mathbf{S}^p_\alpha$ and $\mathbf{F}^p_\alpha$ $(\alpha = 1, 2)$, and choose the parameter fields $\mathbf{F}^a_\alpha$ as

$$\mathbf{F}^p_\alpha = r_\alpha \boldsymbol{\epsilon}^p(\mathbf{M}+\mathbf{f}),\ r_\alpha \mathbf{s}^p_\alpha = 0,\ \ r^2_\alpha = 1,\ \boldsymbol{\epsilon}^p\boldsymbol{\epsilon}^p = 1 \tag{4.46}$$

so that the old $\mathbf{S}^p_\alpha$ fields in (4.45) are related to the new ones $\mathbf{s}^p_\alpha$ in the following way

$$\mathbf{S}^p_\alpha = \mathbf{s}^p_\alpha - r_\alpha \boldsymbol{\epsilon}^p(\mathbf{M}+\mathbf{f}),\ r_\alpha \mathbf{s}^p_\alpha = 0,\ \mathbf{S}^p_\alpha\mathbf{S}^p_\alpha = \mathbf{s}^p_\alpha\mathbf{s}^p_\alpha + (\mathbf{M}+\mathbf{f})^2. \tag{4.47}$$

where $\mathbf{M}$ is a new mass parameter, $\mathbf{f}(x)$ is some Higgs field like function, r_α is again the two-component unit "vector" chosen to be orthogonal to the scalar $\mathbf{s}^p_\alpha$, while $\boldsymbol{\epsilon}^p$ is the unit $SU(N)$ adjoint vector. This parametrization for the old fields $\mathbf{S}^p_\alpha$ formally looks as if they develop the VEV, $\langle \mathbf{S}^p_\alpha \rangle = -r_\alpha \boldsymbol{\epsilon}^p \mathbf{M}$, due to which the related $SO(2) \times SU(N)$ symmetry would be spontaneously violated and corresponding zero modes in terms of the new fields $\mathbf{s}^p_\alpha$ could be consequently produced (indeed, they they never appear in the theory). Eventually, for an appropriate choice of the Higgs field like function $\mathbf{f}(x)$ in (4.47)

$$\mathbf{f} = -\mathbf{M} + \sqrt{\mathbf{M}^2 - \mathbf{s}^p_\alpha\mathbf{s}^p_\alpha} \tag{4.48}$$

we come in (4.43) to the condition

$$\mathbf{A}^p_\mu\mathbf{A}^{p\mu} = \mathbf{M}^2\ . \tag{4.49}$$

leading, as in the QED $U(1)$ symmetry case [25], exclusively to the time-like SLIV.

Remarkably, thanks to a generic high symmetry of the constraint (4.49) one can apply the emergence conjecture with dynamically produced massless gauge modes to any non-Abelian internal symmetry case as well, though SLIV itself could produce only one zero vector mode. The point is that although we only propose Lorentz invariance $SO(1,3)$ and internal symmetry $U(1) \times SU(N)$ of the Lagrangian $\mathcal{L}^{em}$ (4.40), the emerged constraint (4.49) possesses in fact a much higher accidental symmetry $SO(\Upsilon, 3\Upsilon)$ determined by the dimension $\Upsilon = N^2 - 1$ of the $SU(N)$ adjoint representation to which the vector fields $\mathbf{A}^p_\mu$ belong[4]. This

[4] Actually, a total symmetry even higher if one keeps in mind both constraints (4.1) and (4.49) put on the vector fields A_μ and $\mathbf{A}^a_\mu$, respectively. As long as they are independent the related total symmetry is in fact $SO(1,3) \times SO(\Upsilon, 3\Upsilon)$ until it starts breaking.

symmetry is indeed spontaneously broken at a scale $\mathbf{M}$ leading exclusively to the time-like SLIV case, as is determined by the positive sign in the SUSY SLIV constraint (4.49). The emerging pseudo-Goldstone vector bosons may be in fact considered as candidates for non-Abelian gauge fields which together with the true vector Goldstone boson entirely complete the adjoint multiplet of the internal symmetry group $SU(N)$. Remarkably, they remain strictly massless being protected by the simultaneously generated non-Abelian gauge invariance. When expressed in these zero modes, the theory look essentially nonlinear and contains many Lorentz and CPT violating couplings. However, as in the SUSY QED case, they do not lead to physical SLIV effects which due to simultaneously generated gauge invariance appear to be strictly cancelled out.

As in the pure QED case, one can calculate the gauge functions $\boldsymbol{\varphi}^p(x)$ comparing the relation between the old and new vector fields in (4.45) with a conventional SLIV parametrization for non-Abelian vector fields [9]

$$\boldsymbol{A}_\mu^p = \boldsymbol{a}_\mu^p + \boldsymbol{n}_\mu^p \sqrt{\mathbf{M}^2 - \boldsymbol{n}^2 \boldsymbol{a}^2}\ ,\ \ \boldsymbol{n}_\mu^p \boldsymbol{a}^{p\mu} = 0 \quad (\boldsymbol{a}^2 \equiv \boldsymbol{a}_\mu^p \boldsymbol{a}^{p\mu}). \tag{4.50}$$

They are expressed through the non-Abelian Goldstone and pseudo-Goldstone modes $\boldsymbol{a}_\mu^p$

$$\boldsymbol{\varphi}^p = \boldsymbol{\epsilon}^p \int^x d(n_\mu x^\mu) \sqrt{\mathbf{M}^2 - \boldsymbol{n}^2 \boldsymbol{a}^2}\ . \tag{4.51}$$

Here n_μ is the unit Lorentz vector being analogous to the vector introduced in the Abelian case (4.2), which is now oriented in Minkowskian spacetime so as to be ″parallel″ to the vacuum unit $\boldsymbol{n}_\mu^p$ matrix. This matrix can be taken in the ″two-vector″ form

$$\boldsymbol{n}_\mu^p = n_\mu \boldsymbol{\epsilon}^p\ ,\ \ \boldsymbol{\epsilon}^p \boldsymbol{\epsilon}^p = 1 \tag{4.52}$$

where $\boldsymbol{\epsilon}^p$ is the unit $SU(N)$ group vector belonging to its adjoint representation.

4.3.4 Some immediate outcomes

Quite remarkably, an obligatory split symmetry form $U(1) \times SU(N)$ (or $U(1) \times G$, in general) of plausible emergent theories which could exist beyond the prototype QED case, leads us to the standard electroweak theory with an $U(1) \times SU(2)$ symmetry as the simplest possibility. The potential of type (4.21) written for the corresponding superfields requires spontaneous SUSY breaking in the visible sector to avoid the vacuum instability in the theory. Eventually, this requires the SLIV type constraints to be put on the hypercharge and weak isospin vector fields, respectively,

$$B_\mu B^\mu = M^2\ ,\ \ \boldsymbol{W}_\mu^p \boldsymbol{W}^{p\mu} = \mathbf{M}^2 \ \ (p = 1, 2, 3). \tag{4.53}$$

These constraints are independent from each other and possess, as was generally argued above, the total symmetry $SO(1,3) \times SO(3,9)$ which is much higher than the actual Lorentz invariance and electroweak $U(1) \times SU(2)$ symmetry in the theory. Thanks to this fact, one Goldstone and three pseudo-Goldstone zero vector modes b_μ and $\boldsymbol{w}_\mu^p$ are generated to eventually complete the gauge multiplet of the Standard Model

$$B_\mu = b_\mu + n_\mu \sqrt{M^2 - b_\mu b^\mu}\ ,\ n_\mu b_\mu = 0\ ,$$
$$\boldsymbol{W}^p_\mu = \boldsymbol{w}^p_\mu + n_\mu \boldsymbol{\epsilon}^p \sqrt{\mathbf{M}^2 - \boldsymbol{w}^q_\mu \boldsymbol{w}^{q\mu}}\ , n_\mu \boldsymbol{w}^{p\mu} = 0 \tag{4.54}$$

where the unit vectors n_μ and $\boldsymbol{\epsilon}^p$ are defined in accordance with a rectangular unit matrix $\mathbf{n}^p_\mu$ taken in the two-vector form (4.52). The true vector Goldstone boson appear to be some superposition of the zero modes b_μ and $\boldsymbol{w}^3_\mu$. This superposition is in fact determined by the conventional Higgs doublet in the model since just through the Higgs field couplings these modes are only mixed [19]. Thus, when the electroweak symmetry gets spontaneously broken an accidental degeneracy related to the total symmetry of constraints mentioned above is lifted. As a consequence, the vector pseudo-Goldstones acquire masses and only photon, being the true vector Goldstone boson in the model, is left massless. In this sense, there is not much difference for a photon in emergent QED and SM: it emerges as a true vector Goldstone boson in both frameworks.

Going beyond the Standard Model we unavoidably come to the flipped $SU(5)$ GUT [27] as a minimal and in fact distinguished possibility. Indeed, the $U(1)$ symmetry part being mandatory for emergent theories now naturally appears as a linear combination of a conventional electroweak hypercharge and another hypercharge belonging to the standard $SU(5)$. The flipped $SU(5)$ GUT has several advantages over the standard $SU(5)$ one: the doublet-triplet splitting problem is resolved with use of only minimal Higgs representations and protons are naturally long lived, neutrinos are necessarily massive, and supersymmetric hybrid inflation can easily be implemented successfully. Also in string theory, the flipped $SU(5)$ model is of significant interest for a variety of reasons. In essence, the above-mentioned natural solution to the doublet-triplet splitting problem without using large GUT representations is in the remarkable conformity with string theories where such representations are typically unavailable. Also, in weakly coupled heterotic models, the flipped $SU(5)$ allows to achieve gauge coupling unification at the string scale 10^{17} GeV if some extra vector-like particles are added. They are normally taken to transform in the 10 and $\overline{10}$ representations, that is easy to engineer in string theory.

So, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at high energies as the flipped $SU(5)$ GUTs being inspired by superstrings.

4.4 Summary

As we have seen above, spontaneous Lorentz violation in a vector field theory framework may be active as in the composite and potential-based models leading to physical Lorentz violation, or inactive as in the constraint-based models resulting in the nonlinear gauge choice in an otherwise Lorentz invariant theory. Remarkably, between these two basic SLIV versions SUSY unambiguously chooses the inactive SLIV case. Indeed, SUSY theories only admit the bilinear mass term in the vector field potential energy. As a result, without a stabilizing quartic vector field terms, the physical spontaneous Lorentz violation never occurs in SUSY theories. Hence it follows that the composite and potential-based SLIV models can in no way be realized in the SUSY context. This may have far-going consequences in that supergravity and superstring theories could also disfavor such models in general.

Though, even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance. To see how this idea might work we have considered supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield that induces spontaneous SUSY violation in the visible sector, and gauge invariance gets broken as well. Nevertheless, the special gauge invariance is in fact recovered in the broken SUSY phase that universally protects the photon masslessness.

All basic arguments developed in SUSY QED were then generalized to Standard Model and Grand Unified Theories. Remarkably, thanks to a generic high symmetry of the length-fixing SLIV constraint (4.49) put on the vector fields the emergence conjecture with dynamically produced massless gauge modes can be applied to any non-Abelian global internal symmetry case due to which it gets converted into to the local one. For definiteness, we have focused above on the $U(1) \times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case one only has the SUSY invariant phase in the theory that would make it inappropriate for an outgrowth of an emergence process. As we briefly discussed, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at high energies as the flipped $SU(5)$ GUTs inspired by superstrings.

However, their most generic manifestations, as I discussed here in Bled about a year ago [25] (for more details, see also [9]), is related to a spontaneous SUSY violation in the visible sector that seems to open a new avenue for exploring the origin of gauge symmetries. Indeed, the photino emerging due to this violation will be then mixed with another goldstino which stems from a spontaneous SUSY violation in the hidden sector. Eventually, it largely turns into light pseudo-goldstino whose physics seems to be of special interest. Such pseudo-Goldstone photinos might appear typically as the eV scale stable LSP or the electroweak scale long-lived NLSP, being accompanied by a very light gravitinos in both cases. Their observation could shed some light on an emergence nature of gauge symmetries.

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5 Some Potential Problems of OHe Composite Dark Matter

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Abstract. Among composite-dark-matter scenarios, one of the simplest and most predictive is that of O-helium (OHe) dark atoms, in which a lepton-like doubly charged particle O^{--} is bound with a primordial helium nucleus, and is the main constituent of dark matter. This model liberates the physics of dark matter from many unknown features of new physics, and it demands a deep understanding of the details of known nuclear and atomic physics, which are still somewhat unclear in the case of nuclear interacting "atomic" shells. So far the model has relied on the dominance of elastic scattering of OHe with the matter. In view of the uncertainty in our understanding of OHe interaction with nuclei we study the opposite scenario, in which inelastic nuclear reactions dominate the OHe interactions with nuclei. We show that in this case all the OHe atoms bind with extra He nuclei, forming doubly charged O-beryllium ions, which behave like anomalous helium, causing potential problems with overabundance of anomalous isotopes in terrestrial matter.

Povzetek. Avtorji obravnavajo model, v katerem sestavljajo temno snov atomi O-helija (OHe), v katerih se novi lepton O^{--} z dvojnim nabojem veže z jedrom helija. Sila med jedrom in leptonom je tedaj elektromagnetna. Kljub preprostosti modela pa je izračun lastnosti takega atoma pri elastičnem in neelastičnem sipanju na običajni snovi zahteven. Avtorji študirajo v tem prispevku neelastično sipanje teh atomov na običajni snovi s predpostavko, da je to dominanten prispevek temne snovi. Pokažejo, da se tedaj pri sipanju OHe na helijevih jedrih veže OHe s helijem v O-berilij z dvema elektromagnetnima nabojema. Avtorji pridejo do zaključka, da bi lahko tak model napovedal preveliko gostoto anomalnih izotopov na Zemlji.

5.1 Introduction

Direct searches for dark matter have produced surprising results. Since the DAMA collaboration observed a signal, several other collaborations seem to confirm an observation, while many others clearly rule out any detection. The current

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experimental situation is reviewed in [1]. This apparent contradiction comes from the analysis of the data under the assumption that nuclear recoil is the source of the signal.

Starting from 2006 it was proposed [2–6] that the signal may be due to a different source: if dark matter can bind to normal matter, the observations could come from radiative capture of thermalized dark matter, and could depend on the detector composition and temperature. This scenario naturally comes from the consideration of composite dark matter. Indeed, one can imagine that dark matter is the result of the existence of heavy negatively charged particles that bind to primordial nuclei.

Cosmological considerations imply that such candidates for dark matter should consist of negatively doubly-charged heavy ($\sim$ 1 TeV) particles, which we call O^{--}, coupled to primordial helium. Lepton-like technibaryons, technileptons, AC-leptons or clusters of three heavy anti-U-quarks of 4th or 5th generation with strongly suppressed hadronic interactions are examples of such O^{--} particles (see [3–6] for a review and for references).

It was first assumed that the effective potential between OHe and a normal nucleus would have a barrier, preventing He and/or O^{--} from falling into the nucleus, allowing only one bound state, and diminishing considerably the interactions of OHe. Under these conditions elastic collisions dominate in OHe interactions with matter, and lead to a successful OHe scenario. The cosmological and astrophysical effects of such composite dark matter (dark atoms of OHe) are dominantly related to the helium shell of OHe and involve only one parameter of new physics $-$ the mass of O^{--}. The positive results of the DAMA/NaI and DAMA/LIBRA experiments are explained by the annual modulations of the rate of radiative capture of OHe by sodium nuclei. Such radiative capture is possible only for intermediate-mass nuclei: this explains the negative results of the XENON100 experiment. The rate of this capture is proportional to the temperature: this leads to a suppression of this effect in cryogenic detectors, such as CDMS. OHe collisions in the central part of the Galaxy lead to OHe excitations, and de-excitations with pair production in E0 transitions can explain the excess of the positron-annihilation line, observed by INTEGRAL in the galactic bulge [5–10]. In a two-component dark atom model, based on Walking Technicolor, a sparse WIMP-like component of atom-like state, made of positive and negative doubly charged techniparticles, is present together with the dominant OHe dark atom and the decays of doubly positive charged techniparticles to pairs of same-sign leptons can explain the excess of high-energy cosmic-ray positrons, found in PAMELA and AMS02 experiments [11].

These astroparticle data can be fitted, avoiding many astrophysical uncertainties of WIMP models, for a mass of $O^{--} \sim 1$ TeV, which stimulates searches for stable doubly charged lepton-like particles at the LHC as a test of the composite-dark-matter scenario.

In this paper, we want to explore the opposite scenarion, in which OHe dark matter interacts strongly with normal matter: OHe is neutral, but a priori it has an unshielded nuclear attraction to matter nuclei. We first study some effects of inelastic collisions of OHe in the early Universe and in the terrestrial matter and find that such collisions strongly increase the formation of charged nuclear species with O^{--} bound in them. Recombination of such charged species with electrons (even if it is partial) leads to the formation of atoms (or ions) of anomalous isotopes of helium and heavier elements. The atomic size of such atoms (or ions) of anomalous isotopes strongly suppresses their mobility in the terrestrial matter, making them stop near the surface, where anomalous superheavy nuclei are strongly constrained by the experimental searches. In Section 5.2 we study effect of inelastic processes during the period of Big Bang Nucleosynthesis and show that if these processes are not suppressed all the OHe atoms capture additional He nuclei, forming a doubly charged ion of O-beryllium (OBe). In Section 5.3 we briefly examine the problems of an OBe-dominated universe and show that, because the mobility of the anomalous isotopes is greatly suppressed even if they recombine with only one electron, their drift to the center of the Earth is strongly slowed down, and their abundance increases near the terrestrial surface and in the World Ocean with the danger of their overabundance. We stress the importance of solving the open questions of OHe nuclear physics in the Conclusion.

5.2 Inelastic processes with OHe in the early Universe

As soon as all the OHe is formed in the early Universe, inelastic processes between OHe and OHe itself and between OHe and the primordial He take place and start consuming the available OHe. The two relevant reactions are:

$$\mathrm{OHe} + \mathrm{OHe} \to \mathrm{O_2Be} \tag{5.1}$$

$$\mathrm{OHe} + \mathrm{He} \to \mathrm{OBe} \tag{5.2}$$

Note that in these reactions the addition of a He nucleus to the bound OHe system will result in merging the two He nuclei into ^{8}Be, since in the presence of O^{--}, ^{8}Be becomes stable: we calculated, as in Ref. [13], that the energy of OBe is 2.9 MeV smaller than that of OHe+He. The temperature T_0 at which OHe forms depends on its binding energy, which has been accurately evaluated as 1.175 MeV in Ref. [13], and corresponds approximately to $T_0 = 50$ keV. As the cosmological time t is related to the temperature through $t(s) \simeq \frac{1}{T^2(\mathrm{MeV})}$, processes (5.1) and (5.2) start at a time $t_0 \simeq \frac{1}{0.05^2} = 400$ s after the Big Bang and continue until helium freezes out at $t_* \simeq 10$ min $= 600$ s.

During these 200 s, the OHe atoms are consumed at a rate:

$$\frac{dn_{OHe}}{dt} = -3Hn_{OHe} - n_{OHe}^2\sigma_1 v_1 - n_{OHe}n_{He}\sigma_2 v_2, \tag{5.3}$$

where n_{OHe} and n_{He} are the number densities of OHe and He, $H = \frac{1}{2t}$ is the expansion rate of the Universe during the radiation-dominated era, σ_1 and σ_2 are the cross sections of processes (5.1) and (5.2) respectively and v_1 and v_2 are the OHe-OHe and OHe-He mean relative velocities. The first term in the right-hand side of equation (5.3) corresponds to the dilution in an expanding universe. The number of helium nuclei per comoving volume is assumed to be unaffected by reaction (5.2) since the abundance of helium is more than an order of magnitude higher than that of OHe, so that the only effect on n_{He} is due to the expansion:

$$\frac{dn_{He}}{dt} = -3Hn_{He}, \tag{5.4}$$

from which it follows that:

$$n_{He}(t) = n_{He}^0 \left(\frac{t_0}{t}\right)^{3/2}, \tag{5.5}$$

where n_{He}^0 is the number density of He at $t = t_0$ (In the following, we shall use a superscript 0 to denote quantities taken at the time of the decoupling of OHe, $t = t_0$).

To take into account the effect of the expansion and calculate the decrease of the fraction of free OHe atoms due to their inelastic reactions, we study the ratio f of the number density of OHe atoms to the number desity of He nuclei, $f = \frac{n_{OHe}}{n_{He}}$. From (5.3) and (5.4), its evolution is given by:

$$\frac{df}{dt} = -n_{He} f \left(\sigma_1 v_1 f + \sigma_2 v_2\right) \tag{5.6}$$

The capture cross sections σ_1 and σ_2 are of the order of the geometrical ones:

$$\sigma_1 \approx 4\pi \left(2r_{OHe}\right)^2, \tag{5.7}$$

$$\sigma_2 \approx 4\pi \left(r_{OHe} + r_{He}\right)^2, \tag{5.8}$$

where r_{OHe} is the Bohr radius of an OHe atom and r_{He} is the radius of a He nucleus. As both of them are approximately equal to 2 fm, $\sigma_1 \approx \sigma_2 \approx 64\pi\, 10^{-26}$ cm^2. As the OHe and He species are in thermal equilibrium with the plasma at temperature T, the mean relative velocities v_1 and v_2 are obtained from the Maxwell-Boltzmann velocity distributions of OHe and He and are given by:

$$v_1 = \sqrt{\frac{8T}{\pi\mu_1}}, \tag{5.9}$$

$$v_2 = \sqrt{\frac{8T}{\pi\mu_2}}, \tag{5.10}$$

where $\mu_1 = m_{OHe}/2$ and $\mu_2 \simeq m_{He}$ are the reduced masses of the OHe-OHe and OHe-He systems. $m_{OHe} = 1000$ GeV is the mass of an OHe atom, and $m_{He} = 3.73$ GeV that of a He nucleus. Given the time dependence of the temperature during the radiation-dominated era, $Tt^{1/2} = T_0 t_0^{1/2}$, one can use it to express the velocities (5.9) and (5.10) as functions of time and insert the resulting expressions together with (5.5) in equation (5.6) and get:

$$\frac{df}{dt} = -\gamma \frac{1}{t^{7/4}} f \left(\alpha f + \beta\right), \tag{5.11}$$

with

$$\alpha = \frac{\sigma_1}{\sqrt{\mu_1}}, \tag{5.12}$$

$$\beta = \frac{\sigma_2}{\sqrt{\mu_2}}, \tag{5.13}$$

$$\gamma = n^0_{He} t_0^{7/4} \sqrt{\frac{8T_0}{\pi}}. \tag{5.14}$$

The solution of (5.11) corresponding to the initial condition $f(t_0) = f_0$ is given by:

$$f(t) = \frac{\beta f_0}{\exp\left(\frac{4}{3}\beta\gamma\left(t_0^{-3/4} - t^{-3/4}\right)\right)(\alpha f_0 + \beta) - \alpha}. \tag{5.15}$$

The number density of He nuclei at the time of OHe formation, n^0_{He}, can be found from its value n^1_{He} today (In the following, the superscript 1 will denote quantities at the present time). Helium nuclei represent nowadays approximately 10% of all baryons, which have an energy density ρ^1_B of about 5% of the critical density ρ^1_c: $n^1_{He} \simeq 0.1 n^1_B = 0.1 \frac{\rho^1_B}{m_p} \simeq 0.1 \times 0.05 \frac{\rho^1_c}{m_p}$, where m_p is the mass of the proton. The present critical density is measured to be $\rho^1_c = 5.67 \times 10^{-6} m_p/\text{cm}^3$, so that $n^1_{He} \simeq 2.8 \times 10^{-8}$ cm^{-3}. As it was assumed that the He number density was not affected by reaction (5.2), the only effect between t_0 and now has been a dilution due to the expansion, and hence $n_{He} \propto \frac{1}{a^3} \propto T^3$, where a is the scale factor. Knowing that the temperature of the CMB today is $T_1 = 2.7$ K$= 2.33 \times 10^{-7}$ keV, this gives $n^0_{He} = n^1_{He}\left(\frac{T_0}{T_1}\right)^3 \simeq 2.8 \times 10^{-8}\left(\frac{50}{2.33\times 10^{-7}}\right)^3 \simeq 2.8 \times 10^{17}$ cm^{-3}.

At the time of OHe formation, all the O^{--} particles were in the form of OHe, i.e. the number density of O^{--} at $t = t_0$, n^0_O, was equal to that of OHe, n^0_{OHe}. Between t_0 and today, O^{--} particles may have been bound in different structures, but they have not been created or destroyed, so that their number density has only been diluted by the expansion in the same way as that of He nuclei, so that the ratio of the number density of O^{--} particles to the number density of He nuclei remains unchanged: $\frac{n^0_O}{n^0_{He}} = \frac{n^1_O}{n^1_{He}}$. Therefore, the initial fraction f_0 of OHe atoms can be calculated from present quantities: $f_0 = \frac{n^0_{OHe}}{n^0_{He}} = \frac{n^0_O}{n^0_{He}} = \frac{n^1_O}{n^1_{He}}$. n^1_O is obtained from the fact that O^{--} saturates the dark matter energy density, which represents about 25% of the critical density: $n^1_O \simeq 0.25 \frac{\rho^1_c}{m_O} \simeq 1.3 \times 10^{-9}$ cm^{-3}, where $m_O = 1$ TeV is the mass of O^{--}. With the previously calculated value of n^1_{He}, this gives $f_0 \simeq 0.05$.

We can now insert the numerical values into Eq. 5.15 and get the fraction of OHe atoms at the time of helium freeze-out $t = t_* = 600$ s:

$$f(t_*) \simeq 5 \times 10^{-6133} \ll f_0, \tag{5.16}$$

meaning that no OHe survives reactions (5.1) and (5.2). More precisely, most of the OHe atoms have captured He nuclei via process (5.2) and are now in the form of OBe. Indeed, the majority of the suppression of f comes from the exponential term present in (5.15), evaluated to be e^{14127}. The large argument of the exponential represents the number N_2 of reactions (5.2) that happened between t_0 and t_*, per OHe atom:

$$\begin{aligned}
N_2 &= \int_{t_0}^{t_*} n_{He}(t)\sigma_2 v_2(t)dt \\
&= n_{He}^0 t_0^{3/2}\sigma_2\sqrt{\frac{8T_0 t_0^{1/2}}{\pi\mu_2}}\int_{t_0}^{t_*}\frac{1}{t^{7/4}}dt \\
&= n_{He}^0 t_0^{7/4}\sqrt{\frac{8T_0}{\pi}}\frac{\sigma_2}{\sqrt{\mu_2}}\left(-\frac{4}{3}\right)\left(\frac{1}{t_*^{3/4}}-\frac{1}{t_0^{3/4}}\right) \\
&= \frac{4}{3}\beta\gamma\left(\frac{1}{t_0^{3/4}}-\frac{1}{t_*^{3/4}}\right),
\end{aligned}$$

where we have used (5.5), (5.10) and $Tt^{1/2} = T_0 t_0^{1/2}$ to pass from the first to the second line and the definitions (5.13) and (5.14) for the last line.

Therefore, the realization of the scenario of an OHe Universe implies a very strong suppression of reaction (5.2), corresponding to $N_2 \ll 1$. Such a suppression needs the development of a strong dipole Coulomb barrier in OHe-He interaction. The existence of this barrier and its effect is one of the most important open problems of the OHe model.

5.3 Problems of OBe "dark" matter

Due to Coulomb repulsion further helium capture by OBe is suppressed and one should expect that dark matter is mostly made of doubly charged OBe, which recombines with electrons in the period of recombination of helium at the temperature $T_{od} = 2\,\text{eV}$, before the beginning of matter dominance at $T_{RM} = 1\,\text{eV}$. It makes anomalous helium the dominant form of dark matter in this scenario. After recombination the OBe gas decouples from the plasma and from radiation and can play the role of a specific Warmer than Cold dark matter, since the adiabatic damping slightly suppresses density fluctuations at scales smaller than the scale of the horizon in the period of He recombination. The total mass of the OBe gas within the horizon in that period is given by analogy with the case of OHe [2,5] by

$$M_{od} = \frac{T_{RM}}{T_{od}} m_{Pl}\left(\frac{m_{Pl}}{T_{od}}\right)^2 \approx 2\cdot 10^{50}\,\text{g} = 10^{17} M_\odot, \tag{5.17}$$

where $M_\odot$ is the solar mass.

At momentum values of interest, one finds that elastic cross sections are significantly enhanced from their geometrical estimate. In the following, we shall use the estimate of Ref. [14], based on a compilation of results from general quantum mechanical scattering and from detailed quantum computations of hydrogen scattering [15]:

$$\sigma = 4\pi(\kappa r_0)^2,\ \kappa = 3-10, \tag{5.18}$$

with larger values of κ at low momentum.

For a size of OBe atoms equal to that of helium $r_0 = 3 \cdot 10^{-9}$ cm and one obtains an elastic scattering cross section on light elements of the order of $\sigma \approx 10^{-15}-10^{-14}$ cm^2. It makes this "dark matter" follow the ordinary baryonic matter in the process of galaxy formation, and makes it collisional on the scale of galaxies. This causes problems with the explanation of the observations of halo shapes [16]. Presence of OBe in stars can also influence nuclear processes, in particular helium burning in the red giants. The processes in stars can lead to the capture by OBe of additional nuclei, thus creating anomalous isotopes of elements with higher Z. OBe atoms can also be ionized in the Galaxy, but in the following we shall assume that neutral OBe atoms are the dominant part of this "dark matter" on Earth, considering also that slowing down anomalous nuclei in the atmosphere leads to ionization and their neutralisation through electron capture.

Falling down on Earth OBe atoms are slowed down and due to the atomic cross section of their collisions have a very low mobility. After they fall down to the terrestrial surface, the OBe atoms are further slowed down by their elastic collisions with matter. They drift, sinking down towards the center of the Earth with velocity

$$V = \frac{g}{n\sigma v} \leq 2.710^{-11} \text{ cm/s} \approx 270 \text{ fm/s}. \quad (5.19)$$

Here n is the number density of terrestrial atoms, σv is the rate of atomic collisions, taken at room temperature, and $g = 980$ cm/s^2. We assimilated the crust of the Earth as made of SiO_2, and got the number density to be $n = 0.27\ 10^{23}$ molecules/cm^3. Using (5.18), and taking the geometrical radius to be that of SiO_2, *i.e.* $r_0 \approx 2$ Å, we obtained $\sigma \geq 4.5\ 10^{-14}$ cm^2, and for collisions on SiO_2 $v \approx 3\ 10^4$cm/s.

The OBe abundance in the Earth is determined by the equilibrium between the in-falling and down-drifting fluxes. The in-falling O-helium flux from dark matter halo is given by [4]

$$F = \frac{n_0}{8\pi} \cdot |\overline{V_h} + \overline{V_E}|,$$

where V_h is the speed of the Solar System (220 km/s), V_E the speed of the Earth (29.5 km/s) and $n_0 = 3 \cdot 10^{-4}$ cm^{-3} is the assumed local density of OBe dark matter (for an OBe of mass 1 TeV). Furthermore, for simplicity, we didn't take into account the annual modulation of the incoming flux and take $|\overline{V_h} + \overline{V_E}| = u \approx 300$ km/s.

The equilibrium concentration of OBe, which is established in the matter consisting of atoms with number density n, is given by [4]

$$n_{oE} = \frac{2\pi \cdot F}{V}, \quad (5.20)$$

and the ratio of anomalous helium isotopes to the total amount of SiO_2 is given by

$$r_{oE} = \frac{n_{oE}}{n} = \frac{2\pi \cdot F\sigma v}{g} \geq 3.1\ 10^{-9}, \quad (5.21)$$

being independent of the atomic number density of the matter. Note that the migration rate (and the dilution) considered here is of larger than that observed at the Oklo site for heavy elements [18].

The upper limits on the anomalous helium abundance are very stringent [17] $r_{oE} \leq 10^{-19}$, and our rough estimate is ten orders of magnitude too large. Together with other problems of OBe Universe stipulated above, this rules out the OBe scenario.

5.4 Conclusion

The advantages of the OHe composite-dark-matter scenario is that it is minimally related to the parameters of new physics and is dominantly based on the effects of known atomic and nuclear physics. However, the full quantum treatment of this problem turns out to be rather complicated and remains an open.

We have considered here the scenario in which such a barrier does not appear. This leads to a significant role of inelastic reaction of OHe, and strongly modifies the main features of the OHe scenario. In the period of Big Bang Nucleosynthesis, when OHe is formed, it captures an additional He nucleus, so that the dominant ' form of dark matter becomes charged, recombining with electrons in anomalous isotopes of helium and heavier elements. Over-abundance of anomalous isotopes in terrestrial matter seems to be unavoidable in this case.

This makes the full solution of OHe nuclear physics, started in [12], vital. The answer to the possibility of the creation of a dipole Coulomb barrier in OHe interaction with nuclei is crucial. Without that barrier one gets no suppression of inelastic reactions, in which O^{--} binds with nuclei. These charged species form atoms (or ions) with atomic cross sections, and that strongly suppresses their mobility in terrestrial matter, leading to their storage and over-abundance near the Earth's surface and oceans.

Hence, the model cannot work if no repulsive interaction appears at some distance between OHe and the nucleus, and the solution to this open question of OHe nuclear physics is vital for the composite-dark-matter OHe scenario.

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6 Notoph-Graviton-Photon Coupling

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Abstract. In the sixties Ogievetskiĭ and Polubarinov proposed the concept of *notoph*, whose helicity properties are complementary to those of *photon*. Later, Kalb and Ramond (and others) developed this theoretical concept. And, at the present times it is widely accepted. We analyze the quantum theory of antisymmetric tensor fields with taking into account mass dimensions of notoph and photon. It appears to be possible to describe both photon and notoph degrees of freedom on the basis of the modified Bargmann-Wigner formalism for the symmetric second-rank spinor.

Next, we proceed to derive equations for the symmetric tensor of the second rank on the basis of the Bargmann-Wigner formalism in a straightforward way. The symmetric multispinor of the fourth rank is used. It is constructed out of the Dirac 4-spinors. Due to serious problems with the interpretation of the results obtained on using the standard procedure we generalize it, and we obtain the spin-2 relativistic equations, which are consistent with the general relativity. The importance of the 4-vector field (and its gauge part) is pointed out.

Thus, we present the full theory which contains the photon, the notoph (the Kalb-Ramond field) and the graviton. The relations of this theory with the higher spin theories are established. In fact, we deduced the gravitational field equations from relativistic quantum mechanics. The relations of this theory with scalar-tensor theories of gravitation and $f(R)$ are discussed. We estimate possible interactions, fermion-notoph, graviton-notoph, photon-notoph, and we conclude that they will be probably seen in experiments in the next few years.

Povzetek. V šestdesetih letih sta Ogievetskiĭ in Polubarinov predlagala delec z lastnostmi, komplementarnimi fotonu. Poimenovala sta ga *notof*. Lastnosti delca *notof* sta kasneje študirala in dopolnila tudi Kalb in Ramond ter drugi. Zdaj lastnosti tega delca študirajo številni teoretiki.

Avtor prispevka analizira kvantno teorijo antisimetričnih tenzorskih polj in dopusti, da nosita foton in notof od nič različno maso. Izkazalo se je, da nekoliko spremenjen formalizem za simetrične spinorje ranga 2 avtorjev Bargmanna-Wignerja opiše tudi foton in notof.

Uspe mu preprosta izpeljeva enačbe gibanja za simetrični tenzor ranga 2 iz Bargmann-Wignerjevega formalizma. Uporabi simetrični multispinor ranga štiri, ki ga izvede iz Diracovih štiri spinorjev. Standardni postopek posploši, ker so težave z interpretacijo. Posplošitev ga pripelje do relativistične enačbe za spin 2, ki je skladna s splošno teorijo relativnosti. Avtor poudari pomen polja 4-vektorjev (in njegovega umeritvenega dela).

Avtor predstavi celovito teorijo, ki vsebuje foton, notof (Kalb-Ramondovo polje) in graviton. Osvetli relacijo te teorije s teorijami z višjimi spini. Iz relativistične kvantne

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mehanike izpelje enačbe za gravitacijsko polje. Diskutira o povezavi te teorije s skalarno-tenzorskimi in teorijami gravitacije $f(R)$, oceni interakcije med fermionom in notofom, med gravitonom in notofom ter med fotonom in notofom ter napove, da bodo najbrž opazili te interakcije pri poskusih v prihodnjih nekaj letih.

6.1 Introduction

In the series of the papers [1–5], cf. with Refs. [6–8], we tried to find connection between the theory of the quantized antisymmetric tensor (AST) field of the second rank (and that of the corresponding 4-vector field) with the $2(2s+1)$ Weinberg-Tucker-Hammer formalism [9,10].

Several previously published works [11–16], introduced the concept of the notoph (the Kalb-Ramond field) which is constructed on the basis of the antisymmetric tensor "potentials". It represents itself the non-trivial spin-0 field. The well-known textbooks [17–20] did *not* discuss the problems, whether the massless quantized AST field and the quantized 4-vector field are transverse or longitudinal fields (in the sense if the helicity $h = \pm 1$ or $h = 0$)? can the electromagnetic potential be a 4-vector in a quantized theory? how should the massless limit be taken? and many other fundamental problems of the physics of bosons. In my opinion, the most rigorous works are refs. [22,9,23,21], but it is not easy to extract corresponding answers even from them.

First of all, we note that 1) "...In natural units ($c = \hbar = 1$) ... a lagrangian density, since the action is dimensionless, has dimension of [energy]^4"; 2) One can always renormalize the lagrangian density and "one can obtain the same equations of motion... by substituting $L \to (1/M^N)L$, where M is an arbitrary energy scale", cf. [2]; 3) the right physical dimension of the field strength tensor $F^{\mu\nu}$ is [energy]^2; "the transformation $F^{\mu\nu} \to (1/2M)F^{\mu\nu}$ [which was regarded in Ref. [5]] ... requires a more detailed study ... [because] the transformation above changes its physical dimension: it is not a simple normalization transformation". Furthermore, in the first papers on the notoph [12–14][1] the authors used the normalization of the 4-vector F^μ field[2] to [energy]^2 and, hence, the antisymmetric tensor "potentials" $A^{\mu\nu}$, to [energy]^1. We try to discuss these problems on the basis of the generalized Bargmann-Wigner formalism [22]. Thus, the Proca and Maxwell formalisms are generalized, see, e. g., Ref. [24].

In the Sections 6.3 and 6.4 we consider the spin-2 equations. The general scheme for derivation of higher-spin equations has been given in [22]. A field of the rest mass m and the spin $s \geq \frac{1}{2}$ is represented by a completely symmetric multispinor of rank $2s$. The particular cases $s = 1$ and $s = \frac{3}{2}$ have been considered in the textbooks, e. g., Ref. [17]. The spin-2 case can also be of some interest because we can believe that the essential features of the gravitational field are obtained from transverse components of the $(2,0) \oplus (0,2)$ representation of the Lorentz group. Nevertheless, questions of the redundant components of the higher-spin relativistic equations are not yet understood in detail [25].

[1] It is also known as the longitudinal Kalb-Ramond field, but the consideration of Ogievetskiĭ and Polubarinov permits to study the $m \to 0$ procedure.

[2] It is well known that it is related to the third-rank antisymmetric field tensor.

In the last Sections we discuss the questions of interactions.

6.2 Photon-Notoph Equations

For spin 1 we start from[3]

$$[\gamma_{\alpha\beta}p_\alpha p_\beta + Ap_\alpha p_\alpha + Bm^2]\Psi = 0\,, \tag{6.1}$$

where $p_\mu = -i\partial_\mu$ and $\gamma_{\alpha\beta}$ are the Barut-Muzinich-Williams covariantly - defined 6×6 matrices, $\sum_\mu \gamma_{\mu\mu} = 0$. The determinant of $[\gamma_{\alpha\beta}p_\alpha p_\beta + Ap_\alpha p_\alpha + Bm^2]$ is of the 12th order in p_μ. If we are interested in solutions with $E^2 - \mathbf{p}^2 = m^2$, $c = \hbar = 1$, they can be obtained on using the constraints in the above equation:

$$\frac{B}{A+1} = 1\,, \quad \frac{B}{A-1} = 1\,. \tag{6.2}$$

We may also have the tachyonic solutions, etc. The particular cases are:

- $A = 0, B = 1 \Leftrightarrow$ we have the Weinberg's equation for $s = 1$ with 3 solutions $E = +\sqrt{\mathbf{p}^2 + m^2}$, 3 solutions $E = -\sqrt{\mathbf{p}^2 + m^2}$, 3 solutions $E = +\sqrt{\mathbf{p}^2 - m^2}$ and 3 solutions $E = -\sqrt{\mathbf{p}^2 - m^2}$. Tachyonic solutions have been reformulated in various ways, for instance, as the ones leading to the spontaneous symmetry breaking, and to the non-zero quantum vacuum.
- $A = 1, B = 2 \Leftrightarrow$ we have the Tucker-Hammer equation for $s = 1$. The solutions are with $E = \pm\sqrt{\mathbf{p}^2 + m^2}$ *only*.

Thus, the addition of the Klein-Gordon equation to (6.1) may change the physical content even on the *free* level.

What are the corresponding equations for the antisymmetric tensor field? They can be the Proca equations in the massive case, and the Maxwell equations in the massless case. We have shown in Refs. [1,2] that one can obtain *four* different equations for antisymmetric tensor fields from the Weinberg's $2(2s+1)-$ component formalism. First of all, we note that Ψ is, in fact, bivector, $\mathbf{E}_i = -iF_{4i}$, $\mathbf{B}_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$,, or $\mathbf{E}_i = -\frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}$, $\mathbf{B}_i = -i\tilde{F}_{4i}$, or their combinations. One can separate the four cases:

- $\Psi^{(I)} = \begin{pmatrix}\mathbf{E}+i\mathbf{B}\\ \mathbf{E}-i\mathbf{B}\end{pmatrix}$, $P = -1$, where $\mathbf{E}_i$ and $\mathbf{B}_i$ are the components of the tensor.
- $\Psi^{(II)} = \begin{pmatrix}\mathbf{B}-i\mathbf{E}\\ \mathbf{B}+i\mathbf{E}\end{pmatrix}$, $P = +1$, where $\mathbf{E}_i$, $\mathbf{B}_i$ are the components of the tensor.
- $\Psi^{(III)} = \Psi^{(I)}$, but (!) $\mathbf{E}_i$ and $\mathbf{B}_i$ are the corresponding vector and axial-vector components of the *dual* tensor $\tilde{F}_{\mu\nu}$.
- $\Psi^{(IV)} = \Psi^{(II)}$, where $\mathbf{E}_i$ and $\mathbf{B}_i$ are the components of the *dual* tensor $\tilde{F}_{\mu\nu}$.

[3] In the classic works on this formalism the authors worked in the Euclidean metrics. However, there is no any problem to write the equations and other formulas in the pseudo-Euclidean metrics accustomed today; just change the sign of $p_\mu p_\mu$, and other products.

The mappings of the WTH equations are:

$$\partial_\alpha\partial_\mu F^{(I)}_{\mu\beta} - \partial_\beta\partial_\mu F^{(I)}_{\mu\alpha} + \frac{A-1}{2}\partial_\mu\partial_\mu F^{(I)}_{\alpha\beta} - \frac{B}{2}m^2 F^{(I)}_{\alpha\beta} = 0\,, \quad (6.3)$$

$$\partial_\alpha\partial_\mu F^{(II)}_{\mu\beta} - \partial_\beta\partial_\mu F^{(II)}_{\mu\alpha} - \frac{A+1}{2}\partial_\mu\partial_\mu F^{(II)}_{\alpha\beta} + \frac{B}{2}m^2 F^{(II)}_{\alpha\beta} = 0\,, \quad (6.4)$$

$$\partial_\alpha\partial_\mu \tilde{F}^{(III)}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}^{(III)}_{\mu\alpha} - \frac{A+1}{2}\partial_\mu\partial_\mu \tilde{F}^{(III)}_{\alpha\beta} + \frac{B}{2}m^2 \tilde{F}^{(III)}_{\alpha\beta} = 0\,, \quad (6.5)$$

$$\partial_\alpha\partial_\mu \tilde{F}^{(IV)}_{\mu\beta} - \partial_\beta\partial_\mu \tilde{F}^{(IV)}_{\mu\alpha} + \frac{A-1}{2}\partial_\mu\partial_\mu \tilde{F}^{(IV)}_{\alpha\beta} - \frac{B}{2}m^2 \tilde{F}^{(IV)}_{\alpha\beta} = 0\,. \quad (6.6)$$

In the Tucker-Hammer case ($A = 1, B = 2$) we can recover the Proca theory from (6.3):

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = m^2 F_{\alpha\beta}\,, \quad (6.7)$$

($A_\nu = \frac{1}{m^2}\partial_\alpha F_{\alpha\nu}$ should be substituted in $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the result is multiplied by m^2).

We also noted that the massless limit of this theory does *not* coincide with the Maxwell theory in some cases, while it contains the latter as a particular case. In [3,5,30] we showed that it is possible to define various massless limits for the Duffin-Kemmer-Proca theory. Another one is the Ogievetskiĭ-Polubarinov *notoph* (which is also called the Kalb-Ramond field), Ref. [12] in the US literature. The transverse components of the AST field can be removed from the corresponding Lagrangian by means of the "new gauge transformation" $A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu$, with the vector gauge function Λ_μ.

The second (II) case is

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = [\partial_\mu\partial_\mu - m^2]F_{\alpha\beta}\,. \quad (6.8)$$

So, on the mass shell we have $[\partial_\mu\partial_\mu - m^2]F_{\alpha\beta} = 0$, and, hence,

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = 0\,, \quad (6.9)$$

which rather corresponds to the Maxwell-like case. However, if we calculate dispersion relations for the second case, Eq. (6.9), it appears that the equation has solutions even if $m \neq 0$.

Now we are interested in the *parity-violating* equations for antisymmetric tensor fields. We investigate the most general mapping of the Weinberg-Tucker-Hammer formulation to the antisymetric tensor field formulation too. Instead of $\Psi^{(I-IV)}$ we shall try to use now

$$\Psi^{(A)} = \begin{pmatrix}\mathbf{E} + i\mathbf{B}\\ \mathbf{B} + i\mathbf{E}\end{pmatrix} = \frac{1+\gamma^5}{2}\Psi^{(I)} + \frac{1-\gamma^5}{2}\Psi^{(II)}\,. \quad (6.10)$$

As a result, the equation for the AST fields is

$$\partial_\alpha\partial_\mu F_{\mu\beta} - \partial_\beta\partial_\mu F_{\mu\alpha} = \frac{1}{2}(\partial_\mu\partial_\mu)F_{\alpha\beta} + [-\frac{A}{2}(\partial_\mu\partial_\mu) + \frac{B}{2}m^2]\tilde{F}_{\alpha\beta}\,. \quad (6.11)$$

Of course, $\Psi^{(A)'} = \begin{pmatrix} \mathbf{B} - i\mathbf{E} \\ \mathbf{E} - i\mathbf{B} \end{pmatrix} = -i\Psi^{(A)}$, and the equation is unchanged. The different choice is

$$\Psi^{(B)} = \begin{pmatrix} \mathbf{E} + i\mathbf{B} \\ -\mathbf{B} - i\mathbf{E} \end{pmatrix} = \frac{1+\gamma^5}{2}\Psi^{(I)} - \frac{1-\gamma^5}{2}\Psi^{(II)} \,. \tag{6.12}$$

Thus, one has

$$\partial_\alpha \partial_\mu F_{\mu\beta} - \partial_\beta \partial_\mu F_{\mu\alpha} = \frac{1}{2}(\partial_\mu \partial_\mu) F_{\alpha\beta} + [\frac{A}{2}(\partial_\mu \partial_\mu) - \frac{B}{2} m^2] \tilde{F}_{\alpha\beta} \,. \tag{6.13}$$

Of course, one can also use the dual tensor ($\mathbf{E}^i = -\frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}$ and $\mathbf{B}^i = -i\tilde{F}_{4i}$) and obtain analogous equations:

$$\partial_\alpha \partial_\mu \tilde{F}_{\mu\beta} - \partial_\beta \partial_\mu \tilde{F}_{\mu\alpha} = \frac{1}{2}(\partial_\mu \partial_\mu) \tilde{F}_{\alpha\beta} + [-\frac{A}{2}(\partial_\mu \partial_\mu) + \frac{B}{2} m^2] F_{\alpha\beta} \,, \tag{6.14}$$

$$\partial_\alpha \partial_\mu \tilde{F}_{\mu\beta} - \partial_\beta \partial_\mu \tilde{F}_{\mu\alpha} = \frac{1}{2}(\partial_\mu \partial_\mu) \tilde{F}_{\alpha\beta} + [\frac{A}{2}(\partial_\mu \partial_\mu) - \frac{B}{2} m^2] F_{\alpha\beta} \,. \tag{6.15}$$

They are connected with (6.11,6.13) by the dual transformations.

The states corresponding to the new functions $\Psi^{(A)}$, $\Psi^{(B)}$ etc are *not* the parity eigenstates. So, it is not surprising that we have $F_{\alpha\beta}$ and its dual $\tilde{F}_{\alpha\beta}$ in the same equations. In total we have already eight equations.

One can also consider the most general case

$$\Psi^{(W)} = \begin{pmatrix} aF_{4i} + b\tilde{F}_{4i} + c\epsilon_{ijk}F_{jk} + d\epsilon_{ijk}\tilde{F}_{jk} \\ eF_{4i} + f\tilde{F}_{4i} + g\epsilon_{ijk}F_{jk} + h\epsilon_{ijk}\tilde{F}_{jk} \end{pmatrix} \,. \tag{6.16}$$

So, we shall have dynamical equations for $F_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ with additional parameters $a, b, c, d, \ldots \in \mathbf{C}$. We have a lot of antisymmetric tensor fields here. However,

- the covariant form preserves if there are some restrictions on the parameters, only. Alternatively, we have some additional terms of ∂_4^2 or ∇^2;
- both $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ are present in the equations;
- under the definite choice of $a, b, c, d \ldots$ the equations can be reduced to the above equations for the tensor $\mathcal{H}_{\mu\nu}$ and its dual:

$$\mathcal{H}_{\mu\nu} = c_1 F_{\mu\nu} + c_2 \tilde{F}_{\mu\nu} + \frac{c_3}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} + \frac{c_4}{2}\epsilon_{\mu\nu\alpha\beta}\tilde{F}_{\alpha\beta} \,; \tag{6.17}$$

- the parity properties of $\Psi^{(W)}$ are very complicated.

Anther way of constructing the equations of high-spin particles has been given in [22,17].[4] Bargmann and Wigner claimed explicitly that they constructed

[4] On can also obtain the $s = 0$ Kemmer equations on using the Bargmann-Wigner procedure. One should use the *antisymmetric* second-rank multispinor in this case.

$(2s+1)$ states.[5] Below we present the standard Bargmann-Wigner formalism for the spin $s = 1$ (and turn to the standard pseudo-Euclidean metric):

$$[i\gamma^\mu \partial_\mu - m]_{\alpha\beta} \Psi_{\beta\gamma} = 0\,, \tag{6.18}$$
$$[i\gamma^\mu \partial_\mu - m]_{\gamma\beta} \Psi_{\alpha\beta} = 0\,, \tag{6.19}$$

If one has

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta} A_\mu + (\sigma^{\mu\nu} R)_{\alpha\beta} F_{\mu\nu}\,, \tag{6.20}$$

with[6]

$$R = e^{i\varphi} \begin{pmatrix} \Theta & 0 \\ 0 & -\Theta \end{pmatrix} \qquad \Theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{6.25}$$

in the spinorial representation of γ-matrices, we obtain the Duffin-Kemmer-Proca equations:

$$\partial^\alpha F_{\alpha\mu} = \frac{m}{2} A_\mu\,, \tag{6.26}$$
$$2m F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{6.27}$$

In order to obtain these equations one should add the equations (6.18,6.19) and compare functional coefficients at the corresponding commutators, see Ref. [17]. After the corresponding re-normalization $A_\mu \rightarrow 2m A_\mu$ (or $F^{\mu\nu} \rightarrow (1/2m) F^{\mu\nu}$), we obtain the standard textbook set:

$$\partial^\alpha F_{\alpha\mu} = m^2 A_\mu\,, \tag{6.28}$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{6.29}$$

It gives the equation (6.7) for the antisymmetric tensor field. Of course, one can investigate other sets of equations with different normalization of the $F_{\mu\nu}$ and A_μ fields. Are all these sets of equations equivalent? As we see, to answer this question is not trivial. It was argued that the physical normalization is such that in the massless limit the zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). Moreover, we advocate the following approach: the massless limit can and must be taken in the end of all calculations only, i. e., for physical quantities.

How can one obtain other equations following from the Weinberg-Tucker-Hammer approach? The recipe for the third equation is simple: use, instead of $(\sigma^{\mu\nu} R) F_{\mu\nu}$, another symmetric matrix $(\gamma^5 \sigma^{\mu\nu} R) F_{\mu\nu}$.

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[5] The Weinberg-Tucker-Hammer theory has essentially $2(2s+1)$ components.

[6] The reflection operator R has the properties

$$R^T = -R\,, \quad R^\dagger = R = R^{-1}\,, \tag{6.21}$$
$$R^{-1} \gamma^5 R = (\gamma^5)^T\,, \tag{6.22}$$
$$R^{-1} \gamma^\mu R = -(\gamma^\mu)^T\,, \tag{6.23}$$
$$R^{-1} \sigma^{\mu\nu} R = -(\sigma^{\mu\nu})^T\,. \tag{6.24}$$

After taking into account the above observations let us repeat the procedure of derivation of the Proca equations from the Bargmann-Wigner equations for a *symmetric* second-rank spinor. However, we now use

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(c_a m A_\mu + c_f F_\mu) + (\sigma^{\mu\nu} R)_{\alpha\rho}(c_A m(\gamma^5)_{\rho\beta} A_{\mu\nu} + c_F I_{\rho\beta} F_{\mu\nu}) \,, \tag{6.30}$$

with the same R and Θ as above. Matrices γ^μ are again chosen in the Weyl (spinorial) representation, i.e., γ^5 is assumed to be diagonal. Constants c_i are some numerical dimensionless coefficients. The properties of the reflection operator R are necessary for the expansion (6.30) to be possible in such a form, i.e., in order to have the $\gamma^\mu R$, $\sigma^{\mu\nu} R$ and $\gamma^5 \sigma^{\mu\nu} R$ to be *symmetric* matrices.

The substitution of the above expansion into the Bargmann-Wigner equations, Ref. [17], gives us the new Proca-like equations:

$$c_a m(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_f(\partial_\mu F_\nu - \partial_\nu F_\mu) = i c_A m^2 \epsilon_{\alpha\beta\mu\nu} A^{\alpha\beta} + 2 m c_F F_{\mu\nu} \,, \tag{6.31}$$
$$c_a m^2 A_\mu + c_f m F_\mu = i c_A m \epsilon_{\mu\nu\alpha\beta} \partial^\nu A^{\alpha\beta} + 2 c_F \partial^\nu F_{\mu\nu} \,. \tag{6.32}$$

In the case $c_a = 1$, $c_F = \frac{1}{2}$ and $c_f = c_A = 0$ they are reduced to the ordinary Proca equations.[7] In the general case we obtain dynamical equations which connect the photon, the notoph and their potentials. The divergent (in $m \to 0$) parts of field functions and those of dynamical variables should be removed by corresponding gauge (or Kalb-Ramond gauge) transformations. It is well known that the notoph massless field is considered to be the pure longitudinal field after one takes into account $\partial_\mu A^{\mu\nu} = 0$. Apart from these dynamical equations we can obtain a number of constraints by means of the subtraction of the equations of the Bargmann-Wigner system (instead of the addition as for (6.31,6.32)). They read

$$m c_a \partial^\mu A_\mu + c_f \partial^\mu f_\mu = 0 \,, \tag{6.33}$$
$$m c_A \partial^\alpha A_{\alpha\mu} + \frac{i}{2} c_F \epsilon_{\alpha\beta\nu\mu} \partial^\alpha F^{\beta\nu} = 0, \tag{6.34}$$

that suggests $\widetilde{F}^{\mu\nu} \sim i m A^{\mu\nu}$ and $f^\mu \sim m A^\mu$, as in [12].

Thus, after the suitable choice of the dimensionless coefficients c_i the Lagrangian density for the photon-notoph field can be proposed:

$$\mathcal{L} = \mathcal{L}^{\rm Proca} + \mathcal{L}^{\rm Notoph} = -\frac{1}{8} F_\mu F^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \\ + \frac{m^2}{2} A_\mu A^\mu + \frac{m^2}{4} A_{\mu\nu} A^{\mu\nu} \,, \tag{6.35}$$

The limit $m \to 0$ may be taken for dynamical variables, in the end of calculations only.

[7] We still note that the division by m in the first equation is *not* the well-defined operation in the case if someone is interested in the subsequent limiting procedure $m \to 0$. Probably, in order to avoid this obscure point one may wish to write the Dirac equations in the form $[(i\gamma^\mu \partial_\mu)/m - I]\, \psi(x) = 0$, which follows straightforwardly in the derivation of the Dirac equation on the basis of the Ryder relation [7] and the Wigner rules for the boosts of the field functions from the zero-momentum frame.

Furthermore, it is logical to introduce the normalization scalar field $\varphi(x)$, and consider the expansion:

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(\varphi A_\mu) + (\sigma^{\mu\nu}R)_{\alpha\beta}F_{\mu\nu}\,. \tag{6.36}$$

Then, we arrive at the following set

$$2mF_{\mu\nu} = \varphi(\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu\varphi)A_\nu - (\partial_\nu\varphi)A_\mu\,, \tag{6.37}$$

$$\partial^\nu F_{\mu\nu} = \frac{m}{2}(\varphi A_\mu)\,, \tag{6.38}$$

which in the case of the constant scalar field $\varphi = 2m$ can also be reduced to the system of the Proca equations. The additional constraints are

$$(\partial^\mu\varphi)A_\mu + \varphi(\partial^\mu A_\mu) = 0\,, \tag{6.39}$$

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0\,. \tag{6.40}$$

At the moment it is not yet obvious, how can we account for other equations in the $(1,0)\oplus(0,1)$ representation, e.g. [7b], rigorously. For instance, one can wish to seek the generalization of the Proca equations on the basis of the introduction of two mass parameters m_1 and m_2. But, when we apply the BW procedure to the Dirac equations we cannot obtain new physical content. Another equation in the $(1/2,0)\oplus(0,1/2)$ representation was discussed in Ref. [26]. It has the form:

$$\left[i\gamma^\mu\partial_\mu - m_1 - \gamma^5 m_2\right]\Psi(x) = 0\,. \tag{6.41}$$

The Bargmann-Wigner procedure for this system of equations (which include the γ^5 matrix in the mass term) yields:

$$2m_1F^{\mu\nu} + 2im_2\widetilde{F}^{\mu\nu} = \varphi(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\partial^\mu\varphi)A^\nu - (\partial^\nu\varphi)A^\mu\,, \tag{6.42}$$

$$\partial^\nu F_{\mu\nu} = \frac{m_1}{2}(\varphi A_\mu), \tag{6.43}$$

with the constraints

$$(\partial^\mu\varphi)A_\mu + \varphi(\partial^\mu A_\mu) = 0\,, \tag{6.44}$$

$$\partial^\nu\widetilde{F}_{\mu\nu} = \frac{im_2}{2}(\varphi A_\mu)\,. \tag{6.45}$$

In general, we can now use the four different mass parameters in the equations which are analogous to (6.18,6.19). However, the equality of mass factors[8] ($m_1^{(1)} = m_1^{(2)}$ and $m_2^{(1)} = m_2^{(2)}$) is obtained as necessary conditions in the process of calculations in the system of the Dirac-like equations.

In fact, the results of this paper develop the old results of Ref. [12]. According to [12, Eqs.(9,10)] we proceed in constructing the "potentials" for the notoph as follows:[9]

$$A_{\mu\nu}(\mathbf{p}) = N\left[\epsilon_\mu^{(1)}(\mathbf{p})\epsilon_\nu^{(2)}(\mathbf{p}) - \epsilon_\nu^{(1)}(\mathbf{p})\epsilon_\mu^{(2)}(\mathbf{p})\right]\,. \tag{6.46}$$

[8] Here, the superscripts (1) and (2) refers to the first and the second equations, respectively, in the modified Bargmann-Wigner system.

[9] The notation is that of Ref. [12] here.

We use explicit forms for the polarization vectors (e.g., Refs. [21] and [5, formulas(15a,b)]) boosted to the momentum $\mathbf{p}$:

$$\epsilon^{\mu}(\mathbf{0},+1) = -\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}, \epsilon^{\mu}(\mathbf{0},0) = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \epsilon^{\mu}(\mathbf{0},-1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}, \quad (6.47)$$

and ($\widehat{p}_i = p_i/|\mathbf{p}|$, $\gamma = E_p/m$), Ref. [21, p.68] or Ref. [19, p.108],

$$\epsilon^{\mu}(\mathbf{p},\sigma) = L^{\mu}{}_{\nu}(\mathbf{p})\epsilon^{\nu}(\mathbf{0},\sigma)\,, \quad (6.48)$$

$$L^{0}{}_{0}(\mathbf{p}) = \gamma\,, \quad L^{i}{}_{0}(\mathbf{p}) = L^{0}{}_{i}(\mathbf{p}) = \widehat{p}_i\sqrt{\gamma^2-1}\,, \quad (6.49)$$

$$L^{i}{}_{k}(\mathbf{p}) = \delta_{ik} + (\gamma-1)\widehat{p}_i\widehat{p}_k\,. \quad (6.50)$$

N, the normalization factor, should be taken into account for possible analyses of propagators and massless limits. After substitutions in the definition (6.46) one obtains

$$A^{\mu\nu}(\mathbf{p}) = \frac{iN^2}{m}\begin{pmatrix} 0 & -p_2 & p_1 & 0 \\ p_2 & 0 & m+\frac{p_r p_l}{p_0+m} & \frac{p_2 p_3}{p_0+m} \\ -p_1 & -m-\frac{p_r p_l}{p_0+m} & 0 & -\frac{p_1 p_3}{p_0+m} \\ 0 & -\frac{p_2 p_3}{p_0+m} & \frac{p_1 p_3}{p_0+m} & 0 \end{pmatrix}, \quad (6.51)$$

i.e., it coincides with the longitudinal components of the antisymmetric tensor obtained in Refs. [7a,Eqs.(2.14,2.17)] and [5, Eqs.(17b,18b)] within the normalization and different forms of the spin basis. The $A_{\mu\nu}(\mathbf{p})$ potential reduces to zero in the limiting case ($m \to 0$) under appropriate choice of the normalization $N = m^{\alpha}$, $\alpha > 1/2$. If $N = \sqrt{m}$ this reduction of the non-transverse state occurs if a $s = 1$ particle moves along with the third axis OZ.[10] It is also useful to compare Eq. (6.51) with the formula (B2) in Ref. [8] in order to think about correct procedures for taking the massless limits.

Next, the Tam-Happer experiments [27] on two laser beams interaction did not find satisfactory explanation in the framework of the ordinary QED (at least, their explanation is complicated by huge technical calculations). On the other hand, in Ref. [28] a very interesting model has been proposed. It is based on gauging the Dirac field on using the coordinate-dependent parameters $\alpha_{\mu\nu}(x)$ in

$$\psi(x) \to \psi'(x') = \Omega\psi(x)\,, \quad \Omega = \exp\left[\frac{i}{2}\sigma^{\mu\nu}\alpha_{\mu\nu}(x)\right], \quad (6.52)$$

and, thus, the second "photon" was introduced. The compensating 24-component (in general) field $B_{\mu,\nu\lambda}$ reduces to the 4-vector field as follows (the notation of [28] is used here):

$$B_{\mu,\nu\lambda} = \frac{1}{4}\epsilon_{\mu\nu\lambda\sigma}a_{\sigma}(x)\,. \quad (6.53)$$

[10] But, even in this case we cannot have a good behaviour of the 4-vector fields/potentials in the massless limit in the instant form of the relativistic dynamics, cf. [8].

As readily seen, after comparison of these formulas with those of Refs. [12–14], the second photon is nothing more than the Ogievetskiĭ-Polubarinov *notoph* within the normalization. Parity properties are dependent not only on the explicit forms of the momentum-space field functions of the $(1/2, 1/2)$ representation, but also on the properties of corresponding creation/annihilation operators. Helicity properties depend on the normalization.

6.3 The Standard Bargmann-Wigner Formalism Applied for Spin 2

In this Section we use the commonly-accepted procedure for the derivation of higher-spin equations [22]. We begin with the equations for the 4-rank symmetric spinor:

$$[i\gamma^\mu\partial_\mu - m]_{\alpha\alpha'}\Psi_{\alpha'\beta\gamma\delta} = 0\,, \tag{6.54}$$

$$[i\gamma^\mu\partial_\mu - m]_{\beta\beta'}\Psi_{\alpha\beta'\gamma\delta} = 0\,, \tag{6.55}$$

$$[i\gamma^\mu\partial_\mu - m]_{\gamma\gamma'}\Psi_{\alpha\beta\gamma'\delta} = 0\,, \tag{6.56}$$

$$[i\gamma^\mu\partial_\mu - m]_{\delta\delta'}\Psi_{\alpha\beta\gamma\delta'} = 0\,. \tag{6.57}$$

The massless limit (if one needs) should be taken in the end of all calculations.

We proceed expanding the field function in the set of symmetric matrices (as in the spin-1 case, cf. Ref. [5]). In the beginning let us use the first two indices:[11]

$$\Psi_{\{\alpha\beta\}\gamma\delta} = (\gamma_\mu R)_{\alpha\beta}\Psi^\mu_{\gamma\delta} + (\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta}\,. \tag{6.58}$$

We would like to write the corresponding equations for functions $\Psi^\mu_{\gamma\delta}$ and $\Psi^{\mu\nu}_{\gamma\delta}$ in the form:

$$\frac{2}{m}\partial_\mu\Psi^{\mu\nu}_{\gamma\delta} = -\Psi^\nu_{\gamma\delta}\,, \tag{6.59}$$

$$\Psi^{\mu\nu}_{\gamma\delta} = \frac{1}{2m}\left[\partial^\mu\Psi^\nu_{\gamma\delta} - \partial^\nu\Psi^\mu_{\gamma\delta}\right]\,. \tag{6.60}$$

Constraints $(1/m)\partial_\mu\Psi^\mu_{\gamma\delta} = 0$ and $(1/m)\epsilon^{\mu\nu}{}_{\alpha\beta}\,\partial_\mu\Psi^{\alpha\beta}_{\gamma\delta} = 0$ can be regarded as the consequence of Eqs. (6.59,6.60).

Next, we present the vector-spinor and tensor-spinor functions as

$$\Psi^\mu_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\;\mu} + (\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^\mu\,, \tag{6.61}$$

$$\Psi^{\mu\nu}_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\;\mu\nu} + (\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}\,, \tag{6.62}$$

i. e., using the symmetric matrix coefficients in indices γ and δ. Hence, the total function is

$$\begin{aligned}\Psi_{\{\alpha\beta\}\{\gamma\delta\}} = (\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\;\mu} + (\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^\mu +\\ + (\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\;\mu\nu} + (\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}\,,\end{aligned} \tag{6.63}$$

[11] The matrix R can be related to the CP operation in the $(1/2, 0) \oplus (0, 1/2)$ representation.

and the resulting tensor equations are:

$$\frac{2}{m}\partial_\mu T_\kappa{}^{\mu\nu} = -G_\kappa{}^\nu\,, \tag{6.64}$$

$$\frac{2}{m}\partial_\mu R_{\kappa\tau}{}^{\mu\nu} = -F_{\kappa\tau}{}^\nu\,, \tag{6.65}$$

$$T_\kappa{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu G_\kappa{}^\nu - \partial^\nu G_\kappa{}^\mu\right]\,, \tag{6.66}$$

$$R_{\kappa\tau}{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu F_{\kappa\tau}{}^\nu - \partial^\nu F_{\kappa\tau}{}^\mu\right]\,. \tag{6.67}$$

The constraints are re-written to

$$\frac{1}{m}\partial_\mu G_\kappa{}^\mu = 0\,, \quad \frac{1}{m}\partial_\mu F_{\kappa\tau}{}^\mu = 0\,, \tag{6.68}$$

$$\frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha T_\kappa{}^{\beta\nu} = 0\,, \quad \frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha R_{\kappa\tau}{}^{\beta\nu} = 0\,. \tag{6.69}$$

However, we need to make symmetrization over these two sets of indices $\{\alpha\beta\}$ and $\{\gamma\delta\}$. The total symmetry can be ensured if one contracts the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ with *antisymmetric* matrices $R^{-1}_{\beta\gamma}$, $(R^{-1}\gamma^5)_{\beta\gamma}$ and $(R^{-1}\gamma^5\gamma^\lambda)_{\beta\gamma}$, and equate all these contractions to zero (similar to the $s = 3/2$ case considered in Ref. [17, p. 44]. We obtain additional constraints on the tensor field functions:

$$G_\mu{}^\mu = 0\,, \quad G_{[\kappa\,\mu]} = 0\,, \quad G^{\kappa\mu} = \frac{1}{2}g^{\kappa\mu}G_\nu{}^\nu\,, \tag{6.70}$$

$$F_{\kappa\mu}{}^\mu = F_{\mu\kappa}{}^\mu = 0\,, \quad \epsilon^{\kappa\tau\mu\nu}F_{\kappa\tau,\mu} = 0\,, \tag{6.71}$$

$$T^\mu{}_{\mu\kappa} = T^\mu{}_{\kappa\mu} = 0\,, \quad \epsilon^{\kappa\tau\mu\nu}T_{\kappa,\tau\mu} = 0\,, \tag{6.72}$$

$$F^{\kappa\tau,\mu} = T^{\mu,\kappa\tau}\,, \quad \epsilon^{\kappa\tau\mu\lambda}(F_{\kappa\tau,\mu} + T_{\kappa,\tau\mu}) = 0\,, \tag{6.73}$$

$$R_{\kappa\nu}{}^{\mu\nu} = R_{\nu\kappa}{}^{\mu\nu} = R_{\kappa\nu}{}^{\nu\mu} = R_{\nu\kappa}{}^{\nu\mu} = R_{\mu\nu}{}^{\mu\nu} = 0\,, \tag{6.74}$$

$$\epsilon^{\mu\nu\alpha\beta}(g_{\beta\kappa}R_{\mu\tau,\nu\alpha} - g_{\beta\tau}R_{\nu\alpha,\mu\kappa}) = 0 \quad \epsilon^{\kappa\tau\mu\nu}R_{\kappa\tau,\mu\nu} = 0\,. \tag{6.75}$$

Thus, we encountered with the well-known difficulty of the theory of spin-2 particles in the Minkowski space. We explicitly showed that all field functions become to be equal to zero. Such a situation cannot be considered as a satisfactory one (because it does not give us any physical information), and it can be corrected in several ways.[12]

6.4 The Generalized Bargmann-Wigner Formalism for Spin 2

We shall modify the formalism in the spirit of Ref. [30]. The field function (6.58) is now presented as

$$\Psi_{\{\alpha\beta\}\gamma\delta} = \alpha_1(\gamma_\mu R)_{\alpha\beta}\Psi^\mu_{\gamma\delta} + \alpha_2(\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta} + \alpha_3(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}\widetilde{\Psi}^{\mu\nu}_{\gamma\delta}\,, \tag{6.76}$$

[12] The reader can compare our results of this Section with those of Ref. [29]. I became aware about their consideration from Dr. D. V. Ahluwalia (personal communications, May 5, 1998). I consider their discussion of the standard formalism in the Sections I and II, as insufficient.

with

$$\Psi^{\mu}_{\{\gamma\delta\}} = \beta_1(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\mu} + \beta_2(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu} + \beta_3(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu}\,, \quad (6.77)$$

$$\Psi^{\mu\nu}_{\{\gamma\delta\}} = \beta_4(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\mu\nu} + \beta_5(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu} + \beta_6(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu}, \quad (6.78)$$

$$\widetilde{\Psi}^{\mu\nu}_{\{\gamma\delta\}} = \beta_7(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_\kappa{}^{\mu\nu} + \beta_8(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu} + \beta_9(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu} \quad (6.79)$$

Hence, the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ can be expressed as a sum of nine terms:

$$\begin{aligned}\Psi_{\{\alpha\beta\}\{\gamma\delta\}} &= \alpha_1\beta_1(\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\mu} + \alpha_1\beta_2(\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu} + \\ &+ \alpha_1\beta_3(\gamma_\mu R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu} + +\alpha_2\beta_4(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\mu\nu} + \\ &+ \alpha_2\beta_5(\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu} + \alpha_2\beta_6(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu} + \\ &+ \alpha_3\beta_7(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_\kappa{}^{\mu\nu} + \alpha_3\beta_8(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu} + \\ &+ \alpha_3\beta_9(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu}\,. \end{aligned} \quad (6.80)$$

The corresponding dynamical equations are given by[13]

$$\frac{2\alpha_2\beta_4}{m}\partial_\nu T_\kappa{}^{\mu\nu} + \frac{i\alpha_3\beta_7}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\widetilde{T}_{\kappa,\alpha\beta} = \alpha_1\beta_1 G_\kappa{}^{\mu}\,, \quad (6.81)$$

$$\begin{aligned}&\frac{2\alpha_2\beta_5}{m}\partial_\nu R_{\kappa\tau}{}^{\mu\nu} + \frac{i\alpha_2\beta_6}{m}\epsilon_{\alpha\beta\kappa\tau}\partial_\nu\widetilde{R}^{\alpha\beta,\mu\nu} + \frac{i\alpha_3\beta_8}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\widetilde{D}_{\kappa\tau,\alpha\beta} - \\ &- \frac{\alpha_3\beta_9}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta} = \alpha_1\beta_2 F_{\kappa\tau}{}^{\mu} + \frac{i\alpha_1\beta_3}{2}\epsilon_{\alpha\beta\kappa\tau}\widetilde{F}^{\alpha\beta,\mu}\,, \end{aligned} \quad (6.82)$$

$$2\alpha_2\beta_4 T_\kappa{}^{\mu\nu} + i\alpha_3\beta_7\epsilon^{\alpha\beta\mu\nu}\widetilde{T}_{\kappa,\alpha\beta} = \frac{\alpha_1\beta_1}{m}(\partial^\mu G_\kappa{}^{\nu} - \partial^\nu G_\kappa{}^{\mu})\,, \quad (6.83)$$

$$\begin{aligned}&2\alpha_2\beta_5 R_{\kappa\tau}{}^{\mu\nu} + i\alpha_3\beta_8\epsilon^{\alpha\beta\mu\nu}\widetilde{D}_{\kappa\tau,\alpha\beta} + i\alpha_2\beta_6\epsilon_{\alpha\beta\kappa\tau}\widetilde{R}^{\alpha\beta,\mu\nu} \\ &- \frac{\alpha_3\beta_9}{2}\epsilon^{\alpha\beta\mu\nu}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta} = \\ &= \frac{\alpha_1\beta_2}{m}(\partial^\mu F_{\kappa\tau}{}^{\nu} - \partial^\nu F_{\kappa\tau}{}^{\mu}) + \frac{i\alpha_1\beta_3}{2m}\epsilon_{\alpha\beta\kappa\tau}(\partial^\mu\widetilde{F}^{\alpha\beta,\nu} - \partial^\nu\widetilde{F}^{\alpha\beta,\mu})\,. \end{aligned} \quad (6.84)$$

The essential constraints are:

$$\alpha_1\beta_1 G^{\mu}{}_{\mu} = 0\,, \quad \alpha_1\beta_1 G_{[\kappa\mu]} = 0\,, \quad (6.85)$$

$$2i\alpha_1\beta_2 F_{\alpha\mu}{}^{\mu} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{F}_{\kappa\tau,\mu} = 0\,, \quad (6.86)$$

$$2i\alpha_1\beta_3\widetilde{F}_{\alpha\mu}{}^{\mu} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu}{}_{\alpha}F_{\kappa\tau,\mu} = 0\,, \quad (6.87)$$

$$2i\alpha_2\beta_4 T^{\mu}{}_{\mu\alpha} - \alpha_3\beta_7\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{T}_{\kappa,\tau\mu} = 0\,, \quad (6.88)$$

$$2i\alpha_3\beta_7\widetilde{T}^{\mu}{}_{\mu\alpha} - \alpha_2\beta_4\epsilon^{\kappa\tau\mu}{}_{\alpha}T_{\kappa,\tau\mu} = 0\,, \quad (6.89)$$

[13] All indices in this formula are already pure vectorial and have nothing to do with previous notation. The coefficients α_i and β_i may, in general, carry some dimension.

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_6\widetilde{R}_{\kappa\tau,\mu\nu}+\alpha_3\beta_8\widetilde{D}_{\kappa\tau,\mu\nu}\right]+2\alpha_2\beta_5 R^{\mu\nu}{}_{\mu\nu}+2\alpha_3\beta_9 D^{\mu\nu}{}_{\mu\nu}=0\,, \tag{6.90}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_5 R_{\kappa\tau,\mu\nu}+\alpha_3\beta_9 D_{\kappa\tau,\mu\nu}\right]+2\alpha_2\beta_6\widetilde{R}^{\mu\nu}{}_{\mu\nu}+2\alpha_3\beta_8\widetilde{D}^{\mu\nu}{}_{\mu\nu}=0\,, \tag{6.91}$$

$$2i\alpha_2\beta_5 R_{\beta\mu}{}^{\mu\alpha}+2i\alpha_3\beta_9 D_{\beta\mu}{}^{\mu\alpha}+\alpha_2\beta_6\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{R}^{\lambda\mu}{}_{\mu\nu}+\alpha_3\beta_8\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{D}^{\lambda\mu}{}_{\mu\nu}=0\,, \tag{6.92}$$

$$2i\alpha_1\beta_2 F^{\lambda\mu}{}_{\mu}-2i\alpha_2\beta_4 T_{\mu}{}^{\mu\lambda}+\alpha_1\beta_3\epsilon^{\kappa\tau\mu\lambda}\widetilde{F}_{\kappa\tau,\mu}+\alpha_3\beta_7\epsilon^{\kappa\tau\mu\lambda}\widetilde{T}_{\kappa,\tau\mu}=0\,, \tag{6.93}$$

$$2i\alpha_1\beta_3\widetilde{F}^{\lambda\mu}{}_{\mu}-2i\alpha_3\beta_7\widetilde{T}_{\mu}{}^{\mu\lambda}+\alpha_1\beta_2\epsilon^{\kappa\tau\mu\lambda}F_{\kappa\tau,\mu}+\alpha_2\beta_4\epsilon^{\kappa\tau\mu\lambda}T_{\kappa,\tau\mu}=0\,, \tag{6.94}$$

$$\begin{aligned}&\alpha_1\beta_1(2G^{\lambda}{}_{\alpha}-g^{\lambda}{}_{\alpha}G^{\mu}{}_{\mu})-2\alpha_2\beta_5(2R^{\lambda\mu}{}_{\mu\alpha}+2R_{\alpha\mu}{}^{\mu\lambda}+g^{\lambda}{}_{\alpha}R^{\mu\nu}{}_{\mu\nu})+\\&+2\alpha_3\beta_9(2D^{\lambda\mu}{}_{\mu\alpha}+2D_{\alpha\mu}{}^{\mu\lambda}+g^{\lambda}{}_{\alpha}D^{\mu\nu}{}_{\mu\nu})\\&+2i\alpha_3\beta_8(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{D}^{\kappa\lambda}{}_{\mu\nu}-\epsilon^{\kappa\tau\mu\lambda}\widetilde{D}_{\kappa\tau,\mu\alpha})-\\&-2i\alpha_2\beta_6(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{R}^{\kappa\lambda}{}_{\mu\nu}-\epsilon^{\kappa\tau\mu\lambda}\widetilde{R}_{\kappa\tau,\mu\alpha})=0\,,\end{aligned} \tag{6.95}$$

$$\begin{aligned}&2\alpha_3\beta_8(2\widetilde{D}^{\lambda\mu}{}_{\mu\alpha}+2\widetilde{D}_{\alpha\mu}{}^{\mu\lambda}+g^{\lambda}{}_{\alpha}\widetilde{D}^{\mu\nu}{}_{\mu\nu})-2\alpha_2\beta_6(2\widetilde{R}^{\lambda\mu}{}_{\mu\alpha}+2\widetilde{R}_{\alpha\mu}{}^{\mu\lambda}+\\&+g^{\lambda}{}_{\alpha}\widetilde{R}^{\mu\nu}{}_{\mu\nu})++2i\alpha_3\beta_9(\epsilon_{\kappa\alpha}{}^{\mu\nu}D^{\kappa\lambda}{}_{\mu\nu}-\epsilon^{\kappa\tau\mu\lambda}D_{\kappa\tau,\mu\alpha})-\\&-2i\alpha_2\beta_5(\epsilon_{\kappa\alpha}{}^{\mu\nu}R^{\kappa\lambda}{}_{\mu\nu}-\epsilon^{\kappa\tau\mu\lambda}R_{\kappa\tau,\mu\alpha})=0\,,\end{aligned} \tag{6.96}$$

$$\begin{aligned}&\alpha_1\beta_2(F^{\alpha\beta,\lambda}-2F^{\beta\lambda,\alpha}+F^{\beta\mu}{}_{\mu}g^{\lambda\alpha}-F^{\alpha\mu}{}_{\mu}g^{\lambda\beta})-\\&-\alpha_2\beta_4(T^{\lambda,\alpha\beta}-2T^{\beta,\lambda\alpha}+T_{\mu}{}^{\mu\alpha}g^{\lambda\beta}-T_{\mu}{}^{\mu\beta}g^{\lambda\alpha})+\\&+\frac{i}{2}\alpha_1\beta_3(\epsilon^{\kappa\tau\alpha\beta}\widetilde{F}_{\kappa\tau}{}^{\lambda}+2\epsilon^{\lambda\kappa\alpha\beta}\widetilde{F}_{\kappa\mu}{}^{\mu}+2\epsilon^{\mu\kappa\alpha\beta}\widetilde{F}^{\lambda}{}_{\kappa,\mu})-\\&-\frac{i}{2}\alpha_3\beta_7(\epsilon^{\mu\nu\alpha\beta}\widetilde{T}^{\lambda}{}_{\mu\nu}+2\epsilon^{\nu\lambda\alpha\beta}\widetilde{T}^{\mu}{}_{\mu\nu}+2\epsilon^{\mu\kappa\alpha\beta}\widetilde{T}_{\kappa,\mu}{}^{\lambda})=0\,.\end{aligned} \tag{6.97}$$

They are the results of contractions of the field function (6.80) with six antisymmetric matrices, as above. Furthermore, one should recover the relations (6.70-6.75) in the particular case when $\alpha_3=\beta_3=\beta_6=\beta_9=0$ and $\alpha_1=\alpha_2=\beta_1=\beta_2=\beta_4=\beta_5=\beta_7=\beta_8=1$.

As a discussion, we note that in such a framework we have physical content because only certain combinations of field functions can be equal to zero. In general, the fields $F_{\kappa\tau}{}^{\mu}$, $\widetilde{F}_{\kappa\tau}{}^{\mu}$, $T_{\kappa}{}^{\mu\nu}$, $\widetilde{T}_{\kappa}{}^{\mu\nu}$, and $R_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{R}_{\kappa\tau}{}^{\mu\nu}$, $D_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{D}_{\kappa\tau}{}^{\mu\nu}$ can correspond to different physical states and the equations above describe couplings one state with another.

Furthermore, from the set of equations (6.81-6.84) one obtains the *second*-order equation for the symmetric traceless tensor of the second rank ($\alpha_1 \neq 0$, $\beta_1 \neq 0$):

$$\frac{1}{m^2}\left[\partial_\nu \partial^\mu G_\kappa{}^\nu - \partial_\nu \partial^\nu G_\kappa{}^\mu\right] = G_\kappa{}^\mu \,. \tag{6.98}$$

After the contraction in indices κ and μ this equation is reduced to

$$\partial_\mu G^\mu{}_\alpha = F_\alpha \,, \tag{6.99}$$

$$\frac{1}{m^2}\partial_\alpha F^\alpha = 0 \,, \tag{6.100}$$

i. e., to the equations connecting the analogue of the energy-momentum tensor and the analogue of the 4-vector potential (the additional notoph field as opposed to the Logunov theory?). As we showed in our recent work [30] the longitudinal potential may have importance in the construction of electromagnetism (see also the works on the notoph and notivarg concept [31]). Moreover, according to the Weinberg theorem [9] for massless particles it is the gauge part of the 4-vector potential $\sim \partial_\mu \chi$, which is the physical field. The case, when the longitudinal potential is equated to zero, can be considered as a particular case only. This case may be relevant to some physical situation but hardly to be considered as a basis for unification. Further investigations may provide additional foundations to "surprising" similarities of gravitational and electromagnetic equations in the low-velocity limit, Refs. [32–34,36].

6.5 Interactions with Fermions

The possibility of terms as $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ appears to be related to the matters of chiral interactions [38,39]. As we are now convinced, the Dirac field operator can be always presented as a superposition of the self- and anti-self charge conjugate field operators (cf. Ref. [37]). The anti-self charge conjugate part can give the self charge conjugate part after multiplying by the γ^5 matrix, and *vice versa*. We derived[14]

$$[i\gamma^\mu D^*_\mu - m]\psi_1^s = 0 \,, \tag{6.102}$$

or[15]

$$[i\gamma^\mu D_\mu - m]\psi_2^a = 0 \,. \tag{6.104}$$

[14] The anti-self charge conjugate field function ψ_2 can also be used. The equation has then the form:

$$[i\gamma^\mu D^*_\mu + m]\psi_2^a = 0 \,. \tag{6.101}$$

[15] The self charge conjugate field function ψ_1 also can be used. The equation has the form:

$$[i\gamma^\mu D_\mu + m]\psi_1^s = 0 \,. \tag{6.103}$$

As readily seen, in the cases of alternative choices we have opposite charges in the terms of the type $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ and in the mass terms.

Both equations lead to the terms of interaction such as $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ provided that the 4-vector potential is considered as a complex function(al). In fact, from (6.102) we have:

$$i\sigma^\mu \nabla_\mu \chi_1 - m\phi_1 = 0\,, \tag{6.105}$$

$$i\tilde{\sigma}^\mu \nabla^*_\mu \phi_1 - m\chi_1 = 0\,. \tag{6.106}$$

And, from (6.104) we have

$$i\sigma^\mu \nabla^*_\mu \chi_2 - m\phi_2 = 0\,, \tag{6.107}$$

$$i\tilde{\sigma}^\mu \nabla_\mu \phi_2 - m\chi_2 = 0\,. \tag{6.108}$$

The meanings of σ^μ and $\tilde{\sigma}^\mu$ are obvious from the definition of γ matrices. The derivatives are defined:

$$D_\mu = \partial_\mu - ie\gamma^5 C_\mu + eB_\mu\,, \quad \nabla_\mu = \partial_\mu - ieA_\mu\,, \tag{6.109}$$

and $A_\mu = C_\mu + iB_\mu$. Thus, relations with the magnetic monopoles can also be established.

¿From the above system we extract the terms as $\pm e^2 \sigma^i \sigma^j A_i A^*_j$, which lead to the discussed terms [38,39].[16] Furthermore, one can come to the same conclusions not applying to the constraints on the creation/annihilation operators (which we have previously chosen for clarity and simplicity in Ref. [39]). It is possible to work with self/anti-self charge conjugate fields and the Majorana *anzatzen*. Thus, in the considered cases it is the γ^5 transformation which distinguishes various field configurations (helicity, self/anti-self charge conjugate properties etc) in the coordinate representation.

6.6 Boson Interactions

The most general relativistic-invariant Lagrangian for the symmetric 2nd-rank tensor is

$$\begin{aligned} \mathcal{L} = & -\alpha_1(\partial^\alpha G_{\alpha\lambda})(\partial_\beta G^{\beta\lambda}) - \alpha_2(\partial_\alpha G^{\beta\lambda})(\partial^\alpha G_{\beta\lambda}) \\ & - \alpha_3(\partial^\alpha G^{\beta\lambda})(\partial_\beta G_{\alpha\lambda}) + m^2 G_{\alpha\beta}G^{\alpha\beta}\,. \end{aligned} \tag{6.110}$$

It leads to the equation

$$\left[\alpha_2 \partial^2 + m^2\right] G^{\{\mu\nu\}} + (\alpha_1 + \alpha_3)\partial^{\{\mu|}(\partial_\alpha G^{\alpha|\nu\}}) = 0\,. \tag{6.111}$$

In the case $\alpha_2 = 1 > 0$ and $\alpha_1 + \alpha_3 = -1$ it coincides with Eq. (6.98). There is no any problem to obtain the dynamical invariants for the fields of the spin 2 from the above Lagrangian. The mass dimension of $G^{\mu\nu}$ is $[energy]^1$.

[16] I am grateful to Prof. S. Esposito for the e-mail communications (1997-98) on the alternative proof of the considered interaction. We would like to note that the terms of the type $\sigma \cdot [\mathbf{A} \times \mathbf{A}^*]$ can be reduced to $(\sigma \cdot \nabla)V$, where V is the scalar potential.

We now present possible relativistic interactions of the symemtric 2nd-rank tensor. They should be the following ones:

$$\mathcal{L}^{int}_{(1)} \sim G_{\mu\nu} F^\mu F^\nu \,, \tag{6.112}$$

$$\mathcal{L}^{int}_{(2)} \sim (\partial^\mu G_{\mu\nu}) F^\nu \,, \tag{6.113}$$

$$\mathcal{L}^{int}_{(3)} \sim G_{\mu\nu} (\partial^\mu F^\nu) \,. \tag{6.114}$$

The term $\sim (\partial_\mu G^\alpha{}_\alpha) F^\mu$ vanishes due to the constraint of tracelessness. Obviously, these interactions cannot be obtained from the free Lagrangian (6.110) just by the covariantization of the derivative $\partial_\mu \to \partial_\mu + g F_\mu$.

It is also interesting to note that thanks to the possible terms

$$V(F) = \beta_1 (F_\mu F^\mu) + \beta_2 (F_\mu F^\mu)(F_\nu F^\nu) \tag{6.115}$$

we can give the mass to the G_{00} component of the spin-2 field. This is due to the possibility of the Higgs spontaneous symmetry breaking [40]

$$F^\mu(x) = \begin{pmatrix} v + \partial_0 \chi(x) \\ g^1 \\ g^2 \\ g^3 \end{pmatrix} \,, \tag{6.116}$$

with v being the vacuum expectation value, $v^2 = (F_\mu F^\mu) = -\beta_1 / 2\beta_2 > 0$. Other degrees of freedom of the 4-vector field are removed since they can be interpreted as the Goldstone bosons. It was stated that "for any continuous symmetry which does not preserve the ground state, there is a massless degree of freedom which decouples at low energies. This mode is called the Goldstone (or Nambu-Goldstone) particle for the symmetry". As usual, the Higgs mechanism and the Goldstone modes should be important in giving masses to the three vector bosons.[17] As one can easily see, this expression does not permit an arbitrary phase for F^μ, which is possible only if the 4-vector would be the complex one.

Next, due to the Lagrangian interaction of fermions with notoph are of the order e^2 since the beginning (as opposed to the interaction with the 4-vector potential A_μ), it is more difficult to observe it. However, as far as I know the theoretical precision calculus in QED (the Landé factor, the anomalous magnetic moment, the hyperfine splittings in positronium and muonium, and the decay rate of o-Ps and p-Ps) are about the order corresponding to the 4th-5th loops, where the difference may appear with the experiments [41,42].

[17] It is interesting to note the following statement (given without references in wikipedia.org): "In general, the phonon is effectively the Nambu-Goldstone boson for spontaneously broken Galilean/Lorentz symmetry. However, in contrast to the case of internal symmetry breaking, when spacetime symmetries are broken, the order parameter need not be a scalar field, but may be a tensor field, and the corresponding independent massless modes may now be fewer than the number of spontaneously broken generators, because the Goldstone modes may now be linearly dependent among themselves: e.g., the Goldstone modes for some generators might be expressed as gradients of Goldstone modes for other broken generators."

6.7 Conclusions

We considered the Bargmann-Wigner formalism to derive the equations for the AST field and for the symmetric tensor of the 2nd rank. We introduced additional scalar normalization field in the Bargmann-Wigner formalism in order to take into account possible physical significance of the Ogievetskiĭ-Polubarinov–Kalb-Ramond modes. We introduced the additional symmetric matrix in the Bargmann-Wigner expansion ($\gamma^5\sigma^{\mu\nu}R$) in order to take into account the dual fields. The consideration is similar to Ref. [43].

Furthermore, we discussed the interactions of notoph, photon and graviton (and, probably, notivarg[18]). For instance, the interaction notoph-graviton may give the mass to spin-2 particles in the way which is similar to the spontaneous-symmetry-breaking Higgs formalism.

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7 General Majorana Neutrino Mass Matrix from a Low Energy SU(3) Family Symmetry with Sterile Neutrinos

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Abstract. Within the framework of a local SU(3) family symmetry model, we report a general analysis of the mechanism for neutrino mass generation and mixing, including light sterile neutrinos. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. Right-handed and the $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general Majorana neutrino mass matrix with four or five massless neutrinos at tree level. Hence, light fermions, including light neutrinos get masses from radiative corrections mediated by the massive SU(3) gauge bosons. We report the corresponding Majorana neutrino mass matrix up to one loop. Previous numerical analysis of the free parameters show out solutions for quarks and charged lepton masses within a parameter space region where the vector-like fermion masses M_U , M_D , M_E, and the SU(3) family gauge boson masses lie in the low energy region of $\mathcal{O}(1 - 20)$ TeV, with light neutrinos within the correct order of square neutrino mass differences: $m_2^2 - m_1^2 \approx 7 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2 \times 10^{-3}$ eV^2, and at least one sterile neutrino of the order ≈ 0.5 eV. A more precise fit of the parameters is still needed to account also for the quark and lepton mixing.

Povzetek. Avtor pojasnjuje pojav družin pri leptonih tako, da uporabi za opis družin model z lokalno simetrijo SU(3). Trem družinam kvarkov in leptonov doda še družinski triplet desnoročnih nevtrinov, ki nosi samo družinski naboj, levoročni in desnoročni U in prav tak D kvark, ki nosijo poleg barve le hiper naboj, levoročni in desnoročni nevtrino, ki ne nosita nobenega naboja, ter levoročni in desnoročni elektron s hipernabojem (−2). Vsi ti novi delci so masivni. Novi fermioni poskrbijo na drevesnem nivoju samo za maso tretje družine kvarkov in leptonov. Lahkim fermionom, tudi lahkim nevtrinom, priskrbijo maso popravki v naslednjih redih pri interakciji z masivnimi bozoni, ki nosijo družinsko kvantno število. Avtor izračuna masno matriko 8x8 za Majoranine nevtrine do prvega reda. Proste parametre modela določi z izmerjenimi masami in mešalnimi matrikami. Po dosedanjih izračunih so primerne vrednosti za mase fermionov M_U , M_D , M_E in za maso družinskega tripleta umeritvenega bozona v intervalu $\mathcal{O}(1 - 20)$ TeV, za izmerjene masne razlike lahkih nevtrinov $m_2 2 - m_1 2 \approx 7 \times 10^{-5}$ eV2, $m_3 2 - m_1 2 \approx 2 \times 10^{-3}$ eV2 lahko avtor poskrbi s še vsaj enim sterilnim nevtrinom, ki ima maso ≈ 0.5 eV. Avtor pričakuje, da bo z bolj natančnimi izračuni lahko s pomočjo tega modela pojasnil mešalne matrike kvarkov in leptonov.

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7.1 Introduction

Although the standard picture with three light flavor neutrinos has been successful to describe the neutrino oscillation data. On the other hand, there have been recent hints from the LSND and MiniBooNe short-baseline neutrino oscillation experiments[1,2] on the possible existence of at least one light sterile neutrino in the eV scale, which mix with the active neutrinos. On the other hand, an explanation of the strong hierarchy of quark and charged lepton masses is still a big challenge in particle physics. This hierarchy have suggested to many model building theorists that light fermion masses could be generated from radiative corrections, while those of the top and bottom quarks and the tau lepton are generated at tree level. This may be understood as the breaking of a symmetry among families , a horizontal symmetry.

In this report we update the general features of a "Beyond the Standard Model"(BSM) proposal which introduces a $SU(3)$ [3] gauged family symmetry [1] commuting with the Standard Model group. Previous reports[4] within this scenario showed that this model has the features and particle content to account for a realistic spectrum of charged fermion masses and quark mixing. This BSM model introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[5] mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions.

The $SU(3)$ *family symmetry model allows one to address the problem of quark and lepton masses and mixing, including active and light sterile neutrinos.*

7.2 $SU(3)$ flavor symmetry model

7.2.1 Fermion content

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the global symmetry in this limit of all quarks and leptons massless, including R-handed neutrinos, is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R} \tag{7.1}$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3) \tag{7.2}$$

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where Eq.(7.2) defines the $SU(3)$ gauged family symmetry, and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with g_H, g_s, g and g' the corresponding

[1] See [3,4] and references therein for some $SU(3)$ family symmetry models.

coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

$$\psi_q^o = (3,3,2,\frac{1}{3})_L \quad , \quad \psi_u^o = (3,3,1,\frac{4}{3})_R \quad , \quad \psi_d^o = (3,3,1,-\frac{2}{3})_R$$

$$\psi_l^o = (3,1,2,-1)_L \quad , \quad \psi_e^o = (3,1,1,-2)_R \ ,$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions:

- Right handed neutrinos $\Psi_\nu^o = (3,1,1,0)_R$ required to cancel anomalies[6], and
- the $SU(2)_L$ singlet vector-like fermions:

$$U_{L,R}^o = (1,3,1,\frac{4}{3}) \quad , \quad D_{L,R}^o = (1,3,1,-\frac{2}{3}) \tag{7.3}$$

$$N_{L,R}^o = (1,1,1,0) \quad , \quad E_{L,R}^o = (1,1,1,-2)\,, \tag{7.4}$$

which conserve the previous anomaly cancellation. The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U\, \bar{U}_L^o\, U_R^o \,+\, M_D\, \bar{D}_L^o\, D_R^o \,+\, M_E\, \bar{E}_L^o\, E_R^o + h.c.\,, \tag{7.5}$$

and

$$m_D\, \bar{N}_L^o\, N_R^o \,+\, m_L\, \bar{N}_L^o\, (N_L^o)^c \,+\, m_R\, \bar{N}_R^o\, (N_R^o)^c \,+\, h.c \tag{7.6}$$

These $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. M_U , M_D , M_E play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses and mixing, meanwhile m_D , m_L , m_R play a similar role for neutrino masses and lepton mixing, all together with the radiative corrections.

7.3 $SU(3)$ family symmetry breaking

The corresponding $SU(3)$ gauge bosons are defined through their couplings to fermions as

$$\begin{aligned} i\mathcal{L}_{int} = {} & \frac{g_H}{2}\left(\bar{f}_1^o\gamma_\mu f_1^o - \bar{f}_2^o\gamma_\mu f_2^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{f}_1^o\gamma_\mu f_1^o + \bar{f}_2^o\gamma_\mu f_2^o - 2\bar{f}_3^o\gamma_\mu f_3^o\right) Z_2^\mu \\ & + \frac{g_H}{\sqrt{2}}\left(\bar{f}_1^o\gamma_\mu f_2^o\, Y_1^+ + \bar{f}_1^o\gamma_\mu f_3^o\, Y_2^+ + \bar{f}_2^o\gamma_\mu f_3^o\, Y_3^+ + h.c.\right) \end{aligned} \tag{7.7}$$

$f_1^o = u^o, d^o, e^o, \nu_e^o$, $f_2^o = c^o, s^o, \mu^o, \nu_\mu^o$ and $f_3^o = t^o, b^o, \tau^o, \nu_\tau^o$. To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)$, we introduce the flavon scalar fields: $\eta_i = (3,1,1,0),\ i = 1,2,3,$ transforming as the fundamental representation under $SU(3)$ and being standard model singlets, with the "Vacuum Expectation Values" (VEV's):

$$\langle \eta_1 \rangle^T = (\Lambda_1, 0, 0) \quad , \quad \langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \, . \tag{7.8}$$

Actually, let us point out here that only two scalar flavons in the fundamental representation are needed to completely break down the $SU(3)$ *symmetry. The most convenient way to accomplish the spontaneous breaking of the* $SU(3)$ *family symmetry is under current study*. Thus, the contribution to the horizontal gauge boson masses from Eq.(7.8) read

- $\eta_1 :\quad \frac{g_{H_1}^2 \Lambda_1^2}{2} (Y_1^+ Y_1^- + Y_2^+ Y_2^-) + \frac{g_{H_1}^2 \Lambda_1^2}{4} (Z_1^2 + \frac{Z_2^2}{3} + 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\eta_2 :\quad \frac{g_{H_2}^2 \Lambda_2^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4} (Z_1^2 + \frac{Z_2^2}{3} - 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\eta_3 :\quad \frac{g_{H_3}^2 \Lambda_3^2}{2} (Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms

$$(M_1^2 + M_2^2) \, Y_1^+ Y_1^- + (M_1^2 + M_3^2) \, Y_2^+ Y_2^- + (M_2^2 + M_3^2) \, Y_3^+ Y_3^-$$

$$+ \frac{1}{2} (M_1^2 + M_2^2) \, Z_1^2 + \frac{1}{2} \frac{M_1^2 + M_2^2 + 4 M_3^2}{3} \, Z_2^2 + \frac{1}{2} (M_1^2 - M_2^2) \frac{2}{\sqrt{3}} \, Z_1 \, Z_2 \tag{7.9}$$

$$M_1^2 = \frac{g_{H_1}^2 \Lambda_1^2}{2} \quad , \quad M_2^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2} \tag{7.10}$$

	Z_1	Z_2
Z_1	$M_1^2 + M_2^2$	$\frac{M_1^2 - M_2^2}{\sqrt{3}}$
Z_2	$\frac{M_1^2 - M_2^2}{\sqrt{3}}$	$\frac{M_1^2 + M_2^2 + 4 M_3^2}{3}$

Table 7.1. $Z_1 - Z_2$ mixing mass matrix

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix, we obtain the eigenvalues

$$M_-^2 = \frac{2}{3} \left(M_1^2 + M_2^2 + M_3^2 - \sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)} \right)$$

$$M_+^2 = \frac{2}{3} \left(M_1^2 + M_2^2 + M_3^2 + \sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)} \right)$$

$$M_{Y_1}^2\, Y_1^+ Y_1^- + M_{Y_2}^2\, Y_2^+ Y_2^- + M_{Y_3}^2\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2} \tag{7.11}$$

where

$$M_{Y_1}^2 = M_1^2 + M_2^2 \quad , \quad M_{Y_2}^2 = M_1^2 + M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2 \tag{7.12}$$

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{7.13}$$

$$\cos\phi\,\sin\phi = \frac{\sqrt{3}}{4}\,\frac{M_1^2 - M_2^2}{\sqrt{(M_2^2 - M_1^2)^2 + (M_3^2 - M_1^2)(M_3^2 - M_2^2)}}\,,$$

with the hierarchy $M_1, M_2 \gg M_W{}^2$. Due to the $Z_1 - Z_2$ mixing we diagonalize the propagators involving Z_1 and Z_2 gauge bosons according to Eq.(7.13):

$$Z_1 = \cos\phi\; Z_- - \sin\phi\; Z_+ \quad , \quad Z_2 = \sin\phi\; Z_- + \cos\phi\; Z_+$$

$$\langle Z_1 Z_1 \rangle = \cos^2\phi\; \langle Z_- Z_- \rangle + \sin^2\phi\; \langle Z_+ Z_+ \rangle$$

$$\langle Z_2 Z_2 \rangle = \sin^2\phi\; \langle Z_- Z_- \rangle + \cos^2\phi\; \langle Z_+ Z_+ \rangle$$

$$\langle Z_1 Z_2 \rangle = \cos\phi\;\sin\phi\; (\langle Z_- Z_- \rangle - \langle Z_+ Z_+ \rangle)$$

7.4 Electroweak symmetry breaking

Recently ATLAS[7] and CMS[8] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the $SU(3)$ family symmetry model involves the introduction of two triplets of $SU(2)_L$ Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to $U(1)_Q$, we introduce the scalars: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the VEVs:

$$\langle \Phi^u \rangle = \begin{pmatrix} \langle \Phi_1^u \rangle \\ \langle \Phi_2^u \rangle \\ \langle \Phi_3^u \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi_1^d \rangle \\ \langle \Phi_2^d \rangle \\ \langle \Phi_3^d \rangle \end{pmatrix}\,, \tag{7.14}$$

$$\langle \Phi_i^u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_i^d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix}\,, \tag{7.15}$$

[2] **Notice that in the limit** $M_1^2 = M_2^2; \quad \sin\phi = 0,\ \cos\phi = 1$

contribute to the W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+g'^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

7.5 Tree level neutrino masses

Now we describe briefly the procedure to get the masses for ordinary fermions. The analysis for quarks and charged leptons has already discussed in [4]. Here, we introduce the procedure for neutrinos.

7.5.1 Tree level Dirac neutrino masses

With the fields of particles introduced in the model, we may write the Dirac type gauge invariant Yukawa couplings

$$h_D\,\bar{\Psi}_l^o\,\Phi^u\,N_R^o \;+\; h_1\,\bar{\Psi}_\nu^o\,\eta_1\,N_L^o \;+\; h_2\,\bar{\Psi}_\nu^o\,\eta_2\,N_L^o \;+\; h_3\,\bar{\Psi}_\nu^o\,\eta_3\,N_L^o \;+\; M_D\,\bar{N}_L^o\,N_R^o \;+ h.c \quad (7.16)$$

h_D, h_1, h_2 and h_3 are Yukawa couplings, and M_D a Dirac type, invariant neutrino mass for the sterile neutrinos $N_{L,R}^o$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi_{\nu L,R}^{o\,T} = (\nu_e^o, \nu_\mu^o, \nu_\tau^o, N^o)_{L,R}$, the mass terms

$$h_D\left[v_1\,\bar{\nu}_{eL}^o + v_2\,\bar{\nu}_{\mu L}^o + v_3\,\bar{\nu}_{\tau L}^o\right] N_R^o + \left[h_1\Lambda_1\,\bar{\nu}_{eR}^o + h_2\Lambda_2\,\bar{\nu}_{\mu R}^o + h_3\Lambda_3\,\bar{\nu}_{\tau R}^o\right] N_L^o + M_D\,\bar{N}_L^o\,N_R^o + h.c. \quad (7.17)$$

7.5.2 Tree level Majorana masses:

Since $N_{L,R}^o$, Eq.(7.4), are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L\,\bar{\Psi}_l^o\,\Phi^u (N_L^o)^c \;+\; m_L\,\bar{N}_L^o\,(N_L^o)^c + h.c \quad (7.18)$$

and

$$h_{1R}\,\bar{\Psi}_\nu^o\,\eta_1\,(N_R^o)^c \;+\; h_{2R}\,\bar{\Psi}_\nu^o\,\eta_2\,(N_R^o)^c \;+\; h_{3R}\,\bar{\Psi}_\nu^o\,\eta_3\,(N_R^o)^c \;+\; m_R\,\bar{N}_R^o\,(N_R^o)^c + h.c\,, \quad (7.19)$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \left[\nu_1 \, \bar{\nu}^o_{eL} + \nu_2 \, \bar{\nu}^o_{\mu L} + \nu_3 \, \bar{\nu}^o_{\tau L}\right] \, (N^o_L)^c \; + \; m_L \, \bar{N}^o_L \, (N^o_L)^c + h.c. \,, \tag{7.20}$$

$$+ \left[h_{1R} \, \Lambda_1 \, \bar{\nu}^o_{eR} + h_{2R} \, \Lambda_2 \, \bar{\nu}^o_{\mu R} + h_{3R} \, \Lambda_3 \, \bar{\nu}^o_{\tau R}\right] \, (N^o_R)^c \; + m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c. \,, \tag{7.21}$$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L \, \nu_1$	0	0	0	$h_D \, \nu_1$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L \, \nu_2$	0	0	0	$h_D \, \nu_2$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L \, \nu_3$	0	0	0	$h_D \, \nu_3$
$\overline{N^o_L}$	$h_L \, \nu_1$	$h_L \, \nu_2$	$h_L \, \nu_3$	m_L	$h_1 \, \Lambda_1$	$h_2 \, \Lambda_2$	$h_3 \, \Lambda_3$	m_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	$h_1 \, \Lambda_1$	0	0	0	$h_{1R} \, \Lambda_1$
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2 \, \Lambda_2$	0	0	0	$h_{2R} \, \Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	$h_3 \, \Lambda_3$	0	0	0	$h_{3R} \, \Lambda_3$
$\overline{(N^o_R)^c}$	$h_D \, \nu_1$	$h_D \, \nu_2$	$h_D \, \nu_3$	m_D	$h_{1R} \, \Lambda_1$	$h_{2R} \, \Lambda_2$	$h_{3R} \, \Lambda_3$	m_R

Table 7.2. Tree Level Majorana masses

Thus, in the basis

$$\Psi^{o\,T}_\nu = \left(\nu^o_{eL} \,, \, \nu^o_{\mu L} \,, \, \nu^o_{\tau L} \,, \, N^o_L \,, \, (\nu^o_{eR})^c \,, \, (\nu^o_{\mu R})^c \,, \, (\nu^o_{\tau R})^c \,, \, (N^o_R)^c \right) \,, \tag{7.22}$$

the Generic 8×8 tree level Majorana mass matrix for neutrinos $\mathcal{M}^o_\nu$, from Table 7.2, $\bar{\Psi}^o_\nu \; \mathcal{M}^o_\nu \; (\Psi^o_\nu)^c$, read

$$\mathcal{M}^o_\nu = \begin{pmatrix} \mathcal{M}^o_L & \mathcal{M}^o_D \\ & \\ \mathcal{M}^{o\,T}_D & \mathcal{M}^o_R \end{pmatrix} \tag{7.23}$$

where

$$\mathcal{M}^o_L = \begin{pmatrix} 0 & 0 & 0 & \alpha_1 \\ 0 & 0 & 0 & \alpha_2 \\ 0 & 0 & 0 & \alpha_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & m_L \end{pmatrix} \quad , \quad \mathcal{M}^o_R = \begin{pmatrix} 0 & 0 & 0 & \beta_1 \\ 0 & 0 & 0 & \beta_2 \\ 0 & 0 & 0 & \beta_3 \\ \beta_1 & \beta_2 & \beta_3 & m_R \end{pmatrix} \tag{7.24}$$

and

$$\mathcal{M}_D^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ b_1 & b_2 & b_3 & m_D \end{pmatrix} , \tag{7.25}$$

$$\alpha_i = h_L \, v_i \quad , \quad a_i = h_D \, v_i \quad , \quad b_i = h_i \, \Lambda_i \quad , \quad \beta_i = h_{iR} \, \Lambda_i \tag{7.26}$$

Diagonalization of $\mathcal{M}_\nu^{(o)}$, Eq.(7.23), yields four zero eigenvalues, associated to the neutrino fields:

$$\frac{a_2}{ap} \, \nu_{eL}^o - \frac{a_1}{ap} \, \nu_{\mu L}^o \quad , \quad \frac{a_1 \, a_3}{ap \, a} \, \nu_{eL}^o + \frac{a_2 \, a_3}{ap \, a} \, \nu_{\mu L}^o - \frac{a_p}{a} \, \nu_{\tau L}^o , \tag{7.27}$$

$$\frac{b_2}{bp} \, \nu_{eR}^o - \frac{b_1}{bp} \, \nu_{\mu R}^o \quad , \quad \frac{b_1 \, b_3}{bp \, b} \, \nu_{eR}^o + \frac{b_2 \, b_3}{bp \, b} \, \nu_{\mu R}^o - \frac{b_p}{b} \, \nu_{\tau R}^o , \tag{7.28}$$

$$ap = \sqrt{a_1^2 + a_2^2} \; , \; bp = \sqrt{b_1^2 + b_2^2} \; , \; a = \sqrt{a_1^2 + a_2^2 + a_3^2} \; , \; b = \sqrt{b_1^2 + b_2^2 + b_3^2} \; .$$

Assuming for simplicity $\frac{h_{1R}}{h_1} = \frac{h_{2R}}{h_2} = \frac{h_{3R}}{h_3} \equiv c_R$, that is

$$\frac{\alpha_i}{a_i} = \frac{h_L}{h_D} = c_L \quad , \quad \frac{\beta_i}{b_i} = \frac{h_{iR}}{h_i} = c_R \, ,$$

the Characteristic Polynomial for the nonzero eigenvalues of $\mathcal{M}_\nu^o$ reduce to the one of the matrix m_4[3], Eq.(7.29), where

$$m_4 = \begin{pmatrix} 0 & \alpha & 0 & a \\ \alpha & m_L & b & m_D \\ 0 & b & 0 & \beta \\ a & m_D & \beta & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \tag{7.29}$$

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2} \quad , \quad \beta = \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2} \; .$$

$$U_4^T \, m_4 \, U_4 = Diag(m_5^o, m_6^o, m_7^o, m_8^o) \equiv d_4 \quad , \quad m_4 = U_4 \, d_4 \, U_4^T \tag{7.30}$$

Eq.(7.30) impose the constrains

$$u_{11}^2 \, m_5^o + u_{12}^2 \, m_6^o + u_{13}^2 \, m_7^o + u_{14}^2 \, m_8^o = 0 \tag{7.31}$$

$$u_{31}^2 \, m_5^o + u_{32}^2 \, m_6^o + u_{33}^2 \, m_7^o + u_{34}^2 \, m_8^o = 0 \tag{7.32}$$

$$u_{11} u_{31} \, m_5^o + u_{12} u_{32} \, m_6^o + u_{13} u_{33} \, m_7^o + u_{14} u_{34} \, m_8^o = 0 \, , \tag{7.33}$$

[3] The relation $a \, b = \alpha \, \beta$ would yield five massless neutrinos at tree level.

corresponding to the $(m_4)_{11} = (m_4)_{33} = (m_4)_{13} = 0$ zero entries, respectively.

Therefore, $\mathcal{M}_\nu^o$ is diagonalized by the orthogonal matrix

$$U_\nu^o = \begin{pmatrix} \frac{a_2}{a_p} & \frac{a_1}{a}\frac{a_3}{a_p} & 0 & 0 & \frac{a_1}{a}u_{11} & \frac{a_1}{a}u_{12} & \frac{a_1}{a}u_{13} & \frac{a_1}{a}u_{14} \\ -\frac{a_1}{a_p} & \frac{a_2}{a}\frac{a_3}{a_p} & 0 & 0 & \frac{a_2}{a}u_{11} & \frac{a_2}{a}u_{12} & \frac{a_2}{a}u_{13} & \frac{a_2}{a}u_{14} \\ 0 & -\frac{a_p}{a} & 0 & 0 & \frac{a_3}{a}u_{11} & \frac{a_3}{a}u_{12} & \frac{a_3}{a}u_{13} & \frac{a_3}{a}u_{14} \\ 0 & 0 & 0 & 0 & u_{21} & u_{22} & u_{23} & u_{24} \\ 0 & 0 & \frac{b_2}{b_p} & \frac{b_1}{b}\frac{b_3}{b_p} & \frac{b_1}{b}u_{31} & \frac{b_1}{b}u_{32} & \frac{b_1}{b}u_{33} & \frac{b_1}{b}u_{34} \\ 0 & 0 & -\frac{b_1}{b_p} & \frac{b_2}{b}\frac{b_3}{b_p} & \frac{b_2}{b}u_{31} & \frac{b_2}{b}u_{32} & \frac{b_2}{b}u_{33} & \frac{b_2}{b}u_{34} \\ 0 & 0 & 0 & -\frac{b_p}{b} & \frac{b_3}{b}u_{31} & \frac{b_3}{b}u_{32} & \frac{b_3}{b}u_{33} & \frac{b_3}{b}u_{34} \\ 0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \tag{7.34}$$

$$(U_\nu^o)^T \mathcal{M}_\nu^o U_\nu^o = Diag(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o) \tag{7.35}$$

7.6 One loop neutrino masses

After tree level contributions the fermion global symmetry, Eq.(7.1), is broken down to

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R} \ . \tag{7.36}$$

Therefore, in this scenario light neutrinos may get extremely small masses from radiative corrections mediated by the SU(3) heavy gauge bosons.

7.6.1 One loop Dirac Neutrino masses

After the breakdown of the electroweak symmetry, neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 7.1, The internal fermion line in this diagram represent the tree level see-saw mechanisms, Eqs.(7.16-7.21). The vertices read from the SU(3) family symmetry interaction Lagrangian

$$\begin{aligned} i\mathcal{L}_{int} = \frac{g_H}{2}\left(\bar{\nu_e^o}\gamma_\mu\nu_e^o - \bar{\nu_\mu^o}\gamma_\mu\nu_\mu^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{\nu_e^o}\gamma_\mu\nu_e^o + \bar{\nu_\mu^o}\gamma_\mu\nu_\mu^o - 2\bar{\nu_\tau^o}\gamma_\mu\nu_\tau^o\right) Z_2^\mu \\ + \frac{g_H}{\sqrt{2}}\left(\bar{\nu_e^o}\gamma_\mu\nu_\mu^o\, Y_1^+ + \bar{\nu_e^o}\gamma_\mu\nu_\tau^o\, Y_2^+ + \bar{\nu_\mu^o}\gamma_\mu\nu_\tau^o\, Y_3^+ + h.c.\right) \end{aligned} \tag{7.37}$$

The contribution from these diagrams may be written as

$$c_Y \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij} \quad , \quad \alpha_H = \frac{g_H^2}{4\pi} \, , \tag{7.38}$$

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o \, f(M_Y, m_k^o) \, , \tag{7.39}$$

$f(M_Y, m_k^o) = \frac{M_Y^2}{M_Y^2 - m_k^{o\,2}} \ln \frac{M_Y^2}{m_k^{o\,2}}$,

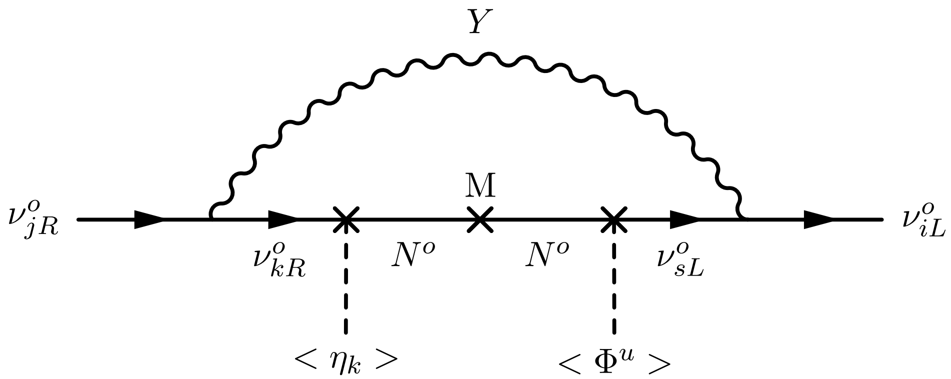

Fig. 7.1. Generic one loop diagram contribution to the Dirac mass term $m_{ij} \, \bar{\nu}_{iL}^o \nu_{jR}^o$. $M = M_D, m_L, m_R$

	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\bar{\nu}_{eL}^o$	$D_{\nu\,11}$	$D_{\nu\,12}$	$D_{\nu\,13}$	0
$\bar{\nu}_{\mu L}^o$	$D_{\nu\,21}$	$D_{\nu\,22}$	$D_{\nu\,23}$	0
$\bar{\nu}_{\tau L}^o$	$D_{\nu\,31}$	$D_{\nu\,32}$	$D_{\nu\,33}$	0
$\bar{N}_L^o$	0	0	0	0

Table 7.3. One loop Dirac mass terms $\frac{\alpha_H}{\pi} D_{\nu\,ij} \, \bar{\nu}_{iL}^o \, \nu_{jR}^o$

$$m_\nu(M_Y)_{i,4+j} = \frac{a_i \, b_j}{a \, b} \mathcal{F}_\nu(M_Y) \tag{7.40}$$

$$\mathcal{F}_\nu(M_Y) = u_{11}u_{31} \, m_5^o \, f(M_Y, m_5^o) + u_{12}u_{32} \, m_6^o \, f(M_Y, m_6^o)$$

$$+ u_{13}u_{33} \, m_7^o \, f(M_Y, m_7^o) + u_{14}u_{34} \, m_8^o \, f(M_Y, m_8^o) \tag{7.41}$$

$$D_{\nu\,11} = \frac{a_1 b_1}{ab}\left[\frac{1}{4}\mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m}\right] + \frac{1}{2}\left[\frac{a_2 b_2}{ab}\mathcal{F}_\nu(M_{Y_1}) + \frac{a_3 b_3}{ab}\mathcal{F}_\nu(M_{Y_2})\right] ,$$

$$D_{\nu\,12} = \frac{a_1 b_2}{ab}\left[-\frac{1}{4}\mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{F}_\nu(M_{Z_2})\right] ,$$

$$D_{\nu\,13} = \frac{a_1 b_3}{ab}\left[-\frac{1}{6}\mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m}\right] ,$$

$$D_{\nu\,21} = \frac{a_2 b_1}{ab}\left[-\frac{1}{4}\mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{F}_\nu(M_{Z_2})\right] ,$$

$$D_{\nu\,22} = \frac{a_2 b_2}{ab}\left[\frac{1}{4}\mathcal{F}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m}\right] + \frac{1}{2}\left[\frac{a_1 b_1}{ab}\mathcal{F}_\nu(M_{Y_1}) + \frac{a_3 b_3}{ab}\mathcal{F}_\nu(M_{Y_3})\right] ,$$

$$D_{\nu\,23} = \frac{a_2 b_3}{ab}\left[-\frac{1}{6}\mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m}\right] ,$$

$$D_{\nu\,31} = \frac{a_3 b_1}{ab}\left[-\frac{1}{6}\mathcal{F}_\nu(M_{Z_2}) - \mathcal{F}_{\nu,m}\right] ,$$

$$D_{\nu\,32} = \frac{a_3 b_2}{ab}\left[-\frac{1}{6}\mathcal{F}_\nu(M_{Z_2}) + \mathcal{F}_{\nu,m}\right] ,$$

$$D_{\nu\,33} = \frac{1}{3}\frac{a_3 b_3}{ab}\mathcal{F}_\nu(M_{Z_2}) + \frac{1}{2}\left[\frac{a_1 b_1}{ab}\mathcal{F}_\nu(M_{Y_2}) + \frac{a_2 b_2}{ab}\mathcal{F}_\nu(M_{Y_3})\right] ,$$

$$\mathcal{F}_\nu(M_{Z_1}) = \cos^2\phi\,\mathcal{F}_\nu(M_-) + \sin^2\phi\,\mathcal{F}_\nu(M_+)$$

$$\mathcal{F}_\nu(M_{Z_2}) = \sin^2\phi\,\mathcal{F}_\nu(M_-) + \cos^2\phi\,\mathcal{F}_\nu(M_+)$$

$$\mathcal{F}_{\nu,m} = \frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{F}_\nu(M_-) - \mathcal{F}_\nu(M_+)] , \tag{7.42}$$

7.6.2 One loop L-handed Majorana masses

Neutrinos also obtain one loop corrections to L-handed and R-handed Majorana masses from the diagrams of Fig. 7.2 and Fig. 7.3, respectively. A similar procedure as for Dirac Neutrino masses leads to the one loop Majorana mass terms

$$m_\nu(M_Y)_{i,j} = \frac{a_i\, a_j}{a^2}\mathcal{G}_\nu(M_Y) \tag{7.43}$$

$$\mathcal{G}_\nu(M_Y) = m_5^o\, u_{11}^2\, f(M_Y, m_5^o) + m_6^o\, u_{12}^2\, f(M_Y, m_6^o) + m_7^o\, u_{13}^2\, f(M_Y, m_7^o)$$

$$+ m_8^o\, u_{14}^2\, f(M_Y, m_8^o) \tag{7.44}$$

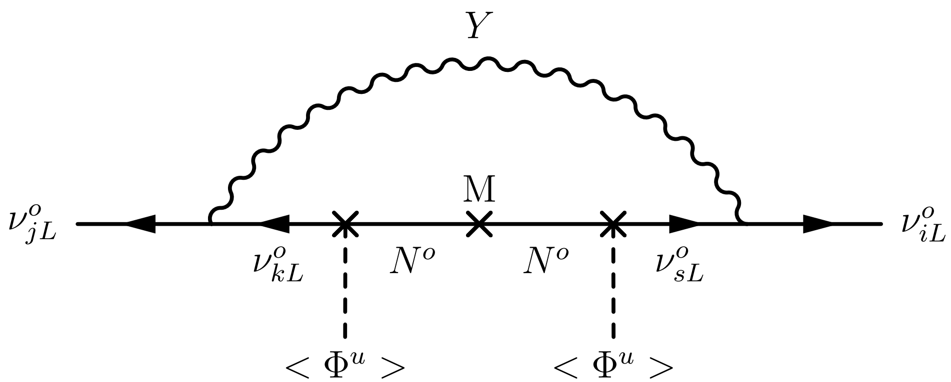

Fig. 7.2. Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\ \bar{\nu}_{iL}^o(\nu_{jL}^o)^T$. $M = M_D, m_L, m_R$

	ν_{eL}^o	$\nu_{\mu L}^o$	$\nu_{\tau L}^o$	N_L^o
ν_{eL}^o	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu_{\mu L}^o$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu_{\tau L}^o$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N_L^o	0	0	0	0

Table 7.4. One loop L-handed Majorana mass terms $\frac{\alpha_H}{\pi}\, L_{\nu\,ij}\ \bar{\nu}_{iL}^o\ (\nu_{jL}^o)^T$

$$L_{\nu\,11} = \frac{a_1^2}{a^2}\left[\frac{1}{4}\mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{G}_\nu(M_{Z_2}) + \mathcal{G}_{\nu,m}\right] ,$$

$$L_{\nu\,22} = \frac{a_2^2}{a^2}\left[\frac{1}{4}\mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{G}_\nu(M_{Z_2}) - \mathcal{G}_{\nu,m}\right] ,$$

$$L_{\nu\,33} = \frac{1}{3}\frac{a_3^2}{a^2}\mathcal{G}_\nu(M_{Z_2}) ,$$

$$L_{\nu\,12} = \frac{a_1\,a_2}{a^2}\left[-\frac{1}{4}\mathcal{G}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{G}_\nu(M_1)\right] ,$$

$$L_{\nu\,13} = \frac{a_1\,a_3}{a^2}\left[-\frac{1}{6}\mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{G}_\nu(M_2) - \mathcal{G}_{\nu,m}\right] ,$$

$$L_{\nu\,23} = \frac{a_2\,a_3}{a^2}\left[-\frac{1}{6}\mathcal{G}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{G}_\nu(M_3) + \mathcal{G}_{\nu,m}\right]$$

$$\mathcal{G}_\nu(M_{Z_1}) = \cos^2\phi\,\mathcal{G}_\nu(M_-) + \sin^2\phi\,\mathcal{G}_\nu(M_+)$$

$$\mathcal{G}_\nu(M_{Z_2}) = \sin^2\phi\,\mathcal{G}_\nu(M_-) + \cos^2\phi\,\mathcal{G}_\nu(M_+)$$

$$\mathcal{G}_{\nu,m} = \frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{G}_\nu(M_-) - \mathcal{G}_\nu(M_+)] , \tag{7.45}$$

7.6.3 One loop R-handed Majorana masses

$$m_\nu(M_Y)_{4+i,4+j} = \frac{b_i\,b_j}{b^2}\mathcal{H}_\nu(M_Y) \tag{7.46}$$

$$\mathcal{H}_\nu(M_Y) = m_5^o\,u_{31}^2\,f(M_Y, m_5^o) + m_6^o\,u_{32}^2\,f(M_Y, m_6^o) + m_7^o\,u_{33}^2\,f(M_Y, m_7^o)$$
$$+ m_8^o\,u_{34}^2\,f(M_Y, m_8^o) \tag{7.47}$$

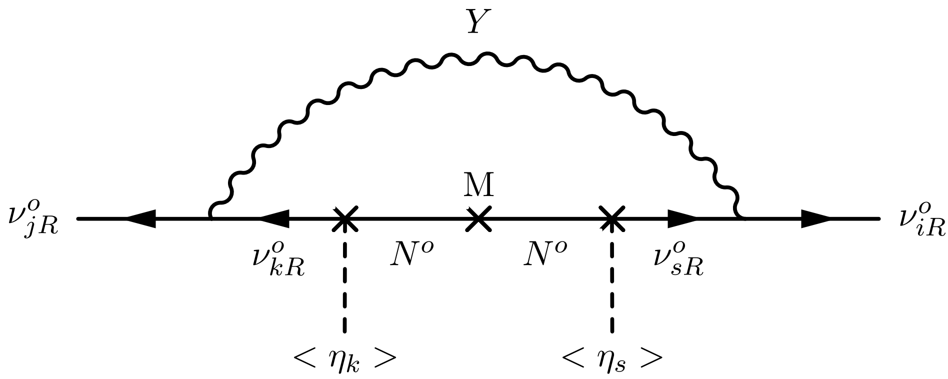

Fig. 7.3. Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\ \bar{\nu}^o_{iR}(\nu^o_{jR})^T$. $M = M_D, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	$R_{\nu\,11}$	$R_{\nu\,12}$	$R_{\nu\,13}$	0
$\nu^o_{\mu R}$	$R_{\nu\,12}$	$R_{\nu\,22}$	$R_{\nu\,23}$	0
$\nu^o_{\tau R}$	$R_{\nu\,13}$	$R_{\nu\,23}$	$R_{\nu\,33}$	0
N^o_R	0	0	0	0

Table 7.5. One loop R-handed Majorana mass terms $\frac{\alpha_H}{\pi} R_{\nu\,ij}\ \bar{\nu}^o_{iR}\ (\nu^o_{jR})^T$

$$R_{\nu\,11} = \frac{b_1^2}{b^2}\left[\frac{1}{4}\mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{H}_\nu(M_{Z_2}) + \mathcal{H}_{\nu,m}\right] ,$$

$$R_{\nu\,22} = \frac{b_2^2}{b^2}\left[\frac{1}{4}\mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{H}_\nu(M_{Z_2}) - \mathcal{H}_{\nu,m}\right] ,$$

$$R_{\nu\,33} = \frac{1}{3}\frac{b_3^2}{b^2}\mathcal{H}_\nu(M_{Z_2}) ,$$

$$R_{\nu\,12} = \frac{b_1 b_2}{b^2}\left[-\frac{1}{4}\mathcal{H}_\nu(M_{Z_1}) + \frac{1}{12}\mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{H}_\nu(M_1)\right] ,$$

$$R_{\nu\,13} = \frac{b_1 b_3}{b^2}\left[-\frac{1}{6}\mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{H}_\nu(M_2) - \mathcal{H}_{\nu,m}\right] ,$$

$$R_{\nu\,23} = \frac{b_2 b_3}{b^2}\left[-\frac{1}{6}\mathcal{H}_\nu(M_{Z_2}) + \frac{1}{2}\mathcal{H}_\nu(M_3) + \mathcal{H}_{\nu,m}\right]$$

$$\mathcal{H}_\nu(M_{Z_1}) = \cos^2\phi\,\mathcal{H}_\nu(M_-) + \sin^2\phi\,\mathcal{H}_\nu(M_+)$$

$$\mathcal{H}_\nu(M_{Z_2}) = \sin^2\phi\,\mathcal{H}_\nu(M_-) + \cos^2\phi\,\mathcal{H}_\nu(M_+)$$

$$\mathcal{H}_{\nu,m} = \frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{H}_\nu(M_-) - \mathcal{H}_\nu(M_+)]\,, \qquad (7.48)$$

where $\mathcal{F}_{\nu,m}$, $\mathcal{G}_{\nu,m}$ and $\mathcal{H}_{\nu,m}$, Eqs.(7.42,7.45,7.48), come from $Z_1 - Z_2$ mixing diagram contributions.

Thus, in the Ψ_ν^o basis, Eq.(7.22), we may write the one loop contribution for neutrinos as $\bar{\Psi_\nu^o}\ \mathcal{M}_{1\,\nu}^o\ (\Psi_\nu^o)^c$,

$$\mathcal{M}_{1\,\nu}^o = \begin{pmatrix} L_{\nu\,11} & L_{\nu\,12} & L_{\nu\,13} & 0 & D_{\nu\,11} & D_{\nu\,12} & D_{\nu\,13} & 0 \\ L_{\nu\,12} & L_{\nu\,22} & L_{\nu\,23} & 0 & D_{\nu\,21} & D_{\nu\,22} & D_{\nu\,23} & 0 \\ L_{\nu\,13} & L_{\nu\,23} & L_{\nu\,33} & 0 & D_{\nu\,31} & D_{\nu\,32} & D_{\nu\,33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{\nu\,11} & D_{\nu\,21} & D_{\nu\,31} & 0 & R_{\nu\,11} & R_{\nu\,12} & R_{\nu\,13} & 0 \\ D_{\nu\,12} & D_{\nu\,22} & D_{\nu\,32} & 0 & R_{\nu\,12} & R_{\nu\,22} & R_{\nu\,23} & 0 \\ D_{\nu\,13} & D_{\nu\,23} & D_{\nu\,33} & 0 & R_{\nu\,13} & R_{\nu\,23} & R_{\nu\,33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi} \qquad (7.49)$$

7.6.4 Neutrino mass matrix up to one loop

Finally, we obtain the general symmetric Majorana mass matrix for neutrinos up to one loop

$$\mathcal{M}_\nu = (U_\nu^o)^T\,\mathcal{M}_{1\,\nu}^o\,U_\nu^o + Diag(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o)\,, \qquad (7.50)$$

where explicitly

$$(U_\nu^o)^T \mathcal{M}_{1\,\nu}^o U_\nu^o = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & N_{33} & N_{34} & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & N_{34} & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55} & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66} & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77} & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88} \end{pmatrix} \frac{\alpha_H}{\pi} \qquad (7.51)$$

Majorana L-handed:

$$N_{11} = \frac{a_1^2 a_2^2}{a_p^2 a^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) \qquad (7.52)$$

$$N_{12} = -\frac{a_1 a_2 a_3}{2a^3} [\frac{a_2^2 - a_1^2}{a_p^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) + \mathcal{G}_2 - \mathcal{G}_3 - 6\mathcal{G}_m] \qquad (7.53)$$

$$N_{22} = \frac{a_3^2}{a^2} \left[\frac{1}{4} \frac{(a_2^2 - a_1^2)^2}{a_p^2\, a^2} (\mathcal{G}_{Z_1} - \mathcal{G}_1) + \frac{a_2^2}{a^2} (\mathcal{G}_2 - \mathcal{G}_3) \right.$$
$$\left. + \frac{a_p^2}{4\,a^2} (\mathcal{G}_1 + 3\mathcal{G}_{Z_2} - 4\mathcal{G}_2) - 3 \frac{a_2^2 - a_1^2}{a^2} \mathcal{G}_m \right] \qquad (7.54)$$

Dirac:

$$N_{13} = \frac{1}{2ap\, bp\, a\, b} \left\{ (a_1^2 b_1^2 + a_2^2 b_2^2) \mathcal{F}_1 + a_3 b_3 (a_2 b_2 \mathcal{F}_2 + a_1 b_1 \mathcal{F}_3) \right.$$
$$\left. + 2 a_1 b_1 a_2 b_2 \mathcal{F}_{Z_1} \right\} \qquad (7.55)$$

$$N_{14} = \frac{1}{2ap\, bp\, a\, b} \frac{b_3}{b} \left\{ b_1 b_2 (a_2^2 - a_1^2) \mathcal{F}_1 + a_3 b_3 (a_2 b_1 \mathcal{F}_2 - a_1 b_2 \mathcal{F}_3) \right.$$
$$\left. + a_1 a_2 (b_1^2 - b_2^2) \mathcal{F}_{Z_1} + 6 a_1 a_2\, bp^2\, \mathcal{F}_m \right\} \qquad (7.56)$$

$$N_{23} = \frac{1}{2ap\, bp\, a\, b} \frac{a_3}{a} \left\{ a_1 a_2 (b_2^2 - b_1^2)\mathcal{F}_1 + a_3 b_3 (a_1 b_2 \mathcal{F}_2 - a_2 b_1 \mathcal{F}_3) \right.$$

$$\left. + b_1 b_2 (a_1^2 - a_2^2)\mathcal{F}_{Z_1} + 6 b_1 b_2\, ap^2\, \mathcal{F}_m \right\} \quad (7.57)$$

$$N_{24} = \frac{1}{ap\, bp\, a^2\, b^2}$$

$$\left\{ a_3 b_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4}(a_1^2 - a_2^2)(b_1^2 - b_2^2)\mathcal{F}_{Z_1} + \frac{3}{4} ap^2\, bp^2\, \mathcal{F}_{Z_2}] \right.$$

$$\left. + \frac{1}{2}(a_3^2 b_3^2 + ap^2\, bp^2)(a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + 3 a_3 b_3 (a_1^2 b_1^2 - a_2^2 b_2^2)\mathcal{F}_m \right\}$$

Majorana R-handed:

$$N_{33} = \frac{b_1^2 b_2^2}{b_p^2 b^2}(\mathcal{H}_{Z_1} - \mathcal{H}_1) \quad (7.58)$$

$$N_{34} = -\frac{b_1 b_2 b_3}{2b^3}[\frac{b_2^2 - b_1^2}{b_p^2}(\mathcal{H}_{Z_1} - \mathcal{H}_1) + \mathcal{H}_2 - \mathcal{H}_3 - 6\mathcal{H}_m] \quad (7.59)$$

$$N_{44} = \frac{b_3^2}{b^2} \left[\frac{1}{4} \frac{(b_2^2 - b_1^2)^2}{b_p^2\, b^2}(\mathcal{H}_{Z_1} - \mathcal{H}_1) + \frac{b_2^2}{b^2}(\mathcal{H}_2 - \mathcal{H}_3) \right.$$

$$\left. + \frac{b_p^2}{4\, b^2}(\mathcal{H}_1 + 3\mathcal{H}_{Z_2} - 4\mathcal{H}_2) - 3\frac{b_2^2 - b_1^2}{b^2}\mathcal{H}_m \right] \quad (7.60)$$

Majorana L-handed and Dirac:

$$N_{15} = \mathcal{G}_{15}\, u_{11} + m_{13}\, u_{31} \qquad ; \qquad N_{16} = \mathcal{G}_{15}\, u_{12} + m_{13}\, u_{32} \tag{7.61}$$

$$N_{17} = \mathcal{G}_{15}\, u_{13} + m_{13}\, u_{33} \qquad ; \qquad N_{18} = \mathcal{G}_{15}\, u_{14} + m_{13}\, u_{34} \tag{7.62}$$

$$\mathcal{G}_{15} = -\frac{a_1\, a_2}{2\, a_p\, a} \left[\frac{a_2^2 - a_1^2}{a^2}(\mathcal{G}_{Z_1} - \mathcal{G}_1) + \frac{a_3^2}{a^2}(\mathcal{G}_3 - \mathcal{G}_2) + 2\, \frac{(2\, a_3^2 - a_p^2)}{a^2}\, \mathcal{G}_m \right]$$

$$m_{13} = \frac{1}{2ap\, a\, b^2} \Big\{ b_1 b_2 (a_2^2 - a_1^2)\mathcal{F}_1 + a_3 b_3 (a_2 b_1 \mathcal{F}_2 - a_1 b_2 \mathcal{F}_3) \\ + a_1 a_2 (b_1^2 - b_2^2)\mathcal{F}_{Z_1} + 2a_1 a_2\, (bp^2 - 2b_3^2)\mathcal{F}_m \Big\}$$

$$N_{25} = \mathcal{G}_{25}\, u_{11} + m_{23}\, u_{31} \qquad ; \qquad N_{26} = \mathcal{G}_{25}\, u_{12} + m_{23}\, u_{32} \tag{7.63}$$

$$N_{27} = \mathcal{G}_{25}\, u_{13} + m_{23}\, u_{33} \qquad ; \qquad N_{28} = \mathcal{G}_{25}\, u_{14} + m_{23}\, u_{34} \tag{7.64}$$

$$\mathcal{G}_{25} = \frac{a_3}{4\, a_p\, a^4} \\ \Big\{ (a_2^2 - a_1^2)^2\, (\mathcal{G}_{Z_1} - \mathcal{G}_1) + 2\, a_2^2 (a_3^2 - a_p^2)\, (\mathcal{G}_3 - \mathcal{G}_2) - a_p^4\, (\mathcal{G}_{Z_2} - \mathcal{G}_1) \\ -2\, a_p^2 (a_3^2 - a_p^2)\, (\mathcal{G}_{Z_2} - \mathcal{G}_2) + 4\, (a_2^2 - a_1^2)\, (a_3^2 - 2a_p^2)\, \mathcal{G}_m \Big\}$$

$$m_{23} = \frac{1}{ap\, a^2\, b^2} \\ \Bigg\{ a_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4}(a_1^2 - a_2^2)(b_1^2 - b_2^2)\mathcal{F}_{Z_1} + \frac{1}{4} ap^2 (bp^2 - 2b_3^2)\mathcal{F}_{Z_2}] \\ + \frac{1}{2} b_3 (a_3^2 - ap^2)(a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + a_3 [a_1^2 (3b_1^2 - b^2) + a_2^2 (b^2 - 3b_2^2)]\mathcal{F}_m \Bigg\}$$

Dirac and Majorana R-handed:

$$N_{35} = m_{31}\, u_{11} + \mathcal{H}_{35}\, u_{31} \quad , \quad N_{36} = m_{31}\, u_{12} + \mathcal{H}_{35}\, u_{32}$$

$$N_{37} = m_{31}\, u_{13} + \mathcal{H}_{35}\, u_{33} \quad , \quad N_{38} = m_{31}\, u_{14} + \mathcal{H}_{35}\, u_{34}$$

$$m_{31} = \frac{1}{2bp\, a^2\, b} \left\{ a_1 a_2 (b_2^2 - b_1^2)\mathcal{F}_1 + a_3 b_3 (a_1 b_2 \mathcal{F}_2 - a_2 b_1 \mathcal{F}_3) \right. \\ \left. + b_1 b_2 (a_1^2 - a_2^2)\mathcal{F}_{Z_1} + 2 b_1 b_2 (ap^2 - 2a_3^2)\mathcal{F}_m \right\}$$

$$\mathcal{H}_{35} = -\frac{b_1\, b_2}{2\, b_p\, b} \left[\frac{b_2^2 - b_1^2}{b^2} (\mathcal{H}_{Z_1} - \mathcal{H}_1) + \frac{b_3^2}{b^2} (\mathcal{H}_3 - \mathcal{H}_2) + 2\, \frac{(2\, b_3^2 - b_p^2)}{b^2}\, \mathcal{H}_m \right]$$

$$N_{45} = m_{32}\, u_{11} + \mathcal{H}_{45}\, u_{31} \quad , \quad N_{46} = m_{32}\, u_{12} + \mathcal{H}_{45}\, u_{32} \tag{7.65}$$

$$N_{47} = m_{32}\, u_{13} + \mathcal{H}_{45}\, u_{33} \quad , \quad N_{48} = m_{32}\, u_{14} + \mathcal{H}_{45}\, u_{34} \tag{7.66}$$

$$m_{32} = \frac{1}{bp\, a^2\, b^2} \\ \left\{ b_3 [a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4}(a_1^2 - a_2^2)(b_1^2 - b_2^2)\mathcal{F}_{Z_1} + \frac{1}{4} bp^2 (ap^2 - 2a_3^2)\mathcal{F}_{Z_2}] \right. \\ \left. + \frac{1}{2} a_3 (b_3^2 - bp^2)(a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + b_3 [b_1^2 (3a_1^2 - a^2) + b_2^2 (a^2 - 3a_2^2)]\mathcal{F}_m \right\}$$

$$\mathcal{H}_{45} = \frac{b_3}{4\, b_p\, b^4} \\ \left\{ (b_2^2 - b_1^2)^2 (\mathcal{H}_{Z_1} - \mathcal{H}_1) + 2\, b_2^2 (b_3^2 - b_p^2) (\mathcal{H}_3 - \mathcal{H}_2) - b_p^4 (\mathcal{H}_{Z_2} - \mathcal{H}_1) \right. \\ \left. - 2\, b_p^2 (b_3^2 - b_p^2) (\mathcal{H}_{Z_2} - \mathcal{H}_2) + 4 (b_2^2 - b_1^2)(b_3^2 - 2b_p^2)\, \mathcal{H}_m \right\}$$

Majorana L-handed, Dirac and Majorana R-handed:

$$N_{55} = \mathcal{G}_{55}\, u_{11}^2 + 2\, m_{33}\, u_{11}\, u_{31} + \mathcal{H}_{55}\, u_{31}^2 \tag{7.67}$$

$$N_{56} = \mathcal{G}_{55}\, u_{11}\, u_{12} + m_{33}\, (u_{11}\, u_{32} + u_{12}\, u_{31}) + \mathcal{H}_{55}\, u_{31}\, u_{32} \tag{7.68}$$

$$N_{57} = \mathcal{G}_{55}\, u_{11}\, u_{13} + m_{33}\, (u_{11}\, u_{33} + u_{13}\, u_{31}) + \mathcal{H}_{55}\, u_{31}\, u_{33} \tag{7.69}$$

$$N_{58} = \mathcal{G}_{55}\, u_{11}\, u_{14} + m_{33}\, (u_{11}\, u_{34} + u_{14}\, u_{31}) + \mathcal{H}_{55}\, u_{31}\, u_{34} \tag{7.70}$$

$$N_{66} = \mathcal{G}_{55}\, u_{12}^2 + 2\, m_{33}\, u_{12}\, u_{32} + \mathcal{H}_{55}\, u_{32}^2 \tag{7.71}$$

$$N_{67} = \mathcal{G}_{55}\, u_{12}\, u_{13} + m_{33}\, (u_{13}\, u_{32} + u_{12}\, u_{33}) + \mathcal{H}_{55}\, u_{32}\, u_{33} \tag{7.72}$$

$$N_{68} = \mathcal{G}_{55}\, u_{12}\, u_{14} + m_{33}\, (u_{14}\, u_{32} + u_{12}\, u_{34}) + \mathcal{H}_{55}\, u_{32}\, u_{34} \tag{7.73}$$

$$N_{77} = \mathcal{G}_{55}\, u_{13}^2 + 2\, m_{33}\, u_{13}\, u_{33} + \mathcal{H}_{55}\, u_{33}^2 \tag{7.74}$$

$$N_{78} = \mathcal{G}_{55}\, u_{13}\, u_{14} + m_{33}\, (u_{14}\, u_{33} + u_{13}\, u_{34}) + \mathcal{H}_{55}\, u_{33}\, u_{34} \tag{7.75}$$

$$N_{88} = \mathcal{G}_{55}\, u_{14}^2 + 2\, m_{33}\, u_{14}\, u_{34} + \mathcal{H}_{55}\, u_{34}^2 \tag{7.76}$$

$$\mathcal{G}_{55} = \frac{a_1^2\, a_2^2}{a^4}\, \mathcal{G}_1 + \frac{a_1^2\, a_3^2}{a^4}\, \mathcal{G}_2 + \frac{a_2^2\, a_3^2}{a^4}\, \mathcal{G}_3 + \frac{(a_2^2 - a_1^2)^2}{4\, a^4}\, \mathcal{G}_{Z_1} + \frac{(2a_3^2 - a_p^2)^2}{12\, a^4}\, \mathcal{G}_{Z_2} + \frac{(a_2^2 - a_1^2)\,(2\, a_3^2 - a_p^2)}{a^4}\, \mathcal{G}_m$$

$$m_{33} = \frac{1}{a^2\, b^2} \Big\{ a_1 b_1 a_2 b_2 \mathcal{F}_1 + \frac{1}{4}(a_1^2 - a_2^2)(b_1^2 - b_2^2)\mathcal{F}_{Z_1} + \frac{1}{12}(ap^2 - 2a_3^2)(bp^2 - 2b_3^2)\mathcal{F}_{Z_2} + a_3 b_3 (a_1 b_1 \mathcal{F}_2 + a_2 b_2 \mathcal{F}_3) + [a_1^2 b_1^2 - a_2^2 b_2^2 + a_3^2(b_2^2 - b_1^2) + b_3^2(a_2^2 - a_1^2)]\mathcal{F}_m \Big\}$$

$$\mathcal{H}_{55} = \frac{b_1^2\, b_2^2}{b^4}\, \mathcal{H}_1 + \frac{b_1^2\, b_3^2}{b^4}\, \mathcal{H}_2 + \frac{b_2^2\, b_3^2}{b^4}\, \mathcal{H}_3 + \frac{(b_2^2 - b_1^2)^2}{4\, b^4}\, \mathcal{H}_{Z_1} + \frac{(2b_3^2 - b_p^2)^2}{12\, b^4}\, \mathcal{H}_{Z_2} + \frac{(b_2^2 - b_1^2)\,(2\, b_3^2 - b_p^2)}{b^4}\, \mathcal{H}_m$$

7.6.5 Quark $(V_{CKM})_{4\times 4}$ and $(V_{PMNS})_{4\times 8}$ mixing matrices

Within this SU(3) family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R\,,$$

and for neutrinos $\Psi_\nu^o = U_\nu^o\, U_\nu\, \Psi_\nu$. Recall now that vector like quarks, Eq.(7.3), are $SU(2)_L$ weak singlets, and hence, they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL}\gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\, [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\, (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4}\, \gamma_\mu \Psi_{dL}\, W^{+\mu}\,, \tag{7.77}$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\, (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4} \tag{7.78}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times 8} = [(V_{eL}^o\, V_{eL}^{(1)})_{3\times 4}]^T\, (U_\nu^o\, U_\nu)_{3\times 8} \tag{7.79}$$

7.7 Conclusions

We reported an updated and general analysis for the generation of neutrino masses and mixing within the SU(3) family symmetry model. The right handed neutrinos $(\nu_e\ \nu_\mu\ \nu_\tau)_R$, and the vector like completely sterile neutrinos $N_{L,R}$, the flavon scalar fields and their VEV's introduced to break the symmetries: Φ^u, Φ^d, η_1, η_2 and η_3, all together, yields a 8×8 general Majorana neutrino mass matrix with four or five massless neutrinos at tree level. Therefore, light neutrinos get tiny masses from radiative corrections mediated by the heavy SU(3) gauge bosons. Neutrino masses and mixing are extremely sensitive to the parameter space region, and a global fit for all quark masses and mixing together with neutrino masses and lepton mixing is in progress.

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8 Gravitational Interactions and Fine-Structure Constant

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Abstract. Electromagnetic and gravitational central-field problems are studied with relativistic quantum mechanics on curved space-time backgrounds. Corrections to the transition current are identified. Analogies of the gravitational and electromagnetic spectra suggest the definition of a gravitational fine-structure constant. The electromagnetic and gravitational coupling constants enter the Einstein–Hilbert–Maxwell Lagrangian. We postulate that the variational principle holds with regard to a global dilation transformation of the space-time coordinates. The variation suggests is consistent with a functional relationship of the form $\alpha_{\rm QED} \propto (\alpha_{\rm G})^{1/2}$, where $\alpha_{\rm QED}$ is the electrodynamic fine-structure constant, and $\alpha_{\rm G}$ its gravitational analogue.

Povzetek. Avtorji obravnavajo Diracov delec v elektromagnem in gravitacijskem centralno simetričnem polju. Poiščejo popravke za emisijo fotona v prisotnosti gravitacijskega polja. Po analogiji s spektrom elektrona v centralnosimetričnem potencialu definirajo tudi konstanto gravitacijske fine strukture. Predpostavijo, da velja variacijsko načelo za transformacijo koordinat prostor-časa z globalno dilatacijo. Predlagana variacija je skladna s funkcijsko zvezo oblike $\alpha_{\rm QED} \propto (\alpha_{\rm G})^{1/2}$, kjer je $\alpha_{\rm QED}$ konstanta elektrodinamične fine strukture, ki ima gravitacijski analog $\alpha_{\rm G}$.

8.1 Introduction

If we are ever to gain a better understanding of the relationship of gravitational interactions and electrodynamics in the quantum world, then a very practical approach is to try to solve a number of important example problems in gravitational theory, whose solution is known in electromagnetic theory, to try to generalize the approach to the gravitational analogue, and to compare. In order to proceed, it is not necessarily required to quantize space-time itself [1]. Indeed, the formulation of quantum mechanics on curved-space backgrounds in itself constitutes an interesting problem [2–6]. A priori, one might think that the simple substitution $\partial/\partial x^i \to \nabla_i$ is the Schrödinger equation might suffice. Here, $\partial/\partial x^i$ is the ith partial derivative with respect to the ith spatial coordinate, whereas ∇_i is the ith covariant derivative. However, this naive approach is destined to fail; the gravitational theory of Einstein and Hilbert inherently is a relativistic theory, and

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the only way to describe quantum particles on curved space-times is to start from a fully relativistic wave function. The Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\,\psi(x) = 0 \tag{8.1}$$

generalizes as follows to a curved space-time background [2–6],

$$(i\gamma^{\mu}(x)(\partial_{\mu} - \Gamma_{\mu}) - m)\,\psi(x) = 0\,. \tag{8.2}$$

The Dirac algebra [7–9] needs to be generalized to the local metric $g^{\mu\nu}(x)$,

$$\{\gamma^{\mu}(x), \gamma^{\nu}(x)\} = 2\,g^{\mu\nu}(x)\,, \qquad \sigma^{\mu\nu}(x) = \frac{i}{2}[\gamma^{\mu}(x), \gamma^{\nu}(x)]\,. \tag{8.3}$$

The spin connection matrix Γ_{μ} is given as

$$\Gamma_{\mu} = -\frac{i}{4}\,g_{\rho\alpha}(x)\left(\frac{\partial b_{\nu}{}^{\beta}(x)}{\partial x^{\mu}}\,a^{\alpha}{}_{\beta}(x) - \Gamma^{\alpha}{}_{\nu\mu}\right)\sigma^{\rho\nu}(x)\,, \tag{8.4}$$

where repeated indices are summed. Finally, the a and b coefficients belong to the square root of the metric,

$$\gamma_{\rho}(x) = b_{\rho}{}^{\alpha}(x)\,\gamma_{\alpha}\,, \qquad \gamma^{\alpha}(x) = a^{\alpha}{}_{\rho}(x)\,\gamma^{\rho}, \tag{8.5}$$

where the γ^{α} are the flat-space Dirac matrices, which are preferentially used in the Dirac representation [9–11,1,12]. The Christoffel symbols are $\Gamma^{\alpha}{}_{\nu\rho} \equiv \Gamma^{\alpha}{}_{\nu\rho}(x)$.

8.2 Central–Field Problem

8.2.1 Foldy–Wouthuysen Method

The Foldy–Wouthuysen method [13,14] is a standard tool for the extraction of the physical, nonrelativistic degrees of freedom, from a fully relativistic Dirac theory. The general paradigm is as follows: The positive and negative energy solutions of a (generalized) Dirac equation are intertwined in the fully relativistic formalism. One has to separate the upper and lower spinors in the bispinor solution, and in order to do so, one eliminates the "off-diagonal couplings" of the upper and lower spinor components order by order in some perturbative parameters, possibly, using iterated (unitary) transformations.

For the plain free Dirac Hamiltonian, a standard method exists to all orders in perturbation theory, while for more difficult problems, one manifestly has to resort to a perturbative formalism [13,14]. A suitable expansion parameter in a general case is the particle's momentum operator. Let us consider a space-time metric of the form

$$\overline{g}_{\mu\nu} = \mathrm{diag}\left(w^2(r), -v^2(r), -v^2(r), -v^2(r)\right)\,. \tag{8.6}$$

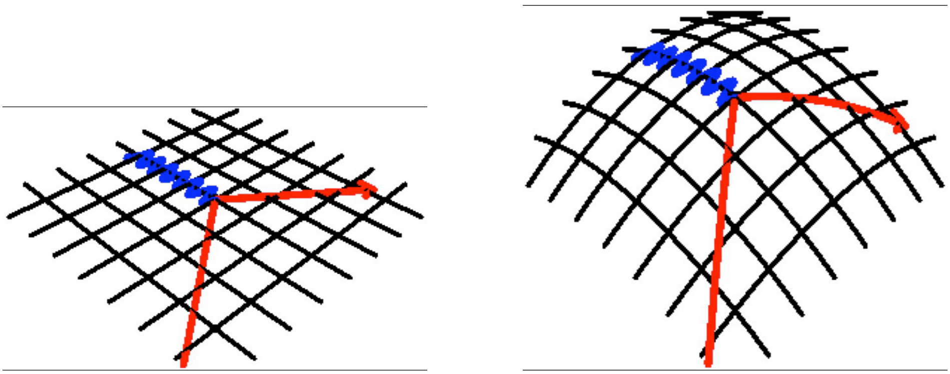

Fig. 8.1. The flat-space photon emission vertex (left figure) is promoted to a curved-space vertex (right figure) in general relativity. The curved background leads to higher-order corrections to the transition current, which are summarized, for the Schwarzschild metric, in Eq. (8.12).

The Schwarzschild metric in isotropic coordinates (see Sec. 43 of Chap. 3 of Ref. [15]), involves the Schwarzschild radius r_s,

$$w = \left(1 - \frac{r_s}{4r}\right)\left(1 + \frac{r_s}{4r}\right)^{-1} = \frac{4r - r_s}{4r + r_s} \approx 1 - \frac{r_s}{2r},$$
$$v = \left(1 + \frac{r_s}{4r}\right)^2 \approx 1 + \frac{r_s}{2r}, \qquad \frac{w}{v} = \frac{16\,r^2\,(4r - r_s)}{(4r + r_s)^3} \approx 1 - \frac{r_s}{r}. \tag{8.7}$$

The Schwarzschild radius reads as $r_s = 2\,G\,M$, where G is Newton's gravitational constant, and M is the mass of the planet (or "black hole"). The Hamiltonian or time translation operator is necessarily "noncovariant" in the sense that the time coordinate needs to be singled out. If we insist on using the time translation with respect to the time coordinate dt in the metric $ds^2 = w^2(r)\,dt^2 - v^2(r)d\vec{r}^2$ and bring the Hamiltonian into Hermitian form [see Ref. [16] and Eqs. (9)–(13) of Ref. [10]], then we obtain

$$H_{DS} = \frac{1}{2}\left\{\vec{\alpha}\cdot\vec{p}, \left(1 - \frac{r_s}{r}\right)\right\} + \beta m \left(1 - \frac{r_s}{2r}\right), \tag{8.8}$$

where $\alpha^i = \gamma^0\gamma^i$ is the Dirac α matrix (we here use the Dirac representation). The Foldy–Wouthuysen transformed Dirac–Schwarzschild Hamiltonian is finally obtained as [10]

$$H_{FW} = \beta\left(m + \frac{\vec{p}^{\,2}}{2m} - \frac{\vec{p}^{\,4}}{8m^3}\right) - \beta\,\frac{m\,r_s}{2\,r} \tag{8.9}$$
$$+ \beta\left(-\frac{3r_s}{8m}\left\{\vec{p}^{\,2}, \frac{1}{r}\right\} + \frac{3\pi r_s}{4m}\delta^{(3)}(\vec{r}) + \frac{3r_s}{8m}\,\frac{\vec{\Sigma}\cdot\vec{L}}{r^3}\right).$$

The parity-violating terms obtained in Refs. [17,16] are spurious.

8.2.2 Transition Current

As we couple the Dirac–Schwarzschild Hamiltonian (8.8) to an electromagnetic field (see Fig. 8.1), it is clear that the transition current in the interaction Hamiltonian is $H_{\rm int} = -\vec{j} \cdot \vec{A}$. takes the form

$$j^i = \frac{1}{2} \left\{ 1 - \frac{r_s}{r}, \alpha^i \, \exp(i\vec{k} \cdot r) \right\} . \tag{8.10}$$

We now employ the multipole expansion

$$\alpha^i \, \exp(i\vec{k} \cdot r) \approx \alpha^i + \alpha^i \, (i\vec{k} \cdot \vec{r}) - \frac{1}{2} \alpha^i (\vec{k} \cdot r)^2 \tag{8.11}$$

A unitary transformation with the same generators are used for the Dirac–Schwarzschild Hamiltonian then yields the result [10],

$$\begin{aligned} j^i_{\rm FW} = & \frac{p^i}{m} - \frac{p^i \, \vec{p}^{\,2}}{2m} - \frac{i}{2m} \left(\vec{k} \times \vec{\sigma} \right)^i + \frac{1}{2} \left\{ \frac{p^i}{m}, \, (i\vec{k} \cdot \vec{r}) \right\} \\ & - \frac{1}{4} \left\{ (\vec{k} \cdot \vec{r})^2, \frac{p^i}{m} \right\} + \frac{1}{2m} \left(\vec{k} \cdot \vec{r} \right) \, (\vec{k} \times \vec{\sigma})^i \\ & - \frac{3}{4} \left\{ \frac{p^i}{m}, \frac{r_s}{r} \right\} + \frac{r_s}{2r} \frac{(\vec{\sigma} \times \vec{r})^i}{m \, r^2} - \frac{1}{2} \left\{ \left(i\vec{k} \cdot \vec{r} \right), \left\{ \frac{p^i}{m}, \frac{r_s}{r} \right\} \right\} \\ & + \frac{3 i r_s}{4r} \frac{(\vec{k} \times \vec{\sigma})^i}{m} + \frac{1}{4} \left\{ \frac{r_s}{r} \, (i\vec{k} \cdot \vec{r}), \frac{p^i}{m} \right\} . \end{aligned} \tag{8.12}$$

This result contains a gravitational kinetic correction, and gravitational corrections to the magnetic coupling, in addition to the known multipole and retardation corrections [14,18].

8.2.3 Spectrum

The bound-state spectrum resulting from the Hamiltonian (8.8) has recently been evaluated as [12],

$$\begin{aligned} E_{n\ell j} = & - \frac{\alpha_G^2 m_e c^2}{2n^2} + \alpha_G^4 m_e c^2 \left(\frac{15}{8n^4} \right. \\ & \left. - \frac{(7j+5) \, \delta_{\ell,j+1/2}}{(j+1) \, (2j+1) \, n^3} - \frac{(7j+2) \, \delta_{\ell,j-1/2}}{j \, (2j+1) \, n^3} \right) \\ = & - \frac{\alpha_G^2 m_e c^2}{2n^2} + \frac{\alpha_G^4 m_e c^2}{n^3} \left(\frac{15}{8n} - \frac{14\,\varkappa + 3}{2\,|\varkappa| \, (2\varkappa + 1)} \right) , \end{aligned} \tag{8.13}$$

where ℓ is the orbital angular momentum, j is the total angular momentum of the bound particle, and $\varkappa$ is the (integer) Dirac angular quantum number,

$$\varkappa = 2(\ell - j) \, (j + 1/2) = (-1)^{j+\ell+1/2} \left(j + \frac{1}{2} \right) . \tag{8.14}$$

For a bound electron-proton system, the coupling constant entering the gravitational spectrum given in Eq. (8.13) reads as

$$\alpha_G = \frac{G\, m_e\, m_p}{\hbar\, c} = 3.21637(39) \times 10^{-42}\,. \tag{8.15}$$

The coupling α_G is much larger than for particles bound to macroscopic objects. By contrast, the electrodynamic coupling parameter

$$\alpha_{QED} = \frac{e^2}{4\pi\hbar\epsilon_0 c} \approx \frac{1}{137.036} \tag{8.16}$$

is just the fine-structure constant.

8.3 Global Dilation Transformation

8.3.1 Lagrangian

The analogy of the leading (Schrödinger) term in Eq. (8.13) for the nonrelativistic contribution to the bound-state energy (under the replacement $\alpha_G \to \alpha_{QED}$) may encourage us to look for connections of gravitational and electromagnetic interactions on a more global scale, possibly, using scaling transformations [19]. Indeed, the first attempts to unify electromagnetism with gravity are almost 100 years old [20,21]. Let us apply a scaling transformation to the boson and fermion fields,

$$A^\mu \to \lambda\, A^\mu\,, \qquad A_\mu \to \lambda\, A_\mu\,, \qquad \psi \to \lambda\, \psi\,, \tag{8.17}$$

combined with a transformation of the coordinates,

$$x^\mu \to \lambda^{-1/2}\, x^\mu\,, \qquad x_\mu \to \lambda^{-1/2}\, x_\mu\,, \tag{8.18}$$

and of the metric

$$g_{\mu\nu} \to \lambda g_{\mu\nu}\,, \qquad g^{\mu\nu} \to \lambda^{-1}\, g^{\mu\nu}\,, \tag{8.19}$$

Under this transformation, the space-time intervals, the integration measure, the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R, transform as follows,

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = g^{\mu\nu}\, dx_\mu\, dx_\nu \to ds^2\,, \tag{8.20a}$$

$$d^4x = d^4x \to \frac{d^4x}{\lambda^2}\,, \qquad \det\, g = \det\, g_{\mu\nu} \to \lambda^4\, \det\, g\,, \tag{8.20b}$$

$$R_{\mu\nu} \to \lambda\, R_{\mu\nu}\,, \qquad R = g^{\mu\nu}\, R_{\mu\nu} \to R\,. \tag{8.20c}$$

The Einstein–Maxwell Lagrangian, with a coupling to the fermion terms, is given as

$$S = \int d^4x\, \sqrt{-\det g}\, \left\{ \frac{R}{16\,\pi\, G} - \frac{1}{4}\, F^{\mu\nu}\, F_{\mu\nu} \right.$$
$$\left. +\overline{\psi}(x)\;\; [i\overline{\gamma}^\mu\;\; (\nabla_\mu - e\, A_\mu) - m]\;\; \psi(x) \right\}\,. \tag{8.21}$$

It transforms into

$$S' = \int \frac{d^4x}{\lambda^2} \sqrt{-\lambda^4 \det g} \left\{ \frac{R}{16\pi G} - \frac{\lambda^2}{4} F^{\mu\nu} F_{\mu\nu} \right. \\ \left. + \lambda^2 \overline{\psi}(x) \left[i \lambda^{-1/2} \overline{\gamma}^\mu \left(\lambda^{1/2} \nabla_\mu - e \lambda A_\mu \right) - m \right] \psi(x) \right\} , \tag{8.22}$$

which can be rearranged into

$$S'' = \frac{S'}{\lambda^2} = \int d^4x \sqrt{-\det g} \left\{ \frac{R}{16\pi G \lambda^2} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right. \\ \left. + \overline{\psi}(x) \left[i \overline{\gamma}^\mu \left(\nabla_\mu - e \lambda^{1/2} A_\mu \right) - m \right] \psi(x) \right\} . \tag{8.23}$$

The Lagrangian S'' is the same S, but with scaled coupling constants,

$$G \to \lambda^2 G , \qquad e^2 \to \lambda e^2 . \tag{8.24}$$

This scaling suggests a deeper connection of the coupling constants of electromagnetic and gravitational interactions, which is explored in further detail in Ref. [19].

8.3.2 Coupling Constants

If we assume that the scaling (8.24) holds globally, with the current Universe "picking" a value of λ, then this scaling might suggest a relationship of the type

$$\alpha_{\rm QED}^2 \propto e^4 \propto \lambda^2 \propto G . \tag{8.25}$$

Indeed, as discussed in Ref. [19], a relationship of the type $\alpha_{\rm QED} \propto \sqrt{G}$ is otherwise suggested by string theory; the rough analogy being that gravitational interactions in string theory correspond to "closed" strings while electromagnetic interactions correspond to "open" strings. The product of two "open" string amplitudes is proportional to $e^2 \propto g_0^2 \propto \alpha_{\rm QED}$, while the "closed"-string amplitude is proportional to $\kappa \propto g_c \propto \sqrt{G}$. According to Eq. (3.7.17) of Ref. [22], the proportionality

$$g_o^2 \propto g_c \qquad \Leftrightarrow \qquad \alpha_{\rm QED}^2 \propto \sqrt{G} \tag{8.26}$$

therefore is suggested by string theory. A simple analytic form of the proportionality factor in the relationship $\alpha_{\rm QED}^2 \propto \sqrt{G}$ has recently been given in Eq. (8) of Ref. [1].

8.4 Conclusions

We have performed an analysis of the gravitationally coupled Dirac equation in the curved space-time surrounding a central gravitating object, which is described by the (static) Schwarzschild metric. The Foldy–Wouthuysen method leads to gravitational zitterbewegung terms and the gravitational spin-orbit coupling, which is also known as the Fokker precession term. In a curved space-time, the photon emission vertex receives additional corrections due to the curved background, which can be given, within the multipole expansion and for a conceptually simple background metric (e.g., the Schwarzschild metric), in closed analytic form (at least for the first terms of the multipole and retardation expansion). The gravitational bound states display a certain analogy for the gravitational as compared to the electromagnetic (Schrödinger) central-field problem. Based on this analogy, one may explore possible connections of the gravitational and electromagnetic coupling constants, based on scaling arguments. Such a scaling transformation gives additional support for the relationship $\alpha_{\rm QED}^2 \propto \sqrt{G}$, which has been suggested by string theory [22].

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9 Can Spin-charge-family Theory Explain Baryon Number Non-conservation?

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Abstract. The *spin-charge-family* theory [1–12,14], in which spinors carry besides the Dirac spin also the second kind of the Clifford object, no charges, is a kind of the Kaluza-Klein theories [13]. The Dirac spinors of one Weyl representation in $d = (13 + 1)$ manifest [1,4,3,10,14,15] in $d = (3 + 1)$ at low energies all the properties of quarks and leptons assumed by the *standard model*. The second kind of spins explains the origin of families. Spinors interact with the vielbeins and the two kinds of the spin connection fields, the gauge fields of the two kinds of the Clifford objects, which manifest in $d = (3 + 1)$ besides the gravity and the known gauge vector fields also several scalar gauge fields. Scalars with the space index $s \in (7, 8)$ carry the weak charge and the hyper charge ($\mp\frac{1}{2}, \pm\frac{1}{2}$, respectively), explaining the origin of the Higgs and the Yukawa couplings. It is demonstrated in this paper that the scalar fields with the space index $t \in (9, 10, \ldots, 14)$ carry the triplet colour charges, causing transitions of antileptons and antiquarks into quarks and back, enabling the appearance and the decay of baryons. These scalar fields are offering in the presence of the right handed neutrino condensate, which breaks the $\mathcal{CP}$ symmetry, the answer to the question about the matter-antimatter asymmetry.

Povzetek. V teoriji *spinov-nabojev-družin* [1–12,14] nosijo spinorji dve vrsti kvantnih števil, ki jih določata dve vrsti operatorjev γ^a: Diracovi operatorji γ^a in avtoričini $\tilde{\gamma}^a$, obe sta povezani z množenjem Cliffordovih objektov, ena vrsta z leve, druga z desne. Obe vrsti spina sta neodvisni in tvorita druga drugi ekvivalentne upodobitve. Analiza Lorentzove grupe $SO(13, 1)$ s podgrupami te grupe pokaže, da vsebuje ena Weylova upodobitev Diracovih spinorjev v $d = (13 + 1)$ vse kvarke in leptone (ter antikvarke in antileptone) s kvantnimi števili kot jih predpiše *standardni model* pred elektrošibko zlomitvijo, le da so desnoročni nevtrini enakopravni partnerji elektronom [1,4,3,10,14,15]. Druga vrsta spina pojasni izvor družin. Spinorji interagirajo s tetradami in s polji dveh vrst spinskih povezav, ki so umeritvena polja obeh vrst operatorjev gamma. Po zlomitvi simetrij, tedaj pri opazljivih nizkih energijah, določajo ta polja, skupaj z vektorskimi svežnji, gravitacijo in vsa znana umeritvena vektorska polja. Določajo pa tudi skalarna polja. Skalarna polja s prostorskim indexom $s = (7, 8)$ so šibki dubleti ($\tau^{13} = \mp\frac{1}{2}, Y = \pm\frac{1}{2}$), kar pojasni izvor Higgsovega skalarnega polja in Yukawinih sklopitev. Skalarna polja s prostorskim indeksom $t \in (9, 10, \ldots, 14)$ pa so barvni tripleti, ki povzročajo prehode antileptonov in antikvarkov v kvarke in obratno, kar omogoči nastanek in razpad barionov. Vsa skalarna polja nosijo glede na kvantna števila, ki jih določajo Diracovi γ^a in družinski $\tilde{\gamma}^a$, tudi družinska in Diracova kvantna števila v adjungirani upodobitvi grup. Lepota te teorije je, da en sam kondenzat iz dveh desnoročnih nevtrinov z družinskimi kvantnimi števili, ki niso družinska kvantna števila spodnjih štirih družin, zlomi diskretno simetrijo $\mathcal{CP}$ in poskrbi za maso vseh skalarnih polj, ter še neopaženega vektorskega polja. Ta skalarna polja ponujajo v prisotnosti kondenzata desnoročnih nevtrinov, ki zlomi simetrijo $\mathcal{CP}$, odgovor na vprašanje kako je v našem vesolju nastala opazljiva asimetrija med snovjo in antisnovjo. Skalarna polja s prostorskim indexom $s = (7, 8)$ zlomijo z neničelno vakuumsko pričakovano vrednostjo še šibki in hipernaboj, in spremenijo tudi lastno maso, ter tako pojasnijo vse privzetke standardnega modela. Ker teorija napoveduje dve ločeni gruči po štiri družine kvarkov in leptonov, pojasni stabilna od zgornjih širih družin izvor temne snovi. Teorija pa napoveduje tudi, da bodo na LHC izmerili četrto k trem že opaženim družinam, izmerili pa bodo tudi več skalarnih polj.

9.1 Introduction

The *spin-charge-family* [1–12,14] theory is offering, as a kind of the Kaluza-Klein like theories, the explanation for the charges of quarks and leptons (right handed neutrinos are in this theory the regular members of a family) and antiquarks and antileptons [15,16], and for the existence of the corresponding gauge vector fields. The theory explains, by using besides the Dirac kind of the Clifford algebra objects also the second kind of the Clifford algebra objects (there are only two kinds [5–7,3,17,18,20,19], associated with the left and the right multiplication of any Clifford object), the origin of families of quarks and leptons and correspondingly the origin of the scalar gauge fields causing the electroweak break. These scalar fields are responsible, after gaining nonzero vacuum expectation values, for the masses and mixing matrices of quarks and leptons [9–11] and for the masses of the weak vector gauge fields. They manifest, carrying the weak charge and the hyper charge equal to ($\pm\frac{1}{2}$, $\mp\frac{1}{2}$, respectively) [14], as the Higgs field and the Yukawa couplings of the *standard model*.

The *spin-charge-family* theory predicts two decoupled groups of four families [3,4,9–11]: The fourth of the lower group will be measured at the LHC [10], while the lowest of the upper four families constitutes the dark matter [12].

This theory also predicts the existence of the scalar fields which carry the triplet colour charges. All the scalars fields carry the fractional quantum numbers with respect to the scalar index $s \geq 5$, either the ones of $SU(2)$ or the ones of $SU(3)$, while they are with respect to other groups in the adjoint representations. Neither these scalar fields nor the scalars causing the electroweak break are the supersymmetric scalar partners of the quarks and leptons, since they do not carry all the charges of a family member.

These scalar fields with the triplet colour charges cause transitions of antileptons into quarks and antiquarks into quarks and back, offering, in the presence of the condensate of the two right handed neutrinos with the family quantum numbers belonging to the upper four families which breaks the CP symmetry, the explanation for the matter-antimatter asymmetry. This is the topic of the present paper.

Let me point out that the *spin-charge-family* theory overlaps in many points with other unifying theories [26–31], since all the unifying groups can be seen as the subgroups of the large enough orthogonal groups, with family groups included. But there are also many differences. While the theories built on chosen groups must for their choice propose the Lagrange densities designed for these groups and representations (which means that there must be a theory behind this effective Lagrange densities), the *spin-charge-family* theory starts with a very simple action, from where all the properties of spinors and the gauge vector and scalar fields follow, provided that the breaks of symmetries occur.

Consequently this theory differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up on different levels of the break of symmetries, in the unification scheme, in the family degrees of freedom and correspondingly also in the evolution of our universe.

It will be demonstrated in this paper that one condensate of two right handed neutrinos makes all the scalar gauge fields and all the vector gauge fields massive on the scale of the appearance of the condensate, except the vector gauge fields which appear in the *standard model* action before the electroweak break as massless fields. The scalar gauge fields, which cause the electroweak break while gaining nonzero vacuum expectation values and changing their masses, then explain masses of quarks and leptons and of the weak bosons.

It is an extremely encouraging fact, that one scalar condensate and the nonzero vacuum expectation values of some scalar fields, those with the weak and the hyper charge equal to by the *standard model* required charges for the Higgs's scalar, can bring the simple starting action in $d = (13+1)$ to manifest in $d = (3+1)$ in the low energy regime the observed phenomena of fermions and bosons, explaining the assumptions of the *standard model* and can possibly answer also the open questions, like the ones of the appearance of family members, of families, of the dark matter and of the matter-antimatter asymmetry.

The paper leaves, however, many a question connected with the break of symmetries open. Although the scales of breaks of symmetries can roughly be estimated, for the trustworthy predictions a careful study of the properties of fermions and bosons in the expanding universe is needed. It stays to be checked under which conditions in the expanding universe, the starting fields (fermions with the two kinds of spins and the corresponding vielbeins and the two kind of the spin connection fields) after the spontaneous breaks manifest in the low energy regime the observed phenomena. This is a very demanding study, a first simple step of which was done in the refs. [12,22]. The present paper is a step towards understanding the matter-antimatter asymmetry within the *spin-charge-family* theory.

In the subsection 9.1.1 I present the *action* and the *assumptions* of the *spin-charge-family* theory, with the comments added.

In sections (9.2, 9.4, 9.5, 9.3) the properties of the scalar and vector gauge fields and of the condensate are discussed. In appendices the discrete symmetries of the *spin-charge-family* theory and the technique used for representing spinors, with the one Weyl representation of $SO(13,1)$ and the families in $SO(7,1)$ included, is briefly presented. The final discussions are presented in sect. 9.7.

9.1.1 The action of the *spin-charge-family* theory and the assumptions

In this subsection all the assumptions of the *spin-charge-family* theory are presented and commented. This subsection follows to some extend a similar subsection of

the ref. [14].
i. The space-time is $d(=(13+1))$ dimensional. Spinors carry besides the internal degrees of freedom, determined by the Dirac γ^a's operators, also the second kind of the Clifford algebra operators [5–7,4], called $\tilde{\gamma}^a$'s.
ii. In the simple action [3,1] fermions ψ carry in $d=(13+1)$ only two kinds of spins, no charges, and *interact correspondingly with only the two kinds of the spin connection gauge fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and the vielbeins, $f^{\alpha}{}_{a}$.*

$$\begin{aligned}
S &= \int d^dx \, E \, \mathcal{L}_f + \\
&\quad \int d^dx \, E \, (\alpha \, R + \tilde{\alpha} \, \tilde{R}) \,, \\
\mathcal{L}_f &= \frac{1}{2} \, (\bar{\psi} \, \gamma^a p_{0a} \psi) + \text{h.c.}, \\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^{\alpha}{}_a\}_-, \\
p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \\
R &= \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]} \, (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \, \omega^c{}_{b\beta}) \} + \text{h.c.} \,, \\
\tilde{R} &= \frac{1}{2} \, f^{\alpha[a} f^{\beta b]} \, (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c{}_{b\beta}) + \text{h.c.} \,.
\end{aligned} \tag{9.1}$$

Here [1] $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. S^{ab} and $\tilde{S}^{ab}$ are generators (Eqs.(9.5, 9.37, 9.37) of the groups $SO(13,1)$ and $\widetilde{SO}(13,1)$, respectively, expressible by γ^a and $\tilde{\gamma}^a$.
iii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (which manifests as $SU(3) \times U(1)$), affecting both internal degrees of freedom, $SO(13+1)$ and $\widetilde{SO}(13+1)$. After this break there are $2^{((7+1)/2-1)}$ massless families, all the rest families get heavy masses [2].
Both internal degrees of freedom, the ordinary $SO(13+1)$ one (where γ^a determine spins and charges of spinors) and the $\widetilde{SO}(13+1)$ (where $\tilde{\gamma}^a$ determine family quantum numbers), break simultaneously with the manifolds.
iv. There are additional breaks of symmetry: The manifold $M^{(7+1)}$ breaks further

[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b$, $e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[2] A toy model [22,23,15] was studied in $d=(5+1)$ with the same action as in Eq.'(9.1). For a particular choice of vielbeins and for a class of spin connection fields the manifold M^{5+1} breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families stay massless and mass protected. Equivalent assumption, although not yet proved that it really works, is made also in the case that $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^{(6)}$. The study is in progress.

into $M^{(3+1)}\times M^{(4)}$.
v. There is a scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale above the unification scale, to all the vector and scalar gauge fields, which interact with the condensate.
vi. There are nonzero vacuum expectation values of the scalar fields with the scalar indices $(7,8)$, which cause the electroweak break and bring masses to the fermions and weak gauge bosons, conserving the electromagnetic and colour charge.

Comments on the assumptions:

i.: There are, as already written above, two (only two) kinds of the Clifford algebra objects. The Dirac one (Eq.(9.35)) (γ^a) will be used to describe spins of spinors (fermions) in $d=(13+1)$, manifesting in $d=(3+1)$ the spin and all the fermion charges, the second one (Eq.(9.35)) ($\tilde{\gamma}^a$) will describe families of spinors. The representations of γ^a's and $\tilde{\gamma}^a$'s are orthogonal to one another [3]. There are correspondingly two groups determining internal degrees of freedom of spinors in $d=(13+1)$: The Lorentz group $SO(13,1)$ and the group $\widetilde{SO}(13,1)$.
One Weyl representation of $SO(13,1)$ contains, if analysed [1,3,4,15] with respect to the *standard model* groups, all the family members, assumed by the *standard model*, with the right handed neutrinos included (the family members are presented in table 9.3). It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), the right handed weakless and $SU(2)_{II}$ charged quarks and leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless colour antitriplet antiquarks and (anti)colourless antileptons, and the left handed weakless and $SU(2)_{II}$ charged antiquarks and antileptons. The reader can easily check the properties of the representations of spinors (table 9.3), presented in the "technique" (appendix 9.9) way, if using Eqs. (9.5, 9.8, 9.9, 9.11, 9.14).
Each family member carries the family quantum numbers, originating in $\tilde{\gamma}^a$'s degrees of freedom. Correspondingly $\tilde{S}^{ab}$ changes the family quantum numbers, leaving the family member quantum number unchanged.
ii.: This starting action enables to represent the *standard model* as an effective low energy manifestation of the *spin-charge-family* theory, which explains all the *standard model* assumptions, with the families included. There are gauge vector fields, massless before the electroweak break: gravity, colour $SU(3)$ octet vector gauge fields, weak $SU(2)$ (it will be named $SU(2)_I$) triplet vector gauge field and "hyper" $U(1)$ (it will be named $U(1)_I$) singlet vector gauge fields. All are superposition of $f^{\alpha}{}_{c}\,\omega_{ab\alpha}$. There are (eight rather than the observed three) families of quarks and leptons, massless before the electroweak break.
These eight families are indeed two decoupled groups of four families, in the fundamental representations with respect to twice $\widetilde{SU}(2)\times\widetilde{SU}(2)$ groups, the

[3] One can learn in Eq. (9.44)of appendix (9.9) that S^{ab} transforms one state of the representation into another state of the same representation, while $\tilde{S}^{ab}$ transforms the state into the state belonging to another representation.

subgroups of $\widetilde{SO}(3,1)\times\widetilde{SO}(4)\in\widetilde{SO}(7,1)$. The scalar gauge fields, determining the mass matrices of quarks and leptons, carry with respect to the scalar index $s\in(7,8)$ the weak and the hyper charge of the scalar Higgs, while they carry if they are the superposition of $f^{\sigma}{}_{s}\,\tilde{\omega}_{ab\sigma}$ two kinds of the family quantum numbers in the adjoint representations, representing two (orthogonal) groups, each of the group contains two triplets (with respect to $\widetilde{SU}(2)_{\widetilde{SO}(3,1)}\times\widetilde{SU}(2)_{\widetilde{SO}(4)}$).
The scalar fields with the quantum numbers (Q, Q', Y', which are the superposition of $f^{\sigma}{}_{s}\,\omega_{ab\sigma}$) are the three singlets, again carrying the weak and the hyper charge of the scalar Higgs. One group of two triplets determine, together with the three singlets, after gaining nonzero expectation values, the Higgs's scalar and the Yukawa couplings of the *standard model*. The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge field and the scalar fields with the space index $s\in(5,6)$ and $t\in(9,10,11,12)$, as well as the auxiliary vector gauge fields expressible (Eqs. (9.56, 9.55) in the appendix 9.10) with vielbeins. They all remain either auxiliary (if there are no spinor sources manifesting their quantum numbers) or become massive after the appearance of the condensate.
iii., iv.: The assumed break from $M^{(13+1)}$ first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting the symmetry $SU(3)\times U(1)_{II}$) explains why the weak and the hyper charge are connected with the handedness of spinors. In the spinor representation of $SO(7,1)$ there are left handed weak charged quarks and leptons with the hyper charges ($\frac{1}{6}, -\frac{1}{2}$, respectively) and the right handed weak chargeless quarks with the hyper charge either $\frac{2}{3}$ or $-\frac{1}{3}$, while the right handed weak chargeless leptons carry the hyper charge either 0 or (−1). A further break from $M^{(7+1)}$ into $M^{(3+1)}$ $\times\, M^{4)}$, manifesting the symmetry $SO(3,1)\times SU(2)_{I}\times SU(2)_{II}\times U(1)_{II}\times SU(3)$, explains the observed properties of the family members - the coloured quarks, left handed weak charged and $SU(2)_{II}$ chargeless and right handed weak chargeless and $SU(2)_{II}$ charged and colourless leptons, again left handed weak charged and $SU(2)_{II}$ chargeless and right handed weak chargeless and $SU(2)_{II}$ charged, quarks with the "spinor" charge $\frac{1}{6}$ and leptons with the "spinor" charge $-\frac{1}{2}$ - and of the observed vector gauge fields and the scalar fields (through Higgs's scalar and Yukawa couplings).
Since the left handed members distinguish from the right handed partners in the weak and the hyper charges, the family members of all the families stay massless and mass protected up to the electroweak break [4]. Antiparticles are accessible from particles by the $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$, as explained in refs. [15,16] and briefly also in the appendix (9.8). This discrete symmetry operator does not contain $\tilde{\gamma}^{a}$'s degrees of freedom. To each family member there corresponds the antimember, with the same family quantum number.
v.: It is a condensate of the two right handed neutrinos with the quantum numbers of the upper four families (table 9.2), appearing in the energy region

[4] As long as the left handed family members and their right handed partners carry different conserved charges, they can not behave as massive particles, they are mass protected. It is the appearance of nonzero vacuum expectation values of the scalar fields, carrying the weak and the hyper charge, which cause non conservation of these two charges, which makes the superposition of the left and the right handed family members possible, and breaks the mass protection.

above the unification scale, which makes all the scalar gauge fields (those with the space index $(5,6,7,8)$, as well as those with the space index $(9,\ldots,14)$) and the vector gauge fields, manifesting nonzero quantum numbers τ^4, τ^{23}, Q ,Y, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{Q}$,$\tilde{Y}$,$\tilde{N}^3_R$ (Eqs. (9.8, 9.9, 9.11, 9.12, 9.13, 9.14)) massive.
vi.: At the electroweak break the scalar fields with the space index $s=(7,8)$, triplets with respect to the family index (originating in $\tilde{\omega}_{abs}$, Eq. (9.16)) and the three singlets carrying the charges (Q, Q', Y') (originating in $\omega_{ts's}$, Eq. (9.15)), all with the weak and the hyper charge equal to $(\mp\frac{1}{2}, \pm\frac{1}{2}$, respectively), get nonzero vacuum expectation values, changing also their masses and breaking the weak and the hyper charge symmetry. These scalars determine mass matrices of twice the four families, as well as the masses of the weak bosons.
All the rest scalar fields keep masses of the condensate scale and are correspondingly unobservable in the low energy regime [5]. The fourth family to the observed three ones will (sooner or later) be observed at the LHC. Its properties are under the consideration [10], while the stable of the upper four families is the candidate for the dark matter constituents.

The above assumptions enable that the starting action (Eq. (9.1)) manifests effectively in $d=(3+1)$ in the low energy regime fermion and boson fields as assumed by the *standard model*.

To see this [3,1,4–8,2,9,10,12,14], let us formally rewrite the Lagrange density for a Weyl spinor of (Eq.(9.1)), as follows

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m (p_m - \sum_{Ai} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
&\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi\} + \\
&\{\sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^t p_{0t}\,\psi\}, \\
p_{0s} &= p_s - \frac{1}{2} S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}, \\
p_{0t} &= p_t - \frac{1}{2} S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt},
\end{aligned} \tag{9.2}
$$

where $m \in (0,1,2,3)$, $s \in 7,8$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within $\in (0,1,2,3)$ and $\in (5,6,7,8)$, $t \in (5,6,9,\ldots,13,14)$, $(t',t") \in (5,6,7,8)$ and $\in (9,10,\ldots,14)$. ψ represents all family members of all the families. The generators of the charge groups τ^{Ai} (expressed in Eqs. (9.3), (9.9), (9.11) in terms of S^{ab}) fulfil the commutation relations

$$
\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}, \\
\{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB} f^{Aijk}\tau^{Ak}.
\end{aligned} \tag{9.3}
$$

[5] Correspondingly $d=(13+1)$ manifests in $d=(3+1)$ spins and charges as if there would be $d=(9+1)$, since the plane $(5,6)$ and the plane in which the vector τ^4 lies, are unobservable at low energies.

The spin generators are defined in Eq. (9.8). These group generators determine all the internal degrees of freedom of one family members as seen from the point of view of $d = (3+1)$: The colour charge ($SU(3)$ with the generators $\vec{\tau}^3$) and the "spinor charge" ($U(1)_{II}$) with the generator τ^4 originating in $SO(6)$, the weak charge $SU(2)_I$ with the generators $\vec{\tau}^1$ and the second $SU(2)_{II}$ charge with the generators $\vec{\tau}^2$ originating in $SO(4)$ ($SU(2)_{II}$ breaks in the presence of the condensate into $U(1)_I$, defining together with τ^4 the hyper charge Y ($= \tau^{23} + \tau^4$) and the spin determined by $SO(3,1)$.
The condensate of two right handed neutrinos with the family quantum numbers of the upper four families bring masses (of the unifying scale $\geq 10^{16}$ GeV or above) to all the scalar and those vector gauge fields which are not observed at so far measurable energies.
The scalar fields causing, when getting nonzero vacuum expectation values, the electroweak phase transitions changing at the transition also their own masses, bring masses to the eight families and to the weak bosons. We shall comment all these fields in what follows.

The first line of Eq. (9.2) describes [1,3] before the electroweak break the dynamics of eight families of massless fermions in interaction with the massless colour $\vec{A}^3_m$, weak $\vec{A}^1_m$ and hyper A^Y_m ($= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m$) gauge fields, all are the superposition of ω_{abm} [6].

The second line of the same equation (Eq. (9.2)) determines the mass term, which after the electroweak break brings masses to all the family members of the eight families and to the weak bosons. The scalar fields responsible - after getting nonzero vacuum expectation values - for masses of the family members and of the weak bosons are namely included in the second line of Eq. (9.2) as $(-\frac{1}{2} S^{s's"} \omega_{s's"s} - \frac{1}{2} \tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s =\in (7,8)\,, (s', s") \in (5,6,7,8), (\tilde{a},\tilde{b}) \in (\tilde{0},\tilde{1},\dots,\tilde{8}))$ [7]. The properties of these scalar fields are discussed in sect. (9.4), where the proof is presented that they all carry the weak charge and the hyper charge as the *standard model* Higgs's scalar, while they are either triplets with respect to the family quantum numbers or singlets with respect to the charges Q, Q' and Y'. While the two triplets ($\vec{\tilde{A}}^1_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$) interact with the lower four families, interact ($\vec{\tilde{A}}^2_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$) with the upper four families. These twice two triplets are superposition of $\frac{1}{2}\, \tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s =\in (7,8)$, Eq. (9.16). The three singlets (A^Q_s, $A^{Q'}_s$ and $A^{Y'}_m$) are superposition of $\omega_{s's"s}$, Eq.(9.15). They interact with the family members of all the families, "seeing" family members charges.

The third line of Eq. (9.2) represents fermions in interaction with all the rest scalar fields. Scalar fields become massive after interacting with the condensate. Those which do not gain nonzero vacuum expectation values, keep the heavy masses of the order of the scale of the condensate up to low energies. The massive

---

[6] These superposition can easily be found by using Eqs. (9.11, 9.9). They are explicitly written in the ref. [3]. The interaction with the condensate makes the fields $A^{Y'}_m$, Eq. (9.14), A^{21}_m and A^{22}_m very massive (at the scale of the condensate).

[7] To point out that S^{ab} and $\tilde{S}^{ab}$ belong to two different kinds of the Clifford algebra objects are the indices (a,b) are in $\tilde{S}^{ab}$ in this paragraph written as $(\tilde{a},\tilde{b})$. Normally only (a,b) will be used for S^{ab} and $\tilde{S}^{ab}$.

scalars with the space index $t \in (5,6)$ transform (table 9.3) u_R-quarks into d_L-quarks and ν_R-leptons into e_L-leptons and back, as well as $\bar{u}_R$-antiquarks into $\bar{d}_L$-antiquarks and back and $\bar{\nu}_R$-antileptons into $\bar{e}_L$-antileptons and back, breaking in the presence of the condensate the Q global symmetry. Those scalar fields with the space index $t = (9, 10, \cdots, 14)$ transform antileptons into quarks and antiquarks into quarks and back. They are offering in the presence of the scalar condensate breaking the $\mathcal{CP}$ symmetry the explanation for the observed matter-antimatter asymmetry, as we shall show in sect. 9.2.

Let us write down the part of the fermion action which in the presence of the condensate offers the explanation for the observed matter/antimatter asymmetry.

$$\mathcal{L}_{f'} = \psi^{\dagger}\gamma^0\gamma^t\{ \sum_{t=(9,10,\dots 14)} [p_t - (\frac{1}{2} S^{s's"}\omega_{s's"t} + \frac{1}{2} S^{t't''}\omega_{t't"t} + \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt})]\}\,\psi\,, \tag{9.4}$$

where $(s', s") \in (5,6,7,8)$, $(t, t', t") \in (9, 10, \dots, 14)$ and $(a, b) \in (0,1,2,3)$ and $\in (5,6,7,8)$, in agreement with the assumed breaks in sect. (9.1). Again operators $\tilde{S}^{ab}$ determine family quantum numbers and S^{ab} determine family members quantum numbers. Correspondingly the superposition of the scalar fields $\tilde{\omega}_{abt}$ and the superposition of the scalar fields ω_{abt} carry the quantum numbers determined by either the superposition of $\tilde{S}^{ab}$ or by the superposition S^{ab} in the adjoint representations, while they carry the colour charge, determined by the space index $t \in (9, 10, \dots, 14)$, in the triplet representation of the $SU(3)$ group, as we shall see. Similarly the scalars with the space index $s \in (7,8)$ carry the weak and the hyper charge in the doublets representations.

The condensate of two right handed neutrinos with the family quantum numbers of the upper four families carries (table 9.2) $\tau^4 = 1$, $\tau^{23} = -1$, $\tau^{13} = 0$, $Y = 0$, $Q = 0$, and the family quantum numbers of the upper four families and gives masses to scalar and vector gauge fields with the nonzero corresponding quantum numbers. The only vector gauge fields which stay massless up to the electroweak break are the hyper charge field (A^Y_m), the weak charge field ($\vec{A}^1_m$) and the colour charge field ($\vec{A}^3_m$).

The *standard model* **subgroups of the $SO(13+1)$ and $\widetilde{SO}(13+1)$ groups and the corresponding gauge fields** This section follows to large extend the refs. cite-JMP,NscalarsweakY2014. To calculate quantum numbers of one Weyl representation presented in table 9.3 in terms of the generators of the *standard model* charge groups τ^{Ai} $(= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab})$ one must look for the coefficients $c^{Ai}{}_{ab}$ (Eq. (9.3)). Similarly also the spin and the family degrees of freedom can be expressed.

The same coefficients $c^{Ai}{}_{ab}$ determine operators which apply on spinors and on vectors. The difference among the three kinds of operators - vector and two kinds of spinor - lies in the difference among S^{ab}, $\tilde{S}^{ab}$ and $\mathcal{S}^{ab}$.

While S^{ab} for spins of spinors is equal to

$$S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)\,, \quad \{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \tag{9.5}$$

and $\tilde{S}^{ab}$ for families of spinors is equal to

$$\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,, \quad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,,$$
$$\{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{9.6}$$

one must take, when $\mathcal{S}^{ab}$ apply on the spin connections ω_{bde} $(= f^{\alpha}{}_e\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_e\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, the operator

$$(\mathcal{S}^{ab})^c{}_e A^{d\dots e\dots g} = i(\eta^{ac}\delta^b_e - \eta^{bc}\delta^a_e)\, A^{d\dots e\dots g}\,. \tag{9.7}$$

This means that the space index (e) of ω_{bde} transforms according to the requirement of Eq. (9.7), and so do b, d and $\tilde{b}, \tilde{d}$. Here I used again the notation $\tilde{b}, \tilde{d}$ to point out that S^{ab} and $\tilde{S}^{ab}$ $(= \tilde{S}^{\tilde{a}\tilde{b}})$ are the generators of two independent groups[14].

One finds [1,3–8,2] for the generators of the spin and the charge groups, which are the subgroups of $SO(13,1)$, the expressions:

$$\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})\,, \tag{9.8}$$

where the generators $\vec{N}_{\pm}$ determine representations of the two $SU(2)$ invariant subgroups of $SO(3,1)$, the generators $\vec{\tau}^1$ and $\vec{\tau}^2$,

$$\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,, \tag{9.9}$$
$$\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78})\,, \tag{9.10}$$

determine representations of the $SU(2)_I \times SU(2)_{II}$ invariant subgroups of the group $SO(4)$, which is further the subgroup of $SO(7,1)$ ($SO(4), SO(3,1)$ are subgroups of $SO(7,1)$), and the generators $\vec{\tau}^3$, τ^4 and $\tilde{\tau}^4$

$$\begin{aligned}
\vec{\tau}^3 :=\ & \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12},\\
& S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13},\\
& S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\},\\
\tau^4 :=\ & -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,,\\
\tilde{\tau}^4 :=\ & -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})\,,
\end{aligned} \tag{9.11}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$, and of $\tilde{\tau}^4$ originating in $\widetilde{SO}(6)$.

One correspondingly finds the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \tag{9.12}$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\begin{aligned}\vec{\tilde{\tau}}^{1} &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}),\\ \vec{\tilde{\tau}}^{2} &:= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}),\end{aligned} \tag{9.13}$$

determine representations of $\widetilde{SU}(2)_{I} \times \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both, $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$, are the subgroups of $\widetilde{SO}(7,1)$.

One further finds [3]

$$\begin{aligned}&Y = \tau^{4} + \tau^{23},\ Y' = -\tau^{4}\tan^{2}\vartheta_{2} + \tau^{23},\ Q = \tau^{13} + Y,\ Q' = -Y\tan^{2}\vartheta_{1} + \tau^{13},\\ &\tilde{Y} = \tilde{\tau}^{4} + \tilde{\tau}^{23},\ \tilde{Y}' = -\tilde{\tau}^{4}\tan^{2}\tilde{\vartheta}_{2} + \tilde{\tau}^{23},\ \tilde{Q} = \tilde{Y} + \tilde{\tau}^{13},\ \tilde{Q}' = -\tilde{Y}\tan^{2}\tilde{\vartheta}_{1} + \tilde{\tau}^{13}.\end{aligned} \tag{9.14}$$

The scalar fields, responsible [1–3] - after getting in the electroweak break nonzero vacuum expectation values - for the masses of the family members and of the weak bosons, and presented in the second line of Eq. (9.2), can be expressed in terms of ω_{abc} fields and $\tilde{\omega}_{abc}$ fields as presented in Eq. (9.15), 9.16).

One can find the below expressions by taking into account Eqs. (9.9, 9.11, 9.12, 9.13) and Eq. (9.14).

$$\begin{aligned}-\frac{1}{2}S^{s's''}\,\omega_{s's''s} &= -(g^{23}\,\tau^{23}\,A^{23}_{s} + g^{13}\,\tau^{13}\,A^{13}_{s} + g^{4}\,\tau^{4}\,A^{4}_{s}),\\ g^{13}\,\tau^{13}\,A^{13}_{s} + g^{23}\,\tau^{23}\,A^{23}_{s} + g^{4}\,\tau^{4}\,A^{4}_{s} &= g^{Q}\,QA^{Q}_{s} + g^{Q'}\,Q'A^{Q'}_{s} + g^{Y'}\,Y'\,A^{Y'}_{s},\\ A^{4}_{s} &= -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s}),\\ A^{13}_{s} &= (\omega_{56s} - \omega_{78s}),\quad A^{23}_{s} = (\omega_{56s} + \omega_{78s}),\\ A^{Q}_{s} &= \sin\vartheta_{1}\,A^{13}_{s} + \cos\vartheta_{1}\,A^{Y}_{s},\\ A^{Q'}_{s} &= \cos\vartheta_{1}\,A^{13}_{s} - \sin\vartheta_{1}\,A^{Y}_{s},\\ A^{Y}_{s} &= \sin\vartheta_{2}\,A^{23}_{s} + \cos\vartheta_{2}\,A^{4}_{s},\\ A^{Y'}_{s} &= \cos\vartheta_{2}\,A^{23}_{s} - \sin\vartheta_{2}\,A^{4}_{s},\\ &(s \in (7,8)).\end{aligned} \tag{9.15}$$

In Eq. (9.15) the coupling constants were explicitly written to see the analogy with the gauge fields in the *standard model*.

$$\begin{aligned}-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\,\tilde{\omega}_{\tilde{a}\tilde{b}s} &= -(\vec{\tilde{\tau}}^{\tilde{1}}\,\vec{\tilde{A}}^{\tilde{1}}_{s} + \vec{\tilde{N}}_{\tilde{L}}\,\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} + \vec{\tilde{\tau}}^{\tilde{2}}\,\vec{\tilde{A}}^{\tilde{2}}_{s} + \vec{\tilde{N}}_{\tilde{R}}\,\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s}),\\ \vec{\tilde{A}}^{\tilde{1}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\ \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{3}s}),\\ \vec{\tilde{A}}^{\tilde{2}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\ \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{3}s}),\\ &(s \in (7,8)).\end{aligned} \tag{9.16}$$

Scalar fields from Eq. (9.16) couple to the family quantum numbers, while those from Eq. (9.15) distinguish among family members.

Expressions for the vector gauge fields in terms of the spin connection fields and the vielbeins, which correspond to the colour charge ($\vec{A}^3_m$), the $SU(2)_{II}$ charge ($\vec{A}^2_m$), the weak charge ($SU(2)_I$) ($\vec{A}^1_m$) and the $U(1)_{II}$ charge originating in $SO(6)$ ($\vec{A}^4_m$), can be found by taking into account Eqs. (9.9, 9.11). Equivalently one finds the vector gauge fields in the "tilde" sector. One really can use just the expressions from Eqs. (9.15, 9.16), if replacing the scalar index s with the vector index m.

9.2 Properties of scalar and vector gauge fields, contributing to transitions of antileptons into quarks

In this - the main - part of the present paper the properties, quantum numbers and discrete symmetries of those scalar and vector gauge fields appearing in the action (Eqs.(9.1, 9.2), 9.4) of the *spin-charge-family* theory [1–9,12] are studied, which cause transitions of antileptons into quarks and back, and antiquarks into quarks and back.

These scalar gauge fields carry the triplet or antitriplet colour charge (see table 9.1) and the fractional hyper and electromagnetic charge.

The Lagrange densities from Eqs. (9.1, 9.2, 9.4) manifest $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance (appendix (9.8)). All the vector and the spinor gauge fields are before the appearance of the condensate massless and reactions creating particles from antiparticles and back goes in both directions equivalently. Correspondingly there is no matter-antimatter asymmetry.

The *spin-charge-family* theory breaks the matter-antimatter symmetry by the appearance of the condensate (sect. 9.3) and also by nonzero vacuum expectation values of the scalar fields causing the electroweak phase transition (sect. 9.4). I shall show that there is the condensate of two right handed neutrinos which breaks this symmetry, giving masses to all the scalar gauge fields and to all those vector gauge fields which would be in contradiction with the observations.

Let us start by analysing the Lagrange density presented in Eq. (9.4) before the appearance of the condensate. The term $\gamma^t \, \frac{1}{2}\, S^{s's"}\, \omega_{s's"t}$ in Eq. (9.4) can be rewritten, if taking into account Eq. (9.42), as follows

$$
\begin{aligned}
\gamma^t \, \frac{1}{2}\, S^{s's"}\, \omega_{s's"t} &= \sum_{+,-} \sum_{(t\,t')} \overset{tt'}{(\oplus)} \; \frac{1}{2}\, S^{s's"}\, \omega_{s"s"\overset{tt'}{(\oplus)}} \,, \\
\omega_{s"s"\overset{tt'}{(\oplus)}} &:= \omega_{s"s"\overset{tt'}{(\pm)}} = (\omega_{s's"t} \mp i\, \omega_{s's"t'}) \,, \\
\overset{tt'}{(\oplus)} &:= \overset{tt'}{(\pm)} = \frac{1}{2}\,(\gamma^t \pm \gamma^{t'}) \,, \\
(t\,t') &\in ((9\,10), (11\,12), (13\,14)) \,.
\end{aligned}
\tag{9.17}
$$

I introduced the notations $\overset{tt'}{(\oplus)}$ and $\omega_{s"s"\overset{tt'}{(\oplus)}}$ to distinguish among different superposition of states in equations below.

Using Eqs. (9.9, 9.11) the expression $\overset{tt'}{(\oplus)}\ \frac{1}{2} S^{s's"}\ \omega_{s"s"\overset{tt'}{(\oplus)}}$ can be further rewritten as follows

$$
\begin{aligned}
&\overset{tt'}{(\oplus)}\quad \frac{1}{2} S^{s's"}\,\omega_{s"s"\overset{tt'}{(\oplus)}} = \\
&\overset{tt'}{(\oplus)}\ \{\tau^{2+} A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-} A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23} A^{23}_{\overset{tt'}{(\oplus)}} + \tau^{1+} A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-} A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13} A^{13}_{\overset{tt'}{(\oplus)}}\},\\
&A^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}} = (\omega_{58\overset{tt'}{(\oplus)}} + \omega_{67\overset{tt'}{(\oplus)}})\ \boxed{\mp}\ i(\omega_{57\overset{tt'}{(\oplus)}} - \omega_{68\overset{tt'}{(\oplus)}}),\\
&A^{23}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} + \omega_{78\overset{tt'}{(\oplus)}}),\\
&A^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}} = \omega_{58\overset{tt'}{(\oplus)}} - \omega_{67\overset{tt'}{(\oplus)}})\ \boxed{\mp}\ i(\omega_{57\overset{tt'}{(\oplus)}} + \omega_{68\overset{tt'}{(\oplus)}}),\\
&A^{13}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} - \omega_{78\overset{tt'}{(\oplus)}}).
\end{aligned}
\tag{9.18}
$$

Equivalently one expresses the term $\gamma^t\ \frac{1}{2}\tilde{S}^{ab}\ \tilde{\omega}_{abt}$ in Eq. (9.4), by using Eqs. (9.12, 9.13), as

$$
\begin{aligned}
&\gamma^t \frac{1}{2}\tilde{S}^{ab}\,\tilde{\omega}_{abt} = \overset{tt'}{(\oplus)}\ \frac{1}{2}\tilde{S}^{ab}\,\tilde{\omega}_{ab\,\overset{tt'}{(\oplus)}} = \\
&\overset{tt'}{(\oplus)}\ \{\tilde{\tau}^{2+} \tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-} \tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23} \tilde{A}^{23}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1+} \tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-} \tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13} \tilde{A}^{13}_{\overset{tt'}{(\oplus)}} +\\
&\tilde{N}^{+}_{R} \tilde{A}^{N_R+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R} \tilde{A}^{N_R-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R} \tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{+}_{L} \tilde{A}^{N_L+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L} \tilde{A}^{N_L-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L} \tilde{A}^{N_L3}_{\overset{tt'}{(\oplus)}}\},\\
&\tilde{A}^{N_R\boxed{\pm}}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{01\overset{tt'}{(\oplus)}})\ \boxed{\mp}\ i(\tilde{\omega}_{31\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{02\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{03\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_L\boxed{\pm}}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{01\overset{tt'}{(\oplus)}})\ \boxed{\mp}\ i(\tilde{\omega}_{31\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{02\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{03\overset{tt'}{(\oplus)}}),
\end{aligned}
\tag{9.19}
$$

with $\tilde{A}^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}}, \tilde{A}^{23}_{\overset{tt'}{(\oplus)}}, \tilde{A}^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$ and $\tilde{A}^{13}_{\overset{tt'}{(\oplus)}}$ following from expressions for $A^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}}, A^{23}_{\overset{tt'}{(\oplus)}}, A^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$ and $A^{13}_{\overset{tt'}{(\oplus)}}$, respectively, in (Eq.(9.18)), if replacing $\omega_{s"s"\overset{tt'}{(\oplus)}}$ by $\tilde{\omega}_{s"s"\overset{tt'}{(\oplus)}}$.

There is the additional term in Eq. (9.4): $\gamma^t\ \frac{1}{2} S^{t't"}\,\omega_{t't"t}$. This term can be written with respect to the generators $S^{t't"}$ as one colour octet scalar field and one $U(1)_{II}$ scalar field (Eq. 9.11)

$$
\begin{aligned}
&\gamma^t \frac{1}{2} S^{t"t'"}\,\omega_{t"t'"t} = \sum_{+,-}\ \sum_{(t\,t')}\ \overset{tt'}{(\oplus)}\ \{\vec{\tau}^3 \cdot \vec{A}^{3}_{\overset{tt'}{(\oplus)}} + \tau^4 \cdot A^{4}_{\overset{tt'}{(\oplus)}}\},\\
&(t\,t') \in ((9\,10), 11\,12), 13\,14))\,.
\end{aligned}
\tag{9.20}
$$

Taking all above equations (9.17, 9.18, 9.19, 9.20) into account Eq. (9.4) can be rewritten, if we leave out $p_{\overset{tt'}{(\oplus)}}$ since in the low energy limit the momentum does not play any role, as follows

$$\begin{aligned}
\mathcal{L}_{f''} = \psi^\dagger \gamma^0 (-) \{ \sum_{+,-} \sum_{(t\,t')} \overset{tt'}{(\oplus)} \cdot \\
[\tau^{2+} A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-} A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23} A^{23}_{\overset{tt'}{(\oplus)}} \\
+ \tau^{1+} A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-} A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13} A^{13}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{\tau}^{2+} \tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-} \tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23} \tilde{A}^{23}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{\tau}^{1+} \tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-} \tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13} \tilde{A}^{13}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{N}^{+}_{R} \tilde{A}^{N_R +}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R} \tilde{A}^{N_R -}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R} \tilde{A}^{N_R 3}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{N}^{+}_{L} \tilde{A}^{N_L +}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L} \tilde{A}^{N_L -}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L} \tilde{A}^{N_L 3}_{\overset{tt'}{(\oplus)}} \\
+ \tau^{3i} A^{3i}_{\overset{tt'}{(\oplus)}} + \tau^{4} A^{4}_{\overset{tt'}{(\oplus)}}] \} \psi ,
\end{aligned} \tag{9.21}$$

where (t, t') run in pairs over $[(9, 10), \ldots (13, 14)]$ and the summation must go over $+$ and $-$ of $\overset{tt'}{(\oplus)}$.

Let us calculate now quantum numbers of the scalar and vector gauge fields appearing in Eq. (9.21) by taking into account that the spin of gauge fields is determined according to Eq. (9.7) ($((S^{ab})^c{}_d A^{d\ldots e\ldots g} = i(\eta^{ac}\delta^b_d - \eta^{bc}\delta^a_d)\, A^{d\ldots e\ldots g}$, for each index ($\in (d\ldots g)$) of a bosonic field $A^{d\cdots g}$ separately). We must take into account also the relation among S^{ab} and the charges (the relations are, of course, the same for bosons and fermions) (Eqs. (9.8, 9.9, 9.11)).

On table 9.1 properties of the scalar gauge fields appearing in Eq. (9.21) are presented.

The scalar fields with the scalar index $s = (9, 10, \cdots, 14)$, presented in table 9.1, carry one of the triplet colour charges and the "spinor" charge equal to twice the quark "spinor" charge, or the antitriplet colour charges and the anti "spinor" charge. They carry in addition the quantum numbers of the adjoint representations originating in S^{ab} or in $\tilde{S}^{ab}$. Although carrying the colour charge in one of the triplet or antitriplet states, these fields can not be interpreted as superpartners of the quarks as required by, let say, the $N = 1$ supersymmetry. The hyper charges and the electromagnetic charges are namely not those required by the supersymmetric partners to the family members.

Let us have a look what do the scalar fields, appearing in Eq. (9.21) and in table 9.1, do when being applied on the left handed members of the Weyl representation presented on table 9.3, containing quarks and leptons and antiquarks and antileptons [4,21,15]. Let us choose the 57^{th} line of table 9.3, which represents in the spinor technique the left handed positron, $\bar{e}^+_L$. If we make, let say, the choice of the term $(\gamma^0 \overset{910}{(+)} \tau^{2\boxminus})\, A^{2\boxminus}_{\overset{9\,10}{(\oplus)}}$ (the scalar field $A^{2\boxminus}_{\overset{9\,10}{(\oplus)}}$ is presented in the 7^{th} line in table 9.1 and in the second line of Eq. (9.21)), the family quantum numbers will

field	prop.	τ^4	τ^{13}	τ^{23}	(τ^{33}, τ^{38})	Y	Q	$\tilde{\tau}^4$	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$A^{1\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	$\boxplus 1$	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}+\boxplus 1$	0	0	0	0	0
$A^{13}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$A^{1\boxplus}_{11\,12}(\oplus)$	scalar	$\oplus\frac{1}{3}$	$\boxplus 1$	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}+\boxplus 1$	0	0	0	0	0
$A^{13}_{11\,12}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$A^{1\boxplus}_{13\,14}(\oplus)$	scalar	$\oplus\frac{1}{3}$	$\boxplus 1$	0	$(0, \oplus\frac{1}{\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}+\boxplus 1$	0	0	0	0	0
$A^{13}_{13\,14}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(0, \oplus\frac{1}{\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$A^{2\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	$\boxplus 1$	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}+\boxplus 1$	$\oplus\frac{1}{3}+\boxplus 1$	0	0	0	0	0
$A^{23}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{1\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	$\boxplus 1$	0	0	0
$\tilde{A}^{13}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{2\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	$\boxplus 1$	0	0
$\tilde{A}^{23}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{N_L\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	$\boxplus 1$	0
$\tilde{A}^{N_L 3}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{N_R\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	$\boxplus 1$
$\tilde{A}^{N_R 3}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$A^{3i}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\boxplus 1+\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$A^{4}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\frac{1}{3}$	0	0	$(\oplus\frac{1}{2}, \oplus\frac{1}{2\sqrt{3}})$	$\oplus\frac{1}{3}$	$\oplus\frac{1}{3}$	0	0	0	0	0
$\vec{A}^3_m$	vector	0	0	0	octet	0	0	0	0	0	0	0
A^4_m	vector	0	0	0	0	0	0	0	0	0	0	0

Table 9.1. Quantum numbers of the scalar gauge fields carrying the space index $t = (9, 10, \cdots, 14)$, appearing in Eq. (9.21), are presented. To the colour charge of all these scalar fields the space degrees of freedom contribute one of the triplets values. These scalars are with respect to the two $SU(2)$ charges, ($\vec{\tau}^1$ and $\vec{\tau}^2$), and the two $\widetilde{SU}(2)$ charges, ($\vec{\tilde{\tau}}^1$ and $\vec{\tilde{\tau}}^2$), triplets (that is in the adjoint representations of the corresponding groups), and they all carry twice the "spinor" number (τ^4) of the quarks. The quantum numbers of the two vector gauge fields, the colour and the $U(1)_{II}$ ones, are added.

not be affected and they can be any. The state carries the "spinor" (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = \frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hyper charge $(Y(= \tau^4 + \tau^{23}))$ is 1 and the electromagnetic charge $Q(= Y + \tau^{13})$ is 1.

So, what does the term $\gamma^0 \stackrel{910}{(+)} \tau^{2\boxminus} A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ make on this spinor? Making use of Eqs. (9.44, 9.46, 9.54) of appendix 9.9 one easily finds that operator $\gamma^0 \stackrel{910}{(+)} \tau^{2-}$ transforms the left handed positron into $\stackrel{03}{(+i)} \stackrel{12}{(+)} | \stackrel{56}{[-]} \stackrel{78}{[-]} || \stackrel{9\,10}{(+)} \stackrel{11\,12}{(-)} \stackrel{13\,14}{(-)}$, which is d_R^{c1}, presented on line 3 of table 9.3. Namely, γ^0 transforms $\stackrel{03}{[-i]}$ into $\stackrel{03}{(+i)}$, $\stackrel{910}{(+)}$ transforms $\stackrel{9\,10}{[-]}$ into $\stackrel{9\,10}{(+)}$, while τ^{2-} $(= - \stackrel{56}{(-)} \stackrel{78}{(-)})$ transforms $\stackrel{56}{(+)} \stackrel{78}{(+)}$ into $\stackrel{56}{[-]} \stackrel{78}{[-]}$. The state d_R^{c1} carries the "spinor" (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hyper charge is $(Y = \tau^4 + \tau^{23} =) -\frac{1}{3}$ and the electromagnetic charge $(Q = Y + \tau^{13} =) -\frac{1}{3}$. The scalar field $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ carries just the needed quantum numbers as we can see in the 7^{th} line of table 9.1.

If the antiquark $\bar{u}_L^{\bar{c}2}$, from the line 43 (it is not presented, but one can very easily construct it) in table 9.3, with the "spinor" charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$, the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hyper charge $Y(= \tau^4 + \tau^{23} =) -\frac{2}{3}$ and the electromagnetic charge $Q(= Y + \tau^{13} =) -\frac{2}{3}$ submits the $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ scalar field, it transforms into u_R^{c3} from the line 17 of table 9.3, carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{2}{3}$ and $Q = \frac{2}{3}$. These two quarks, d_R^{c1} and u_R^{c3} can bind together with u_R^{c2} from the 9^{th} line of the same table (at low enough energy, after the electroweak transition, and if they belong in a superposition with the left handed partners to the first family) into the colour chargeless baryon - a proton. This transition is presented in figure 9.1.

The opposite transition at low energies would make the proton decay.

Let us look at one more example. The 63^{th} line of table 9.3 represents in the spinor technique the right handed positron, $\bar{e}_R^{+}$. Since we shall again not have a look on a transition, in which scalar fields with the nonzero family quantum numbers are involved, the family quantum number of this positron is not important. The state carries the "spinor" (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = \frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$ and the colour charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hyper charge $(Y = \tau^4 + \tau^{23})$ is $\frac{1}{2}$ and the electromagnetic charge $Q = Y + \tau^{13}$ is 1.

What does, let say, the term $\gamma^0 \stackrel{910}{(+)} \tau^{1\boxminus} A^{1\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ (the scalar field $A^{1\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ is presented in the first line of table 9.1) make on $\bar{e}_R^{+}$? Making use of Eqs. (9.44, 9.46, 9.54) of appendix 9.9 one easily finds that the right handed positron transforms under the application of $\gamma^0 \tau^{1-} \stackrel{910}{(+)}$ into $\stackrel{03}{[-i]} \stackrel{12}{(+)} | \stackrel{56}{[-]} \stackrel{78}{(+)} || \stackrel{9\,10}{(+)} \stackrel{11\,12}{(-)} \stackrel{13\,14}{(-)}$, which is d_L^{c1} presented on line 5 of table 9.3. Namely, γ^0 transforms $\stackrel{03}{(+i)}$ into $\stackrel{03}{[-i]}$, $\stackrel{910}{(+)}$

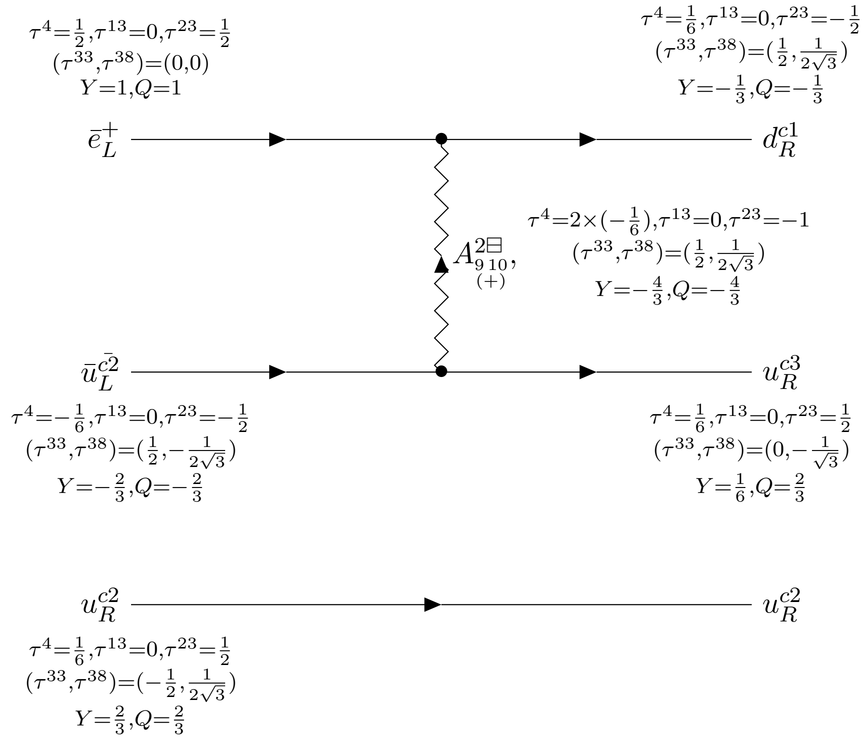

Fig. 9.1. The birth of a proton out of an positron $\bar{e}_L^{+}$, antiquark $\bar{u}_L^{\bar{c}2}$ and quark (spectator) u_R^{c2}. The family quantum number can be any.

transforms $\stackrel{9\,10}{[-]}$ into $\stackrel{9\,10}{(+)}$, while $\tau^{1\boxminus}$ $(=\stackrel{56}{(-)}\stackrel{78}{(+)})$ transforms $\stackrel{56}{(+)}\stackrel{56}{[-]}$ into $\stackrel{56}{[-]}\stackrel{56}{(+)}$. The state d_L^{c1} carries the "spinor" (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$ and the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hyper charge is $(Y = \tau^4 + \tau^{23} =)\ \frac{1}{6}$ and the electromagnetic charge $(Q = Y + \tau^{13} =)\ -\frac{1}{3}$. The scalar field $A^{1\boxminus}_{\substack{9\,10\\(\oplus)}}$ carries all the needed quantum numbers, as one can see in figure 9.1.

If the antiquark $\bar{u}_R^{\bar{c}2}$, from the line 47 in table 9.3 (the reader can easily find the expression $\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \parallel \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{[+]}$), with the "spinor" charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$, the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hypercharge $(Y = \tau^4 + \tau^{23} =)\ -\frac{1}{6}$ and the electromagnetic charge $(Q = Y + \tau^{13} =)\ -\frac{2}{3}$, submits the $A^{1\boxminus}_{\substack{9\,10\\(\oplus)}}$ scalar field, it transforms into u_L^{c3} from the line 23 of table 9.3 ($\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \parallel \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{[+]}$), carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = \frac{1}{2}$, $\tau^{23} = 0$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{1}{6}$ and $Q = \frac{2}{3}$. These two quarks, d_L^{c1} and u_L^{c3}, can bind (at low enough energy, when making after the electroweak transition the superposition with the right handed partners) together with u_L^{c2} from the 15^{th} line of the same table, into the colour chargeless baryon - a proton. This transition is presented in figure 9.2.

The opposite transition would make the proton decay.

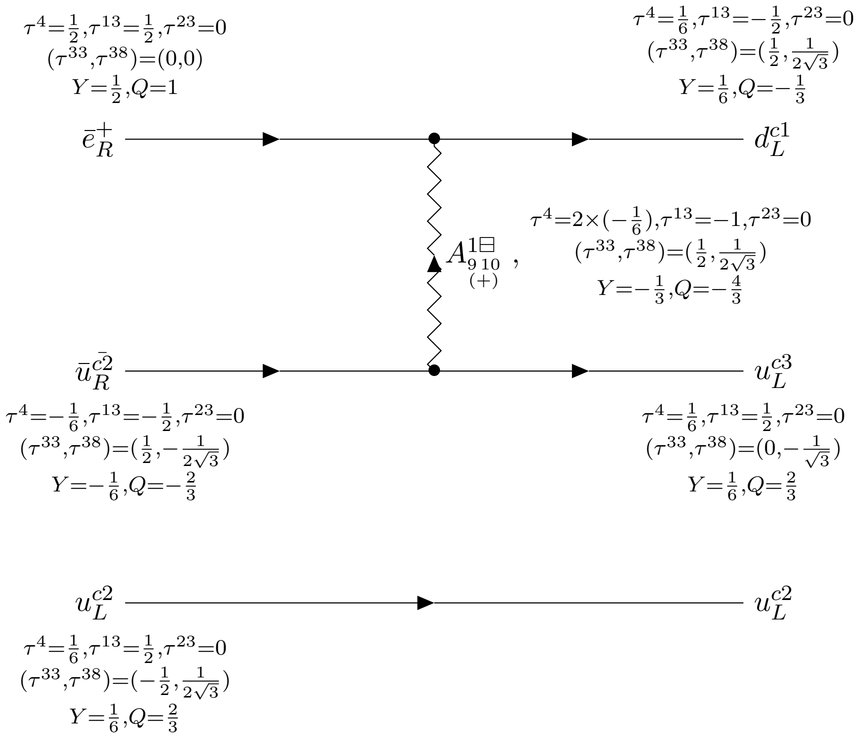

Fig. 9.2. The birth of a proton out of a positron $\bar{e}^{+}_{R}$, antiquark $\bar{u}^{\bar{c2}}_{R}$ and quark (spectator) u^{c2}_{L}. The family quantum number can be any.

Similar transitions go also with other scalars from Eq. (9.21) and table 9.1. The $\vec{\tilde{A}}^{1}_{\stackrel{t' t''}{(+)}}$, $\vec{\tilde{A}}^{2}_{\stackrel{t' t''}{(+)}}$, $\vec{\tilde{A}}^{N_L}_{\stackrel{t' t''}{(+)}}$ and $\vec{\tilde{A}}^{N_L}_{\stackrel{t' t''}{(+)}}$ fields cause transitions among the family members, changing a particular member into the antimember of another colour and of another family. The term $\gamma^0 \stackrel{910}{(+)} \tilde{N}^{-}_{R} \, A^{\tilde{N}_R -}_{\stackrel{9\,10}{(\oplus)}}$ transforms $\bar{e}^{+}_{R}$ into u^{c1}_{L}, changing the family quantum numbers.

The action from Eqs. (9.1, 9.2, 9.4) manifests $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance. All the vector and the spinor gauge fields are massless.

Since no one of the scalar fields from table 9.1 have been observed and also no vector gauge fields like $\vec{A}^{2}_{m}$, A^{4}_{m} and other scalar and vector fields, we shall discuss this topic in sect. 9.5, it must exist a mechanism, which makes the non observed scalar and vector gauge fields massive enough.

Scalar fields from table 9.1 carry the colour and the electromagnetic charge. Therefore their nonzero vacuum expectation values would not be in agreement with the observed phenomena. One, however, notices that all the scalar gauge fields from table 9.1 and several other scalar and vector gauge fields (see sect. 9.5) couple to the condensate with the nonzero quantum number τ^4 and τ^{23} and nonzero family quantum numbers.

It is not difficult to recognize that the desired condensate must have spin zero, $Y = \tau^4 + \tau^{23} = 0$, $Q = Y + \tau^{13} = 0$ and $\vec{\tau}^1 = 0$ in order that in the low energy limit the *spin-charge-family* theory would manifest effectively as the *standard model*.

I make a choice of the two right handed neutrinos of the VIII^{th} family coupled into a scalar, with $\tau^4 = -1$, $\tau^{23} = 1$, correspondingly $Y = 0$, $Q = 0$ and $\vec{\tau}^1 = 0$, and with family quantum numbers (Eqs. (9.13, 9.12)) $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, and correspondingly with $\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23} = 0$, $\tilde{Q} = \tilde{Y} + \tilde{\tau}^{13} = 0$, and $\vec{\tilde{\tau}}^1 = 0$. The condensate carries the family quantum numbers of the upper four families.

The condensate made out of spinors couples to spinors differently than to antispinors - "anticondensate" would namely carry $\tau^4 = 1$, and $\tau^{23} = -1$ - breaking correspondingly the $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ symmetry: The reactions creating particles from antiparticles are not any longer symmetric to those creating antiparticle from particles.

Such a condensate leaves the hyper field A^Y_m $(= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m)$ (for the choice that $\sin\vartheta_2 = \cos\vartheta_2$ and $g^4 = g^2$, there is no justification for such a choice, $A^Y_m = \frac{1}{\sqrt{2}}\,(A^{23}_m + A^4_m)$) massless, while it gives masses to $A^{2\pm}_m$ and $A^{Y'}_m$ $(= \frac{1}{\sqrt{2}}\,(A^4_m - A^{23}_m)$ for $\sin\vartheta_2 = \cos\vartheta_2)$ and it gives masses also to all the scalar gauge fields from table 9.1, since they all couple to the condensate through τ^4.

The weak vector gauge fields, $\vec{A}^1_m$, the hyper charge vector gauge fields, A^Y_m, and the colour vector gauge fields, $\vec{A}^1_m$, stay massless.

The scalar fields with the scalar space index $s = (7, 8)$ - those which couple to all eight families, those which couple only to the upper and those which couple only to the lower four families - carrying the weak and the hyper charges of the Higgs's scalar - wait for getting nonzero vacuum expectation values to change their masses while causing the electroweak break.

The condensate does what is needed so that in the low energy regime the *spin-charge-family* manifests as an effective theory which agrees with the *standard model* to the extend that it is in agreement with the observed phenomena, explaining the *standard model* assumptions and predicting new fermion and boson fields.

It also may hopefully explain also the observed matter-antimatter asymmetry if the conditions in the expanding universe would be appropriate (9.6). The work needed to check these conditions in the expanding universe within the *spin-charge-family* theory is very demanding. Although we do have some experience with following the history of the expanding universe [12], this study needs much more efforts, not only in the calculations, but also in understanding the mechanism of appearing the condensate, relations among the velocity of the expansion, the temperature and the dimension of space-time in the period of the appearance of the condensate. This study has not yet been really started.

9.3 Properties of the condensate

In table 9.2 the properties of the condensate of the two right handed neutrinos $(|\nu^{\text{VIII}}_R >_1 |\nu^{\text{VIII}}_R >_2)$, one with spin up and another with spin down (table 9.3, line

25 and 26), carrying the family quantum numbers of the VIII^{th} family (table 9.4), are presented. The condensate carries the quantum numbers of $SU(2)_{II}$, $\tau^{23}=1$ (Eq. (9.9)), of $U(1)_{II}$ originating in $SO(6)$, $\tau^4 = -1$ (Eq.9.11), correspondingly $Y=0$, $Q=0$, and the family quantum numbers (table 9.4) $\tilde{\tau}^4 = -1$ (Eq. (9.11)), $\tilde{\tau}^{23} = 1$ (Eq. (9.13)), and $\tilde{N}^3_R = 1$ (Eq. (9.12)). Each of the two neutrinos could belong to a different family of the upper four families. In this case the family quantum numbers of the condensate change.

The condensate is presented in the first line of table 9.2 as a member of a triplet of the group $SU(2)_{II}$ with the generators τ^{2i}. Correspondingly the condensate couples to all the vector gauge fields which carry nonzero τ^{2i}, τ^4, $\tilde{\tau}^{2i}$, $\tilde{N}^i_R$ and $\tilde{\tau}^4$. The fields $A^Y_m, \vec{A}^3_m$ and $\vec{A}^1_m$ stay massless. The condensate couples also to all the scalar gauge fields with the scalar indices $s \in (5,6,7,8,9,\dots,14)$, since they all carry nonzero either τ^4 or τ^{23}.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\lvert\nu^{\mathbf{VIII}}_{\mathbf{1R}}>_{\mathbf{1}}\ \lvert\nu^{\mathbf{VIII}}_{\mathbf{2R}}>_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\lvert\nu^{VIII}_{1R}>_1\ \lvert e^{VIII}_{2R}>_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\lvert e^{VIII}_{1R}>_1\ \lvert e^{VIII}_{2R}>_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 9.2. The condensate of the two right handed neutrinos ν_R, with the VIII^{th} family quantum number, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The condensate carries $\vec{\tau}^1 = 0$, $\tau^{23}=1$, $\tau^4=-1$ and $Q=0=Y$. The triplet carries $\tilde{\tau}^4=-1$, $\tilde{\tau}^{23}=1$ and $\tilde{N}^3_R=1$, $\tilde{N}^3_L=0$, $\tilde{Y}=0$, $\tilde{Q}=0$. The family quantum numbers are presented in table 9.4.

Coupling of the scalar gauge fields to the condensate is proportional to

$$
\begin{aligned}
&(<\nu^{VIII}_{1R}|_1\ <\nu^{VIII}_{2R}|)\,(\gamma^0\ \overset{tt'}{(\oplus)}\ \tau^{Ai}\,A^{Ai}_{\substack{tt'\\(\oplus)}})^\dagger(\gamma^0\ \overset{tt'}{(\oplus)}\ \tau^{Ai}\,A^{Ai}_{\substack{tt'\\(\oplus)}})(|\nu^{VIII}_{1R}>_1\ |\nu^{VIII}_{2R}>)\\
&\propto\ (A^{Ai}_{\substack{tt'\\(\oplus)}})^\dagger\,(A^{Ai}_{\substack{tt'\\(\oplus)}})\,,\\
&(tt') \in [(56),(78),(9\,10),\dots,(13\,14)]\,. \qquad (9.22)
\end{aligned}
$$

The condensate does break the $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}$ symmetry. (The "anticondensate" would namely carry $\tau^{23}=-1$ and $\tau^4=1$).

The condensate gives masses to all the scalars from table 9.1, either because they couple to the condensate due to τ^4 or due to τ^4 and τ^{23} quantum numbers. It gives masses also to all the scalar fields with $s \in (5,6,7,8)$, since they couple to the condensate due to the nonzero τ^{23}. The scalar fields with the quantum numbers of the upper four families couple in addition through their family quantum numbers.

The condensate couples also to all the vector gauge fields except to the gauge colour octet field $\vec{A}^3_m$, the hyper charge vector fields A^Y_m and the weak charge vector triplet fields $\vec{A}^1_m$, since they carry zero τ^{23}, τ^4 and Y quantum numbers.

The spin connection fields, of either "tilde" ($\tilde{S}^{ab}$) or "nontilde" (S^{ab}) origin, which do not couple to the spinor condensate, are auxiliary fields, expressible with vielbeins fields (abstract (9.10)).

Below the scalar and vector gauge fields are presented, which get masses through the interaction with the condensate.

$$
\begin{aligned}
&A^{2\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ A^{23}_{\substack{tt'\\(\oplus)}}\,,\quad A^{1\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ A^{13}_{\substack{tt'\\(\oplus)}}\,,\quad \vec{A}^{3}_{\substack{tt'\\(\oplus)}}\,,\\
&\tilde{A}^{2\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ \tilde{A}^{23}_{\substack{tt'\\(\oplus)}}\,,\quad \tilde{A}^{1\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ \tilde{A}^{13}_{\substack{tt'\\(\oplus)}}\,,\\
&\tilde{A}^{N_L\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ \tilde{A}^{N_L 3}_{\substack{tt'\\(\oplus)}}\,,\quad \tilde{A}^{N_R\boxplus}_{\substack{tt'\\(\oplus)}}\,,\ \tilde{A}^{N_R 3}_{\substack{tt'\\(\oplus)}}\,,\\
&(tt') \in [(9\,10),(11\,12),(13\,14)]\,,\\
&A^{2\boxplus}_{\substack{ss'\\(\oplus)}}\,,\quad A^{Y'}_{\substack{ss'\\(\oplus)}} = \frac{1}{\sqrt{2}}\,(A^{23}_{\substack{ss'\\(\oplus)}} - A^{4}_{\substack{ss'\\(\oplus)}})\,,\\
&(ss') \in [(56),(78)]\,,\\
&A^{2\boxplus}_{m}\,,\ A^{Y'}_{m} = \frac{1}{\sqrt{2}}\,(A^{23}_{m} - A^{4}_{m})\,,\\
&\vec{\tilde{A}}^{2}_{m}\,,\tilde{A}^{4}_{m}\,,\vec{\tilde{A}}^{N_R}_{m}\,,\\
&m \in (0,1,2,3)\,.
\end{aligned}
\tag{9.23}
$$

In expression for $A^{Y'}_{m,s}$ $\vartheta_2 = \frac{\pi}{4}$ is chosen, just for simplicity, with no justification so far.

It stays as an open question what does make the right handed neutrinos to form such a condensate in the history of the universe.

Since $A^{Ai}_{s}\,, s \in (5,6)$ couple to the condensate and get masses, while (by assumption) they do not get nonzero vacuum expectation values during the electroweak break (what changes the masses of the scalar fields $A^{Ai}_{s}\,, s \in (7,8)$) the restriction in the sum in Eq. (9.2) is justified.

The scalar fields, causing the birth of baryons, have the triplet colour charges. They resemble the supersymmetric partners of the quarks, but since they do not carry all the quantum numbers of the quarks, they are not.

9.4 Properties of scalar fields which determine mass matrices of fermions

This section is a short overview of the ref. [14].

There are two kinds of the scalar gauge fields, which gain at the electroweak break nonzero vacuum expectation values and determine correspondingly the masses of the families of quarks and leptons and to the masses of gauge weak bosons: The kind originating in $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ and the kind originating in $\omega_{tt's}$, ω_{56s} and ω_{78s}, both kinds have the space index $s = (7,8)$ and both carry the weak and the hyper charge as the Higgs's scalar. These scalar fields are presented in the Lagrange density for fermions (Eq. (9.2)) in the second line. The "tilde" kind influences the family quantum numbers of fermions, the "Dirac" kind influences the family members quantum numbers.

The two triplets ($\vec{\tilde{A}}^{1}_{s}$, $\vec{\tilde{A}}^{N_L}_{s}$) influence the lower four families (the lowest three already observed), while ($\vec{\tilde{A}}^{2}_{s}$, $\vec{\tilde{A}}^{N_R}_{s}$) influence the upper four families, the stable of which constitute the dark matter. Recognizing that $\vec{\tilde{\tau}}^{1}\,\vec{\tilde{A}}^{1}_{s} + \vec{\tilde{N}}_{L}\,\vec{\tilde{A}}^{N_L}_{s} + \vec{\tilde{\tau}}^{2}\,\vec{\tilde{A}}^{2}_{s} + \vec{\tilde{N}}_{R}\,\vec{\tilde{A}}^{N_R}_{s} = \frac{1}{2}\,\tilde{S}^{ab}\,\tilde{\omega}_{abs}$, $s=(7,8)$, one easily finds, taking into account Eqs. (9.12, 9.13), the expressions

$$
\begin{aligned}
\vec{\tilde{A}}^{1}_{s} &= (\tilde{\omega}_{58s} - \tilde{\omega}_{67s}, \tilde{\omega}_{57s} + \tilde{\omega}_{68s}, \tilde{\omega}_{56s} - \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^{N_L}_{s} &= (\tilde{\omega}_{23s} + i\,\tilde{\omega}_{01s}, \tilde{\omega}_{31s} + i\,\tilde{\omega}_{02s}, \tilde{\omega}_{12s} + i\,\tilde{\omega}_{03s})\,,\\
\vec{\tilde{A}}^{2}_{s} &= (\tilde{\omega}_{58s} + \tilde{\omega}_{67s}, \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{\omega}_{56s} + \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^{N_R}_{s} &= (\tilde{\omega}_{23s} - i\,\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\,\tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i\,\tilde{\omega}_{03s})\,,\\
s &= (7,8)\,,
\end{aligned}
\tag{9.24}
$$

presented already in Eq. (9.16). Similarly one finds, taking into account Eqs. (9.8, 9.9, 9.11, 9.14), the expressions for A^{Q}_{s}, A^{Y}_{s} and $A^{Y'}_{s}$, presented in Eqs. (9.15).

The scalar fields A^{Q}_{s}, A^{Y}_{s} and $A^{Y'}_{s}$ distinguish among the family members, coupling to the family members quantum numbers through $Q\ (=\tau^{13}+Y)$, Y $(=\tau^{23}+\tau^{4})$ and $Y' = \tau^{23} - \tan\vartheta_2\,\tau^{4}$, $\tau^{4} = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$. The scalars originating in $\tilde{\omega}_{abs}$ and distinguishing among families, couple the family quantum numbers through ($\vec{\tilde{\tau}}^{1}$ and $\vec{\tilde{N}}_{L}$), or through ($\vec{\tilde{\tau}}^{2}$ and $\vec{\tilde{N}}_{R}$), all in the adjoint representations of the corresponding groups.

Let us now prove that all the scalar fields with the space (scalar with respect to $d=(3+1)$) index $s=(7,8)$ carry the weak and the hyper charge (τ^{13}, Y) equal to either $(-\frac{1}{2},\frac{1}{2})$ or to $(\frac{1}{2},-\frac{1}{2})$. Let us first simplify the notation, using a common name A^{Ai}_{s} for all the scalar fields with the scalar index $s=(7,8)$

$$
A^{Ai}_{s} = (A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}_{s}, \tilde{A}^{4}_{s}, \vec{\tilde{A}}^{2}_{s}, \vec{\tilde{A}}^{1}_{s}, \vec{\tilde{A}}^{N_R}_{s}, \vec{\tilde{A}}^{N_L}_{s})\,,
\tag{9.25}
$$

and let us rewrite the term $\sum_{s=7,8} \bar{\psi}\,\gamma^{s}\,p_{0s}\,\psi$ in Eq. (9.2) as follows

$$
\begin{aligned}
&\sum_{s=7,8} \bar{\psi}\,\gamma^{s}\,p_{0s}\,\psi\,,\\
&= \bar{\psi}\{\stackrel{78}{(+)}\,p_{0+} + \stackrel{78}{(-)}\,p_{0-}\}\psi\,,\\
&p_{0\pm} = (p_{07} \mp i p_{08})\,,\\
&(p_{07} \mp i p_{08}) = (p_{7} \mp i p_{8}) - \tau^{Ai}\,(A^{Ai}_{7} \mp i A^{Ai}_{8})\\
&\stackrel{78}{(\pm)} = \frac{1}{2}\,(\gamma^{7} \pm i\gamma^{8})\,.
\end{aligned}
\tag{9.26}
$$

Let us now apply the operators Y, Q, Eq. (9.14), and $\tau^{13} = \frac{1}{2}\,(S^{56} - S^{78})$, Eq. (9.9), on the fields $A^{Ai}_{\stackrel{78}{(\pm)}} = (A^{Ai}_{7} \mp i\,A^{Ai}_{8})$. One finds

$$
\begin{aligned}
\tau^{13}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \pm\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Y\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \mp\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Q\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= 0\,,
\end{aligned}
\tag{9.27}
$$

This is, with respect to the weak, the hyper and the electromagnetic charge, just what the standard model assumes for the Higgs' scalars. The proof is complete.

One can check also, using Eq. (9.44), that $\gamma^0 \overset{78}{(-)}$ transforms the u_R^{c1} from the first line of table 9.3 into u_L^{c1} from the seventh line of the same table, or ν_R from the 25^{th} line into the ν_L from the 31^{th} line of the same table.

The scalars $A^{Ai}_{\overset{78}{(-)}}$ obviously bring the weak charge $\frac{1}{2}$ and the hyper charge $-\frac{1}{2}$ to the right handed family members (u_R, ν_R), and the scalars $A^{Ai}_{\overset{78}{(+)}}$ bring the weak charge $-\frac{1}{2}$ and the hyper charge $\frac{1}{2}$ to (d_R, e_R).

Let us now prove that the scalar fields of Eq. (9.25) are either triplets with respect to the family quantum numbers ($\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$, $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^1$; Eqs. (9.12, 9.13)) or singlets as the gauge fields of $Q = \tau^{13} + Y$, $Q' = \tau^{13} - Y \tan^2\vartheta_1$ and $Y' = \tau^{23} - \tan^2\vartheta_2\, \tau^4$. One can prove this by applying $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^1$, $\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$ and Q, Q', Y' on their eigenstates. Let us calculate, as an example, $\tilde{N}^3_L$ and Q on $\tilde{A}^{N_L 3}_{\overset{78}{(\pm)}}$ and on $A^{Q}_{\overset{78}{(\pm)}}$, taking into account Eqs. (9.12, 9.11,9.9, 9.7)

$$
\begin{aligned}
\tilde{N}^3_L\, \tilde{A}^{N_L \boxplus}_{\overset{78}{(\pm)}} &= \boxplus\, \tilde{A}^{N_L \boxplus}_{\overset{78}{(\pm)}}\,, \quad \tilde{N}^3_L\, \tilde{A}^{N_L 3}_{\overset{78}{(\pm)}} = 0\,,\\
Q\, A^{Q}_{\overset{78}{(\pm)}} &= 0\,,\\
\tilde{A}^{N_L \boxplus}_{\overset{78}{(\pm)}} &= \{(\tilde{\omega}_{23 \overset{78}{(\pm)}} + i\, \tilde{\omega}_{01 \overset{78}{(\pm)}}) \boxplus i\, (\tilde{\omega}_{31 \overset{78}{(\pm)}} + i\tilde{\omega}_{02 \overset{78}{(\pm)}})\}\,,\\
\tilde{A}^{N_L 3}_{\overset{78}{(\pm)}} &= (\tilde{\omega}_{12 \overset{78}{(\pm)}} + i\tilde{\omega}_{03 \overset{78}{(\pm)}})\\
A^{Q}_{\overset{78}{(\pm)}} &= \sin\vartheta_1\, A^{13}_{\overset{78}{(\pm)}} + \sin\vartheta_1 (-) (\omega_{9\,10 \overset{78}{(\pm)}} + \omega_{11\,12 \overset{78}{(\pm)}} + \omega_{13\,14 \overset{78}{(\pm)}})\,,
\end{aligned}
\tag{9.28}
$$

with $Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, and with τ^4 defined in Eq. (9.11)).

Nonzero vacuum expectation values of the scalar fields (Eq. (9.25)), which carry the scalar index $s = (7, 8)$, and correspondingly the weak and the hyper charges, break the mass protection mechanism of quarks and leptons of the lower and the upper four families. In the loop corrections contribute to all the matrix elements of mass matrices of any family members besides $\tilde{A}^{Ai}_s$ and the scalar fields which are the gauge fields of Q, Q', Y' also the vector gauge fields.

The gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute only to masses of the upper four families, while the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute only to masses of the lower four families. The triplet scalar fields with the scalar index $s = (7, 8)$ and the family charges $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ transform any family member belonging to the group of the upper four families into the same family member belonging to another family of the same group of four families, changing the right handed member into the left handed partner, while those triplets with the family charges $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ transform any family member of particular handedness and belonging to the lower four families into its partner of opposite handedness, belonging to another family of the lower four families.

The scalars $A^{Q}_{\substack{78\\(\pm)}}$ (Eq. (9.28)), $A^{Q'}_{\substack{78\\(\pm)}}$ $(= \cos\vartheta_1\, A^{13}_{\substack{78\\(\pm)}} - \sin\vartheta_1 A^{4}_{\substack{78\\(\pm)}})$ and $A^{Y'}_{\substack{78\\(\pm)}}$ (Eq. (9.23)) contribute to all eight families, distinguishing among the family members and not among the families.

The mass matrix of any family member, belonging to any of the two groups of the four families, manifests - due to the $\widetilde{SU}(2)_{(R,L)} \times \widetilde{SU}(2)_{(II,I)}$ (either (R, II) or (L, I)) structure of the scalar fields, which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$ - the symmetry presented in Eq. (9.29)

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha} . \tag{9.29}$$

In the ref. gn2014 the mass matrices for quarks, which are in the agreement with the experimental data, are presented and predictions made.

9.5 Condensate and nonzero vacuum expectation values of scalar fields make spinors and most of scalar and vector gauge fields massive

Let us shortly overview properties of the scalar and the vector gauge fields after **a.** two right handed neutrinos (coupled to spin zero and with the family quantum numbers (table 9.4) of the upper four families) make a condensate (table 9.2) at the scale $\geq 10^{16}$ GeV and after **b.** the electroweak break, when the scalar fields with the space index $s = (7, 8)$ get nonzero vacuum expectation values.

All the scalar gauge fields $A^{Ai}_{t}, t \in (5, 6, 7, 8, 9, \ldots, 14)$ (Eqs. (9.2, 9.21, 9.23), table 9.1) interact with the condensate through the quantum numbers τ^4 and τ^{23}, those with the family quantum numbers of the upper four families interact also through the family quantum numbers $\vec{\tilde{\tau}}^{2}$ or $\vec{\tilde{N}}_{R}$, getting masses of the order of the condensate scale (Eq.(9.23)).

At the electroweak break the scalar fields A^{Ai}_{s}, $s \in (7, 8)$, from Eqs. (9.25, 9.25) get nonzero vacuum expectation values, changing correspondingly their own masses and determining masses of quarks and leptons, as well as of the weak vector gauge fields.

The vector gauge fields $A^{2\boxplus}_{m}$, $A^{Y'}_{m}$, $\vec{\tilde{A}}^{2\boxplus}_{m}$, $\tilde{A}^{Y'}_{m}$ and $\vec{\tilde{A}}^{N_R}_{m}$ (Eq. (9.23)) get masses due to the interaction with the condensate through τ^{23} and τ^4, the first two, and/or also due to the family quantum numbers of the upper four families, the last three, respectively.

The vector gauge fields $\vec{A}^{3}_{m}$, $\vec{A}^{1}_{m}$, and A^{Y}_{m} stay massless up to the electroweak break when the scalar gauge fields, which are weak doublets with the hypercharge making electromagnetic charge Q equal to zero, give masses to the weak bosons ($A^{1\boxplus}_{m} = \frac{1}{\sqrt{2}}\,(A^{11}_{m} \mp i A^{11}_{m})$ and $A^{Q'}_{m} = \cos\vartheta_1\, A^{13}_{m} - \sin\vartheta_1\, A^{4}_{m}$), while the electromagnetic vector field ($A^{Q}_{m} = \sin\vartheta_1\, A^{13}_{m} + \cos\vartheta_1\, A^{4}_{m}$) and the colour vector gauge field stay massless.

At the electroweak break, when the nonzero vacuum expectation values of the scalar fields break the weak and the hypercharge global symmetry, also all the eight families of quarks and leptons get masses. Until the electroweak break the families were mass protected, since the right handed partners distinguished from the left handed ones in the weak and hyper charges, what disabled them to make the superposition manifesting masses.

9.6 Sakharov conditions as seen in view of the *spin-charge-family* theory

The condensate of the right handed neutrinos, as well as the nonzero vacuum expectation values of the scalar fields $A^{Ai}_{\stackrel{78}{(\pm)}}$ - if leading to the complex matrix elements of the mixing matrices - cause the $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ violation terms, which generate the matter-antimatter asymmetry.

It is the question whether both generators of the matter-antimatter asymmetry - the condensate and the complex phases of the mixing matrices of quarks and leptons (this last alone can not with one complex phase and also very probably not with the three complex phases of the lower four families) - can explain at all the observed matter-antimatter asymmetry of the "ordinary" matter, that is the matter of mostly the first family of quarks and leptons.

The lowest of the upper four families determine the dark matter. For the dark matter any relation among matter and antimatter is so far experimentally allowed.

Both origins of the matter-antimatter asymmetry - the condensate and the nonzero vacuum expectation values of the scalar fields carrying the weak and the hyper charge - (are assumed to) appear spontaneously.

Sakharov [24] states that for the matter-antimatter asymmetry three conditions must be fulfilled:

a. ($\mathcal{C}_{\mathcal{N}}$ and) $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ must not be conserved.

b. Baryon number non conserving processes must take place.

c. Thermal non equilibrium must be present not to equilibrate the number of baryons and antibaryons.

Sakharov uses for **c.** the requirement that CPT must be conserved and that $\{CPT, H\}_- = 0$. In a thermal equilibrium the average number of baryons $< n_B >= Tr(e^{-\beta H} n_B) = Tr(e^{-\beta H} CPT n_B (CPT)^{-1}) = < \bar{n}_B >$. Therefore $< n_B > - < \bar{n}_B >= 0$ at the thermal equilibrium and there is no excess of baryons with respect to antibaryons. In the expanding universe, however, the temperature is changing with time. It is needed that the discrete symmetry $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ is broken to break the symmetry between matter and antimatter, if the universe starts with no matter-antimatter asymmetry.

The *spin-charge-family* theory starting action (Eq.(9.1)) is invariant under the $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ symmetry. The scalar fields (Eq.(9.21)) of this theory cause transitions, in which a quark is born out of a positron (figures (9.1, 9.2)) and a quark is born out of antiquark, and back. These reactions go in both directions with the same probability, until the spontaneous break of the $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ symmetry is caused by the appearance of the condensate of the two right handed neutrinos (table 9.2).

But after the appearance of the condensate (and in addition of the appearance of the non zero vacuum expectation values of the scalar fields with the space index $s \in (7, 8)$), family members "see" the vacuum differently than the antimembers. And this *might* explain the matter-antimatter asymmetry. It is also predicting the proton decay.

It is, of course, the question whether both phenomena can at all explain the observed matter-antimatter asymmetry. I agree completely with the referee of this paper that before answering the question whether or not the *spin-charge-family* theory explains this observed phenomena, one must do a lot of additional work to find out: i. Which is the order of phase transition, which leads to the appearance of the condensate. ii. How strong is the thermal nonequilibrium, which leads to the matter-antimatter asymmetry during the phase transition. iii. How rapid is the appearance of the matter-antimatter asymmetry in comparison with the expansion of the universe. iv. Does the later history of the expanding universe enable that the produced asymmetry survives up to today.

Although we do have some experience with solving the Boltzmann equations for fermions and antifermions [12] to follow the history of the dark matter within the *spin-charge-family* theory, the study of the history of the universe from the very high temperature to the baryon production within the same theory in order to see the matter-antimatter asymmetry in the present time is much more demanding task. These is under consideration, but still at a very starting point since a lot of things must be understood before starting with the calculations.

What I can conclude is that the *spin-charge-family* theory does offer the opportunity also for the explanation for the observed matter-antimatter asymmetry.

9.7 Conclusions

The *spin-charge-family* [1,3–8,2,9,12,14,15] theory is a kind of the Kaluza-Klein theories in $d = (13 + 1)$ but with the families introduced by the second kind of gamma operators - the $\tilde{\gamma}^a$ operators in addition to the Dirac γ^a in $d = (13 + 1)$. The theory assumes a simple starting action (Eq. (9.1)) in $d = (13+1)$. This simple action manifests in the low energy regime, after the breaks of symmetries (subsection 9.1.1), all the degrees of freedom assumed in the *standard model*, offering the explanation for all the properties of quarks and leptons (right handed neutrinos are in this theory the regular members of each family) and antiquarks and antileptons. The theory explains the existence of the observed gauge vector fields. It explains the origin of the scalar fields (the Higgs and the Yukawa couplings) responsible for the quark and lepton masses and the masses of the weak bosons and carrying the weak and the hyper charge of the *standard model* Higgs ([14]).

The theory is offering the explanation also for the matter-antimatter asymmetry and for the appearance of the dark matter.

The *spin-charge-family* theory predicts two decoupled groups of four families [3,4,9,12]: The fourth of the lower group of four families will be measured at the LHC [10] and the lowest of the upper four families constitutes the dark matter [12] and was already seen. It also predicts that there might be several scalar fields observable at the LHC. The upper four families manifest, due to their high masses, a new ”nuclear force” among their baryons.

All these degrees of freedom are already contained in the simple starting action. The scalar fields with the weak and the hyper charges equal to $(\mp\frac{1}{2}, \pm\frac{1}{2})$, respectively (section 9.4), have the space index $s = (7, 8)$, while they carry in addition to the weak and the hyper charges also the family quantum numbers, originating in $\tilde{S}^{ab}$ (they form two groups of twice $SU(2)$ triplets), or the family members quantum numbers, originating in S^{ab} (they form three singlets with the quantum numbers (Q, Q', Y')). These scalar fields cause the transitions of the right handed family members into the left handed partners and back. Those with the family quantum numbers cause at the same time transitions among families within each of the two family groups of four families. They all gain in the electroweak break nonzero vacuum expectation values, giving masses to both groups of four families of quarks and leptons and to weak bosons (changing also their own masses).

There are in this theory also the scalar fields with the space index $s = (5, 6)$; They carry with respect to this degree of freedom they the weak charge equal to the hyper charge $(\mp\frac{1}{2}, \mp\frac{1}{2}$, respectively). They carry also additional quantum numbers Eq.(9.23) like all the scalar fields: The family quantum numbers, originating in $\tilde{S}^{ab}$ and the family members quantum numbers originating in S^{ab}.

And there are also the scalar fields with the scalar index $s = (9, 10, \cdots, 14)$. These scalars carry the triplet colour charge with respect to the space index and the additional quantum numbers (table 9.1), originating in family quantum numbers $\tilde{S}^{ab}$ and in family members quantum numbers S^{ab}.

There are no additional scalar gauge fields.

There are the vector gauge fields with respect to $d = (3 + 1)$: A^{Ai}_{m}, with Ai staying for the groups $SU(3)$ and $U(1)$ (both originating in $SO(6)$ of $SO(13, 1)$), for the groups $SU(2)_{II}$ and $SU(2)_{I}$ (both originating in $SO(4)$ of $SO(7, 1)$) and for the groups $SU(2) \times SU(2)$ ($\in SO(3, 1)$), in both sectors, the S^{ab} and $\tilde{S}^{ab}$ ones.

The condensate of the two right handed neutrinos with the family charges of the upper four families (table 9.2) gives masses to all the scalar and vector gauge fields, except to the colour octet vector, the hyper singlet vector and the weak triplet vector gauge fields, to which the condensate does not couple. It gives masses also to all the vector gauge fields to which the condensate couples. Those vector gauge fields of either S^{ab} or $\tilde{S}^{ab}$ origin, which do not couple to the condensate, are expressible with the corresponding vielbeins (appendices 9.55, 9.56). The condensate breaks the $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ symmetry (sections (9.3, 9.8)).

There are no additional vector gauge fields in this theory.

Nonzero vacuum expectation values of the scalar gauge fields with the space index $s = (7, 8)$ and the quantum numbers as explained in the fourth paragraph of this section change in the electroweak break their masses, while all the other scalars or vectors either stay massless (the colour octet, the electromagnetic field), or keep the masses of the scale of the condensate. The only before the electroweak massless vector fields, which become at the electroweak break massive, are the heavy bosons.

It is extremely encouraging that the simple starting action of the spin-charge-family *offers at low energies the explanations for so many observed phenomena,* although the starting assumptions (section 9.1.1) wait to be derived from the initial and boundary conditions of the expanding universe.

This paper is a step towards understanding the matter-antimatter asymmetry within the *spin-charge-family* theory, predicting also the proton decay. The theory obviously offers the possibility that the scalar gauge fields with the space index $s = (9, 10, \cdots, 14)$ explain, after the appearance of the condensate, the matter-antimatter asymmetry. To prove, however, that this indeed happen, requires the additional study: Following the universe through the phase transitions which breaks the $\mathbb{C}_{\mathcal{N}}\, \mathcal{P}_{\mathcal{N}}$ symmetry at the level of the condensate and further through the electroweak phase transition up to today, to check how much of the matter-antimatter asymmetry is left. The experience when following the history of the expanding universe to see whether the *spin-charge-family* theory can explain the dark matter content [12] is of some help. However, answering the question to which extend this theory can explain the observed matter-antimatter asymmetry requires a lot of additional understanding and a lot of work.

Let me conclude with the recognition, pointed out already in the introduction, that the *spin-charge-family* theory overlaps in many points with other unifying theories [26–31], since all the unifying groups can be recognized as the subgroups of the large enough orthogonal groups, with family groups included. But there are also many differences: The *spin-charge-family* theory starts with a very simple action, from where all the properties of spinors and the gauge vector and scalar fields follow, provided that the breaks of symmetries occur in the desired way. Consequently it differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up on different levels of the break of symmetries, in the unification scheme, in the family degrees of freedom and correspondingly also in the evolution of our universe.

9.8 APPENDIX: Discrete symmetry operators [15]

I present here the discrete symmetry operators in the second quantized picture, for the description of which the Dirac sea is used. I follow the reference [15]. The discrete symmetry operators of this reference are designed for the Kaluza-Klein like theories, in which the total angular momentum in higher than $(3+1)$ dimensions manifest as charges in $d = (3+1)$. The dimension of space-time is even, as it is in the case of the *spin-charge-family* theory.

$$\begin{aligned}
\mathcal{C}_{\mathcal{N}} &= \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^6, x^8, \ldots, x^d}\,, \\
\mathcal{T}_{\mathcal{N}} &= \prod_{\Re\gamma^m, m=1}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^0}\, I_{x^5, x^7, \ldots, x^{d-1}}\,, \\
\mathcal{P}_{\mathcal{N}} &= \gamma^0\, \Gamma^{(3+1)}\, \Gamma^{(d)}\, I_{\vec{x}_3}\,.
\end{aligned} \tag{9.30}$$

The operator of handedness in even d dimensional spaces is defined as

$$\Gamma^{(d)} := (i)^{d/2} \prod_{a} (\sqrt{\eta^{aa}}\, \gamma^a)\,, \tag{9.31}$$

with products of γ^a in ascending order. We choose γ^0, γ^1 real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, alternating imaginary and real up to γ^d real. Operators I operate as follows:

$$\begin{aligned}
I_{x^0} x^0 &= -x^0;\\
I_{x} x^a &= -x^a;\\
I_{x^0} x^a &= (-x^0, \vec{x});\\
I_{\vec{x}} \vec{x} &= -\vec{x};\\
I_{\vec{x}_3} x^a &= (x^0, -x^1, -x^2, -x^3, x^5, x^6, \dots, x^d);\\
I_{x^5, x^7, \dots, x^{d-1}} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &=\\
(x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \dots, -x^{d-1}, x^d);\\
I_{x^6, x^8, \dots, x^d} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &=\\
(x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \dots, x^{d-1}, -x^d),\, d = 2n.
\end{aligned}$$

$\mathcal{C}_{\mathcal{N}}$ transforms the state, put on the top of the Dirac sea, into the corresponding negative energy state in the Dirac sea.

The operator, it is named [1,16,15] $\mathbb{C}_{\mathcal{N}}$, is needed, which transforms the starting single particle state on the top of the Dirac sea into the negative energy state and then empties this negative energy state. This hole in the Dirac sea is the antiparticle state put on the top of the Dirac sea. Both, a particle and its antiparticle state (both put on the top of the Dirac sea), must solve the Weyl equations of motion.

This $\mathbb{C}_{\mathcal{N}}$ is defined as a product of the operator [1,16] "$emptying$", (making transformations into a completely different Fock space)

$$\text{"}emptying\text{"} = \prod_{\Re\gamma^a} \gamma^a\, K = (-)^{\frac{d}{2}+1} \prod_{\Im\gamma^a} \gamma^a\, \Gamma^{(d)} K\,, \tag{9.32}$$

and $\mathcal{C}_{\mathcal{N}}$

$$\begin{aligned}
\mathbb{C}_{\mathcal{N}} &= \prod_{\Re\gamma^a, a=0}^{d} \gamma^a \;\; K \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^6, x^8, \dots, x^d}\\
&= \prod_{\Re\gamma^s, s=5}^{d} \gamma^s \;\; I_{x^6, x^8, \dots, x^d}\,.
\end{aligned} \tag{9.33}$$

We shall need inded only the product of operators $\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}$, $\mathcal{T}_{\mathcal{N}}$ and $\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\, \mathcal{T}_{\mathcal{N}}$, since either $\mathbb{C}_{\mathcal{N}}$ or $\mathcal{P}_{\mathcal{N}}$ have in even dimensional spaces with $d = 2(2n+1)$ an odd number of γ^a operators, transforming accordingly states from the representation of one handedness in $d = 2(2n+1)$ into the Weyl of another handedness.

$$\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}} = \gamma^0 \prod_{\Im\gamma^s, s=5}^{d} \gamma^s \; I_{\vec{x}_3} \, I_{x^6, x^8, \dots, x^d} \,,$$

$$\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\mathcal{T}_{\mathcal{N}} = \prod_{\Im\gamma^a, a=0}^{d} \gamma^a \; K \, I_x \,. \tag{9.34}$$

9.9 APPENDIX: Short presentation of technique [6,18,20]

I make in this appendix a short review of the technique [18,20], initiated and developed [5–8] when proposing the *spin-charge-family* theory [5,6,8,4,1,2,12,9] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anticommuting (Grassmann) coordinates [6], if the dimension of ordinary space is d. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra, which anticommute with one another. The technique was further developed in the present shape together with H.B. Nielsen [18,20] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\begin{aligned}
&\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,,\\
&\tilde{\gamma}^a B := i(-)^{n_B} B\gamma^a \, |\psi_0>,\\
&B = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\dots\gamma^{a_d})|\psi_0>
\end{aligned} \tag{9.35}$$

for any d, even or odd. I is the unit element in the Clifford algebra. The two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects B. In Eq. (9.35) B is expressed as a polynomial of γ^a, $(-)^{n_B} = +1, -1$, when the object B has a Clifford even (+1) or odd (+1) character, respectively. $|\psi_0>$ is a vacuum state on which the operators γ^a apply.

If B is a Clifford algebra object, let say a polynomial of γ^a, then one finds

$$\begin{aligned}
&(\tilde{\gamma}^a B := i(-)^{n_B} \, B\gamma^a \,) \, |\psi_0>,\\
&\qquad B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,,
\end{aligned} \tag{9.36}$$

where $|\psi_0>$ is a vacuum state, defined in Eq. (9.50) and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's.

In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$
\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,,\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,,
\end{aligned}
\tag{9.37}
$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$
\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{9.38}
$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (9.35) and formally unitary, i.e. $\gamma^{a\,\dagger}\,\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(9.38) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(9.37) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, \quad \text{if} \quad d = 2n \ge 4,\\
&S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, \quad \text{if} \quad d = 2n \ge 4\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, \quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned}
\tag{9.39}
$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$
\begin{aligned}
\Gamma^{(d)} &:= (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,\\
\Gamma^{(d)} &:= (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,.
\end{aligned}
\tag{9.40}
$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, subtituting γ^a's by $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(9.38) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$
\begin{aligned}
&\overset{ab}{(k)} := \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \overset{ab}{[k]} := \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,\\
&\overset{+}{\circ} := \frac{1}{2}(1 + \Gamma)\,, \qquad \overset{-}{\bullet} := \frac{1}{2}(1 - \Gamma),
\end{aligned}
\tag{9.41}
$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then

$$\begin{aligned}
\gamma^a &= \stackrel{ab}{(k)} + \stackrel{ab}{(-k)}\,, \qquad \gamma^b = ik\eta^{aa}\,(\stackrel{ab}{(k)} - \stackrel{ab}{(-k)})\,,\\
S^{ab} &= \frac{k}{2}(\stackrel{ab}{[k]} - \stackrel{ab}{[-k]})
\end{aligned} \tag{9.42}$$

One can easily check by taking into account the Clifford algebra relation (Eq.9.35) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.9.37) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$\begin{aligned}
S^{ab}\,\stackrel{ab}{(k)} &= \frac{1}{2}\,k\,\stackrel{ab}{(k)}\,, \qquad S^{ab}\,\stackrel{ab}{[k]} = \frac{1}{2}\,k\,\stackrel{ab}{[k]}\,,\\
\tilde{S}^{ab}\,\stackrel{ab}{(k)} &= \frac{1}{2}\,k\,\stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab}\,\stackrel{ab}{[k]} = -\frac{1}{2}\,k\,\stackrel{ab}{[k]}\,,
\end{aligned} \tag{9.43}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\begin{aligned}
&\gamma^a\,\stackrel{ab}{(k)} = \eta^{aa}\,\stackrel{ab}{[-k]},\ \gamma^b\,\stackrel{ab}{(k)} = -ik\,\stackrel{ab}{[-k]},\ \gamma^a\,\stackrel{ab}{[k]} = \stackrel{ab}{(-k)},\ \gamma^b\,\stackrel{ab}{[k]} = -ik\eta^{aa}\,\stackrel{ab}{(-k)}\,,\\
&\tilde{\gamma}^a\,\stackrel{ab}{(k)} = -i\eta^{aa}\,\stackrel{ab}{[k]},\ \tilde{\gamma}^b\,\stackrel{ab}{(k)} = -k\,\stackrel{ab}{[k]},\ \tilde{\gamma}^a\,\stackrel{ab}{[k]} = i\,\stackrel{ab}{(k)},\ \tilde{\gamma}^b\,\stackrel{ab}{[k]} = -k\eta^{aa}\,\stackrel{ab}{(k)}
\end{aligned} \tag{9.44}$$

From Eq.(9.44) it follows

$$\begin{aligned}
S^{ac}\,\stackrel{ab}{(k)}\stackrel{cd}{(k)} &= -\frac{i}{2}\eta^{aa}\eta^{cc}\,\stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac}\,\stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc}\,\stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
S^{ac}\,\stackrel{ab}{[k]}\stackrel{cd}{[k]} &= \frac{i}{2}\,\stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac}\,\stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2}\,\stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
S^{ac}\,\stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa}\,\stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac}\,\stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\,\stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
S^{ac}\,\stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc}\,\stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac}\,\stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\,\stackrel{ab}{(k)}\stackrel{cd}{[k]}\;.
\end{aligned} \tag{9.45}$$

From Eqs. (9.45) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\,\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\,\stackrel{ab}{[-k]}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\;.
\end{aligned} \tag{9.46}$$

We recognize in the first equation of the first line and the first and the second equation of the second line the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$\stackrel{ab}{(\tilde{\pm}i)}= \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{(\tilde{\pm}1)}= \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{9.47}$$

one recognizes that

$$\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(\tilde{-k})}\stackrel{ab}{(k)}= -i\eta^{aa}\,\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]}= i\,\stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]}= 0\,. \tag{9.48}$$

Recognizing that

$$\stackrel{ab}{(k)}^{\dagger} = \eta^{aa}\,\stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,, \tag{9.49}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned} &< \stackrel{ab}{(k)}^{\dagger}\,\stackrel{ab}{(k)} >= 1\,,\\ &< \stackrel{ab}{[k]}^{\dagger}\,\stackrel{ab}{[k]} >= 1\,. \end{aligned} \tag{9.50}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of d/2, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(9.39)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{array}{c}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0\\
\vdots\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{[-k_{d-1\;d-2}]}\;\psi_0\\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;\psi_0\\
\vdots
\end{array} \tag{9.51}$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ}:= \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet}:= \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (9.39)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (9.39)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq.(9.39)) of the algebra S^{ab} and $\tilde{S}^{ab}$ a left handed ($\Gamma^{(13,1)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;|\psi\rangle = \frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)|| (\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi\rangle\,. \tag{9.52}$$

This state is an eigen state of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (9.39)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (9.39)), generate families from the starting u_R quark, transforming u_R quark from Eq. (9.52) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$ from Eq. (9.52) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}\;. \tag{9.53}$$

Below some useful relations [4] are presented

$$
\begin{aligned}
N^{\pm}_{+} &= N^{1}_{+} \pm i\, N^{2}_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\, N^{2}_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
\tilde{N}^{\pm}_{+} &= -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
\tau^{1\pm} &= (\mp)\,\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\,\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
\tilde{\tau}^{1\pm} &= (\mp)\,\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\,\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\,.
\end{aligned}
\tag{9.54}
$$

I present at the end one Weyl representation of $SO(13+1)$ and the family quantum numbers of the two groups of four families.

One Weyl representation of $SO(13+1)$ contains left handed weak charged and the second $SU(2)$ chargeless coloured quarks and colourless leptons and right handed weak chargeless and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from the reference [15].

i		$\vert ^a\psi_i>$ Octet, $\Gamma^{(1,7)}=1$, $\Gamma^{(6)}=-1$, of quarks and leptons	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
17	u_R^{c3}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
25	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$ -	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$

Continued on next page

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
		Octet, $\Gamma^{(1,7)}=1,\ \Gamma^{(6)}=-1$, of quarks and leptons										
36	$\bar{u}^{\bar{c}1}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}^{\bar{c}1}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}^{\bar{c}1}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}^{\bar{c}1}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40	$\bar{u}^{\bar{c}1}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41	$\bar{d}^{\bar{c}2}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
...												
49	$\bar{d}^{\bar{c}3}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
...												
57	$\bar{e}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 9.3. The left handed ($\Gamma^{(13,1)} = -1$) multiplet of spinors - the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, is presented in the massless basis using the technique presented in Appendix 9.9. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^{13} = \pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks of three colours ($c^i = (\tau^{33}, \tau^{38})$) with the "spinor" charge ($\tau^4 = \frac{1}{6}$) and the colourless left handed weak charged leptons and the right handed weak chargeless leptons with the "spinor" charge ($\tau^4 = -\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. The vacuum state $|vac>_{fam}$, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in the refs. [21,3]. Left handed antiquarks and anti leptons are weak chargeless and carry opposite charges.

The eight families of the first member of the eight-plet of quarks from Table 9.3, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 9.4 [3]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,2}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{3}$
II	$u^{c1}_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{[+]}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$	$\nu_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\parallel\overset{9\,10}{(+)}\;\overset{11\,12}{(+)}\;\overset{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 9.4. Eight families of the right handed u^{c1}_R (9.3) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ (9.2) are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (9.54). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (9.54)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

9.10 APPENDIX: Expressions for the spin connection fields in terms of vielbeins and the spinor sources [14]

The expressions for the spin connection of both kind, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ in terms of the vielbeins and the spinor sources of both kinds are presented, obtained by the variation of the action Eq.(9.1). The expression for the spin connection $\omega_{ab\alpha}$ is taken from the ref. [32].

$$\begin{aligned}
\omega_{ab\alpha} = &-\frac{1}{2E}\Big\{ e_{e\alpha}e_{b\gamma}\,\partial_\beta(Ef^{\gamma[e}f^{\beta}{}_{a]}) + e_{e\alpha}e_{a\gamma}\,\partial_\beta(Ef^{\gamma}{}_{[b}f^{\beta e]}) \\
&\qquad - e_{e\alpha}e^{e}{}_{\gamma}\,\partial_\beta\left(Ef^{\gamma}{}_{[a}f^{\beta}{}_{b]}\right)\Big\} \\
&- \frac{e_{e\alpha}}{4}\Big\{\bar{\Psi}\left(\gamma_e\,S_{ab} + \frac{3i}{2}\,(\delta^e_b\gamma_a - \delta^e_a\gamma_b)\right)\Psi\Big\} \\
&- \frac{1}{d-2}\Big\{e_{a\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{b]}\right) + \frac{1}{2}\bar{\Psi}\gamma^d S_{db}\,\Psi\right] \\
&\qquad - e_{b\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{a]}\right) + \frac{1}{2}\bar{\Psi}\gamma^d S_{da}\,\Psi\Big\}\right]\,. \qquad (9.55)
\end{aligned}$$

One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then $\omega_{ab\alpha}$ is completely determined by the vielbeins.

Equivalently one obtains expressions for the spin connection fields carryin family quantum numbers

$$\begin{aligned}
\tilde{\omega}_{ab\alpha} = &-\frac{1}{2E}\Big\{ e_{e\alpha}e_{b\gamma}\,\partial_\beta(Ef^{\gamma[e}f^{\beta}{}_{a]}) + e_{e\alpha}e_{a\gamma}\,\partial_\beta(Ef^{\gamma}{}_{[b}f^{\beta e]}) \\
&\qquad - e_{e\alpha}e^{e}{}_{\gamma}\,\partial_\beta\left(Ef^{\gamma}{}_{[a}f^{\beta}{}_{b]}\right)\Big\} \\
&- \frac{e_{e\alpha}}{4}\Big\{\bar{\Psi}\left(\gamma_e\,\tilde{S}_{ab} + \frac{3i}{2}\,(\delta^e_b\gamma_a - \delta^e_a\gamma_b)\right)\Psi\Big\} \\
&- \frac{1}{d-2}\Big\{e_{a\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{b]}\right) + \frac{1}{2}\bar{\Psi}\gamma^d\,\tilde{S}_{db}\,\Psi\right] \\
&\qquad - e_{b\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta\left(Ef^{\gamma}{}_{[d}f^{\beta}{}_{a]}\right) + \frac{1}{2}\bar{\Psi}\gamma^d\,\tilde{S}_{da}\,\Psi\Big\}\right]\,. \qquad (9.56)
\end{aligned}$$

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10 The *Spin-charge-family* Theory Explains Why the Scalar Higgs Carries the Weak Charge $\pm\frac{1}{2}$ and the Hyper Charge $\mp\frac{1}{2}$

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Abstract. One Weyl representation of $SO(13+1)$ contains [1–7], if analysed with respect to the charge and the spin groups of the *standard model*, left handed weak ($SU(2)_{I}$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons, and right handed weakless and $SU(2)_{II}$ charged quarks and leptons (neutrinos and electrons). In the *spin-charge-family* theory [1–12] spinors carry also the family quantum numbers, explaining the origin of families and correspondingly the masses of fermions and weak bosons and the origin of the scalar Higgs and Yukawa couplings. I am demonstrating in this paper that all the fields appearing in the simple starting action of the *spin-charge-family* theory in $d = (13+1)$ with the scalar index with respect to $d = (3+1)$ and determining masses of quarks and leptons (and correspondingly also of the weak boson fields) carry the weak and the hyper charge required by the *standard model* for the scalar Higgs.

Povzetek. Ena Weylova upodobitev $SO(13+1)$ vsebuje [1–7], če jo analiziramo glede na grupe nabojev in spinov *standardnega modela*, levoročne kvarke z barvnim tripletnim nabojem in brezbarvne leptone s šibkim nabojem ($SU(2)_{I}$), ki nimajo naboja $SU(2)_{II}$ ter desnoročne barvne triplete kvarkov in brezbarvnih leptonov, ki ne nosijo šibkega naboja, nosijo pa naboj $SU(2)_{II}$. V teoriji *spinov-nabojev-družin* [1–12] nosijo spinorji tudi kvantna števila družin, kar pojasni izvor družin in tudi mase fermionov in šibkih bozonov ter izvor Higgsovega skalarja in Yukawinih sklopitev. V tem prispevku pokažem, da nosijo vsa polja s skalarnim indeksom glede na $d = (3+1)$ $s = (7,8)$, ki nastopajo v enostavni začetni akciji teorije *spinov-nabojev-družin* v $d = (13+1)$ in določajo mase kvarkov in leptonov, s tem pa tudi mase šibkih bozonov, šibki in hiper naboj tak, kot ju zahteva *standardni model* za skalarno Higgsovo polje. Teorija tako ponudi razlago za izmerjene lastnosti Higgsovega skalarja ter Yukawinih sklopitev.

10.1 Introduction

The *standard model* assumed and the LHC confirmed the existence of the Higgs scalar - the only so far observed bosons with the charge in the fundamental representation.

I am demonstrating in this paper that the *spin-charge-family* theory explains the appearance of the scalar fields with the charges of the Higgs scalar fields. There are, namely, in this theory, in its simple starting action in $d = (13+1)$, the fields with the scalar index with respect to $d = (3 + 1)$, which have the properties of the higgs and explain masses of quarks and leptons together with the Yukawa couplings, and correspondingly also the masses of the weak vector boson fields.

Let me add that all the scalars, that is all the gauge fields of this theory with the space index ≥ 5, have in the starting action the corresponding charges with respect to the scalar index in the fundamental representations: They are either doublets with respect to the two $SU(2)$ groups (the weak $SU(2)_{I}$ and the second $SU(2)_{II}$, which correspondingly result in the properties of the Higgs scalars with respect to the weak and the hyper charge) or they are colour triplets (the properties of these triplets are discussed in a separate paper [13]). All these scalar fields carry the additional charges (the charges not originating in the space index - the family charges, for example) in adjoint representations.

The referee of this paper stated that it is not at all remarkable that there are the scalar fields which are doublets with respect to the weak charge after starting with so many independent fields.

It is, of course, true that a large enough orthogonal group can contain any desired subgroups. *But this is not what the spin-charge-family theory proposes*: It starts with an (very simple) action for spinors and the corresponding gauge fields, manifesting very limited properties, and it is not at all self-evident that some of these fields manifest the desired properties in the low energy regime while all the other spinors and vector and scalar gauge fields - unobserved in the low energy regime - get high masses through the interaction with only one scalar condensate, what is happening in the *spin-charge-family* theory.

On the contrary, it is an extremely encouraging fact that one scalar condensate makes all the vector and the scalar gauge fields appearing in the *spin-charge-family* theory, except those which are observable at the low energy regime (the gravity, the colour vector gauge field, the weak and the hyper charge vector gauge fields, and the eight families of quarks and leptons, decoupled into two times four families), very massive with respect to the weak scale and correspondingly unobservable in the low energy regime. Several scalar gauge fields, however, which when gaining nonzero vacuum expectation values (changing in this case also their masses) cause the electroweak break, have the weak charge equal to $\pm\frac{1}{2}$ and the hyper charge correspondingly $\mp\frac{1}{2}$, as the scalar Higgs in the *standard model*, while they have all the other quantum numbers in the adjoint representations. All the rest of the scalar fields are colour triplets with respect to the scalar space index.

Those who are proposing unifying theories, must offer for the chosen groups and the chosen representations of these groups also the Lagrange densities, designed for those groups and representations, what calls for the theory beyond those effective actions. I am proposing a simple starting action, out of which - after the breaks of symmetries triggered by boundary conditions in a complicated many body problem - manifests in the low energy regime the observable phenomena.

Let me make in this introduction make a short overview of the *spin-charge-family* theory [1,2,7,6,3–5,8–12], pointing out that this theory is offering the explanation for the assumptions of the *standard model*: For the properties of quarks and leptons (right handed neutrinos are in this theory the regular members of a family) and antiquarks and antileptons, and for the existence of the gauge vector fields [1] of the charges. It is offering also the explanation for the existence of the families of quarks and leptons and correspondingly for the scalar gauge fields, which are responsible for masses of quarks and leptons and of weak gauge fields and for Yukawa couplings.

There are, namely, (only) two kinds [3–5,7,16–18] of the Clifford algebra objects (connected by the left and the right multiplication of any Clifford object): the Dirac γ^a's and the second $\tilde{\gamma}^a$'s, respectively. These two Clifford algebra objects (Eq. (9.35)) anticommute, forming the equivalent representations with respect to each other. If using the Dirac γ^a's in $d = (13+1)$ to describe in $d = (3+1)$ the spin and all the charges, then $\tilde{\gamma}^a$'s describe families.

All predictions of the *spin-charge-family* theory in the low energy regime (after the break of the starting symmetry) follow from the simple starting action (Eq.(10.1)) in $d = (13+1)$ for spinors carrying two kinds of a spin (no charges) and for the vielbeins and the two kinds of spin connection fields, with which spinors interact.

Let us first tell that one Weyl representation of $SO(13,1)$ contains [1,2,7,6,14], if analysed with respect to the subgroups $SO(3,1)\times$ $SU(2)_I\times$ $SU(2)_{II}$ $\times$ $SU(3)$ $\times U(1)$, all the family members, required by the *standard model*, with the right handed neutrinos in addition: It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and right handed weakless and $SU(2)_{II}$ charged quarks and leptons, as well as right handed weak charged and $SU(2)_{II}$ chargeless colour antitriplet antiquarks and (anti)colourless antileptons, and left handed weakless and $SU(2)_{II}$ charged antiquarks and antileptons. The antifermions are reachable from the fermions by the application of the discrete symmetry operator $\mathcal{C_N}$ $\mathcal{P_N}$, presented in ref. [14].

The theory accordingly explains how and why is the weak charge connected with the handedness determined by the spin degrees of freedom in $d = (3+1)$. One Weyl (one family) representation of spinors of the group $SO(13,1)$ is presented in table 9.3. Each state is written as a product of nilpotents and projectors defined in the "technique" [4,16,18,17], short version of which can be found in appendix 9.9. Quantum numbers of each of the family members, all are presented in table 9.3 together with the quantum numbers, are defined in Eqs. (10.8, 10.9, 10.10).

The symmetry of both kinds of groups, $SO(13,1)$ and $\widetilde{SO}(13,1)$ (are assumed to) break simultaneously, influencing family members and families of spinors, as well as the gauge fields. After the break of symmetries from the manifold $M^{(13+1)}$ to $M^{(7+1)} \times M^{(6)}$, which makes all the families, except the $2^{\frac{7+1}{2}-1}$ ones determined by the group $\widetilde{SO}(7,1)$, massive [2], carries each family member the family

[1] In this sense the *spin-charge-family* is the Kaluza-Klein like theory [15].

[2] In this paper the break of symmetries in the way that only $2^{\frac{(7+1)}{2}-1}$ families stay massless, while all the others get high masses of the order above the unifying scale, is just assumed. This assumption, however, is supported by several works on the toy model with the same starting action (Eq. (10.1)) but with $d = (5+1)$, ref. [20,23], while the preliminary work on this more complex case is in progress.

quantum numbers, belonging in $\widetilde{SO}(7,1)$ to two times $\widetilde{SU}(2) \times \widetilde{SU}(2)$ groups, originating in $\widetilde{SO}(3,1)$ and in $\widetilde{SO}(4)$, respectively (where $\widetilde{SO}(n)$ represent the subgroups the generators of which are expressed by $\tilde{\gamma}^a$). The families correspondingly decouple to two times four families.

The generators of the corresponding subgroups of the family group $\widetilde{SO}(7,1)$, are defined in Eqs. (10.11, 10.12). To each family member of each family the antimember corresponds, accessible from the member by the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$, which does not depend on $\tilde{\gamma}^a$'s, as explained in the ref. [14,25].

Let us add that since all the charges, with the family charge included, emerge from the spins, correspondingly all the charges are quantized.

Quarks and leptons have the "spinor" quantum number (τ^4, originating in $SO(6)$, Eq. (10.10)) $\frac{1}{6}$ and $-\frac{1}{2}$, respectively [3], with the sum of both equal to $3 \times \frac{1}{6} + (-\frac{1}{2}) = 0$.

The *spin-charge-family* theory therefore predicts that there are two decoupled groups of four families: The fourth [1,7,6,9] to the already observed three families of quarks and leptons should (sooner or later) be measured at the LHC [11], while the lowest of the upper four families constitute the dark matter [10].

Let me *summarize* this first part of the introduction with the *statement*: The *spin-charge-family theory is offering the explanation for the assumptions of the standard model*, having correspondingly a chance to be the right step beyond the *standard model*.

This paper presents in section 10.2 that the properties of the scalar field, the weak and the hyper charge of the scalar Higgs, which are in the *standard model* just assumed to properly "dress" the right handed members by the weak and the hyper charge, *appear in the spin-charge-family* naturally, offering the explanation for the appearance of the scalar fields, observed so far as the scalar Higgs.

In the subsection of this introductory section the simple starting action of the *spin-charge-family* theory is presented, as well as all the assumptions made to achieve that the theory manifests at low energies the observed phenomena.

In section 10.3 the resume and conclusions are presented. In the first appendix 9.9 a short review of the technique, used in this paper to manifest properties of the spinor states, as well as the expressions for the two kinds of spin connection fields, in terms of vielbeins and the spinor sources, are added.

In section 10.3 the resume and conclusions are presented. In appendix a short review of the technique, used in this paper to manifest properties of the spinor states, as well as the expressions for the two kinds of spin connection fields, in terms of vielbeins and the spinor sources, are added.

10.1.1 The action of the *spin-charge-family* theory and the assumptions

Let me present the *assumptions* on which the theory is built, starting with the (simple) action in $d = (13+1)$:

[3] In the Pati-Salam model [21] this "spinor" quantum number is named $\frac{B-L}{2}$ quantum number and is twice the "spinor" quantum number, for quarks equal to $\frac{1}{3}$ and for leptons to -1.

i. In the simple action [7,1] fermions ψ carry in $d = (13+1)$ as the *internal degrees of freedom only two kinds of spins, no charges*, and *interact correspondingly with only the two kinds of the spin connection gauge fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and the vielbeins,* $f^{\alpha}{}_{a}$.

$$
\begin{aligned}
S &= \int d^dx\, E\, \mathcal{L}_f + \\
&\quad \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
\mathcal{L}_f &= \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + \text{h.c.}, \\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\, \{p_{\alpha}, E f^{\alpha}{}_a\}_-, \\
p_{0\alpha} &= p_{\alpha} - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \\
R &= \frac{1}{2}\, \{f^{\alpha[a} f^{\beta b]}\, (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta})\} + \text{h.c.}\,, \\
\tilde{R} &= \frac{1}{2}\, f^{\alpha[a} f^{\beta b]}\, (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^{c}{}_{b\beta}) + \text{h.c.}\,.
\end{aligned}
\tag{10.1}
$$

Here [4] $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. S^{ab} and $\tilde{S}^{ab}$ are generators of the groups $SO(13,1)$ and $\widetilde{SO}(13,1)$, respectively, expressible by γ^a and $\tilde{\gamma}^a$.

ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (which manifests as $SU(3) \times U(1)$), affecting both internal degrees of freedom - the one represented by γ^a and the one represented by $\tilde{\gamma}^a$, leading to $2^{((7+1)/2-1)}$ massless families, all the rest families get heavy masses [5]. Both internal degrees of freedom, the ordinary $SO(13+1)$ one (where γ^a determine spins and charges of spinors) and the $\widetilde{SO}(13+1)$ (where $\tilde{\gamma}^a$ determine family quantum numbers), break simultaneously with the manifolds.

iii. There are additional breaks of symmetry: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

iv. There is a scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale above the unification scale, to all the vector and scalar gauge fields, which interact with the condensate.

---

[4] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b$, $e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[5] A toy model [23,24,14] was studied in $d = (5+1)$ with the same action as in Eq.'(10.1). For a particular choice of vielbeins and for a class of spin connection fields the manifold M^{5+1} breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families stay massless and mass protected. Equivalent assumption, although not yet proved that it really works, is made also in the case that $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^{(6)}$. The study is in progress.

v. There are nonzero vacuum expectation values of the scalar fields with the scalar indices $(7,8)$, which cause the electroweak break and bring masses to the fermions and weak gauge bosons, conserving the electromagnetic and colour charge.

Comments on the assumptions:

i.: This starting action enables to represent the *standard model* as the effective low energy manifestation of the *spin-charge-family* theory, explaining all the *standard model* assumptions, with the families included. There are (before the electroweak break all massless) observable gauge fields: gravity, colour ($SU(3)$, from $SO(6)$) octet vector gauge fields, weak ($SU(2)_I$ from $SO(4)$) triplet vector gauge field and "hyper" ($U(1)$ from $SO(6)$) singlet vector gauge fields. All are superposition of ${f^\alpha}_c\, \omega_{ab\alpha}$. And there are (before the electroweak break all massless) observable (eight rather than observed three) families of quarks and leptons. (There are indeed two decoupled groups of four families, in the fundamental representations of twice $\widetilde{SU}(2)\times \widetilde{SU}(2)$ groups, the subgroups of $\widetilde{SO}(3,1)\times \widetilde{SO}(4)$. There are correspondingly the scalar fields with the weak and the hyper charge of the scalar Higgs and with either two kinds of the family quantum numbers in the adjoint representations - they are two times two triplets, emerging from the superposition of ${f^\sigma}_s\tilde{\omega}_{ab\sigma}$ with $s \in (7,8)$, in accordance with twice $\widetilde{SU}(2)\times \widetilde{SU}(2)$ groups, the subgroups of $\widetilde{SO}(3,1)\times \widetilde{SO}(4)$ - or with the quantum numbers (Q, Q', Y') emerging from the superposition of ${f^\sigma}_s\tilde{\omega}_{ab\sigma}$. Both determine the Yukawa couplings.) The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge field and the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10,11,12)$, as well as the auxiliary vector gauge fields expressible with vielbeins, which are the superposition of ${f^\mu}_m\tilde{\omega}_{ab\mu}$. They all remain either auxiliary or become massive after the appearance of the condensate.

ii., iii.: The assumed breaks explain why the weak and the hyper charge are connected with the handedness of spinors, manifesting the observed properties of the family members - the quarks and the leptons, left and right handed - and of the observed vector gauge fields. Since the left handed members are weak charged while the right handed are weak chargeless, the family members stay massless and mass protected up to the electroweak break. Antiparticles are accessible from particles by the $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$, as explained in refs. [14,25]. This discrete symmetry operator does not contain $\tilde{\gamma}^a$'s degrees of freedom. To each family member there corresponds the antimember, with the same family quantum number.

iv.: It is the condensate of two right handed neutrinos with the quantum numbers of the upper four families, which makes all the scalar gauge fields (with the index $(5,6,7,8)$, as well as those with the index $(9,\ldots,14)$) and the vector gauge fields, manifesting nonzero τ^4, τ^{23}, Q ,Y, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{Q}$,$\tilde{Y}$,$\tilde{N}^3_R$ (Eqs. (10.8, 10.9, 10.10, 10.11, 10.12, 10.13)) massive [13].

v.: At the electroweak break the scalar fields with the space index $s = (7,8)$, originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{s's''s}$, those which conserve the electromagnetic charge, get nonzero vacuum expectation values, what changes also their masses. They determine mass matrices of twice the four families, as well as the masses of the weak bosons. All the rest scalar fields keep masses of the condensate scale and are correspondingly (so far) unobservable in the low energy regime [6]. The fourth family to the observed three ones will (sooner or later) be observed at the LHC. Its properties are under the consideration [11], while the stable of the upper four families is the candidate for the dark matter.

The above assumptions enable that the starting action (Eq. (10.1)) manifests effectively in $d = (3+1)$ in the low energy regime by the *standard model* required degrees of freedom of fermions and bosons [1,2,7,6,3–5,8–12], that is the quarks and the leptons, left and right handed, the families of quarks and leptons and all the known gauge fields, with (several, explaining the Yukawa couplings) scalar fields included.

To see this let us rewrite formally the action for the Weyl spinor of (Eq.(10.1)) as follows

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
&\quad \{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi\} + \\
&\quad \{\sum_{t=5,6,9,\dots,14} \bar{\psi}\gamma^t p_{0t}\,\psi\}, \\
p_{0s} &= p_s - \frac{1}{2}S^{s's''}\omega_{s's''s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}, \\
p_{0t} &= p_t - \frac{1}{2}S^{t't''}\omega_{t't''t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt},
\end{aligned} \tag{10.2}$$

where $m \in (0,1,2,3)$, $s \in 7,8$, $(s', s'') \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within $(0,1,2,3)$ and $(5,6,7,8)$, $t \in (5,6,9,\dots,13,14)$.

The first line of Eq. (10.2) determines the kinematics and dynamics of spinor fields in $d = (3+1)$, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients ${c^{Ai}}_{ab}$, as presented in Eqs. (10.9, 10.10, 10.13)

$$\tau^{Ai} = \sum_{a,b} {c^{Ai}}_{ab}\, S^{ab}, \tag{10.3}$$

and the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk}\tau^{Ak}. \tag{10.4}$$

The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}, with (s,t) either $\in (5,6,7,8)$ or $\in (9,10,\dots,13,14)$, in agreement with the assumptions **ii.** and **iii.**. Before the electroweak break the vector gauge fields appearing in the first line of Eq. (10.2) are all massless: $\vec{A}^3_m$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^1_m$ carries the weak charge $SU(2)_I$ ($SU(2)_I$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and $A^Y_m = \sin\vartheta_2 A^{23}_m + \cos\vartheta_2 A^4_m$ (Y is defined in Eq. (10.13), τ^4 in Eq. (10.10), the corresponding $U(1)$ group originates in $SO(6)$), A^4_m is defined in Eq. (10.15), if the scalar space index s is replaced by

[6] Correspondingly $d = (13+1)$ manifests in $d = (3+1)$ spins and charges as if one would start with $d = (9+1)$ instead of with $d = (13+1)$, since the plane $(5,6)$ and the plane in which the vector τ^4 lies, are unobservable at low energies.

the space vector index m, A^{23}_m is the third component of the second $SU(2)_{II}$ field $\vec{A}^2_m$. The corresponding charges $(\vec{\tau}^3, \vec{\tau^1}, Y)$ are the conserved charges.

Before the appearance of the condensate of the two right handed neutrinos with the quantum numbers of the upper four families (properties of the condensate are presented in table 10.1) at the scale far about the electroweak scale, all the three components of the field $\vec{A}^2_m$ are massless. The condensate gives the mass of the order of the scale of the appearance of the condensate to $A^{Y'}_m = \cos\vartheta_2 A^{23}_m - \sin\vartheta_2 A^4_m$, and to all the scalar gauge fields, presented in the second and the third line of Eq. (10.2), leading to $A^{Ai}_s, s \in (5, 6, \ldots, 13, 14)$ and $\tilde{A}^{Ai}_t$, $t \in (5, 6, \ldots, 13, 14)$.

Vector gauge fields $A^Y_m, \vec{A}^1_m$ and $\vec{A}^3_m$ do not couple to the condensate (table 10.1).

In Eqs. (10.15, 10.14) the expressions for the scalars with the scalar index $(7, 8)$ in terms of both kinds of the spin connection fields are presented. These scalar fields (the second line in Eq. (10.2)) determine after the electroweak break the mass matrices of the two decoupled groups of four families. Getting nonzero vacuum expectation values they cause the electroweak break, changing also their own masses. These scalar fields determine also the masses of the gauge bosons.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_1 \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_2)$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 10.1. The condensate of the two right handed neutrinos ν_R, with the $VIII^{th}$ family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in table 9.4, taken from the ref. [13].

Among the vector gauge fields $\vec{A}^3_m$ and $\vec{\tilde{A}}^3_m$ and the corresponding vielbeins only one of these three vector gauge fields is the propagating one, while the rest two are the auxiliary fields as one can learn from Eqs. (9.55, 9.56) of the second appendix section 9.10, if taking into account that there is no spinor (fermion) sources with the corresponding quantum numbers. Equivalently, also only one of the three vector gauge fields $\vec{A}^1_m$, $\vec{\tilde{A}}^1_m$ and the corresponding vielbein field is the propagating field, the other two are the auxiliary fields, as well as only one of the three vector gauge fields $\vec{A}^{N_L}_m$, $\vec{\tilde{A}}^{N_L}_m$ and the corresponding vielbein field is the propagating field, while $\vec{A}^{N_R}_m$ is massive due to the interaction with the condensate of the two right handed neutrinos through quantum numbers $\vec{\tilde{N}}_R$, presented in Eqs. (10.8, 10.11).

Let me *summarize* this subsection: The starting action (Eq.(10.1)) of the *spin-charge-family* theory manifests under the assumptions **i.-v.** in the low energy regime properties of the *standard model*, explaining the *standard model* assumptions: Before the electroweak break all the scalar gauge fields and the vector gauge fields -

except the colour, the weak and the hyper vector fields (and the gravity), which stay massless - are massive, due to the interaction with the scalar condensate of the two right handed neutrinos with the family quantum numbers of the upper four families. There are also the two decoupled massless groups of four families.

At the electroweak break the scalar gauge fields, carrying the scalar space index and keeping the electromagnetic charge conserved and changing their own masses, bring masses to all the fermions and all the gauge fields, except to the gavity, electromagnetic and the colour ones.

Let me comment that in the presence of the spinor fields (as it is the condensate, for example) all three gauge fields - the vielbeins and the two kinds of the spin connection fields - are in general the propagating fields. If there are no spinors present, only one of the three fields is the propagating field, the other two are expressible with the propagating one (as it is well known). In the second appendix 9.10 the expressions for the spin connection fields of both kinds in terms of the vielbeins and the spinor sources are presented, taken from the ref. [20].

The assumed breaks should occur spontaneously, determined by the starting action and the boundary conditions. To prove that this really can happen is a very difficult (many body) problem. Although several studies made so far, for either a toy model in $d = (5+1)$ or for the $d = (13+1)$ case, support these assumptions, yet several additional studies are needed to justify the assumptions and to clarify further the properties of the scalar and vector gauge fields and of the spinor families, appearing in the starting action. Also the comparison with all the other works made on the unifying theories are needed to see to which extend predictions of this theory coincide with the other theories in the literature, in which sense and what one can learn out of them.

The *standard model* subgroups of the $SO(13+1)$ and of the $\widetilde{SO}(13+1)$ group and the corresponding gauge fields To calculate quantum numbers of one Weyl representation presented in table 9.3 in terms of the generators of the *standard model* groups τ^{Ai} $(= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab})$ one must look for the coefficients $c^{Ai}{}_{ab}$ (Eq. (10.4)). The generators τ^{Ai} are the generators of the charge groups. Similarly one expresses also the spin and the family degrees of freedom.

The same coefficients $c^{Ai}{}_{ab}$ determine operators which apply on spinors and on vectors. The difference among the three kinds of operators - vectors and two kinds of spinors - lies in S^{ab}.

While S^{ab} for spins of spinors is equal to

$$S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)\,, \tag{10.5}$$

and $\tilde{S}^{ab}$ for families of spinors is equal to

$$\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)\,, \tag{10.6}$$

one must take, when $\mathcal{S}^{ab}$ apply on the spin connections ω_{bde} $(= f^{\alpha}{}_{e}\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_{e}\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, the operator

$$(\mathcal{S}^{ab})^c{}_e A^{d\ldots e\ldots g} = i(\eta^{ac}\delta^b_e - \eta^{bc}\delta^a_e)\, A^{d\ldots e\ldots g}\,. \qquad (10.7)$$

This means that the space index (e) of ω_{bde} transforms according to the requirement of Eq. (10.7), and so do b, d and $\tilde{b}$, $\tilde{d}$. I used the notation $\tilde{b}$, $\tilde{d}$ to point out that S^{ab} and $\tilde{S}^{ab}$ $(= \tilde{S}^{\tilde{a}\tilde{b}})$ are generators of two independent groups.

One finds [1,7,6,3–5,8,12] for the generators of the spin and the charge groups, which are the subgroups of $SO(13,1)$, the expressions:

$$\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})\,, \qquad (10.8)$$

where the generators $\vec{N}_{\pm}$ determine representations of the two $SU(2)$ subgroups of $SO(3,1)$, generators $\vec{\tau}^1$ and $\vec{\tau}^2$,

$$\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,,$$
$$\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78}), \qquad (10.9)$$

determine representations of the $SU(2)_I \times SU(2)_{II}$ invariant subgroups of the group $SO(4)$, which is further the subgroup of $SO(7,1)$ ($SO(4), SO(3,1)$ are subgroups of $SO(7,1)$), and the generators $\vec{\tau}^3$, τ^4 and $\tilde{\tau}^4$

$$\begin{aligned}\vec{\tau}^3 :=\; & \frac{1}{2}\{S^{9\;12} - S^{10\;11}, S^{9\;11} + S^{10\;12}, S^{9\;10} - S^{11\;12},\\ & S^{9\;14} - S^{10\;13}, S^{9\;13} + S^{10\;14}, S^{11\;14} - S^{12\;13},\\ & S^{11\;13} + S^{12\;14}, \frac{1}{\sqrt{3}}(S^{9\;10} + S^{11\;12} - 2S^{13\;14})\},\\ \tau^4 :=\; & -\frac{1}{3}(S^{9\;10} + S^{11\;12} + S^{13\;14})\,,\\ \tilde{\tau}^4 :=\; & -\frac{1}{3}(\tilde{S}^{9\;10} + \tilde{S}^{11\;12} + \tilde{S}^{13\;14})\,,\end{aligned} \qquad (10.10)$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$, and of $\tilde{\tau}^4$ originating in $\widetilde{SO}(6)$.

One correspondingly finds the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \qquad (10.11)$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\vec{\tilde{\tau}}^1 := \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67},\, \tilde{S}^{57} + \tilde{S}^{68},\, \tilde{S}^{56} - \tilde{S}^{78})\,,$$
$$\vec{\tilde{\tau}}^2 := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67},\, \tilde{S}^{57} - \tilde{S}^{68},\, \tilde{S}^{56} + \tilde{S}^{78})\,, \qquad (10.12)$$

determine representations of $\widetilde{SU}(2)_I \times \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both, $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$ are the subgroups of $\widetilde{SO}(7,1)$.

One further finds

$$Y = \tau^4 + \tau^{23}, \quad Y' = -\tau^4 \tan^2 \vartheta_2 + \tau^{23}, \quad Q = \tau^{13} + Y, \quad Q' = -Y \tan^2 \vartheta_1 + \tau^{13},$$
$$\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}, \quad \tilde{Y}' = -\tilde{\tau}^4 \tan^2 \tilde{\vartheta}_2 + \tilde{\tau}^{23}, \quad \tilde{Q} = \tilde{Y} + \tilde{\tau}^{13}, \quad \tilde{Q}' = -\tilde{Y} \tan^2 \tilde{\vartheta}_1 + \tilde{\tau}^{13}. \tag{10.13}$$

The scalar fields, responsible [1,2,7] - after getting in the electroweak break nonzero vacuum expectation values - for masses of the family members and of the weak bosons, are presented in the second line of Eq. (10.2). These scalar fields are included in the covariant derivatives as $-\frac{1}{2}\, S^{s's"} \omega_{s's"s} - \frac{1}{2} \tilde{S}^{\tilde{a}\tilde{b}} \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s \in (7,8)$, $(\tilde{a}, \tilde{b}), \in (\tilde{0}, \tilde{1}, \ldots, \tilde{8})$, where $\tilde{a}, \tilde{b}$ is again used to point out that (a, b) belong in this case to the "tilde" space.

One finds, by taking into account Eqs. (10.11, 10.12) and Eq. (10.13), for the choice of the $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ scalar gauge fields, contributing to the electroweak break, the expressions

$$\begin{aligned}
-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s} &= -(\vec{\tilde{\tau}}^{\tilde{1}}\, \vec{\tilde{A}}^{\tilde{1}}_{s} + \vec{\tilde{N}}_{\tilde{L}}\, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} + \vec{\tilde{\tau}}^{\tilde{2}}\, \vec{\tilde{A}}^{\tilde{2}}_{s} + \vec{\tilde{N}}_{\tilde{R}}\, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s})\,,\\
\vec{\tilde{A}}^{\tilde{1}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s},\, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s},\, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s})\,,\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{1}s},\, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{2}s},\, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{3}s})\,,\\
\vec{\tilde{A}}^{\tilde{2}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s},\, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s},\, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s})\,,\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{1}s},\, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{2}s},\, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{3}s})\,,\\
&(s \in (7,8))\,.
\end{aligned} \tag{10.14}$$

Among ω_{abs}, which contribute to the mass matrices of quarks and leptons, one finds when using Eqs. (10.9, 10.10, 10.13), the expressions

$$\begin{aligned}
-\frac{1}{2}S^{s's"}\, \omega_{s's"s} &= -(g^{23}\, \tau^{23}\, A^{23}_{s} + g^{13}\, \tau^{13}\, A^{13}_{s} + g^{4}\, \tau^{4}\, A^{4}_{s})\,,\\
g^{13}\, \tau^{13}\, A^{13}_{s} + g^{23}\, \tau^{23}\, A^{23}_{s} + g^{4}\, \tau^{4}\, A^{4}_{s} &= g^{Q}\, Q A^{Q}_{s} + g^{Q'}\, Q' A^{Q'}_{s} + g^{Y'}\, Y'\, A^{Y'}_{s}\,,\\
A^{4}_{s} &= -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s})\,,\\
A^{13}_{s} &= (\omega_{56s} - \omega_{78s})\,, \quad A^{23}_{s} = (\omega_{56s} + \omega_{78s})\,,\\
A^{Q}_{s} &= \sin\vartheta_1\, A^{13}_{s} + \cos\vartheta_1\, A^{Y}_{s}\,,\\
A^{Q'}_{s} &= \cos\vartheta_1\, A^{13}_{s} - \sin\vartheta_1\, A^{Y}_{s}\,,\\
A^{Y'}_{s} &= \cos\vartheta_2\, A^{23}_{s} - \sin\vartheta_2\, A^{4}_{s}\,,\\
&(s \in (7,8))\,.
\end{aligned} \tag{10.15}$$

Scalar fields from Eq. (10.14) couple to the family quantum numbers, while those from Eq. (10.15) distinguish among family members. In Eq. (10.15) the coupling constants were explicitly written to see the analogy with the gauge fields in the *standard model*.

Expressions for the vector gauge fields in terms of the spin connection fields and the vielbeins, which correspond to the colour charge ($\vec{A}^{3}_{m}$), the $SU(2)_{II}$ charge ($\vec{A}^{2}_{m}$), the weak charge ($SU(2)_{I}$) ($\vec{A}^{1}_{m}$) and the $U(1)$ charge originating in $SO(6)$

($\vec{A}^{4}_{m}$), can be found by taking into account Eqs. (10.9, 10.10). Equivalently one finds the vector gauge fields in the "tilde" sector. One really can use just the expressions from Eqs. (10.15, 10.14), if replacing the scalar index s with the vector index m.

Let me *summarize* this subsubsection: The expressions for the operators τ^{Ai} are presented, either in terms of S^{ab} (Eq. (10.5)) or (in this case we name them $\tilde{\tau}^{Ai}$) in terms of $\tilde{S}^{ab}$ (Eq. (10.6)), valid also in terms of $\mathcal{S}^{ab}$ (Eq. (10.7)), affecting correspondingly spinors spin and charges quantum numbers, spinors families quantum numbers and scalar or vector gauge fields, respectively. Also the expressions for those scalar gauge fields, which contribute to the electroweak break by getting nonzero vacuum expectation values, in terms of the corresponding spin connection fields are presented (Eqs.(10.15, 10.14)). When the scalar index s is replaced by the vector index m, the expressions for the vector gauge fields in terms of spin connection fields follow.

10.2 Scalar fields contributing to the electroweak break are weak charge doublets

In this main part of the paper is demonstrated that all the scalar gauge fields with the scalar index $s \in (7, 8)$, which get nonzero vacuum expectation values causing the electroweak break, carry the weak and the hyper charge as does the scalar Higgs of the *standard model*.

All the scalars (the gauge fields with the scalar index with respect to $d = (3 + 1)$) of the action (Eq. 10.1) contribute charges in the fundamental representations: The scalars with the space indices $s \in (7, 8)$ and $s \in (5, 6)$ are, with respect to this scalar space degree of freedom, before the appearance of the condensate (table 10.1), the weak ($SU(2)_{I}$) and the second $SU(2)_{II}$ *doublets*. After the appearance of the condensate only the weak and the hyper charge Y remain the conserved charges, so that it is the third component of τ^{23}, which determines the hyper charge ($Y = \tau^{23} + \tau^{4}$, Eq. (10.13)) of these scalar fields, since τ^{4} applied on the scalar index of these scalar fields gives zero, according to Eqs. (10.9, 10.10, 10.7).

The scalars with the space indices $s \in (9, 10, \ldots, 13, 14)$ are, again with respect to this scalar space degree of freedom, colour triplets [13]. There are no additional scalar indices and therefore no additional corresponding scalars with respect to the scalar indices in this theory.

The scalars, however, carry additional quantum numbers Ai, the states of which belong to the adjoint representations with respect to either $\tilde{\tau}^{Ai}$ or τ^{Ai}. While, to reproduce the low energy phenomena, the scalar fields of all the family quantum numbers are allowed, only those τ^{Ai} are acceptable, which conserve after the electroweak break the electromagnetic charge. The scalar fields with nonzero vacuum expectation values carrying nonzero weak charge also due to $\vec{\tau}^{1}$ would cause nonconservation of the electromagnetic charge (see the assumption **v.** and the corresponding comments in subsection 10.1.1).

The colour triplet scalars contribute to transition from antileptons into quarks and antiquarks into quarks and back, unless the scalar condensate of the two right handed neutrions, presented in table 10.1, breaks matter-antimatter symmetry [13]. This condensate breaks also the $SU(2)_{II}$ symmetry, leaving massless (besides gravity) only the colour, weak and the hyper charge vector gauge fields. Also all the scalar fields get masses through the interaction with the condensate.

When at the electroweak break the scalar fields with the scalar indices $s \in (7,8)$ originating either in $\tilde{\omega}_{abs}$ or in those superposition of $\omega_{s's''s}$ which conserve the electromagnetic charge (Eq. (10.16)) get nonzero vacuum expectation values, changing also their own masses, they bring masses to all the massless fermions (spinors), breaking their mass protection, and to weak bosons.

Let us recognize, by taking into account Eq. (9.44) and table 9.3, that $\gamma^0\,\gamma^s$ appearing in $\{\sum_{s=7,8}\ \bar{\psi}\gamma^s p_{0s}\ \psi\}$ in the second line of Eq. (10.2), transform for either $s = 7$ or $s = 8$ the right handed u-quark (u_R^{c1}), weak chargeless, with the hyper charge $Y = \frac{2}{3}$ from the first line of table 9.3 to the left handed weak charged u-quark (u_L^{c1}) with the hyper charge $\frac{1}{6}$ from the seventh line of the same table, or that $\gamma^0\,\gamma^s$ transform the right handed ν-lepton (ν_R), weak chargeless, with the hyper charge $Y = 0$ from the 25^{th} line of the same table 9.3 to the left handed weak charged ν-lepton (ν_L) with the hyper charge $-\frac{1}{2}$ from the 31^{st} line of the same table.

Now is the time to prove that the scalar fields with the scalar index $s \in (7,8)$ from the second line of Eq. (10.2) and with quantum numbers of Eq. (10.16) really carry the weak and the hyper charge as required by the *standard model*. I introduce in Eq. (10.16) common notation A_s^{Ai} for all these scalar fields, independently of whether they origin in ω_{abs} - in this case they do not carry the additional weak or hyper charge due to $\vec{\tau}^A$ - or $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ fields.

$$
\begin{aligned}
&A_s^{Ai} \supset (\,A_s^{Q}\,, A_s^{Y}\,, A_s^{Y'}\,, \vec{\tilde{A}}_s^{\tilde{1}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{L}}}\,, \vec{\tilde{A}}_s^{\tilde{2}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{R}}}\,)\,,\\
&\tau^{Ai} \supset (Q,\, Y,\, Y' = -\tan^2\vartheta_2\tau^4 + \tau^{23},\, \vec{\tilde{\tau}}^1,\, \vec{\tilde{N}}_L,\, \vec{\tilde{\tau}}^2,\, \vec{\tilde{N}}_R)\,.
\end{aligned}
\tag{10.16}
$$

These scalars, the gauge scalar fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$ (Eqs. (10.11, 10.12, 10.9, 10.10)), are expressible in terms of the spin connection fields (Eqs. (10.14, 10.15)).

One expects that the solutions with nonzero momentum lead to higher masses of the fermion fields in $d = (3+1)$ [23,24]. We shall correspondingly pay no attention to the momentum p_s, $s \in (4,8)$, when having in mind the lowest energy solutions, manifesting at low energies.

Scalars, which do not get nonzero vacuum expectation values, keep masses on the condensate scale.

Let me now, by taking into account Eqs. (10.7, 10.9), calculate properties of all scalar fields A_s^{Ai} of Eq. (10.13).

To do this let us first recognize

$$
\begin{aligned}
&\tau^{1\boxplus} = \frac{1}{2}[(S^{58} - S^{67})\ \boxminus\ i\,(S^{57} + S^{68})]\,, \tau^{13} = \frac{1}{2}(S^{56} - S^{78})\,,\\
&Y = \tau^{23} + \tau^4\,, \quad Q = Y + \tau^{13}\,,
\end{aligned}
$$

and rewrite the scalar fields A_s^{Ai}, which determine masses of fermions and weak bosons in Eq. (10.2), appearing in the second line of Eq. (10.2), as follows (the momentum p_s is left out)

$$\sum_{s=(7,8),Ai} \bar{\psi}\gamma^s\,(-\tau^{Ai}\,A^{Ai}_s\,) =$$

$$-\psi^\dagger\gamma^0\{\overset{78}{(+)}\;\tau^{Ai}\,(A^{Ai}_7 - i\,A^{Ai}_8) + \overset{78}{(-)}\;(\tau^{Ai}\,(A^{Ai}_s + i\,A^{Ai}_8)\,\psi\},$$
$$\overset{78}{(\pm)} = \frac{1}{2}\,(\gamma^7 \pm i\gamma^8\,)\,, \tag{10.17}$$

with the summation over Ai performed, since A^{Ai}_s represent the scalar fields (A^Q_s, A^Y_s, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$ and $\vec{\tilde{A}}^{N_L}_s$).

Application of the operators Y and τ^{13} on the fields ($A^{Ai}_7 \mp i\,A^{Ai}_8$), leads after using Eq. (10.7) for $\mathcal{S}^{ab}$ and expressions for τ^{13} and Y (Eq. (10.17)), to

$$\tau^{13}\,(A^{Ai}_7 \mp i\,A^{Ai}_8) = \pm\frac{1}{2}\,(A^{Ai}_7 \mp i\,A^{Ai}_8)\,,$$
$$Y\,(A^{Ai}_7 \mp i\,A^{Ai}_8) = \mp\frac{1}{2}\,(A^{Ai}_7 \mp i\,A^{Ai}_8)\,,$$
$$Q\,(A^{Ai}_7 \mp i\,A^{Ai}_8) = 0\,. \tag{10.18}$$

Since Y and τ^{13} give zero, if applied on the upper indices (Q, Y, Y') of (A^Q_s, A^Y_s and $A^{Y'}_s$), as one can read from Eq. (10.15), and since Y and τ^{13} commute with the family quantum numbers, one sees that the scalar fields A^{Ai}_s (A^Q_s, A^Y_s, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\tilde{A}^{\tilde{Q}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$), rewritten as follows, $A^{Ai}_{\pm} = (A^{Ai}_7 \mp i\,A^{Ai}_8)$, are eigen states of τ^{13} and Y having the quantum numbers of the *standard model* Higgs' scalars.

Let us make the notation

$$A^{Ai}_{\underset{(\pm)}{78}} = (A^{Ai}_7 \mp i\,A^{Ai}_8)\,, \tag{10.19}$$

and let us calculate what does the operator $\tau^{1\boxplus}$ (Eq. (10.17)) make if applied on $A^{Ai}_{\underset{(\pm)}{78}}$. Taking into account Eqs. (10.7, 10.9) one finds that

$$\tau^{1\boxplus}\,A^{Ai}_{\underset{(\pm)}{78}} = (A^{Ai}_5 \mp i\,A^{Ai}_6) =: A^{Ai}_{\underset{(\pm)}{56}}\,,$$
$$\tau^{1\boxminus}\,A^{Ai}_{\underset{(\pm)}{78}} = 0\,. \tag{10.20}$$

The scalar fields $A^{Ai}_{\underset{(\pm)}{56}}$ are all massive fields with the masses of the condensate scale (table 10.1), while the scalar fields $A^{Ai}_{\underset{(\pm)}{78}}$ change masses at the electroweak break.

Using Eqs. (9.46, 9.44, 9.54) one finds that $\gamma^0\;\overset{78}{(-)}\,A^{Ai}_{\underset{(-)}{78}}$ transforms the right handed u^{c1}_R quark from the first line of table 9.3 into the leftt handed u^{c1}_L quark from the seventh line of the same table, which can be also interpreted in the *standard model* way, namely, that $A^{Ai}_{\underset{(-)}{78}}$ "dress" u^{c1}_R giving it the weak and the hyper charge of the left handed u^{c1}_L quark, while γ^0 changes handedness. Equivalently happens to ν_R from the 25^{th} line, which the action of $\gamma^0 \stackrel{78}{(-)} A^{Ai}_{\stackrel{78}{(-)}}$ on it transforms into the ν_L from the 31^{th} line, which again can be interpreted in the *standard model* way: With the action of γ^0 and the "dressing" of $A^{Ai}_{\stackrel{78}{(-)}}$ on ν_R, transforming it into ν_L.

The action of $\gamma^0 \stackrel{78}{(+)} A^{Ai}_{\stackrel{78}{(+)}}$ transform d^{c1}_R from the third line of the same table into d^{c1}_L from the fifth line of this table, or e_R from the 27^{th} line into the e_L from the 29^{th} line. One can use in this two cases, knowing the properties of the scalar fields (Eq. (10.18)), again the *standard model* interpretation, in which the scalar fields $A^{Ai}_{\stackrel{78}{(+)}}$ take care of the weak and the hyper charges of the right handed members d^{c1}_R and e_R by "dressing" them with the appropriate weak and the hyper charges, while γ^0 changes handedness. In the *standard model* there is the scalar Higgs and the Yukawa couplings, which take care of fermion and also of the weak boson properties.

In the *spin-charge-family* theory there are several scalar fields, which determine the mass matrices of the two groups of four families.

When the scalar fields ($A^{Q}_{\stackrel{78}{(\pm)}}, A^{Y}_{\stackrel{78}{(\pm)}}, A^{Y'}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{1}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{2}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{\stackrel{78}{(pm)}}$) from Eq. (10.16) get nonzero vacuum expectation values, they determine mass matrices of family members - of quarks and leptons - of the lower (carrying the family quantum numbers ($\vec{\tilde{\tau}}^1, \vec{\tilde{N}}_L$)) and the upper (carrying the family quantum numbers ($\vec{\tilde{\tau}}^2, \vec{\tilde{N}}_R$)) four families, since they carry the weak and the hyper charge (Eqs. (10.9, 10.10)) which breaks the mass protection mechanism of quarks and leptons.

We clearly see that all the scalars $A^{Ai}_{\stackrel{78}{(\pm)}}$ have the following properties:

$$(\tau^{13}, Y)\, A^{Ai}_{\stackrel{78}{(\pm)}} = \pm\,(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\stackrel{78}{(\pm)}}\,. \tag{10.21}$$

The scalars $A^{Ai}_{\stackrel{78}{(-)}}$ obviously bring the right quantum numbers to the right handed partners (u_R, ν_R), and the scalars $A^{Ai}_{\stackrel{78}{(+)}}$ give the right quantum numbers to (d_R, e_R).

The scalar fields $A^{Ai}_{\stackrel{78}{(\pm)}}$ are in the *spin-charge-family* theory *triplets with respect to the family quantum numbers* ($\vec{\tilde{N}}_R, \vec{\tilde{N}}_L, \vec{\tilde{\tau}}^2, \vec{\tilde{\tau}}^1$; Eqs. (10.11, 10.12)) or singlets as the gauge fields of $Q = \tau^{13} + Y$, $Y = \tau^{23} + \tau^4$ and $Y' = -\tan^2\vartheta_2 \tau^4 + \tau^{23}$.

One can prove this by applying $\tilde{\tau}^{23}$, $\tilde{\tau}^{13}$, $\tilde{N}^3_R$ and $\tilde{N}^3_L$ on their eigen states. Let us do this for $\tilde{A}^{N_L i}_{\stackrel{78}{(\pm)}}$ and for $A^{Q}_{\stackrel{78}{(\pm)}}$, taking into account Eqs. (10.11), and recognizing that $\tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}} = \tilde{A}^{N_L 1}_{\stackrel{78}{(\pm)}} \boxplus i\, \tilde{A}^{N_L 2}_{\stackrel{78}{(\pm)}}$ (Eq. (10.7)).

$$\begin{aligned}
&\tilde{N}^3_L\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \boxplus\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}}\,, \quad \tilde{N}^3_L\, \tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = 0\,,\\
&\tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \{(\tilde{\omega}_{\tilde{2}\tilde{3}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{1}\stackrel{78}{(\pm)}}) \boxplus i\, (\tilde{\omega}_{\tilde{3}\tilde{1}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{2}\stackrel{78}{(\pm)}})\}\,,\\
&\tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = (\tilde{\omega}_{\tilde{1}\tilde{2}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{3}\stackrel{78}{(\pm)}})\\
&Q\, A^{Q}_{\stackrel{78}{(\pm)}} = 0\,, A^{Q}_{\stackrel{78}{(\pm)}} = \omega_{56\stackrel{78}{(\pm)}} - (\omega_{9\,10\stackrel{78}{(\pm)}} + \omega_{11\,12\stackrel{78}{(\pm)}} + \omega_{13\,14\stackrel{78}{(\pm)}})\,,
\end{aligned} \tag{10.22}$$

with $Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, and with τ^4 defined in Eq. (10.10)).

To masses of the lower four families only the scalar fields, which are the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute. (To masses of the upper four families only the gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute.) The three scalar fields $A^{Q}_{\stackrel{78}{(\pm)}}$, $A^{Y}_{\stackrel{78}{(\pm)}}$ and $A^{4}_{\stackrel{78}{(\pm)}}$ "see" the family members quantum numbers and contribute correspondingly to all the families.

The scalar fields, with the weak and the hyper charge in the fundamental representations (Eq. (10.21)) and the family charges in the adjoint representations, transform any family member of the lower four families into the same family member belonging to one of the lower four families (while those with the family charges of the upper four families transform any family member into the same family member belonging to one of the upper four families).

In loop corrections all the scalar and vector gauge fields which couple to fermions contribute.

The mass matrix of any family member, belonging to any of the two groups of the four families, manifests - due to the $\widetilde{SU}(2)_{(L,R)} \times \widetilde{SU}(2)_{(I,II)}$ (either (L, I) or (R, II)) structure of the scalar fields, which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$ - the symmetry presented in Eq. (10.23) [7].

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha} . \tag{10.23}$$

Let us *summarize* this section: It is proven that all the scalar fields with the scalar index $s \in (7, 8)$, which gain nonzero vacuum expectation values and keep the electromagnetic charge conserved, carry the weak and the hyper charge quantum numbers as required by the *standard model* for the scalar Higgs (Eq. (10.21)):

[7] Since the upper four families interact with the condensate of the two right handed neutrinos, which carry the family quantum numbers of the upper four families, the symmetry of the mass matrix presented in Eq. (10.23) is the symmetry of the upper four families.

$(\tau^{13}, Y)\, A^{Ai}_{\substack{78\\(\pm)}} = \pm(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\substack{78\\(\pm)}}$. These are the only scalar fields in this theory with the quantum numbers of Higgs' field.

These scalar fields carry additional quantum numbers: The family quantum numbers. The nonzero vacuum expectation values of the scalars with the space index $s = \in (7, 8)$ determine on the tree level the mass matrices of the two groups of four families. While the scalars with the family quantum numbers $(\vec{\tilde{1}}, \vec{\tilde{N}}_L)$ contribute to mass matrices of the lower four families, contribute those with the family quantum numbers $(\vec{\tilde{2}}, \vec{\tilde{N}}_R)$ to masses of the upper four families and those with the family members quantum numbers (Q, Y, Y') to any of these two groups of four families. In loop corrections in all orders the mass matrices of the two groups of four families follow.

All the other scalar fields: A^{Ai}_{s} , $s \in (5, 6)$ and $A^{Ai}_{tt'}$, $(t, t') \in (9, \ldots, 14)$ have masses of the order of the condensate scale and contribute to matter-antimatter asymmetry.

10.3 Conclusions

The *spin-charge-family* [1,2,7,6,3–5,8,12,9,10] theory, a kind of the Kaluza-Klein theories [15] with the families introduced by the second kind of gamma operators ($\tilde{\gamma}^a$ in addition to the Dirac γ^a), is offering the explanation for the properties of quarks and leptons (right handed neutrinos are in this theory regular members of each family) and antiquarks and antileptons, for the appearance of the gauge vector fields and of the scalar Higgs and Yukawa couplings. All these are in the *standard model* just assumed.

The theory offers the explanation why are the weak and hyper charges of fermions connected with their handedness (table 9.3) and where do the scalar fields originate (Eqs. (10.14, 10.15)).

It also explains why do the scalar fields carry the weak and the hyper charges as assumed by the *standard model* (Eq. (10.18)): $(\tau^{13}, Y)\, A^{Ai}_{\substack{78\\(\pm)}} = \pm(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\substack{78\\(\pm)}}$, where τ^{13} denotes the third component of the weak charge, Y the hyper charge, Ai denotes (Q, Y, Y') (originating in the first kind γ^a of the Clifford algebra objects) and all the family quantum numbers (originating in the second kind of the Clifford algebra objects $\tilde{\gamma}^a$). While γ^a, through S^{ab}, determine all the spin and the charges of families, determine $\tilde{\gamma}^a$, through $\tilde{S}^{ab}$, the family quantum numbers.

The *spin-charge-family* therefore, starting with the simple action (Eq.(10.1)) in $d = (13 + 1)$ for spinors (carrying two kinds of gamma operators) and interacting with the gravity only (with the vielbeins and the two kinds of the spin connection fields), differs essentially from the unifying theories of Pati and Salam [21], Georgi and Glashow [27] and other $SO(10)$ and $SU(n)$ theories [28], although all these unifying theories are answering some of the open questions of the *standard model* and accordingly have many things in common - among themselves and with the *spin-charge-family* theory.

The *spin-charge-family* theory predicts two decoupled groups of four families [7,6,9,10]: The fourth of the lower group of families will be measured at the LHC [11] and the lowest of the upper four families constitute the dark matter [10].

It also predicts that there will be several scalar fields observed sooner or later at the LHC.

Besides the scalar fields with the space index $s \in (7, 8)$, which by getting non zero vacuum expectation values cause the electroweak break and take care of massless of fermions and the weak bosons, all the other scalar fields get, through the interaction with the scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, masses of the condensate scale. There are also only weak, hyper and the colour vector gauge bosons which stay massless up to the condensate scale, since they do not interact with the condensate. The scalar fields with the scalar space index $s =\in (9, \ldots, 14)$ are colour triplets with respect to the scalar space index and cause, after interacting with the condensate, matter-antimatter asymmetry [13].

All the scalar fields are in the fundamental representations (Eq. ([13])) with respect to the space index. They resemble the supersymmetry particles, although they are not, since they do not meet all the requirements for the bosonic partners of fermions.

Starting with few assumptions, presented in the introduction 10.1 (**i.- iv.**), I show that the *spin-charge-family* theory is not only offering the explanation for the so far measured phenomena, with the origin of the dark matter and the scalar fields included, but offers also the predictions for new families (the fourth to the observed three families will be measured at the LHC, the fifth - the lowest of the upper four families - forming baryons [10] explains the appearance of the dark matter) and new scalar fields (there are two triplets and three singlets: A^{Q}_{s}, A^{Y}_{s}, $A^{Y'}_{s}$, $\vec{\tilde{A}}^{\tilde{1}}_{s}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{s}$, Eqs. (10.14, 10.15, 10.16), which determine properties of the four lower families - the Higgs and the Yukawa couplings of the *standard model* [2,1]). The theory might be able also to answer questions about the (ordinary, mainly made out of the first family) matter/antimatter asymmetry, which is discussed in a separate paper [13]. The quantum numbers of the condensate, responsible for breaking $\mathcal{CP}$ symmetry, are presented in this paper (table 10.1). The same condensate makes massive scalar and vector gauge fields which would otherwise be as massless observed at low energies.

Although the *spin-charge-family* theory starts in $d = (13 + 1)$ dimensional space with the spin connection fields of two kinds (having the origin in γ^a and in $\tilde{\gamma}^a$) and with the vielbeins - all these look like having a very large number of degrees of freedom - it leads under the assumption that there is a condensate of two right handed neutrinos carrying the quantum numbers of the upper four families and that there are scalar fields, which obtain nonzero vacuum expectation values causing the electroweak break, naturally (what means that all unobserved fields of both origins get masses without additional requirements) at the low energy regime to the observed fermion and vector gauge boson fields.

This paper presents, by explaining that in this theory there are the scalar fields, which carry the quantum numbers of the scalar Higgs scalars and correspondingly offering the explanation for the appearance of the scalar Higgs and the Yukawa couplings, a further step towards understanding the properties of quarks and leptons and in particular of those scalar fields (section 10.2), which determine mass matrices of quarks and leptons.

It stays to be solved, why and how does the condensate of the two right handed neutrinos with the family quantum numbers of the upper four families appear and why do scalars, with the weak and the hyper charge required by the *standard model*, gain non zero vacuum expectation values.

Acknowledgments

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11 Deriving Veneziano Model in a Novel String Field Theory Solving String Theory by Liberating Right and Left Movers*

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Abstract. Bosonic string theory with the possibility for an arbitrary number of strings —i.e. a string field theory— is formulated by a Hilbert space (a Fock space), which is just that for massless noninteracting scalars. We earlier presented this novel type of string field theory, but now we show that it leads to scattering just given by the Veneziano model amplitude. Generalization to strings with fermion modes would presumably be rather easy. It is characteristic for our formulation /model that: 1) We have thrown away some null set of information compared to usual string field theory, 2)Formulated in terms of our "objects" (= the non-interacting scalars) there is no interaction and essentially no time development(Heisenberg picture), 3) so that the S-matrix is in our Hilbert space given as the unit matrix, $S = 1$, and 4) the Veneziano scattering amplitude appear as the overlap between the initial and the final state described in terms of the "objects". 5) The integration in the Euler beta function making up the Veneziano model appear from the summation over the number of "objects" from one of the incoming strings which goes into a certain one of the two outgoing strings.

Povzetek. Novo bozonsko teorijo strun sta avtorja formulirala na Hilbertovem (Fockovem) prostoru brezmasnih skalarjev, ki ne interagirajo. Teorija dopušča posplošitev na poljubno število strun, tedaj na strunsko teorijo polja. Avtorja v tem prispevku pokažeta, da je njuna sipalna amplituda enaka amplitudi v Venezianovem modelu. Kaže, da je posplošitev njune bozonske teorije strun na fermionsko enostavna. Bistveno za njuno teorijo strun je: 1) V primerjavi z običajno strunsko teorijo ima njuna teorija manjše število privzetkov. 2) Njuno struno sestavljajo enaki skalarni "objekti", med katerimi ni nobene interakcije in v bistvu tudi nobenega časovnega razvoja (Heisenbergova slika). 3) Sipalna matrika S je v njunem Hilbertovem prostoru kar enotska matrika, $S = 1$. 4) Venezianova sipalna amplituda sledi iz prekrivanja med začetnim in končnim stanjem njunih skalarnih "objektov". 5) Integriranje Eulerjeve beta funkcije Venezianovega modela sledi v njunem primeru iz seštevanja po skalarnih "objektih" ene od prihajajočih strun, ki gre v eno od dveh odhajajočih strun.

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11.1 Introduction. . .

We have already earlier put forward [1–3] ideas towards a novel string field theory (meaning a second quantized theory of strings from string theory [4–7,9]), which of course means the theory[10–18] in which you can describe several strings like one describes several particles at a time in in quantum field theory. It may be understood that we as other string field theories in or theory have a Hilbert space or Fock space the vectors of which describe states of the whole universe in the string theory. Our model is similar to the formulation of Thorn [19] and also use a discretization like ourselves, but we discretize after having separated right and left movers as we shall see below. This Hilbert space, which describes the states of the universe, turns in our model/formulation out to be really surprisingly simple in as far as it is simply the second quantized Fock space of a non-interacting massless scalar 25 +1 dimensional particle theory in the bosonic string case!

Let it be immediately be stated that although our formulation/model is supposed just a rewriting of string theory - and thus in its goal there is nothing new fundamentally - it is definitely new because we throw away compared to usual string theory and usual string field theory as Kakku and Kikkawa's and Witten's a *null set of information*. The information, which we throw away is the one about how the different pieces of strings hang together. That is to say we rather only keep the information about where in target space time you will find a string and where not. Due to this throwing away of information and other technically doubtful treatment of the string theory by us it is a priori no longer guaranteed that our string field theory appearing as just the non-interacting massless scalar theory in 25+1 dimensions is indeed just a rewriting of string theory. Rather one should see our progresses such as the derivation of the string spectrum [3] in reproducing usual properties of string theory from our model/formulation as tests that indeed our model is in spite of the null set of information thrown away indeed the full string theory.

The major achievement in the present work is also such a test, namely testing that our model/formulation leads to the Veneziano model scattering amplitude for scattering of strings formulated in our novel string field theory.

The particles that formally occurs in the construction of our Hilbert space or Fock space of our model or formulation of string theory we call "even objects" and each such "even object" has in our formulation a kind of momentum variable set J^{μ} (it is proportional to a contribution to the total momentum of the string to which it belongs). Really this J^{μ} has as some technical details got its longitudinal momentum (in target space time of 25 +1 dimensions) component $J^{+} = J^{0} + J^{25}$ fixed by what corresponds to a gauge choice in the string parameterization to be

$$J^{+} = \frac{a\alpha'}{2}, \tag{11.1}$$

(We shall below that we end up being driven to also allow $J^{+} = -\frac{a\alpha'}{2}$) and its infinite momentum frame energy proportional component $J^{-} = J^{0} - J^{25}$ is written just by the mathematical expression ensuring the light-likeness

$$(J^{\mu})^{2} = \eta_{\mu\nu} J^{\mu} J^{\nu} = 0 \tag{11.2}$$

of the "even object" momentum-like J^μ variable. Thus the only genuine degrees of freedom components of this even object variables J^μ are the "transverse" components corresponding to the first 24 components, namely those having $\mu = i$ where $i = 1, 2, 3, ..., 24$, i.e. J^i. In addition the "even objects" have 24 conjugate momenta Π^i, conjugate to the J^i's, so that

$$[\Pi^i, J^j] = i\delta_{ij} \tag{11.3}$$

for Π^i and J^j belonging to the same even object of course.

Our Hilbert space for states of the universe corresponds now simply to a set of harmonic oscillators, one for every set of J^i-value combinations (of 24 real numbers), and the creation operator for an "even object" with its J^i's being J^i is denoted $a^\dagger(J^i)$. Since there is a calculational relation between the set J^i of the transverse components and the full 26-vector J^μ given by adding the equations (11.1, 11.2), we could equally well use as the symbol in the creation and annihilation operators J^μ as the symbol J^i, and so we have by just allowing both notations $a^\dagger(J^i) = a^\dagger(J^\mu)$, where it is understood that the J^μ is calculated from the only important transverse components J^i. Similarly the destruction operators are $a(J^\mu) = a(J^i)$ and we shall think of the Hilbert space describing the states of the Universe (in a string theory world) as having basis vectors of the type

$$a^\dagger(J^i(1))a^\dagger(J^i(2)) \cdots a^\dagger(J^i(L))|0> . \tag{11.4}$$

To tell the truth we though better reveal the little technical detail, that this simple situation with only one type of " even objects" that can exist in the states described by (J^i, Π^i) is only true for the case of a string theory model *with open strings*, while we for the case of a string theory with only closed strings must have *two kind of even objects* that can be put into the 24 or 26=25+1 dimensional (Minkowski) space, one right denoted by R and one left denoted by L. So in the only closed string case we could even naturally consider it that the two types of even objects "live" in two different Minkowski spaces - one R and one L-. The figure 11.1 illustrates these two slightly different cases. The connection between the strings present in a given state of the universe and the even objects corresponding to a set of strings is a priori not completely trivial and has to be described. It is not hundred percent true that the strings consist of even objects, but there is so much about it that there is an actually infinite number of even objects corresponding to each string present. This divergent number of many even objects in a string is given as a function of the small parameter a already mentioned in formula (11.1).

11.2 Correspondence from Strings to Objects

The crux of the matter in the formulation of our string field theory model or formulation is to put forward the rule for how a given string state is translated into a state described in terms of a state of what we call "even objects":

In the case of a theory with only closed strings we shall make use of the *solution in the conformal gauge for the 26-position fields* $X^\mu(\sigma, \tau)$ *in terms of right and left movers*. Remember that the time-development of a string in string theory is

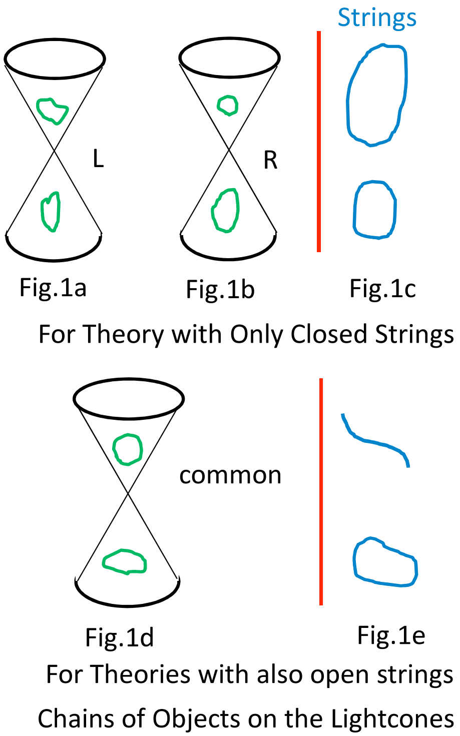

Fig. 11.1.

described by letting its timetrack - which of course becomes a two-dimensional surface in the 25+1 dimensional "target space" - be parameterized by the two real variables called σ and τ. At first one may think of these parameters as parameterizing the timetrack surface in an arbitrary way and therefore one even has to have an action - the Nambu(-Goto) action - chosen so as to be invariant under reparameterization, meaning that one goes over to a new set of coordinates parameterizing the timetrack $(\sigma', \tau') = (\sigma'(\sigma, \tau), \tau'(\sigma, \tau))$. This requirement of reparametrization invariance fixes up to an overall constant the action to be given by the area of the timetrack surface

$$\text{Single string action} = S_{Nambu} \propto \text{area} = \int \sqrt{\det \begin{pmatrix} (\dot{X}^{\mu})^2 & \dot{X}^{\mu} \cdot X'^{\mu} \\ \dot{X}^{\mu} \cdot X'^{\mu} & (X'^{\mu})^2 \end{pmatrix}} d\sigma d\tau, \tag{11.5}$$

where we have as usual denoted

$$\dot{X}^{\mu}(\sigma, \tau) \stackrel{def}{=} \frac{\partial X^{\mu}(\sigma, \tau)}{\partial \tau} \tag{11.6}$$

$$X'^{\mu}(\sigma, \tau) \stackrel{def}{=} \frac{\partial X^{\mu}(\sigma, \tau)}{\partial \sigma}. \tag{11.7}$$

For an open string the timetrack is like a band extending in the time direction, while for a closed string the track is topologically like a tube/cylinder also extending roughly in time direction.

Now one usually in steps fix the "gauge" meaning the parameterization, i.e. the choice of a new set of coordinates which we again may call (σ, τ)(leaving out the prime on (σ', τ')). The first step in the gauge choosing is what is called conformal gauge choice and corresponds to arranging the coordinate equal constant curves to be orthogonal seen from the external/target space of 25+1 dimensions. Often in literature one works with Euclideanized σ and τ as if the string timetrack were a two dimensional Euclidean space, but thinking physically on a true string the space felt by a being living attached onto the string would be a 1+1 dimensional space time with one time dimension and one spatial dimension. For the thinking of the present article and our foregoing works on our novel string field theory we shall take this latter - more physical - point of view that the internal space time is indeed a space-*time*. We think of τ as the time coordinate and of σ as the spatial coordinate along the string.

After having chosen the "conformal gauge" the equation of motion derived from the Nambu action at first simplifies and together with the constraints appearing due to the reparameterization symmetry of the original action we can summarize the equations in the conformal gauge:

$$\Box X^{\mu}(\sigma, \tau) = 0(\text{equation of motion}) \tag{11.8}$$

$$(\dot{X}^{\mu}(\sigma, \tau))^2 - (X'^{\mu}(\sigma, \tau))^2 = 0 \text{ (constraint)} \tag{11.9}$$

$$\dot{X}^{\mu}(\sigma, \tau) \cdot X'^{\mu}(\sigma, \tau) = 0(\text{constraint also}). \tag{11.10}$$

Here the D'Alambertian

$$\Box = \partial_{\tau}^2 - \partial_{\sigma}^2 = (\partial_{\tau} - \partial_{\sigma})(\partial_{\tau} + \partial_{\sigma}), \tag{11.11}$$

and the eqations of motion are easily *solved* by the ansatz

$$X^{\mu}(\sigma, \tau) = X_R^{\mu}(\tau - \sigma) + X_L^{\mu}(\tau + \sigma), \tag{11.12}$$

which is importance for our novel string field theory in as far as it is actually the τ-derivatives of the 26-vectorial functions in the solution $X_R^{\mu}(\tau - \sigma)$ and $X_L^{\mu}(\tau + \sigma)$, which are going to be identified as we shall see soon by our "objects". Note immediately, that these right and left mover variables X_R^{μ} and X_L^{μ} only depend on *one* variable each, namely respectively on $\tau_R \stackrel{def}{=} \tau - \sigma$ and $\tau_L \stackrel{def}{=} \tau + \sigma$, so that the equations of motion with τ conceived of as the time have indeed been solved. The ansatz functions X_R^{μ} and X_L^{μ} are more like initial conditions for the solution.

In terms of these initial condition variables X_R^{μ} and X_L^{μ} the constraints take the very simple form

$$(\dot{X}_R^{\mu}(\tau_R))^2 = (\dot{X}_R^{\mu}(\tau - \sigma))^2 = 0(\text{constraint}) \tag{11.13}$$

$$(\dot{X}_R^{\mu}(\tau_L))^2 = (\dot{X}_R^{\mu}(\tau + \sigma))^2 = 0(\text{constraint}) \tag{11.14}$$

The overview of description of our object rewriting of the string theory is that we let there be an object for every point in (a period for) the coordinates τ_R and

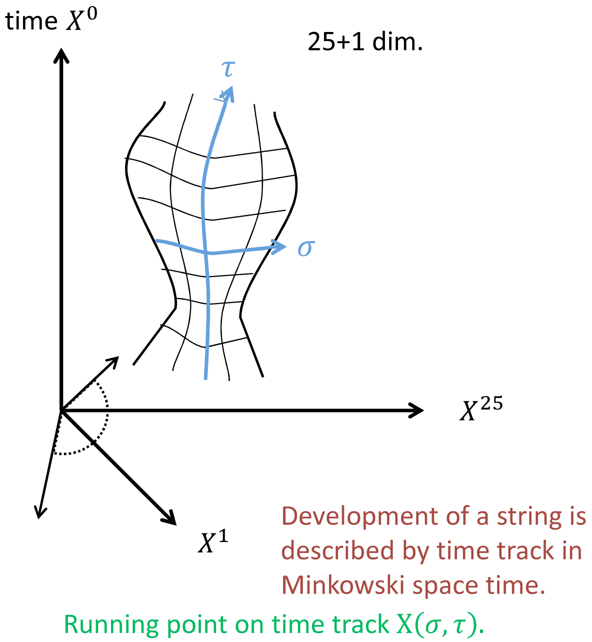

Fig. 11.2.

τ_L in the case of only closed strings, and that the objects are closely related to the variables $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$. For continuity of these variables as functions of respectively τ_R and τ_L the images of these functions $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ are - except for fluctuations at least - smooth curves, because of the constraints (11.14) these curves must lie on the lightcone(s).

Since we can also consider the variables τ_R and τ_L as σ-variables for constant τ the periodicity w.r.t. σ of the position variables etc. - for in fact both open and closed strings- but at least clearly for the closed strings, comes to imply that the just mentioned images for $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ become closed curves on the light cone.

Two points further are illustrated by the figure 11.3: 1) We “discretize”, so that we replace the in principle continuum infinity of τ_R or τ_L values by a series of discrete points with a “distance between these points” being proportional to a small quantity a later taken to go to zero. 2) We treat the even numbered and the odd numbered “discretized points” differently, as is on the figure illustrated by them being denoted differently by dots and crosses.

11.2.1 Open String Case

The open string case has a tiny technical complication:

At the boundaries of the open string one has boundary conditions which are translated into our model favorite language of $\dot{X}^\mu_R$ and $\dot{X}^\mu_L$ implies

$$\dot{X}^\mu_L(\tau) = \dot{X}^\mu_R(\tau) \text{from boundary at } \sigma = 0$$
$$\dot{X}^\mu_L(\tau) = \dot{X}^\mu_R(\tau - 2\pi) \text{from the } \sigma = \pi \text{ end boundary.,} \qquad (11.15)$$

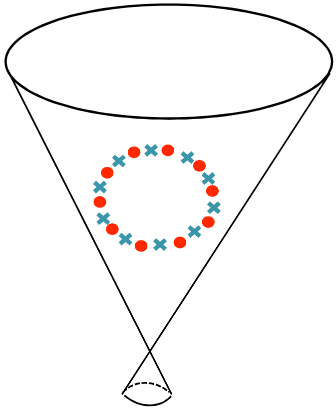

Fig. 11.3.

where we assumed the notation that the length of the string in σ-parameter language is π. These two boundary conditions (11.15) imply that we can identify, for the open string, the $\dot{X}_R = \dot{X}_L$ and have that this common (differentiated)"initial condition variable" must be periodic with the period 2π meaning twice the σ-variable range corresponding to the string length. Thus in the beginning announced we got the fact that while we for only closed string theories have to distinguish $\dot{X}_R$ and $\dot{X}_L$, this is no longer needed for an open string.

11.2.2 More Precise Correspondence between Strings and "even objects"

More precisely we shall divide up into "discretized" pieces the σ range around a closed string or the tour forward and backward along an open string into, let us say, N pieces. What we really want, is to divide up a period for say $\dot{X}_R^\mu$ in its argument τ_R (and in the closed case the same for $\dot{X}_L$, while in the open string case just identify the left and right mover variables because of boundary conditions). The precise way of dividing up could be thought of as dividing in equal steps in the variable, say τ_R, but there is still some coordinate specification/gauge choice left even after the conformal gauge choice. In fact one can still as such a rudimentary freedom of choosing coordinates select any (increasing) function $\tau_R'(\tau_R)$ and any (increasing) function $\tau_L'(\tau_L)$ as a new set of coordinates (having in the background of the mind the identifications $\tau' = \tau - \sigma'$ and $\tau_L = \tau + \sigma$). By discretizing we replace essentially a variable as τ_R by an integer valued variable - counted modulo N if N corresponds to the period -, so that the τ_R value corresponding to the integer I is denoted $\tau_R(I)$. Interpolating we can easily make an approximate sense of even τ_R defined for non-integer values of I. Thus we formally associate any string with a series something we call "objects"- and which is something only defined in our model -, which are characterized each by a set of degrees of freedom (as if it were particles): J_R^μ and the conjugate variables Π^μ or better only Π^i where the i only runs over the transverse coordinates $i = 1, 2, ..., 24$. The reader may crudely think of these objects as a kind of partons, but really we simply have to define them by their relation to the $\dot{X}_R^\mu$ (and for the closed string case also to the separate $\dot{X}_L^\mu$). To every discretization point on the τ_R axis say, let us say discretization point number I, we associate an "object" for which the dynamical variables $J_R^\mu(I)$ are given as

$$J_R^\mu(I) = X_R^\mu(\tau_R(I + 1/2)) - X_R^\mu(\tau_R(I - 1/2)). \tag{11.16}$$

Notice that since the difference between the two argument values $\tau_R(I - 1/2)$ and $\tau_R(I + 1/2)$ is small this definition of $J_R^\mu(I)$ for a discretization point on the τ_R-axis in reality means that

$$J_R^\mu(I) \approx \dot{X}_R^\mu \frac{d\tau_R}{dI}, \tag{11.17}$$

and so indeed as announced our variables J_R^μ assigned to the "objects" are "essentially" the τ_R-derivative $\dot{X}_R^\mu$ of the right mover X_R part of the solution.

11.2.3 The Even Odd Detail

Now there is an important technical detail in the setup of our model:

We have the problem that if one shall make creation and annihilation operators for some "objects" in a way analogous to how one in usual quantum field theory have creation and annihilation operators for particles, one shall describe these creation and annihilation operators by having an argument for a set of variable describing the "object", a set of variables which *commute with each other*. It is indeed well-known that one must in quantum field theory *either* take the creation and annihilation operators to be functions of the spatial momenta of the particles created/annihilated *or* one can use instead position variables, and that corresponds to working with the second quantized fields $\phi(x)$. But the usual simple mutual commutation rules for creation and annihilation operators could not be obtained if one would attempt to construct them to correspond to a combination of dynamical variables for the particles that did not commute with each other. What could also a creation operator depending on mutually complementary variables for a single particle correspond to creating ? It could not create a particle with the specified quantum numbers in such a case because that would be against the Heisenberg uncertainty relation. In the corresponding way we must choose whatever variables we let our "objects", to be associated with creation, and annihilation operators depend on be arranged so as to commute. But then we have problem, because does our $\dot{X}_R^\mu$'s which are proportional to the object-variables J_R^μ commuting? No, they do not commute in as far as the theory of a single starting from the Nambu Lagrancian e.g. in the conformal gauge leads to

$$[\dot{X}_R^\mu(\tau_R'), \dot{X}_R^\nu(\tau_R)] \propto g^{\mu\nu}\delta'(\tau_R' - \tau_R). \tag{11.18}$$

Thinking discretizing, such a derivative of a delta-function commutator means that in the discretized chain the $\dot{X}_R$ or equivalently J_R 's which are *next neighbors do NOT commute*.

So we had to invent a trick to avoid to have to make creation and annihilation operators for "objects" sitting in the chains of "objects" along the variable τ_R as neighbors in the discretization.

The trick which we have chosen consists in *only using in the creation and annihilation operators every second of the by discretization by (11.16) defined "object"-variables* J_R. That is to say we choose to *only construct creation and annihilation operators* for those "objects", which in the discretized series of objects for a given string have got an *even number* I. One could say that we in our model construct our Hilbert space only in terms of such "even objects", and one could almost say only consider these "even objects" as "really existing" in our basic Hilbert space description.

But then we are coming to the problem that we need to a full description of the string states also the "odd objects": what to do about them? We say that when you have a series of the "even objects" on a string, we make the rule to construct in between any two next to neighboring "even objects" (i.e. two "even objects " deviating in number by just 2) an "odd object" from the conjugate momenta Π^i say of the neighboring "even objects". (There is another technical detail connected with the + and - components in the infinite momentum frame we have chosen to work with, so we shall avoid discussing conjugate momenta to other than the transverse components J^i_R - the first 24 components -. Therefore we only consider these first 24 components of conjugate momenta to the J_R's). In fact we have to take the following rule for constructing the "odd object" J^i_R components for the "odd object" number I (where I then is an odd integer (modulo the even number N)),

$$J^i_R = -\pi\alpha'(\Pi^i_R(I+1) - \Pi_R(I-1)), \tag{11.19}$$

in order to obtain the commutation rule corresponding to the derivative of delta function commutation rule (11.18) discretized.

The reader should check and understand that with this construction of the "odd objects" any quantum state of the string expressed as a state of the variables $\dot{X}^i_R$ (and for the closed string also $\dot{X}^i_R$) can be expressed as a corresponding quantum state of a set of N/2 (N must be even) "even objects", because the even object commutation rules

$$[J^i_R(I), \Pi^k_R(K)] = i\delta^{ik}\delta_{IK} \tag{11.20}$$

corresponds just to the commutation rules for the $\dot{X}^i_R$ (and $\dot{X}^i_L$). There is though one little technical detail to be studied in later works: The absolute position of the string were differentiated away from our discussion by dotting the X_R and $X_L$and correspondingly the formula for the "odd objects" does *not* make use of the sum over all the "even object" conjugate variables Π^i_R around the closed chain. So there is suggestively the possibility of identifying the average position of the string proportional to this sum over all the conjugate to even object variables.

11.2.4 Several Strings

So far we should have now given the prescription for constructing a cyclically ordered chain of "even objects" corresponding to a given quantum state of a single open string. (If one wants a closed string one shall construct two cyclically ordered chains of "even objects" one for right movers consisting of J_R "even objects" and consisting J_L left mover "even objects"). Since the commutators were arranged to isomorphic to the discretized $\dot{X}_R$ (and $\dot{X}_L$) it should be possible to construct such an "even object" state. It is then of course also trivial and completely analogous to usual quantum field theory construction of a state with $N/2$ particles to construct a Hilbert space (Fock space) state for $N/2$ of our "even objects". Corresponding to a single open string we thus simply have Hilbert space state with the large (divergent in the limit of $a \to 0$) infinitely many ($= N/2$) "even objects" sitting approximately in a cyclic chain on the light cone in a 25 +1 dimensional (Minkowskian) J_R^μ-space.

But now it is the main point of a model being a string field theory (SFT), that such a model *can describe several strings in one Hilbert space state*. Once we have made our formulation of one string in our "even object" formulation it is, however, rather trivial to construct states with an arbitrary number of strings. One can just act on the "zero even object" with all the product of creation operators corresponding to the various strings - we want to have in the state to be described - each creating the "even objects" associated with the string in question. So to speak if string number 1 is described by the product

$$C_1|0> = \int \Psi_1(J_R^I(0), J_R^i(2), ..., J_R^i(N-2)) \cdot \\ \Pi_{I=0,2,4,...,N-2} a^\dagger(J_R^i(I)) \\ \cdot \Pi_{I=0,2,...,N-2} \Pi_{i=1,2,...,24} dJ_R^i(I)|0>, \quad (11.21)$$

where $\Psi_1(J_R^I(0), J_R^i(2), ..., J_R^i(N-2))$ is the wave function for the state of the single string 1 described in terms of "even objects", and the string number 2 by an analogous expression, then a state with both string 1 and string 2 (say they are open) is given as

$$C_1 C_2|0> = \int \Psi_1(J_R^I(0), J_R^i(2), ..., J_R^i(N-2)) \\ \Pi_{I=0,2,4,...,N-2} a^\dagger(J_R^i(I)) \Pi_{I=0,2,...,N-2} \Pi_{i=1,2,...,24} dJ_R^i(I) \\ \cdot \int \Psi_2(J_R^I(0), J_R^i(2), ..., J_R^i(N-2)) \Pi_{I=0,2,4,...,N-2} a^\dagger(J_R^i(I)) \\ \Pi_{I=0,2,...,N-2} \Pi_{i=1,2,...,24} dJ_R^i(I)|0> . \quad (11.22)$$

Luckily the commutation of the creation operators for "even objects" makes it unnecessary to specify any order in which the creation operator products corresponding to different strings have to be written.

We thus have a scheme for constructing Hilbert space states - in the Hilbert space which is really that of massless scalar "even objects" conceived of as particles in an ordinary Fock space - corresponding to any number of strings wanted. In this sense we have a string field theory.

11.2.5 Final Bit of Gauge Choice

As already mentioned the choice of parameterization (often called gauge choice) were not finished by the conformal gauge, since we could still transform the variables τ_R and τ_L by replacing them by some increasing function of themselves. In the "infinite momentum frame gauge" the choice is to fix this freedom by requiring the density of $P^+ = P^0 + P^{25}$ (longitudinal momentum we can say) momentum is constantly measured in say $\tau_R = \tau - \sigma$ or say in σ. When we discretize as described above and take it that the τ_R distance between neighboring "objects" along the τ_R-axis should be the same all along, then this gauge choice comes to mean that each object gets the same P^+ momentum. We can therefore describe this gauge choice - which is essentially the usual one in infinite momentum frame, just discretized our way - by saying that we impose some fixed small value for the + component of our "object"(-variables)

$$J_R^+ = \frac{a\alpha'}{2} \text{ in our first attempt (later problem comes).} \tag{11.23}$$

With this gauge choice we have made the number of objects N and thus of "even objects" $N/2$ proportional to the $P^+ = P^0 + P^{25}$ component of the 26-momentum of the string in question. So e.g. the conservation of this component of momentum corresponds to the conservation of the number of say "even objects". After this choice of gauge extremely little is left to be chosen for the reparameterization: you can still for the closed string shift the starting point called $\sigma = 0$, but that is all. Corresponding to this extremely little reparameterization left unfixed you can still cyclically shift along the topological circles on which the objects of a string sits, and that turn out due to the possibility for adding a constant to τ also to be true for the open string. The objects corresponding to a cycle for a string are cyclically order but the starting point is still an unchosen gauge ambiguity. To an open string we have one such loop or cycle, and to a closed one we have two.

11.3 Comparing Our String Field Theory to Other Ones

It should be stressed that our "novel" string field theory really is novel/new, since it deviates from earlier ones like Kakku and Kikkawa or Wittens string field theories in important ways even if some calculations should soon turn out similar:

- 1. The information kept in our formalism is *not* the full one kept by the theories by Kaku Kikkawa or by Witten, but deviates by having relative to these other string field theories thrown out - actually only a null set of - information. It is the information about how the strings hang together, that is thrown out. We could say that we - Ninomiya and Nielsen - only in our rewritten string states keep track of where in the space time you may see a piece of string, but not of how one piece hangs together with another piece. If a couple of strings cross each other there is a point in target space wherein four pieces of string meet, two belonging to each of the crossing strings. In usual string field theories, such as Kaku and Kikkawa [10] and Witten's[11], it is part of the information kept in the Hilbert space vector describing the state of the universe which of these 4 pieces are connected to which. In our formulation, however, *this information has been dropped*.

- 2. A further consequence of this drop of information is that if two strings scattering by just exchanging tails - as one must think scattering should typically happen classically - then really nothing have to happen at all in our formalism. Indeed it is a second characteristic property of our string field theory model, that in the scattering counted in terms of our "even objects" (which are the ones truly represented in the Hilbert space; the odd ones are just mathematical constructions from the conjugate variables for the "even" ones) *nothing happens!* The scattering process is not represented in the Hilbert space formalism of ours.
- 3. A consequence of item2. is that the S-matrix gets calculated formally as an *overlap* of the initial with the final state.
- 4. And this fact is also connected with that the Hilbert space or Fock space of our formulation is the extremely simple free massless scalar Fock space. Actually though there is gauge fixing, that makes the states of the "even objects" even have their J^+ components fixed by (11.1). This is contrary to the other string field theories which have much more complicated structures.
- 5. But perhaps the most important distinction for the other string field theories is that *we use a description in terms of something quite different from the strings themselves, namely our 'even objects"*, while the other string field theories have quite clearly all through their formalism the strings one started from. In ours the string has been hit to the extent that we at the end must ask: What happened to the string? The answer is roughly that there is no string sign left in the Hilbert space structure of being only that of free massless scalars. Rather *the string in our formalism only finds way into the calculations via the initial and final states put in!* That is to say that in our formalism it looks that the whole story of the strings only will appear because there is an extra "stringy" assumption put in about the initial state - and presumably it is necessary even to put it in for the final state - so that the whole string story is not part of the structure of the theory nor of the equation of motion, but rather on an equal level with the cosmological start of the Universe, or the initial conditions of low entropy allowing there to be a second law of thermodynamics. If it should turn out that indeed even extra assumptions about *the final states* are needed to make our formalism function as a string theory, then one could say that in our formalism an influence from future is required.

With all these deviations from the usual string field theories, one may worry whether our rewriting truly is a rewriting and thus can count as a true string field theory, because does it indeed describe the conventional string theory, or could it be that we had thrown away too much (even though only a null set)?

Because of this possibility that our model does not truly represent string theory at the end it becomes important - also for the purpose of testing if our model is string theory - to check the various wellknown features of string theory. We have not long ago published an article [3] in which we showed that the mass spectrum of the strings in our string field theory became the usual one. This is one such little check that our model/string field theory is on the right track. In the succession of this article we shall concentrate on sketching the calculation of the scattering amplitude for two ground state strings (tachyons) scattering elastically into two also tachyonic ground state open strings.

Actually it turned out that we were not quite right in the first run, because we only get one term out of three terms that should be present in the correct Veneziano model. This little shock we sought to repair by modifying our gauge fixing condition and allowing "even objects" also with negative J^+. As we shall see later we think it reflects a more general problem with infinite momentum frame.

11.4 Yet More Technical Details

11.4.1 The + and − components of J_R

Especially if one wants to get an idea about our work [3] checking the spectrum of our strings it is necessary to keep in mind that it is only the components J_R^i for $i = 1, 2, ..., 24$, which are simply independent dynamical variables for the "even object". The remaining two components are not independent. Rather:

- +: The J_R^+ components of actually both even and odd objects are fixed to $\pm\frac{a\alpha'}{2}$ as a remaining gauge choice after the conformal gauge has been used to gauge fix to the largest extend. This would have been the infinite momentum frame choice basically, once we assumed that the distances in say σ-variable per object were (put) equal for all the objects. It really means that number of "objects" represent the P^+-momentum of the string associated with those objects.
- −: Next the components J_R^- are fixed from the requirement gotten from the constraints in string theory, namely that $(J_R^\mu)^2 = 0$. This condition fixes the −component (essentially energy) in terms of the 24 transverse components J_R^i and the gauge fixed J_R^+. Remembering that the "odd objects" are constructed from the even ones by means of (11.19) we can write the −components as :

For even objects: (11.24)

$$J_R^-(even\ I) = \frac{\sum_{i=1,2,...,24}(J_R^i)^2}{2\cdot a\alpha'/2} \tag{11.25}$$

For odd I object(constructed): (11.26)

$$J_R^-(odd\ I) = \frac{\pi^2\alpha'\sum_{i=1,2,...,24}(\Pi_R^i(I+1) - \Pi_R^i(I-1))^2}{a} \tag{11.27}$$

It may be interesting to have in mind that from the point of view of our Hilbert space description with a Fock space only having "even objects", and even those only with their transverse - the 24 components - the odd objects as well as both the + and the - components are just "mathematical constructions" simply put up as mathematicians definitions. In this manner the two of the 26 dimensions are pure "construction"! as well as half the number of objects.

It were basically by means of these "constructions" for a cyclical chain of first even, then filled out by odd ones in between, that we in our previous article[3] checked the spectrum of masses. We ran, however, into a slight species doubler problem: Because of our discretization of the τ_R-variable we were seeking a spectrum of latticized theory (in one spatial dimension, the τ_R), and thus we got according to our theorem that one gets species doublers when seeking to make only right mover in fact a species doubler[23]. In order to get rid of that we propose to impose a continuity rule as a postulate.

11.4.2 The Non-Parity Invariant Continuity Rule

The continuity rule which we saw earlier we had to impose to avoid a doubling of the usual string spectrum in our model is actually just the continuity rule, which you would any expect. Crudely it just is that you require the variation of the object J_R^μ or J_R^i to vary slowly from object to the next object. So physically it is extremely reasonable to assume this continuity rule. But we assume it - and have to assume it so - for both *even* and *odd* "objects", and then because of the antisymmetry of the definition of the odd J_R^i in terms of the conjugate of the even ones, we obtain a condition that *is not symmetric under the shift of sign of the object enumeration number* I. Intuitively you expect that if a chain of numbers J_R^i say, enumerated by I vary smoothly with I counted in positive direction, then it should also vary smoothly, if we count in the opposite direction. Because of our 'strange" definition of the odd object J_R-values, however, the continuity concept we are driven towards does *not* have this intuitive property of being inversion invariant. Let us in fact write our smooth variation or continuity requirement for three successive "objects" in the chain - with an odd one in the middle say -

$$J_R^i(I+1) \approx -\pi\alpha'(\Pi_R^i(I+1) - \Pi_R^i(I-1)) \approx J_R^i(I-1) \tag{11.28}$$

Imposing this non-reflection invariant continuity rule not only is a way to at least assume away the species doubler from the lattice, but it also gives an orientation to the τ_R-variable. For instance when we below shall match wave functions for strings in initial and final states to calculate the overlap, this oriented continuity condition can let us ignore possible overlaps, if the two, to be matched, chains of "objects" are not oriented - in terms of the continuity condition - in the matching way. This rule reduces significantly the possibilities for forming overlap contributions. From a symmetry point of view it may be quite natural that working with only right mover say there should be some asymmetry under reflection.

Thinking, however, on our model as the fundamental theory representing a seeming world with a string theory, it means that this rather strange "continuity principle" not being reflection invariant has somehow to be imposed by the laws of nature. But now as already stated the Hilbert space structure and the dynamics in terms of "even objects" are just the free massless scalar theory, and there is no place for such a reflection non-invariant continuity condition, except in initial and "final state conditions" So in terms of our "even objects" we must have a truly rather funny initial state assumption: The "even objects" sit in chains that are continuous or smooth in our special sense in one direction, but therefore cannot be it in the opposite direction!

Of course in some way this continuity is a description of the continuity of the strings, their hanging together.

11.5 Sketch of Calculation

As one - and perhaps the most important - tests of whether our string field theory in fact leads to the Veneziano model scattering amplitude (at least up to some overall factor, which we shall leave for later works, and modulo a rather short treatment only of the rather important appearance of the Weyl anomaly in 2 dimensions, which happens to be where the dimension of 26 is needed in our calculation). We shall also reduce the troubles of calculation by choosing a very special Lorentz frame, something that would not in principle have mattered provided the theory of ours had been known to be Lorentz invariant. However, since we use infinite momentum frame - which is not manifestly Lorentz invariant - it is in principle dangerous to choose a special frame.

11.5.1 The Veneziano Model to be Derived

Let us shortly - and especially with also a purpose of the extra factor in the integrand, for which we shall need the anomaly for the Weyl symmetry - recall what Veneziano model amplitude we shall derive, if we shall claim that it is a success for supporting that our model/our string field theory is indeed describing string theory of the bosonic 25+1 dimensional type, the most simple string theory having though as a little problem, a tachyon. Since it is the simplest and historically the first to have a Veneziano amplitude for four external particles [20], firstly later we generalized to larger number of external particles [4], we shall start by deriving the Veneziano model for four external particles, although not in the phenomenologically supports case of Veneziano, $3\pi + \omega$. Rather we consider here just four external tachyons each having mass square

$$m^2 = -\frac{1}{\alpha'} \tag{11.29}$$

where α' is slope of the - before inclusion of loops - assumed "linear Regge trajectories", the leading one of which has the expression

$$\alpha(t) = \alpha(0) + \alpha' t, \tag{11.30}$$

where

$$\alpha(0) = -\alpha' m^2 = 1. \tag{11.31}$$

The four point Veneziano model is basically given by the Euler Beta function, which can be defined by the integral

$$B(x, y) = \int_0^1 z^{x-1}(1-z)^{y-1} dz \tag{11.32}$$

being used say for (11.33)

$$B(-\alpha(t), -\alpha(s)) = \int_0^1 z^{-\alpha(t)-1}(1-z)^{-\alpha(s)-1} dz. \tag{11.34}$$

In writing such four point amplitudes one uses normally the Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \quad (11.35)$$
$$t = (p_1 - p_4)^2 = (p_2 - p_3)^2 \quad (11.36)$$
$$u = (p_1 - p_3)^2 = (p_2 - p_4)^2 \quad (11.37)$$
$$\text{obeying the relation} \quad (11.38)$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 = 4m^2 = -4/\alpha'. \quad (11.39)$$

Here the four- or rather 26-momenta $p_i = p_i^\mu$ $(i = 1, 2, 3, 4)$ are the external momenta for the tachyonic string states we consider as scattering states in the simplest case considered here, all four counted the physical way, i.e. with positive energies p_i^0 for the process

$$1 + 2 -> 3 + 4, \quad (11.40)$$

being considered. Since we consider the case of pure strings without any Chan-Paton factor giving quarks at the ends, the full scattering amplitude becomes a sum over three terms of the betafunction form. In front there is a factor g^2 involving the coupling constant g for the string or Veneziano theory being its square g^2. We shall, however, postpone the presumably a bit complicated but very interesting question of the overall normalization in our theory to a later article. Thus the full amplitude expected is

$$A(s, t, u) = g^2\{B(-\alpha(s), -\alpha(t)) \quad (11.41)$$
$$+B(-\alpha(s), -\alpha(u)) + B(-\alpha(t), -\alpha(u))\} \quad (11.42)$$
$$= g^2(\int_0^1 z^{-2\alpha' p_1\cdot p_2 - 4}(1 - z)^{-\alpha' p_1\cdot(-p_4)-4} dz + \quad (11.43)$$
$$\int_0^1 z^{-2\alpha' p_1\cdot p_2 - 4}(1 - z)^{-\alpha' p_1\cdot(-p_3)-4} dz + \quad (11.44)$$
$$\int_0^1 z^{-2\alpha' p_1\cdot(-p_4) - 4}(1 - z)^{-\alpha' p_1\cdot(-p_3)-4} dz). \quad (11.45)$$

Since we have chosen to set up our model in what deserves to be called infinite momentum frame and to use the gauge that each object carries the same p^+ or rather having the fixed value $J^+ = a\alpha'/2$ according to (11.1), our formalism is *a priori highly non-Lorentz invariant*, and it almost requires a miracle for it to turn out at the end Lorentz invariant. It is therefore non-trivial and a priori dangerous only as we have chosen in the beginning to compared our model to the Veneziano model in the special case that the four external particles have the same p^+ components,

$$p_1^+ = p_2^+ = p_3^+ = p_4^+ \quad (11.46)$$
$$\text{and consequently} \quad (11.47)$$
$$N_1 = N_2 = N_3 = N_4, \quad (11.48)$$

where the (even) integers N_i (i=1,2,3,4) denote the numbers of "objects" attached to the four external particles. This choice of a special coordinate frame leads to a simplification of the term without poles in the s-channel:

$$g^2 B(-\alpha(t), -\alpha(u)) = \tag{11.49}$$

$$g^2 B(-1 - \alpha'(p_1 - p_4)^2, -1 - \alpha'(p_1 - p_3)^2) = \tag{11.50}$$

$$g^2 B(-1 + \alpha'(\vec{p}_{T1} - \vec{p}_{T4})^2, -1 + \alpha'(\vec{p}_{T1} - \vec{p}_{T3})^2) = \tag{11.51}$$

$$g^2 \int_0^1 z^{-2+\alpha'(\vec{p}_{T1}-\vec{p}_{T4})^2} (1-z)^{-2+\alpha'(\vec{p}_{T1}-\vec{p}_{T3})^2} dz \quad . \tag{11.52}$$

Here we have denoted the "transverse" parts - meaning the first 24 components by

$$\vec{p}_T = \{p^i\}_{i=1,2,\ldots,24}. \tag{11.53}$$

The simplification comes about because the +- term in the contraction with the metric in, say, $(p_1 - p_4)^2$ drops out because of our very special frame choice so that $(p_1 - p_4)^+ = 0$,and so the $(p_1 - p_4)^-$ does not matter, and

$$(p_1 - p_4)^2 = -(\vec{p}_{T1} - \vec{p}_{T4})^2. \tag{11.54}$$

11.5.2 Amplitude in Our Model, Principle of No Interaction!

Whereas in string theory there seems to be an interaction between the strings, it is rather surprising - and a hallmark for our theory - that in the formulation of ours in terms of the object states the S-matrix elements, that shall give the Veneziano amplitude as we shall show, is simply equal to the overlap! That is to say it is calculated as if the genuine S-matrix is just the unit operator. More precisely the S-matrix $< 1 + 2|S|3 + 4 >$, that shall describe the scattering of say, two incoming open strings 1+2 into two outgoing 3+4 is obtained by writing the states in our formalism - in terms of even "objects" - corresponding or representing the two string state 1+2, say $|1 + 2 >_{eo}$ and also to the two string state 3+4 corresponding state in even object space, say $|3 + 4 >_{eo}$, and then simply one takes the overlap of these incoming and outgoing states:

$$< 1 + 2|S|3 + 4 > = < 1 + 2|_{eo}|3 + 4 >_{eo} . \tag{11.55}$$

Here the subindex $_{eo}$ stands for "even objects" and means the state described in our even object notation. This means that in terms of our string field theory = "even object formulation" a scattering goes on without anything happening (whatever might happen in reality must have been thrown out in the construction of our string field theory model). Symbolically this formula for the S-matrix is shown on the figure 11.4:

11.5.3 Procedure

The main tasks in order to evaluate the scattering amplitude are

- A. First we must evaluate in some useful notation the wave functions for the incoming and outgoing strings - we shall in this article only consider scattering of two open strings coming in and two open strings coming-out, all in the tachyonic ground states.

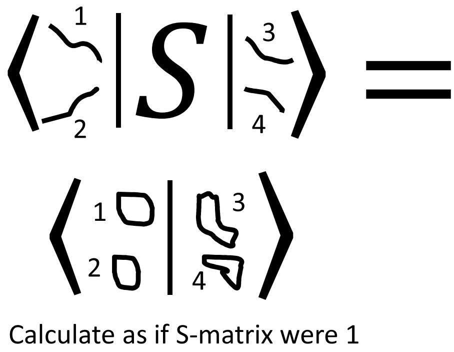

Fig. 11.4.

- B. We must figure out in how many different ways the "even objects" associated to the strings 1 and 2 in the initial state of the scattering can be *identified* with "even objects" associated with the final state strings.
- C. For each way of identification of every "even object" in the initial state(i.e. associated with one of the incoming strings/particles) with an "even object" in the final state (i.e. associated with one of these outgoing particles), we have in principle two wave functions for the "even objects" and shall compute the overlap of these two wave functions.
- D. Then we have to find the total overlap by summing over all the different ways of identifications, considered under B.
- E. This summation under D. will turn out to be approximated by an integral and we shall indeed see, that it becomes essentially the integration in the Euler Beta function definition thus providing the Veneziano model.

In performing this procedure we make some important approximations and simplifications:

- a. We shall assume that due to the continuity of the object series it is by far more profitable for obtaining a big overlap contribution to keep as many of the pairs of neighboring "objects" in the initial state, say strings 1 and 2, remain neighbors again in the final state. This means that we assume that the as contributions to the overlap dominating identification - in the sense of B. - are those in which the largest unbroken series of "even objects" go from one initial state string to one of the outgoing strings. This means the most connected or simplest pattern of identification.
  In fact the not yet quite confirmed though speculation is, that the successively more and more broken up pattern of identification of initial and final "even objects" will turn out to correspond to higher and higher (unitarity correction) loops in dual models(=Veneziano models). Thus we expect, that considering only the least broken transfer of the "even objects" from the initial to the final strings shall give us the lowest order Veneziano model (the original Veneziano model without unitarity corrections).

- b. We shall of course use, that we take the limit $a \to 0$ and correspondingly, that the numbers N_1, N_2, N_3, N_4 of "objects" associated with the various strings go to infinity. Thus we can integrate over the number of objects in a chain going some definite way, say from string 1 to string 4.
- c. To simplify our calculations we choose the very special case of the four strings - the two incoming 1 and 2 and the two outgoing 3 and 4 - all are associated with *same number of "even objects"* $N/2$ (and then of course N 'objects" altogether). This assumption is with our letting the number of "objects" be proportional to the P^+-component of the 26-momentum of the string in question, i.e. just the choice of Lorentz frame, so as to have all the four external strings/particles have the same P^+. So it looks like just being a coordinate choice, but there is the little problem strictly speaking: that our use of infinite momentum frame makes our theory not guaranteed to be Lorentz invariant. Anyway we do it only this non-invariant way in the present article and leave it for later, either to prove Lorentz invariance of our model, or to do it in a more general frame.
- d. As a further strengthening of point b. above about the chains coming in as long pieces as possible being dominant we remember, that our continuity condition (11.28) was *not reflection invariant*. It would therefore be extremely little overlap, if we should attempt to identify the "even objects" of a series in the initial state with a series in the final state in the opposite order. That is to say we require, that the orientation in the pieces of series going over as hanging together from initial to final state are kept. Otherwise the contribution is assumed negligible.
  Together b. and this item d. means that the dominant contributions come when possibly the longest connected pieces go over from one initial to one final without changing orientation of the piece.

We shall in the following seek a way to progress, that relatively quickly leads to string-theory-like expressions and thinking. But the reader shall have in mind that even, if we shall approach string-theory-like expressions, we have at the outset had a formulation - namely our string field theory - in which at first the stringyness is far from obvious. Rather it seemed that the stringy structure only comes in with the initial and final states, while the structure of our free massless scalar Hilbert space or Fock space is too trivial to contain any sign of being a string theory. It is therefore still interesting to calculate the results of our theory, even if it quickly should go into to run along lines extremely similar to usual string theory.

11.5.4 Construction of Wave Functions for Cyclically Ordered Chains Corresponding to Strings

The wave functions for open strings were in fact investigated in our previous article [3] in as far as the quantum system of N objects forming a cyclically ordered chain corresponding to an open string were resolved into harmonic oscillators and thus a Gaussian wave function were obtained in a high (of order N) dimensional space. The trick we shall use here is to write the wave function of this character by means of a functional integral so reminiscent of the Feynman-Dirac-Wentzel functional integral for a string propagation already put into the conformal gauge, that we can say that we already managed to "sneak in" the string by this technology.

In fact one considers in single string description functional integrals of the type:

$$\int \exp(-\int_A (\vec{\partial}\phi(\sigma^1, \sigma^2))^2 d\sigma^1 d\sigma^2)\mathcal{D}\phi, \tag{11.56}$$

with some boundary conditions along the edge of the region A say in (σ^1, σ^2) space, over which the integral in the exponent is performed. We shall for our purpose of making an expression for the wave function in terms of our "even objects" for a string state consider that the region A is taken to be a unit disk and at the edge we imagine putting a series of "even objects" each being assigned a small interval along the circular boundary. Then we identify for example the object J_R^i with the difference of the values of a ϕ^i taken at the two end points of the little interval on the circle surrounding A assigned to the object in question. That is to say for say object number I (here I is even) having as its interval, say, the little region between the points on the circle marked by the angles

$$\theta_{beg} = 2\pi * \frac{I-1}{N} \tag{11.57}$$

$$\theta_{end} = 2\pi * \frac{I+1}{N} \tag{11.58}$$

we identify (e.g.) the difference

$$J_R^i(I) \stackrel{ident.}{=} \phi^i(\exp(i\theta_{end})) - \phi^i(\exp(i\theta_{beg})), \tag{11.59}$$

where we have of course taken a new ϕ^i for each of the 24 i-marked components of J_R and where we have identified the (σ^1, σ^2)- space with the complex plane by considering ϕ^i a function of $\sigma^1 + i\sigma^2$.

The idea, which we seek to use here is that - possibly by some minor modifications, which we must state - we should imagine, that we want to construct a prescription for obtaining a wave function of the type $\Psi(J_R^i(0), J_R^i(2), ..., J_R^i(N-2))$ as used in the expression (11.21), describing say the ground state of a string in our formalism by imposing a boundary condition - depending on a set of values for all the "even objects" in a chain - on the functional integral(s). With these boundary conditions imposed at the end the functional integral become the wave function value $\Psi(J_R^i(0), J_R^i(2), ..., J_R^i(N-2))$ for the in the boundary condition used $J_R^i(i)$ values.

Let us before fixing the details immediately reveal that we shall have an extra boundary condition in the center of the disk A at which point we shall cut off an infinitesimally little disk and use the thereby opened boundary conditions to "let in the (transverse components of) the 26-momentum of the string in question". This "letting in" means in principle that we put on the inside of the little circle a series of J_R arranged to correspond to string with the right 26-momentum, but due to the smallness of the little circle the details except for this total momentum does not matter. In the figure we illustrate this situation on which we think: The line ending at the center and ascribed a P symbolize the just mentioned "in-let" in this center. The small tags on the edge of the disk symbolize the attachments of the "even objects", the values of which are used to fix the boundary conditions for the functional integral. Crudely the idea behind this procedure could be considered

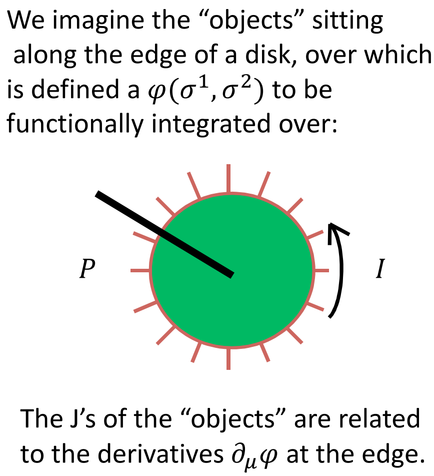

Fig. 11.5.

to be that *we let a (here open) string propagate along during an imaginary time (say an imaginary* τ*), whereby only the lowest mass state survives.* The heavier eigenstates of mass decay in amplitude faster that the lightest state by such an imaginary being spent. Thus one gets after infinite imaginary "time" the ground state selected out. Thus investigating the wave function reached after such an infinite imaginary "time" it should turn out being the ground state wave function, and so we should be able to use it as the Ψ we want, if we want the wave function for a ground state string (the tachyon). Then the idea is of course to write the infinite imaginary "time" development by means of Wentzel-Dirac-Feynman path way integration.

Thus we get into our way of presenting the wavefunction

$$\Psi(J^i_R(0), J^i_R(2), ..., J^i_R(N-2))$$

a functional integral with at first having a region, like A, being an infinite half cylinder. The axis along the half infinite cylinder is the imaginary part of the infinite imaginary 'time", while the coordinate around the cylinder is rather the parameter, τ_R, enumerating the objects in the cyclically ordered chain of objects associated with an open string.

Then the type of functional integral here considered is "essentially" (meaning except for an anomaly becoming very important at the end) invariant under conformal transformations of the region A. Thus ignoring - or seeing that they are not there in the case considered - anomalies we can transform the infinite half cylinder into the unit disk with the little hole in the middle, through which we "let in" the momentum of the string.

Note how the string here comes in (only): We got to a functional integral strongly related to what one usually work with in string theory, *just with the purpose of constructing a wave function* $\Psi(J^i_R(0), J^i_R(2), ..., J^i_R(N-2))$ *describing the string state in terms of our objects.* But there is nothing "stringy" in our Hilbert space structure of our object-theory. The string only comes in via this wave function.

But of course it still means, that after we have got this wave function in, we get our calculations being so similar to usual string theory, that we can almost stop our article there, and the string theorist may have exercised the rest so often, that we do not need to repeat. But logically we have to repeat because we are logically doing something else:

There is in our formulation in terms of the 'objects" a priori no strings. We are on the way to see, that after all the strings must be there, because otherwise it would be strange, that we just get the Veneziano amplitude for scattering.

11.5.5 Adjustment of the Details of the Functional Integral

A few details about the functional integral may be good or even rather important to have in mind:

- I. As long we - as here - just seek to write the exponential for the wave function (which as we know for harmonic oscillators have the Gaussian form - of an exponential of a quadratic expression in the $J^i_R(I)$'s (even I) -) we could use the old proposal by David Fairlie and one of us (HBN) of evaluating the exponential as the heat production in a resistance constructed as the surface region A as a conducting sheet with specific resistance $\pi(2?)\alpha'$. Then one shall identify the boundary conditions by letting the current running out at the interval assigned to a certain "even object" be equal to the $J^i_R(I)$ for that "even object".
- II. There a is little problem, which we have to solve one way or the other with getting the "continuity condition" (11.28) discussed in 11.4.2. Having fixed only the boundary condition to the "*even* objects" through their $J^i_R(I)$ but not involving the conjugate variables $\Pi^i_R(I)$ there is of course no way in which the strange non-reflection symmetric continuity condition of our could be imposed. Concerning the classical approximation one may actually find out that one easily can find the classical ϕ^i solution over the complex plane introduced above after the formula (11.59) which reflects the continuity condition as well as you can require for a classical solution by *extracting only the analytical part of the saddle point for* $\phi^i(\sigma^1 + i\sigma^2)$.
  Indeed one might - and we probably ought to do it - construct a model, in which we use both even and odd J^i_R's on the boundary, in the sense that we assign only half as long intervals on the border for each object - meaning that we replace (11.58) by

$$\theta_{beg} = 2\pi * \frac{I - 1/2}{N} \tag{11.60}$$

$$\theta_{end} = 2\pi * \frac{I + 1/2}{N} \tag{11.61}$$

and use it for both even I and odd I.
But now what are we to impose for the odd object intervals on the disk border? We want to obtain a wave function $\Psi(J^i_R(0), J^i_R(2), ..., J^i_R(N-2))$ expressed *only* as a function of the "*even* object" J^i_R's, while *no* Π^i_R *are accessible among the variables, on which the wave function depends*.
However, in functional integrals one can easily extract what corresponds to the conjugate variable; they are so to speak related to the time derivatives, by relations of the type that the conjugate to a variable q in a general Lagrangian theory is given by

$$p = \frac{\partial L}{\partial \dot{q}}. \tag{11.62}$$

On the other hand the continuity condition tells us that approximately the odd J^i_R's can be replaced by their even neighbors. Thus the proposal is being pointed out that we identify the appropriate time derivatives with the values of the neighboring even $J^i_R(I)$'s. Putting up this proposal is rather easily seen to correspond to, that the boundary condition relating ϕ^i near the boundary to the even object J_R's, which we are allowed to use, get decoupled from say the anti-analytic component in ϕ^i. So with such a boundary inspired by the non-reflection invariant continuity condition would lead to an arbitrary solution for say the anti-analytical part, while the analytical part would get coupled. We should like to develop this approach in further paper(s), but it may not really be needed.
Instead of seeking to put our continuity condition (11.28) into the functional integral formalism, we here shall use it is as a rule for which pieces of cyclically ordered chains can be identified, and then we shall get only oriented two dimensional surfaces - looking formally like string-surfaces for closed oriented strings although what we are talking about are *open strings* (but remember that we get the diagrams for open look like the ones say Mandelstam have for closed) -.

- III. Although it is in fact functional integrals like (11.56), that we basically need, it is so that such a functional integral has divergences. These divergences must in principle be cut off. But now it turns out that the cut off necessarily comes to depend on a metric. Therefore we should rather write our functional integral (11.56) as if depending on a metric tensor $g_{\alpha\beta}(\sigma^1, \sigma^2)$ in the 2-dimensional space time, although it formally would look that there *is actually no such dependence on the metric, at least as long as we just scale it up or down by Weyl transformations*. This seemingly metric dependent functional integral looks like

$$\int \exp(\int g^{\alpha\beta} \partial_\alpha \phi(\sigma^1, \sigma^2) \sqrt{g} d\sigma^1 d\sigma^2) \mathcal{D}\phi, \tag{11.63}$$

where then boundary conditions and region of the (σ^1, σ^2)-parameterization must be further specified. The cut off procedure should also be specified; it could for instance be a lattice curt off, a lattice in the (σ^1, σ^2) variables, say. Then the importance of the metric is that you need the metric to describe the lattice spacing. Note though also that formally a scaling of the metric/Weyl transformation

$$g_{\alpha\beta} \rightarrow \exp 2\omega g_{\alpha\beta}, \tag{11.64}$$

even when the scaling function $\exp 2\omega$ depends on the $(\sigma 1, \sigma^2)$ does *seemingly* not change anything, because the determinant g of the two by two matrix $g_{\alpha\beta}$ just scales with $\exp 4\omega$ so that the square root just compensates for the scaling of the upper index $g^{\alpha\beta}$ metric. The Weyl transformation symmetry is *only* broken by the cut off (the lattice) depending on $g_{\alpha\beta}$. It is via this cut off the anomaly can come in.

11.5.6 Overlap Contributions

The crucial step in calculating the Veneziano model amplitude in our model/string field theory is to see what are the possibilities for identifying all the even objects associated with the initial strings/particles 1 and 2 to the ones associated with the final state strings/particles 3 and 4 in a way that to the largest extend keep neighboring (or better next to neighboring, since we only consider the "*even* objects") "even objects" going into neighboring ones in the same order(same succession).

To simplify the possibilities, we have to consider what we have chosen to assume - basically by appropriate choice of coordinate system - namely that each of the four strings or particles are associated with the same number of "objects". We may remember that by our gauge choice the number of "objects" N associated with say an open string is proportional to the P^+ component of its momentum, so that choosing a frame, wherein all the four external particles have equal P^+ implies that they have an equal number of associated "objects" also.

Now to keep the "objects" most in the succession they already have in the initial state also in the final state we must let connected pieces of "even objects" pass from say string 1 to string 4. Then the rest of the "even objects" associated with string 1 must go to string 3. Now the "even object" numbers on string 2 that must go to respectively to 3 and to 4 is already fixed for what happened for string 1. Since they have to sit in succession and a cyclic rotation of the cyclically ordered chains is the very last rudiment of gauge choice, there is no physically significant freedom in the identification except for the starting number of how many objects go from 1 to 4.

On the figure it is illustrated how different series of "even objects" from 1 or 2 marked with some signature are refound - with same signature - in 3 or 4. The idea of course is that each of the four series marked by the four different signatures are refound in both initial (1+2) and final (3+4) states, and really are the same. It is understood that the series of "even objects" identified to be in both initial and final states are identified "even object" for "even object" in same succession.

To get the contributions to the overlap - and thereby amplitude - from all the physically different "identification ways" one shall sum over the various values, a non-negative integer number, of "even objects" from 1 that are refound in 4. Since such numbers are of order N - which means it goes to infinity as our cut off parameter $a \to 0$ - the actual overlap contribution from each separate value of the number summed over varies slowly and smoothly (we may check by our calculation) and we can replace it by an integral over say the fraction of the "even objects" in 1 (i.e. associated with 1) that are identified with 'even objects"

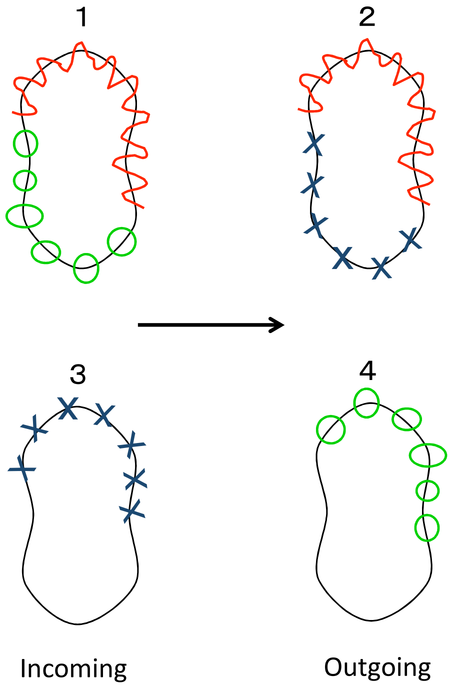

Fig. 11.6.

associated with 4. It is this integration that shall turn out to be the integration in the integration in the Euler Beta function making up the Veneziano model.

11.5.7 The Overlap for One Identification

But before integrating or summing we have to write down the overlap as obtained, if one only includes the possibility of one single "identification" (correspondence between the "even objects" in the initial state 1+2 with them in the final 3+4). This overlap of two states 1+2 and of 3+4 with a fixed "identification" is of course simply the Hilbert product of the two states of the set of $N/2 + N/2$ "even objects" - at least if one ignores the low probability of two "even objects" in say 1 and 2 being in the same state - so that we calculate it as an inner product in an $N/2+N/2$ particle/"even object" system. It becomes an inner product of the form

$$\int \Psi^*_{\substack{3+4,\\ \text{with}\\ \text{identification}I}}((J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_1, (J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_2)$$
$$\times\Psi_{1+2}((J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_1, (J^i_R(0), J^i_R(2), ..., J^i_R(N-2))|_2)$$
$$\times\Pi_{i,I=0,2,...,N-2,k=1,2} dJ^i_R(I)|_k. \tag{11.65}$$

Now the crucial point of our technique is that this inner product integration over the J^i_R-values for all the "even objects" associated with 1 or 2 (and identified with "even objects" in 3 or 4) when the wave functions are written as the functional integrals, we use, can be interpreted as just gluing functional integral regions together. The point is that the functional integrals basically are just - when cut off - integrals over ϕ^i values in all the different "lattice" points along the region A boarder say. At the boarders there is specified linear relations of the ϕ^i values there - or rather the derivatives, but they are also linear relations - to the $J^i_R(i)$'s assigned places on this border. One now has to argue that apart from an overall constant factor we can consider the integration over the $J^i_R(I)$'s in (11.65) going in as part of the functional integration in a functional integral in which the regions A for the two sides (initial and final) are glued together to one big functional integral. Since the integration over the "even object"-variables $J^i_R(I)|_k$ have now been interpreted as part of the functional integration, the new resulting functional integral has no longer any boundary conditions associated with such $J^i_R(I)|_k$'s. Rather the 'big' functional resulting - and expressing the overlap for a specific "identification" - only has as boundary conditions the inlets of the external 26-momenta(or rather their transverse components only),P_1, P_2, P_3, P_4.

One should notice how this picturing by functional integrals come to look really extremely analogous to gluing together strings. There is though a little deviation from the usual open string theory at first, because we have cyclically ordered chains topologically of form as circles as would be closed strings to represent the open strings. Corresponding to this little deviation we get at first that final contribution from "identification" of "even objects" between final and initial states, becomes conformally equivalent to a Riemann sphere with the four inlets from the four external strings being attached to this Rieman sphere. This is what you would expect for closed string scattering in the usual string theory, but *we* obtain this for *open* strings! It turns, however, out that all our four "inlets" - where the momentum boundary conditions are imposed - come by a calculation we shall sketch - to sit on a circle on the Riemann sphere. Thus there is "reflection" symmetry between the two sides of this sphere and mathematically our overlap for the fixed identification come to be equal to a functional integral as usually used for open strings. In this way our model has the possibility of agreeing exactly with usual string theory.

11.5.8 Seeing the Hope

A bit of imagination of how our topologically infinite half cylinders can glue together would reveal, that we could arrange to get them pressed down in a plane but with - we must stress though - in two layers. In such a form we could have arranged that the result would look like a *double* layer four string bands meeting along intervals with their neighbors but only in one point with their opposite string. In order that we could bring it to look like this, we should put the two incoming strings opposite and the two outgoing strings opposite to each other. This would be the usual string gluing picture for the open strings - just doubled, but that essentially does not matter - for the $B(-\alpha(t), -\alpha(u))$ term. This means that it is extremely promising that we should obtain this term of the Veneziano model.

But !:

- 1. What happens to the other two terms $B(-\alpha(s), -\alpha(t))$ and $B(-\alpha(s), -\alpha(u))$, which we should also have gotten, to get the full Veneziano model?
- 2. We have in principle to check that our model predicts the correct weight factor on the integrand in the Veneziano model. We mean that the integrand, which we obtain does not only have the right dependence on the external momenta, but also the right dependence as a function of the integration variable - which in our model comes from the summation over the different "identifications".

  Since in our model this integration comes from the simple summation over "identifications" our model has a very clear rule for what weighting to obtain and one just has to calculate carefully not remembering the anomaly in the functional integral evaluation etc.
- 3. So far we were sloppy about the + and - components, or rather we only started calculating the factors in the integrand coming from the transverse momenta or transverse J^i_R components so far.

11.5.9 Integrand weighting Calculation

If we want to evaluate the integrand meaning the contribution from one specific identification more carefully, we have to be specific about how we for such an "identification" make the construction of the full surface on which the functional integration ϕ^i at the end gets defined. We obtain - using the idea that the overlap integration can be absorbed into the functional integration - that we must glue together four (either infinite half cylinders or) disks corresponding to the four external strings. To be concrete it is easiest to represent the two final state strings 3 and 4 by *exteriors* of a disk rather than by a disk as we represent the initial strings 1 and 2. The inlets for the final state strings are then at infinity of the Riemann surface, while those of the initial strings are at zero.

Now however, we have two incoming and two outgoing, and so we are forced to work with a complex plane with *two* layers.

We take say one layer where the complement of the unit disk is put to be the essential disk for string 3, while the other layer belongs then to string 4. Similarly we must have two layers for the initial strings, but now the important point is that the initial and the final ones are to be glued together in a slightly complicated way, depending also on the integration variable, which is essentially given by the number of "even objects" going from 1 to 4 say.

Having settled on giving 3 and 4 each their layer in the complex plane in the complement of the unit disk, we have let these outside unit disk layers be glued to the inside the unit disk ones associated with the two incoming particles 1 and 2 along the unit circle of course. But now the length measured say in angle - or in number of "even objects" proportional to the angle - along which say the layer in the inside assigned to string 1 has to be glued together with complement of disk region layer assigned to string 4 along a piece of circle proportional to the amount/number of "even objects" passing from string 1 to string 4 in the "identification" we consider. along the rest of the unit circle then of course the disk assigned to string 1 is identified with the outside disk layer connected to string 3. Similarly along the first piece of circle - where 1 is connected to 4 - of course the layer of string 2 shall be connected along the circle to the layer of string 3 (in the outside). Correspondingly along the "rest" of the unit circle the layer assigned to string 2 (inside) is attached (identified with) the layer of string 4. In the figure 11.7 you may see an attempt to give an idea of what to do before having put on the final state strings associated with the complements of the unit disk in their two layers. But on the figure the inner layers are prepared for the gluing together. Now modulo the anomaly - i.e. naively - the functional integrals

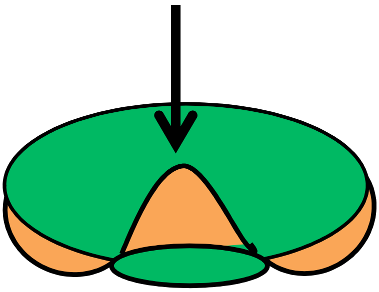

Fig. 11.7.

considered are conformally invariant, so that we are postponing the anomaly allowed to perform a conformal transformation of the combined region - now lying in two layers - of the four disks or complements of disks associated with the four external strings, and the result of the functional integral being proportional to the overlap contribution from the "identification" in question should not be changed.

Since the angle θ (circle piece length) along which say layer of string 1 is identified with the layer of string 4 is proportional to the *number* of "even objects" we shall simply integrate to get the an expression proportional to the full overlap (and thus to the Veneziano amplitude hopefully) integrate simply with the measure $d\theta$.

At first it looks that we have a little problem by only having wave functions as functions of the 24 transverse coordinates so that seemingly the + and - components of the 26-momenta cannot appear in our hoped for Veneziano model integral. However, luckily for the term we actually obtain $B(-\alpha(u), \alpha(t))$ we found above in equation (11.52) that in fact all the terms in the exponents for z and $(1-z)$ that depend on the external 26-momenta could be arranged *to come only from the transverse momenta* provided we have made the very special frame choice that the four external particles have the same P^+ components. So in this our simplifying the contributions from the + and - components turn out not needed. The point really was that just having the gauge choice and the frame choice arranging the + component for the $p_1 - p_4$ and for the $p_1 - p_3$ become zero the character of the u and t of having a + component multiplied with a - one made it enough to ignore but the transverse contributions.

11.5.10 The Conformal Transformation

The way to evaluate the contribution to the overlap of $|1+2>$ with $|3+4>$ from one "identification" is to rewrite it into a functional integral the region of which is composed from the four disks or disk complements corresponding to the four external particles/strings. We obtain at first a manifold described as double layered in the Riemann sphere. It has two branch points on the unit circle corresponding to the points where the "even objects" on say 1 shifts from going to 3 to going to 4 (or opposite). Basically we choose to map this doubled layered region by a map with two square root singularities at the two branch points.

11.5.11 Anomaly

The anomaly that gives us an extra factor mutiplying the contribution from a single "identification" is usually written formulated as the trace anomaly

$$< T^{\alpha}_{\alpha} >= -\#\mathtt{fields} * \frac{1}{48\pi}\sqrt{g} * R, \tag{11.66}$$

(in the notation of our article with Habara wherein $\sqrt{g}R = -2\partial_{\alpha}\partial^{\alpha}\Omega$ for the metric tensor of the form $g_{\alpha\beta} = \exp(2\Omega)\eta_{\alpha\beta}$,and) where R is the scalar curvature of the metrical space given by the metric tensor (in two dimensions enumerated by $\alpha = 1, 2$;). Here the energy momentum tensor is denoted $T^{\alpha\beta}$ is indeed for the theory of the field(s) ϕ^i which had been Weyl invariant as it formally looks like, and so the trace $T^{\alpha}_{\alpha} = 0$ would be zero. The symbol $\#\mathtt{fields}$ denotes the number of fields ϕ_i; it would in the 26=25+1 theory be 24.

This anomaly can be seen to come in by having in mind that we want to perform a conformal transformation - in fact the one corresponding to the analytical function

$$f(z) = \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}} - \tag{11.67}$$

(here we used the notation that the end points of the cut along the unit circle seperating where sheet 1 connects to 4 from where it connects to the sheet associated with particle 3 were arranged to be $\exp(i\delta)$ and $\exp(-i\delta)$.) and then the anomaly gives rise to corrections to the "naive" result that the functional integral is invariant under a conformal transformation. In fact we may first have in mind that we shall evaluate the functional integral (11.56) with a lattice or other cut off only depending on the internal geometry so that it only can give variations depending on the metric tensor $g_{\alpha\beta}$, which under a conformal transformation only changes its scale locally as under a Weyl transformation (11.64). So what we only need to calculate to obtain the effect of the anomaly is how the overall factor on the metric tensor varies under the conformal transformation, we shall use (11.67). Such a scaling is given by the numerical value of the derivative of the function (here f) representing the conformal transformation,

$$g_{\alpha\beta} \to \Omega g_{\alpha\beta} \text{ where then } \Omega = |\frac{\partial f}{\partial z}|^2. \tag{11.68}$$

This is to be understood, that the metric tensor describing the complex plane metric in the f-plane is $\exp 2\Omega g_{\alpha\beta}$ when the metric induced from the z-plane usual metric is $g_{\alpha\beta}$.

It is easy to see that scaling the metric tensor (locally) with an infinitesimal scaling factor $\exp 2\Omega$ with $\Omega << 1$ leads to a correction to the logarithm of the functional integral by $\int \Delta\omega T^{\alpha}_{\alpha} d^2\sigma$. Since the trace T^{α}_{α} of the energy momentum tensor is only non-zero according to (11.66) where there is a non-zero curvature, and our two layered surface lies mostly in the flat complex plane, we only get contributions to this Weyl transformation local change of scale from the two (singular) branche points $\exp(i\delta)$ and $\exp(-i\delta)$, where the curvature R has delta-function contributions.

We can without any change in value of the functional integral make a formal reparametrization from say the double sheeted complex plane to the single layered one by means of $f = \sqrt{\frac{z-\exp(i\delta)}{z-\exp(-i\delta)}}$ provided one then use after the transformation the *transformed metric tensor*. With a conformal transformation the transformed metric inherited from the z-plane into the f-plane will only deviate from the flat metric $\eta_{\alpha\beta}$ in the f-plane by a Weyl transformation. We know that there only shall be curvature - of delta function type - at those points in the f-plane that are the images of the branch points $z = \exp(i\delta)$ and $z = \exp(-i\delta)$, and so the (Weyl transformed) metric reflecting the metric space from the z-plane into the f-plane, $\exp(2\Omega)\eta_{\alpha\beta}$, i.e. $R = 0$ outside these two points $f = 0 \text{and} \infty$.

This means that the Ω outside those two points in the f-plane must be a harmonic function of f, meaning the real part of an anlytical function. This outside the two points harmonic function shall though have *singularities at the two points* on the f-plane (or the corresponding Riemann sphere rather) delivering the delta-function contributions,

$$R = 4\pi\delta(Re(f))\delta(Im(f))\text{at f=0 say.}, \tag{11.69}$$

At the branch points, we have points with the property that going around one of them in the z-plane or system of sheets one get a return angle θ being 2π more than after the mapping into the f-plane (or Riemann space). Thus the integral over the curvature delivering this extra amount of parallel transport extra shift angle should in an infinitesimal region around the image of a branch point - say the point $f = 0$ - be 2π. So with a notation with the rule of such an (excess) angle being given as

$$\int_{area} R\sqrt{g}d^2\sigma = 2\theta \tag{11.70}$$

with θ the extra angle of rotation on return, [1] there will in the metric inherited from the z-plane in the f-plane be delta function contribution to the curvature scalar R at the points corresponding to the branch points $z = \exp(i\delta)$ and $z = \exp(-i\delta)$ in the z-plane, and thus $f = 0$ and $f = \infty$ in the f-plane. One can easily see that because there is just 2π extra angle to go around such a branch point in the sheeted z-plane the delta-function contribution becomes e.g. for the $f = 0$ point

$$R = 4\pi\delta^2(f). \tag{11.72}$$

If r is the distance to the image of the branch point, say the $f = 0$ point, so that $r = |f|$, the solution to $R = -2\partial_\alpha\partial^\alpha\Omega$ for this delta function R is a logarithm of the form

$$\Omega(r) = \ln(r/K). \tag{11.73}$$

(Here K is some constant in the sense of not depending on r) That implies that taken at the point $r = 0$ the $\Omega(0)$ is logarithmically divergent so that the integral to which the anomaly of the logarithm of the correction to the integrand is proportioanal becomes divergent. However, we have anyway given up caculating in this article the overall normalization of the Veneziano model, we hope to derive. We shall therefore be satisfied with only calculating the contribution in Ω *that varies with the angle* δ over which we (finally) integrate. Now the conformal transformation mapping the two-sheeted z-plane into the one-sheeted f-plane is

$$f(z) = \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}}, \tag{11.74}$$

and so its logarithmic derivative

$$\frac{df}{fdz} = \frac{1}{2}\left(\frac{1}{z - \exp(i\delta)} - \frac{1}{z - \exp(-i\delta)}\right), \tag{11.75}$$

and the derivative proper

$$\frac{df}{dz} = \frac{1}{2} * \sqrt{\frac{z - \exp(i\delta)}{z - \exp(-i\delta)}} * \left(\frac{1}{z - \exp(i\delta)} - \frac{1}{z - \exp(-i\delta)}\right). \tag{11.76}$$

We are interested in a hopefully finite term in the change in going from the z-plane simple metric to the one in the f-plane, which is the part of

$$\Omega_{z \text{ to } f} = \ln(|\frac{df}{dz}|) \tag{11.77}$$

depending on the "integration variable" δ. This means that make precise the cut off by saying that we must make the cut off by somehow smoothing out the branch

[1] In our notation we have the rule that going around an area and thereby obtaining for for a parallel transported vector on return a rotation by an angle θ, that the integral over this area

$$\int_{area} R\sqrt{g}d^2\sigma = 2\theta \tag{11.71}$$

point singularity in a fixed way in the *z-plane*. This means that we perform a regularization by putting into our transformation a fixed distance ϵ in the z-plane marking the distance of z to one of the branch points. That is to say we consider a little circle say of points around the exact branch point counted in the z-plane

$$z_{\text{on little circle}} = \epsilon \exp(i\chi) + \exp(i\delta) \tag{11.78}$$

(or analogously using $\exp(-i\delta)$ instead of $\exp(i\delta)$.) On this little circle we find that the scaling - Weyl transformation - going from the z-plane to the f-plane using (11.76, 11.77)

$$\Omega_{z \text{ to } f} = \ln(|df/dz|_{circle}) \approx \ln(\frac{1}{2\sqrt{\epsilon}\sqrt{2 sin(\delta)}}) \tag{11.79}$$

for the $z \approx \exp(i\delta)$ case. The δ-dependent part is of course $-\frac{1}{2} * \ln(\sin(\delta))$. This is the δ-dependent part of $\Omega_{z \text{ to } f}$ which comes into the anomaly correction for the logarithm of the full (product over the 24 values of i of the) functional integral, according to (11.66) and (11.72) of course with a coefficient proportional to the number of truly present dimensions in the functional integral - which is only the transverse dimensions 24 -.

Thus the δ dependent part of the anomaly ends up being in the logarithm of the contribution to the overlap from one value of δ (meaning one "identification"):

$$\begin{aligned} \Delta_{anomaly} \ln \text{integrand} &= \delta \text{ independent } + (d-2) * \frac{1}{48\pi} * \frac{1}{2} \ln \sin\delta * (4\pi + 4\pi) \\ &= \frac{d-2}{6} * \ln \sin\delta + ... \end{aligned} \tag{11.80}$$

Now we should remember that we have decided in this article to go for the form of the amplitude but have left for further studies the over all normalization of the amplitude. This means that the terms in the logarithm of the integrand of the hopefully to appear Veneziano amplitude which do not depend on the integration variable δ (which is proportional to the number of (even) objects from string 1 that goes into string 4) but only so on the cut off parameter ϵ are neglected.

Now the conformal mapping (11.74) brings the inlet points for external momenta for the four external particles into the positions sketched on the figure 11.8: Imagining on this figure that one varies the integration variable δ, then the "inlet" points for the two incomming strings 1 and 2 will remain sitting opposite to each other one the unit circle and analogously the two final state string inlet points 3 and 4. So the distances between 1 and 2 or between 3 and 4 are constant as function of δ and so we can ignore the terms comming proportional to in fact $s = -(p_1^i + p_2^i)^2 + ... = -(p_3^i + p_4^i)^2 + ...$ from the heat production in the analogue model from the current running co as to depend on the distance between 1 and 2 or analogously between 3 and 4. In a similar way we are allowed with our decission to only keep the δ dependent terms to ignore terms involving only one of the four inlet points. There are such contributions but they depend on the inlet momentum squared (with a divergent coefficient), but since only on one point the δ-dependence is not there provided we cut off in a δ independent way of course.

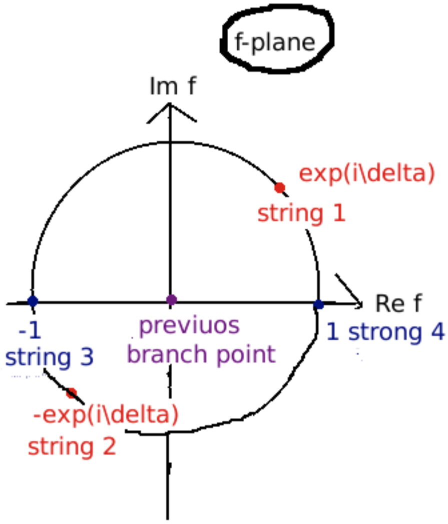

Fig. 11.8.

The divergences form cut off of the anomaly correction connected with originallly - in z-plane - branch points come only in to the extend that they get their cut off small circles scaled to a differnt degree depending on δ.

It is not difficult to see that to seek identification of the δ dependent terms with the integration variable z (not to be confused with our complex plane z which is of course something different) in the Veneziano model we must identify

$$z = \sin^2(\frac{1}{2} * \delta) \tag{11.81}$$

$$1 - z = \cos^2(\frac{1}{2} * \delta). \tag{11.82}$$

Very important for stressing how successful our model/rewritting is to reproduce the Veneziano model integration measure in the z-integration correctly. In our model this integration correponds to the summation over the discrete variable being the number of (even) objects going from string 1 to string 4 and it is proportional to δ, and thus we at first simply the measure of integration is $d\delta$. But now to compare with the usual Veneziano formula expressions or our slight rewrittings of it we must of course relate $d\delta$ to dz:

$$dz = \cos(\frac{1}{2}\delta)\sin(\frac{1}{2}\delta)d\delta \propto \sin(\delta)d\delta \propto \sqrt{z(1-z)}d\delta. \tag{11.83}$$

This happens to show that the correction factor from the anomaly needed to just compensate the factor comming from

$$d\delta = \frac{dz}{\sin(\delta)} \tag{11.84}$$

would be just cancelled by the anomaly provided

$$\frac{d-2}{6} = 1. \tag{11.85}$$

(Apart from a little calculational mistake above by a factor 4) this means that we get precisely the right Veneziano model when the number of transverse dimensions $d - 2 = 6 * 1 = shouldbe24$. The famous result that the bosonic string must exist in d=26 space time dimesions.

11.6 Our Shock; Only One Term

When we went through the above sketched calculation and arrived at *only* the one term proportional to $B(-\alpha(t), -\alpha(u))$, which is the term without poles in the s-channel, it were somewhat unexpected at first. After all we had made up a model essentially written out so as to *make it* the string theory and thereby the Veneziano model. Then it gave only one out of the three terms it should have given. May be even more strangely, if we imagine investigating crossing symmetry it looks we would get a different term after what particles are incoming and which outgoing. So the term we got is not even properly crossing symmetry invariant. Nevertheless it were very encouraging that we got something so reminiscent of the Veneziano model as simply one of the terms.

We believe we have found a way to get the two missing terms also come out:

In fact we think that it is in a way the infinite momentum frame gauge, which we used, that is the reason for the surprising problem for our model: Really one may say that the infinite momentum frame is a method for avoiding having to think about the vacuum, which in quantum field theories is usually an enormously complicated state. In the infinite momentum frame type calculations you imagine an approximation in which the particles have so high energy that they manage not to "feel" the vacuum. But such an approximation may not be a good one. So we thought it might be best somehow to introduce at least some rudimentary effects of a vacuum even though we want to continue to work with an infinite momentum frame formalism, especially an infinite momentum frame gauge/parameterization choice.

The idea, which we here propose, and which actually seems to help to obtain the lacking two terms in the full Veneziano model amplitude, is to allow not only as we did at first for "objects" with the +components $J^+_R = a\alpha'/2$ (a positive number), *but also allow "negative objects"* having rather their $J^+_R = -a\alpha'/2$. At least with inclusion of such negative "objects" you make it at least a possibility to have not totally trivial state with the property of the vacuum of having the "longitudinal" momentum $P^+ = 0$. The vacuum could so to speak consist of a compensating number of usual positive say "even objects" and corresponding number of "negative even objects".

In fact it looks that we with such "negative" "objects" can imagine some of our strings represented by an "extended" cyclically ordered chain(replacement). Hereby we mean that it contains in the "extended" cyclically ordered chain not only usual positive J^+_R objects, but also one or more series of negative J^+_R objects, arranged so that the excess of positive ones over negative ones is proportional to the total P^+ component of the 26-momentum for the string in question. With such "extended" cyclically ordered chains representing some open string we obtain the possibility of the negative part of say string 2 annihilating with part of the cyclically ordered chain of string 1. Similarly one of the final state strings could be produced with content in its cyclically ordered chain of some series of negative objects having been produced together with some positive ones in another final state string.

By very similar procedure to the one used above to the term $B(-\alpha(u), -\alpha(t))$, but now including the negative objects we seem to be able to produce the two missing terms. The detail of the calculation to obtain the full Veneziano amplitude/model will appear soon by the authors [21].

11.7 Conclusion and Outlook

We have in the present article sketched how using our string field theory formalism in which strings are rewritten into be described by states of "even objects" we can obtain the scattering amplitude to be the usual Veneziano model amplitude. It must though be immediately admitted that we at first got *only one out of the three terms expected*. However, introducing "objects" that can have negative J_R^+-components and can function as a kind of holes for objects, we though believe, that it is promising to obtain the whole Veneziano amplitude. Our model or string field theory has previously been shown [3] to lead to the usual mass square spectrum for strings. In this way we collect increasing evidence that our formalism is indeed another representation of all of string theory.

The way we constructed our formalism working from string theory and only throwing away though a null set of information, it is of course a priori expected, that our formalism should be string theory. In so far there sufficient holes in the "derivation" of our formalism from string theory to be equivalent to the latter, that we still need the more indirect support from rederiving features of string theory such as the Veneziano amplitude from our model.

Our model is a formulation in terms of what we called "objects", and they "sit" in circular "cyclically ordered chains", to an open string is assigned one such circular chain of objects, to a closed string two. The "objects" are supposed to "sit" as smoothly as they can from quantum fluctuations - which put severe constraints though, since *the odd numbered "objects" in cyclically ordered chain are not independent dynamical variables, but rather given in terms of (the conjugate variables Π_R^i for) the neighboring "even numbered objects" by equation (11.19).*

Actually we even stressed that the smoothness or continuity condition because of the dependence of the odd objects on *differences* of the conjugate momenta of neighboring even ones become non-reflection invariant. That is to say that a cyclically ordered chain being smooth would not remain smooth, if one puts the objects in the opposite order! The crux of the matter is that we have a genuine string field theory in the sense that we construct a state space of Hilbert vectors describing a whole universe in a string theory governed world. Then of course there can in the various states of this Hilbert space exist different numbers of strings, well this is not hundred percent true, because contrary to other string field theories: our Hilbert space is described in terms of the "even objects" and the number of strings perfectly accurately given once you have a Hilbert space state. The "even objects" can namely be associated to strings in slightly different ways, so that the number of strings *only approximately* can be derived from a given state; even there are no exact eigenstates for the number of strings. But in practice we believe the approximate access to the number of strings in our description is sufficient. But that the number of strings is *not* cleanly defined feature of a state in our Hilbert space, is clear from the fact that we have scattering even scattering that change the number of strings, such as if two strings scattered and became three, but that nothing happen in our formalism under a scattering. We just obtained the Veneziano model scattering amplitude as an *overlap* of initial and final state just corresponding to that nothing happens in the object formulation. In this sense the strings resulting from the scattering must have been there all the time.

One may look at our model as *solution* of string theory in the sense that we have "even object " description that does not even develop with time so that the "even object" state is more like a system of initial data to a solution of string theory.

11.7.1 Outlook

We foresee that there must be really very much it would be reasonable to do in our formalism, which is in many ways simpler than usual string theory especially than usual string field theory.

Presumably it will be very easy to make the superstring version; if nothing else should work one could in principle bosonize the fermionic modes and then treat the resulting bosons similar to the way we treated in our model of the bosonic modes.

Of course we should really also properly finish getting the Veneziano model calculation remaining details. A special interest might be connected with the overall normalization, which we left completely out here, since our formalism has no obvious candidate for the string coupling g, so the latter should come out from whatever parameters such as our cut off parameter a and α' and possible vacuum characteristic, but we did not use openly vacuum properties in the calculation sketched.

Most interesting might be to use our formalism to obtain a better understanding of the Maldacena conjecture by developing our formalism for the Ads space and then see that the corresponding CFT can also be written by our formalism.

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Discussion Section

The discussion section is meant to present in the workshop discussed open problems, which might start a collaboration among participants or at least stimulate them to start to think about possible solutions in a different way. Since the time between the workshop and the deadline for the contributions for the proceedings is very short and includes for most of participants also their holidays, it is not so easy to prepare besides their presentation at the workshop also the common contributions to the discussion section. However, the discussions, even if not presented as a contribution to this section, influenced participants' contributions, published in the main section.

This year quite a lot of started discussions have not succeeded to appear in this proceedings. Organizers hope that they will be developed enough to appear among the next year talks. Consequently this year discussion section has three contributions only.

Most of discussions concerned searches for the theory which would offer a trustable step beyond the *standard models* and the need to find the overlaps among all possible searches for the theory beyond the *standard model*. Participants were trying to understand, as deeply as possible, all the assumptions of different approaches: How much have they in common, as well as how far have the assumptions of different models, first of all of the *standard model*, influenced the experimental results and also how is the choice of the groups of symmetries, through which the universe went in its evolution, connected with smallness of the group representations? And others.

Discussions about the direct dark matter measurements are presented at the end of the talk of Rita Bernabei. She is answering the participants' questions.

The three written contributions concern: a. The toy model following the *spin-charge-family* theory within $d = (5+1)$ (instead of $(13+1)$), from where one can learn how the break of symmetries, global, local and discrete, occur and how do they influence the properties of families and of the scalar and vector gauge fields. b. The idea that the future influences the past and the such an influence is already observed. c. The space can emerge from randomness and diffeomorphism symmetry.

All discussion contributions are arranged alphabetically with respect to the authors' names.

Udeleženci velikega dela diskusij niso uspeli zapisati pravočasno v obliko, da bi ga lahko vključili v letošnji zbornik. Organizatorji upamo, da bodo te diskusije dozorele do oblike, da jih bo mogoče predstaviti na naslednji delavnici. Tako ima ta sekcija samo tri prispevke.

Največ diskusije je teklo okoli vprašanja, kaj mora ponuditi teorija, da ji bo mogoče zaupati, da ponuja pravi naslednji korak k razumevanju lastnosti osnovnih delcev in in evolucije vesolja in razloži ne le predpostavke teh dveh modelov ampak tudi opažene pojave, ki jih modela ne vključujeta. Udeleženci so poskušali razumeti in razčleniti vse predpostavke različnih predlogov, pa tudi, kako so ti predlogi, predvsem pa *standardni model*, vplivali na rezultate meritev, ter tudi, denimo, kako je vesolje „izbiralo" grupe simetrij v svoji evoluciji in kako je ta izbor povezan z velikostjo upodobitev grup.

Diskusija o tem, kako skrbno so očiščeni vseh nečistoč njihovi eksperimeni, kako so poskrbeli, da med delce temne snovi ne prištejejo že poznane delce ter kakšen je vzrok, da ostali eksperimenti (še) niso potrdili njihovih meritev, najde bralec kot dodatek k prispevku Rite Bernabei, v katerem odgovarja vodja laboratorija DAMA/LIBRA na zahtevna vprašanja udeležencev.

Trije prispevki te sekcije obravnavajo: a. Teorijo *spina-nabojev-družin* pri predpostavki, da je prostor primarno $(5 + 1)$-razsežen in ne $(13 + 1)$-razsežen, ki naj pomaga razumeti, kako pride do spontane zlomitve simetrij, globalnih, lokalnih in diskretnih in kako te zlomitve spremenijo lastnosti spinorjev ter lokalnih skalarnih in vektorskih polj. b. Idejo, kako vgraditi v teorijo vpliv prihodnosi na preteklost, ki je opazljiv. c. Kako prostor vznikne iz neurejenosti, ki ji dodamo simetrijo na difeomorfizme.

Prispevki v tej sekciji so, tako kot prispevki v glavnem delu, urejeni po abecednem redu priimkov avtorjev.

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12 Properties of Families of Spinors in $d = (5+1)$ with Zweibein of an Almost S^2 and Two Kinds of Spin Connection Fields, Allowing Massless and Massive Solutions in $d = (3+1)$

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Abstract. We studied in the refs. [1,2] properties of spinors in a toy model in $d = (5+1)$, when $\mathcal{M}^{(5+1)}$ breaks to an infinite disc with a zweibein which makes a disc curved on an almost S^2 and with a spin connection field which allows on such a sphere only one massless spinor state, as a step towards realistic Kaluza-Klein theories in non compact spaces. In the ref.[3] we allow on S^2 two kinds of the spin connection fields, those which are gauge fields of spins in and those which are the gauge fields of the family quantum numbers, both as required for this toy model by the *spin-charge-family* theory [4,5]. This time we study, by taking into account families of spinors interacting with several spin connection fields, properties of massless and massive solutions of equations of motion, with the discrete symmetries [9,10] ($\mathbb{C}_{\mathcal{N}}$, $\mathcal{P}_{\mathcal{N}}$, $\mathcal{T}_{\mathcal{N}}$) included. We also allow nonzero vacuum expectation values of the spin connection fields and study the masses.

Povzetek. Da bi bolje razumeli zlomitve simetrij v teoriji *spinov-nabojev-družin* in njhove posledice, študirata avtorja zlomitve na preprostem modelu v prostoru-času $d = (5+1)$. V tem modelu zvije vektorski svezenj neskončen disk v peti in šesti dimenziji v skoraj sfero S^2, spinske povezave pa poskrbijo za to, da je v opazljivem prostoru-času (d=(3+1)) sodo število brezmasnih družin. Ko dovolita spinskim povezavam, da imajo neničelno pričakovano vrednost, naboj fermionov (S^{56}) ni več dobro kvantno število. Družine pridobijo maso, ki jim jo določijo skalarna polja. Študirata tudi diskretne simetrije brezmasnih in masivnih družin fermionov. Študij ponudi globji vpogled v skalarna polja, ki določajo lastnosti fermionov pred in po zlomitvi simetrije naboja. Medtem ko nosijo skalarna polja v teoriji ″*spini-naboji-družine*″, kadar je dimenzija $d = (13+1)$, polštevilčni šibki in hipernaboj, tak kot Higgsovo polje v *standardnem modelu*, pa je v preprostem modelu naboj skalarnih polj celoštevilčen.

12.1 Introduction

The *spin-charge-family* theory [4,5], proposed by one of us (N.S.M.B.), is offering the explanation for the appearance of families of fermions in any dimension. Starting in $d = (13+1)$ with a simple action for massless fermions interacting with the gravitational interaction only - that is with the vielbeins and the two kinds of the spin connection fields, the ones originating in the Dirac kind of spin (γ^a's) and the others originating in the second kind of the Clifford operators ($\tilde{\gamma}^a$'s) - the theory manifests effectively at low energies the observed properties of fermions and bosons, offering the explanation for all the assumptions of the *standard model*: For the appearance of families, for the appearance of the Higgs's scalar [6] with the weak and the hyper charges ($\mp\frac{1}{2}$, $\pm\frac{1}{2}$, respectively), for the Yukawa couplings, for the charges of the family members, for the vector gauge fields, for the dark matter content, for the matter-antimatter asymmetry [7].

The theory predicts the fourth family, which will soon be observed at the LHC, and several scalar fields, manifesting in the observed Higgs's scalar [8] and the Yukawa couplings, some superposition of which will also be observed at the LHC.

A simple toy model [1–3], which includes also families in the way proposed by the *spin-charge-family* theory [4,5]), is expected to help to better understand mechanisms causing the breaks of symmetries needed in the case of $d = (13 + 1)$, where a simple starting action leads in the low energy regime after the breaks to the observable phenomena.

This contribution is a small further step in understanding properties of the families after the breaks of symmetries, caused by the scalar fields which are the gauge fields of the charges of spinors and the scalar fields which are the gauge fields of the family groups. The discrete symmetries of fermions and bosons in the case of only one family are studied already in the ref. [10]. Here the discrete symmetries are studied when the families are taken into account. We allow also that the spin connection fields gain nonzero vacuum expectation values and study solutions of the equations of motion for massive spinors.

We start with massless spinors [1–3,10] in a flat manifold $\mathcal{M}^{(5+1)}$, which breaks into $\mathcal{M}^{(3+1)}$ times an infinite disc. The vielbein on the disc curves the disc into (almost) a sphere S^2

$$e^s{}_\sigma = f^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, f^\sigma{}_s = f \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{12.1}$$

with

$$\begin{aligned} f &= 1 + (\frac{\rho}{2\rho_0})^2 = \frac{2}{1+\cos\vartheta}, \\ x^{(5)} &= \rho\cos\phi, \quad x^{(6)} = \rho\sin\phi, \quad E = f^{-2}. \end{aligned} \tag{12.2}$$

The angle ϑ is the ordinary azimuthal angle on a sphere. The last relation follows from $ds^2 = e_{s\sigma}e^s{}_\tau dx^\sigma dx^\tau = f^{-2}(d\rho^2 + \rho^2 d\phi^2)$. We use indices $(s,t) \in (5,6)$ to describe the flat index in the space of an infinite plane, and $(\sigma,\tau) \in ((5),(6))$, to describe the Einstein index. Rotations around the axis through the two poles of a sphere are described by the angle ϕ, while $\rho = 2\rho_0\sqrt{\frac{1-\cos\vartheta}{1+\cos\vartheta}}$. The volume of this non compact sphere is finite, equal to $V = \pi(2\rho_0)^2$. The symmetry of S^2 is a symmetry of $U(1)$ group.

We take into account that there are two kinds of the Clifford algebra operators: Beside the Dirac γ^a also $\tilde{\gamma}^a$, introduced in [4,5,12]. Correspondingly the covariant momentum of a spinor on an almost S^2 sphere is

$$p_{0a} = f^{\alpha}{}_{a}\, p_{\alpha} + \frac{1}{2E}\{p_{\alpha}, f^{\alpha}{}_{a} E\}_{-} - \frac{1}{2} S^{cd}\omega_{cda} - \frac{1}{2}\tilde{S}^{cd}\tilde{\omega}_{cda}\,,$$
$$S^{ab} = \frac{i}{4}(\gamma^{a}\gamma^{b} - \gamma^{b}\gamma^{a})\,, \qquad \tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^{a}\tilde{\gamma}^{b} - \tilde{\gamma}^{b}\tilde{\gamma}^{a})\,, \tag{12.3}$$

with $E = \det(e^{a}{}_{\alpha})$ and with vielbeins $f^{\alpha}{}_{a}$ [1], the gauge fields of the infinitesimal generators of translation, and with the two kinds of the spin connection fields: **i.** $\omega_{ab\alpha}$, the gauge fields of S^{ab} and **ii.** $\tilde{\omega}_{ab\alpha}$, the gauge fields of $\tilde{S}^{ab}$.

We make a choice of the spin connection fields of the two kinds on the infinite disc as follows (assuming that there must be some fermion sources causing these spin connections, the study of such sources of the scalar fields $\omega_{st\sigma}$ and $\tilde{\omega}_{ab\sigma}$ are in progress)

$$f^{\sigma}{}_{s'}\,\omega_{st\sigma} = iF_{56}\, f\,\varepsilon_{st}\,\frac{e_{s'\sigma}x^{\sigma}}{(\rho_0)^2} = -\frac{1}{2E}\{p_{\sigma}, Ef^{\sigma}{}_{s'}\}_{-}\,\varepsilon_{st}\,4F_{56}\,,$$
$$f^{\sigma}{}_{s'}\,\tilde{\omega}_{st\sigma} = i\tilde{F}_{56}\, f\,\varepsilon_{st}\,\frac{e_{s'\sigma}x^{\sigma}}{(\rho_0)^2} = -\frac{1}{2E}\{p_{\sigma}, Ef^{\sigma}{}_{s'}\}_{-}\,\varepsilon_{st}\,4\tilde{F}_{56}\,,$$
$$f^{\sigma}{}_{s}\,\tilde{\omega}_{mn\sigma} = -\frac{1}{2E}\{p_{\sigma}\,, Ef^{\sigma}{}_{s}\}_{-}\,4\tilde{F}_{mn}\,,\; \tilde{F}_{mn} = -\tilde{F}_{nm}\,,$$
$$s = 5, 6, \quad \sigma = (5), (6)\,. \tag{12.4}$$

We take the starting action in agreement with the *spin-charge-family* theory for this toy model in $d = (5+1)$, that is the action for a massless spinor ($\mathcal{S}_f$) with the covariant momentum p_{0a} from Eq. (12.3) interacting with gravity only and for the vielbein and the two kinds of the spin connection fields ($\mathcal{S}_b$)

$$\mathcal{S} = \mathcal{S}_b + \mathcal{S}_f\,, \qquad \mathcal{S}_f = \int d^d x\, E\,\mathcal{L}_f$$
$$\mathcal{S}_b = \int d^d x\, E\,(\alpha\mathcal{R} + \tilde{\alpha}\tilde{\mathcal{R}})\,, \qquad \mathcal{L}_f = \psi^{\dagger}\,\gamma^{0}\gamma^{a}\,p_{0a}\,\psi\,. \tag{12.5}$$

The two Riemann scalars, $\mathcal{R} = \mathcal{R}_{abcd}\,\eta^{ac}\eta^{bd}$ and $\tilde{\mathcal{R}} = \tilde{\mathcal{R}}_{abcd}\,\eta^{ac}\eta^{bd}$, are determined by the Riemann tensors

$$\mathcal{R}_{abcd} = \frac{1}{2}\,f^{\alpha}{}_{[a}f^{\beta}{}_{b]}\,(\omega_{cd\beta,\alpha} - \omega_{ce\alpha}\omega^{e}{}_{d\beta})\,,$$
$$\tilde{\mathcal{R}}_{abcd} = \frac{1}{2}\,f^{\alpha}{}_{[a}f^{\beta}{}_{b]}\,(\tilde{\omega}_{cd\beta,\alpha} - \tilde{\omega}_{ce\alpha}\tilde{\omega}^{e}{}_{d\beta})\,, \tag{12.6}$$

where $[a\ b]$ means that the anti-symmetrization must be performed over the two indices a and b.

[1] $f^{\alpha}{}_{a}$ are inverted vielbeins to $e^{a}{}_{\alpha}$ with the properties $e^{a}{}_{\alpha}f^{\alpha}{}_{b} = \delta^{a}{}_{b},\ e^{a}{}_{\alpha}f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \ldots, -1\}$.

We assume no gravity in $d = (3+1)$: $f^{\mu}{}_{m} = \delta^{\mu}_{m}$ and $\omega_{mn\mu} = 0$ for $m, n = (0,1,2,3)$, $\mu = (0,1,2,3)$. Accordingly $(a, b, \dots)$ run in Eq. (12.5) only over $s \in (5,6)$. Taking into account the subgroup structure of the operators $\tilde{S}^{mn}$

$$
\begin{aligned}
\vec{\tilde{N}}_{(L,R)} &= \vec{\tilde{N}}^{\oplus} = \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \\
\tilde{N}_{L}{}^{\genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} &= \tilde{N}^{\oplus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} = \tilde{N}^{\oplus 1} \pm i\,\tilde{N}^{\oplus 2}\,, \quad \tilde{N}_{R}{}^{\genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} = \tilde{N}^{\ominus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} = \tilde{N}^{\ominus 1} \pm i\,\tilde{N}^{\ominus 2}\,,
\end{aligned}
\tag{12.7}
$$

we can rewrite the $\frac{1}{2}\tilde{S}^{cd}\tilde{\omega}_{cda}$ part of the covariant momentum (Eq. 12.3) as follows

$$
\begin{aligned}
-\frac{1}{2}\, f\, \tilde{S}^{mn}\tilde{\omega}_{mn\pm} &= \sum_{i} \tilde{N}^{\oplus i}\tilde{A}^{\oplus i}_{\pm} + \sum_{i} \tilde{N}^{\ominus i}\tilde{A}^{\ominus i}_{\pm} \\
&= \tilde{N}^{\oplus\boxplus}\tilde{A}^{\oplus\boxplus}_{\pm} + \tilde{N}^{\oplus\boxminus}\tilde{A}^{\oplus\boxminus}_{\pm} + \tilde{N}^{\oplus 3}\tilde{A}^{\oplus 3}_{\pm} \\
&\quad + \tilde{N}^{\ominus\boxplus}\tilde{A}^{\ominus\boxplus}_{\pm} + \tilde{N}^{\ominus\boxminus}\tilde{A}^{\ominus\boxminus}_{\pm} + \tilde{N}^{\ominus 3}\tilde{A}^{\ominus 3}_{\pm}\,, \\
\tilde{\omega}_{mn\pm} &= \tilde{\omega}_{mn5} \mp i\tilde{\omega}_{mn6}\,.
\end{aligned}
\tag{12.8}
$$

The notation was used

$$
\begin{aligned}
\tilde{A}^{\oplus i}_{s} = f^{\sigma}{}_{s}\, \tilde{A}^{\oplus i}_{\sigma} &= -f^{\sigma}{}_{s}\{(\tilde{\omega}_{23\sigma} \mp i\tilde{\omega}_{01\sigma}), (\tilde{\omega}_{31\sigma} \mp i\tilde{\omega}_{02\sigma}), (\tilde{\omega}_{12\sigma} \mp i\tilde{\omega}_{03\sigma})\} \\
&= \delta^{\sigma}_{s}\; \frac{1}{2E}\{p_{\sigma}, Ef\}_{-}\; 4\left(\tilde{F}^{\oplus 1}, \tilde{F}^{\oplus 2}, \tilde{F}^{\oplus 3}\right)\,, \\
\tilde{A}^{\oplus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}}_{s} &= \frac{1}{2}\,(\tilde{A}^{\oplus 1}_{s} \mp i\,\tilde{A}^{\oplus 2}_{s})\,, \quad \tilde{A}^{\ominus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}}_{s} = \frac{1}{2}\,(\tilde{A}^{\ominus 1}_{s} \mp i\,\tilde{A}^{\ominus 2}_{s})\,, \\
\tilde{F}^{\oplus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} &= (\tilde{F}^{23} \mp \tilde{F}^{02}) - i(\pm\tilde{F}^{31} + \tilde{F}^{01})\,, \quad \tilde{F}^{\oplus 3} = (\tilde{F}^{12} - i\tilde{F}^{03})\,, \\
\tilde{F}^{\ominus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}} &= (\tilde{F}^{23} \pm \tilde{F}^{02}) + i(\mp\tilde{F}^{31} + \tilde{F}^{01})\,, \quad \tilde{F}^{\ominus 3} = (\tilde{F}^{12} + i\tilde{F}^{03})\,, \\
\sigma &= ((5),(6))\,, \quad s = (5,6)\,,
\end{aligned}
\tag{12.9}
$$

with ω_{abc} and $\tilde{\omega}_{abc}$ defined in Eq. (12.4).

We looked in the ref. [3] for the chiral fermions on this sphere, that is for the fermions of only one handedness in $d = (3+1)$ and accordingly mass protected, without including any extra fundamental gauge fields to the action from Eq.(12.5).

In this contribution we study the influence of several spin connection fields on the properties of families, looking for the intervals within which the parameters of both kinds of the spin connection fields (F_{56}, $\tilde{F}_{56}$, $\tilde{F}^{\oplus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}}$, $\tilde{F}^{\oplus 3}$, $\tilde{F}^{\ominus \genfrac{}{}{0pt}{}{\boxplus}{\boxminus}}$, $\tilde{F}^{\ominus 3}$) allow massless solutions of the equation

$$
\begin{aligned}
&\{\gamma^{0}\gamma^{m}p_{m} + f\gamma^{0}\gamma^{s}\delta^{\sigma}_{s}(p_{0\sigma} + \frac{1}{2Ef}\{p_{\sigma}, Ef\}_{-})\}\psi = 0, \quad \text{with} \\
&p_{0\sigma} = p_{\sigma} - \frac{1}{2}S^{st}\omega_{st\sigma} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\sigma}\,,
\end{aligned}
\tag{12.10}
$$

for several families of spinors.

We also allow nonzero vacuum expectation values of the scalar (with respect to $d = (3+1)$) gauge fields and study properties of spinors.

The discrete symmetries of the equations of motion and of solutions are studied in sections (12.3.1, 12.3).

We look for the properties of spinors and gauge fields, scalars and vectors with respect to $d = (3 + 1)$.

In section 12.2 we present spinor states in "our technique" (see appendix in the ref. [7]). In section 12.2.1 we discuss massless and massive states of families of spinors. In section 12.3.1 we present discrete symmetry operators introduced in the refs. [9,10], in section 12.3 we discuss the properties of spinors and the gauge fields, the zweibein and the two kinds of the spin connection fields, under the discrete symmetry operators.

12.2 Solutions of equations of motion for families of spinors

We first briefly explain, following the refs. [5,1–3], the appearance of families in our toy model, using what is called the technique [12].

There are $2^{d/2-1} = 4$ families in our toy model, each family with $2^{d/2-1} = 4$ members. In the technique [12] the states are defined as a product of nilpotents and projectors

$$\stackrel{ab}{(\pm i)}: = \frac{1}{2}(\gamma^a \mp \gamma^b),\ \stackrel{ab}{[\pm i]}:= \frac{1}{2}(1 \pm \gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = -1,$$
$$\stackrel{ab}{(\pm)}: = \frac{1}{2}(\gamma^a \pm i\gamma^b),\ \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a \gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = 1, \qquad (12.11)$$

which are the eigen vectors of S^{ab} as well as of $\tilde{S}^{ab}$ as follows

$$S^{ab} \stackrel{ab}{(k)} = \frac{k}{2} \stackrel{ab}{(k)},\ S^{ab} \stackrel{ab}{[k]} = \frac{k}{2} \stackrel{ab}{[k]},\ \tilde{S}^{ab} \stackrel{ab}{(k)} = \frac{k}{2} \stackrel{ab}{(k)},\ \tilde{S}^{ab} \stackrel{ab}{[k]} = -\frac{k}{2} \stackrel{ab}{[k]}\,, \qquad (12.12)$$

with the properties that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$

$$\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]},\ \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]},\ \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)},\ \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)}\,,$$
$$\tilde{\gamma}^a \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]},\ \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]},\ \tilde{\gamma}^a \stackrel{ab}{[k]} = i \stackrel{ab}{(k)},\ \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)} \qquad (12.13)$$

After making a choice of the Cartan subalgebra, for which we take: (S^{03}, S^{12}, S^{56}) and $(\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56})$, the four spinor families, each with four vectors, which are eigen vectors of the chosen Cartan subalgebra with the eigen values from Eq. (12.12) [3], follow

$$\varphi^{1I}_1 = \stackrel{56}{(+)}\stackrel{03}{(+i)}\stackrel{12}{(+)}\ \psi_0, \qquad \varphi^{1II}_1 = \stackrel{56}{(+)}\stackrel{03}{[+i]}\stackrel{12}{[+]}\ \psi_0,$$
$$\varphi^{1I}_2 = \stackrel{56}{(+)}\stackrel{03}{[-i]}\stackrel{12}{[-]}\ \psi_0, \qquad \varphi^{1II}_2 = \stackrel{56}{(+)}\stackrel{03}{(-i)}\stackrel{12}{(-)}\ \psi_0,$$
$$\varphi^{2I}_1 = \stackrel{56}{[-]}\stackrel{03}{[-i]}\stackrel{12}{(+)}\ \psi_0, \qquad \varphi^{2II}_1 = \stackrel{56}{[-]}\stackrel{03}{(-i)}\stackrel{12}{[+]}\ \psi_0,$$
$$\varphi^{2I}_2 = \stackrel{56}{[-]}\stackrel{03}{(+i)}\stackrel{12}{[-]}\ \psi_0, \qquad \varphi^{2II}_2 = \stackrel{56}{[-]}\stackrel{03}{[+i]}\stackrel{12}{(-)}\ \psi_0,$$

$$
\begin{aligned}
\varphi_1^{1III} &= \stackrel{56}{[+]}\stackrel{03}{[+i]}\stackrel{12}{(+)}\,\psi_0, & \varphi_1^{1IV} &= \stackrel{56}{[+]}\stackrel{03}{(+i)}\stackrel{12}{[+]}\,\psi_0,\\
\varphi_2^{1III} &= \stackrel{56}{[+]}\stackrel{03}{(-i)}\stackrel{12}{[-]}\,\psi_0, & \varphi_2^{1IV} &= \stackrel{56}{[+]}\stackrel{03}{[-i]}\stackrel{12}{(-)}\,\psi_0,\\
\varphi_1^{2III} &= \stackrel{56}{(-)}\stackrel{03}{(-i)}\stackrel{12}{(+)}\,\psi_0, & \varphi_1^{2IV} &= \stackrel{56}{(-)}\stackrel{03}{[-i]}\stackrel{12}{[+]}\,\psi_0,\\
\varphi_2^{2III} &= \stackrel{56}{(-)}\stackrel{03}{[+i]}\stackrel{12}{[-]}\,\psi_0, & \varphi_2^{2IV} &= \stackrel{56}{(-)}\stackrel{03}{(+i)}\stackrel{12}{(-)}\,\psi_0,
\end{aligned}
\tag{12.14}
$$

where ψ_0 is a vacuum for the spinor state. One can reach from the first member φ_1^{1I} of the first family the same family member of all the other families by the application of $\tilde{S}^{ab}$. One can reach all the family members of each family by applying the generators S^{ab} on one of the family member. If we write the operators of handedness in $d = (5+1)$ as $\Gamma^{(5+1)} = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^5\gamma^6$ $(= 2^3 i S^{03}S^{12}S^{56})$, in $d = (3+1)$ as $\Gamma^{(3+1)} = -i\gamma^0\gamma^1\gamma^2\gamma^3$ $(= 2^2 i S^{03}S^{12})$ and in the two dimensional space as $\Gamma^{(2)} = i\gamma^5\gamma^6$ $(= 2S^{56})$, we find that all the states of all the families are left handed with respect to $\Gamma^{(5+1)}$, with the eigen value -1, the first two states of the first family, and correspondingly the first two states of any family, are right handed and the second two states are left handed with respect to $\Gamma^{(2)}$, with the eigen values 1 and -1, respectively, while the first two are left handed and the second two right handed with respect to $\Gamma^{(3+1)}$ with the eigen values -1 and 1, respectively.

Having the rotational symmetry around the axis perpendicular to the plane of the fifth and the sixth dimension we require that $\psi^{(6)}$ is the eigen function of the total angular momentum operator $M^{56} = x^5p^6 - x^6p^5 + S^{56} = -i\frac{\partial}{\partial\phi} + S^{56}$

$$
M^{56}\psi^{(6)} = (n + \frac{1}{2})\,\psi^{(6)}\,. \tag{12.15}
$$

Accordingly we write, when taking into account Eq. (12.14), the most general wave function $\psi^{(6)}$ obeying Eq. (12.10) in $d = (5+1)$ as

$$
\psi^{(6)} = \mathcal{N} \sum_{i=I,II,III,IV} (\mathcal{A}^i_n \stackrel{56}{(+)}{}^i\,\psi^{(4i)}_{(+)} + \mathcal{B}^i_{n+1}\,e^{i\phi}\,\stackrel{56}{[-]}{}^i\,\psi^{(4i)}_{(-)})\,e^{in\phi}. \tag{12.16}
$$

where $\mathcal{A}^i_n$ and $\mathcal{B}^i_n$ depend on x^σ, while $\psi^{(4i)}_{(+)}$ and $\psi^{(4i)}_{(-)}$ determine the spin and the coordinate dependent parts of the wave function $\psi^{(6)}$ in $d = (3+1)$ in accordance with the definition in Eq.(12.14), for example,

$$
\begin{aligned}
\psi^{(4I)}_{(+)} &= \alpha^I_+\ \stackrel{03}{(+i)}\stackrel{12}{(+)} + \beta^I_+\ \stackrel{03}{[-i]}\stackrel{12}{[-]},\\
\psi^{(4I)}_{(-)} &= \alpha^I_-\ \stackrel{03}{[-i]}\stackrel{12}{(+)} + \beta^I_-\ \stackrel{03}{(+i)}\stackrel{12}{[-]}.
\end{aligned}
\tag{12.17}
$$

$\stackrel{56}{(+)}{}^i = \stackrel{56}{(+)}$, for $i = I, II$ and $\stackrel{56}{(+)}{}^i = \stackrel{56}{[+]}$ for $i = III, IV$, while $\stackrel{56}{[-]}{}^i = \stackrel{56}{[-]}$ for $i = I, II$ and $\stackrel{56}{[-]}{}^i = \stackrel{56}{(-)}$ for $i = III, IV$. Using $\psi^{(6)}$ in Eq. (12.10) and separating dynamics in $(1+3)$ and on S^2, the following relations follow, from which we recognize the mass term m^I: $\frac{\alpha^i_+}{\alpha^i_-}(p^0 - p^3) - \frac{\beta^i_+}{\alpha^i_-}(p^1 - ip^2) = m^i$, $\frac{\beta^i_+}{\beta^i_-}(p^0 + p^3) - \frac{\alpha^i_+}{\beta^i_-}(p^1 + ip^2) = m^i$,

$\frac{\alpha^i_-}{\alpha^i_+}(p^0+p^3)+\frac{\beta_-}{\alpha_+}(p^1-ip^2) = m^i$, $\frac{\beta^i_-}{\beta^i_+}(p^0-p^3)+\frac{\alpha^i_-}{\beta^i_+}(p^1-ip^2) = m^i$. (One notices that for massless solutions ($m^i = 0$) $\psi^{(4i)}_{(+)}$ and $\psi^{(4i)}_{(-)}$, for each $i = I, II, III, IV$, decouple.)

For a spinor with the momentum $p^m = (p^0, 0, 0, p^3)$ in $d = (3+1)$ the spin and coordinate dependent parts for four families are: $\psi^{(4I)}_{(+)} = \alpha \stackrel{03}{(+i)} \stackrel{12}{(+)}$, $\psi^{(4II)}_{(+)} = \alpha \stackrel{03}{[+i]} \stackrel{12}{[+]}$, $\psi^{(4III)}_{(+)} = \alpha \stackrel{03}{[+i]} \stackrel{12}{(+)}$, $\psi^{(4IV)}_{(+)} = \alpha \stackrel{03}{(+i)} \stackrel{12}{[+]}$.

Taking the above derivation into account (Eqs. (12.16, 12.2, 12.4, 12.17, 12.7, 12.8, 12.9)) the equation of motion for spinors follows [3] from the action (12.5)

$$\begin{aligned}
if \ \ &\{e^{i\phi 2S^{56}} [(\frac{\partial}{\partial\rho} + \frac{i\,2S^{56}}{\rho}(\frac{\partial}{\partial\phi})) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 - 2F_{56}\,2S^{56} - 2\tilde{F}_{56}\,2\tilde{S}^{56}\\
&-2\tilde{F}^{\ominus\boxplus}\,2\tilde{N}^{\ominus\boxplus} - 2\tilde{F}^{\ominus\boxminus}\,2\tilde{N}^{\ominus\boxminus} - 2\tilde{F}^{\ominus 3}\,2\tilde{N}^{\ominus 3}\\
&-2\tilde{F}^{\oplus\boxplus}\,2\tilde{N}^{\oplus\boxplus} - 2\tilde{F}^{\oplus\boxminus}\,2\tilde{N}^{\oplus\boxminus} - 2\tilde{F}^{\oplus 3}\,2\tilde{N}^{\oplus 3})]\}\psi^{(6)}\\
+\ &\gamma^0\gamma^5\, m\,\psi^{(6)} = 0. \qquad (12.18)
\end{aligned}$$

One easily recognizes that, due to the break of $\mathcal{M}^{(5+1)}$ into $\mathcal{M}^{(3+1)}\times$ an infinite disc, which concerns (by our assumption) both, S^{ab} and $\tilde{S}^{ab}$ sector, there are two times two coupled families: The first and the second, and the third and the fourth, while the first and the second remain decoupled from the third and the fourth. We end up with two decoupled groups of equations of motion [3] (which all depend on the parameters F_{56} and $\tilde{F}_{56}$):

i. The equations for the first and the second family

$$\begin{aligned}
&-if\{[(\frac{\partial}{\partial\rho} - \frac{n}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 - 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3})]\,\mathcal{A}^{I}_{n}\\
&- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,2\tilde{F}^{\ominus\boxplus}\,\mathcal{A}^{II}_{n}\} + m\,\mathcal{B}^{I}_{n+1} = 0\,,
\end{aligned}$$

$$\begin{aligned}
&-if\{[(\frac{\partial}{\partial\rho} + \frac{n+1}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 + 2F_{56} - 2\tilde{F}_{56} - 2\tilde{F}^{\ominus 3})]\,\mathcal{B}^{I}_{n+1}\\
&- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,2\tilde{F}^{\ominus\boxplus}\,\mathcal{B}^{II}_{n+1}\} + m\,\mathcal{A}^{I}_{n} = 0\,, \qquad (12.19)
\end{aligned}$$

$$\begin{aligned}
&-if\{[(\frac{\partial}{\partial\rho} - \frac{n}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 - 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3})]\,\mathcal{A}^{II}_{n}\\
&- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,2\tilde{F}^{\ominus\boxminus}\,\mathcal{A}^{I}_{n}\} + m\,\mathcal{B}^{II}_{n+1} = 0\,,
\end{aligned}$$

$$\begin{aligned}
&-if\{[(\frac{\partial}{\partial\rho} + \frac{n+1}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 + 2F_{56} - 2\tilde{F}_{56} + 2\tilde{F}^{\ominus 3})]\,\mathcal{B}^{II}_{n+1}\\
&- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,2\tilde{F}^{\ominus\boxminus}\,\mathcal{B}^{I}_{n+1}\} + m\,\mathcal{A}^{II}_{n} = 0\,.
\end{aligned}$$

ii. The equations for the third and the fourth family

$$-if\{[(\frac{\partial}{\partial\rho} - \frac{n}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 - 2F_{56} + 2\tilde{F}_{56} - 2\tilde{F}^{\oplus 3})]\,\mathcal{A}^{III}_{n}$$
$$- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,(-2\tilde{F}^{\oplus\boxplus})\,\mathcal{A}^{IV}_{n}\} + m\,\mathcal{B}^{III}_{n+1} = 0\,,$$

$$-if\{[(\frac{\partial}{\partial\rho} + \frac{n+1}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 + 2F_{56} + 2\tilde{F}_{56} - 2\tilde{F}^{\oplus 3})]\,\mathcal{B}^{III}_{n+1}$$
$$- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,(-2\tilde{F}^{\oplus\boxplus})\,\mathcal{B}^{IV}_{n+1}\} + m\,\mathcal{A}^{III}_{n} = 0\,, \tag{12.20}$$

$$-if\{[(\frac{\partial}{\partial\rho} - \frac{n}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 - 2F_{56} + 2\tilde{F}_{56} + 2\tilde{F}^{\oplus 3})]\,\mathcal{A}^{IV}_{n}$$
$$- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,(-2\tilde{F}^{\oplus\boxminus})\,\mathcal{A}^{III}_{n}\} + m\,\mathcal{B}^{IV}_{n+1} = 0\,,$$

$$-if\{[(\frac{\partial}{\partial\rho} + \frac{n+1}{\rho}) - \frac{1}{2f}\frac{\partial f}{\partial\rho}(1 + 2F_{56} + 2\tilde{F}_{56} + 2\tilde{F}^{\oplus 3})]\,\mathcal{B}^{IV}_{n+1}$$
$$- \frac{1}{2f}\frac{\partial f}{\partial\rho}\,(-2\tilde{F}^{\oplus\boxminus})\,\mathcal{B}^{III}_{n+1}\} + m\,\mathcal{A}^{IV}_{n} = 0\,.$$

Let us look for possible normalizable [1,2] massless solutions for each of the two groups in dependence on the parameters which determine the strength of the spin connection fields. Both groups, although depending on different parameters of the spin connection fields, can be treated in an equivalent way. Let us therefore study massless solutions of the first group of equations of motion.

For $m = 0$ the equations for $\mathcal{A}^{I}_{n}$ and $\mathcal{A}^{II}_{n}$ in Eq. (12.19) decouple from those for $\mathcal{B}^{I}_{n+1}$ and $\mathcal{B}^{II}_{n+1}$. We get for massless solutions

$$\begin{aligned}
\mathcal{A}^{I\pm}_{n} &= a_{\pm}\,\rho^{n}\,f^{\frac{1}{2}(1-2F_{56}-2\tilde{F}_{56})}\,f^{\pm\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}}\,,\\
\mathcal{A}^{II\pm}_{n} &= \frac{\pm\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}+\tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus\boxplus}}\,\mathcal{A}^{I\pm}_{n}\,,\\
\mathcal{B}^{I\pm}_{n+1} &= b_{\pm}\,\rho^{-n-1}\,f^{\frac{1}{2}(1+2F_{56}-2\tilde{F}_{56})}\,f^{\pm\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}}\,,\\
\mathcal{B}^{II\pm}_{n+1} &= \frac{\pm\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}+\tilde{F}^{\ominus 3}}{\tilde{F}^{\ominus\boxplus}}\,\mathcal{B}^{I\pm}_{n+1}\,,
\end{aligned} \tag{12.21}$$

n is a positive integer. The solutions $(\mathcal{A}^{I+}_{n}, \mathcal{A}^{II+}_{n})$ and $(\mathcal{A}^{I-}_{n}, \mathcal{A}^{II-}_{n})$ are two independent solutions, a general solution is any superposition of these two. Similarly is true for $(\mathcal{B}^{I\pm}_{n+1}, \mathcal{B}^{II\pm}_{n+1})$.

In the massless case also $\mathcal{A}^{I,II\pm}_{n}$ decouple from $\mathcal{B}^{I,II\pm}_{n+1}$.

One can easily write down massless solutions of the second group of two families, decoupled from the first one, when knowing massless solutions of the first group of families. It follows

$$
\begin{aligned}
\mathcal{A}_n^{III\pm} &= a_\pm\, \rho^n\, f^{\frac{1}{2}(1-2F_{56}+2\tilde{F}_{56})}\; f^{\pm\sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}}\,,\\
\mathcal{A}_n^{IV\pm} &= \frac{\pm\sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}+\tilde{F}^{\oplus 3}}{-\tilde{F}^{\oplus\boxplus}}\; \mathcal{A}_n^{III\pm}\,,\\
\mathcal{B}_{n+1}^{III\pm} &= b_\pm\, \rho^{-n-1}\, f^{\frac{1}{2}(1+2F_{56}+2\tilde{F}_{56})}\; f^{\pm\sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}}\,,\\
\mathcal{B}_{n+1}^{IV\pm} &= \frac{\pm\sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}+\tilde{F}^{\oplus 3}}{-\tilde{F}^{\oplus\boxplus}}\; \mathcal{B}_{n+1}^{III\pm}\,,
\end{aligned}
\tag{12.22}
$$

n is a positive integer, $a_\pm$ and $b_\pm$ are normalization factors.

Requiring that only normalizable (square integrable) solutions are acceptable

$$
2\pi \int_0^\infty E\, \rho d\rho\, (\mathcal{A}_n^{i\star}\mathcal{A}_n^i + \mathcal{B}_n^{i\star}\mathcal{B}_n^i) \;<\infty\,,
\tag{12.23}
$$

$i \in \{I, II, III, IV\}$, one finds that $\mathcal{A}_n^i$ and $\mathcal{B}_n^i$ are normalizable [1,2] under the following conditions

$$
\begin{aligned}
\mathcal{A}_n^{I,II} &:\; -1 < n < 2(F_{56}+\tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}\;)\,,\\
\mathcal{B}_n^{I,II} &:\; 2(F_{56}-\tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}\;) < n < 1\,,\\
\mathcal{A}_n^{III,IV} &:\; -1 < n < 2(F_{56}-\tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}\;)\,,\\
\mathcal{B}_n^{III,IV} &:\; 2(F_{56}+\tilde{F}_{56} \pm \sqrt{(\tilde{F}^{\oplus 3})^2+\tilde{F}^{\oplus\boxplus}\tilde{F}^{\oplus\boxminus}}\;) < n < 1\,.
\end{aligned}
\tag{12.24}
$$

One immediately sees that for $F_{56} = 0 = \tilde{F}_{56}$ there is no solution for the zweibein from Eq. (12.2). Let us first assume that $\tilde{F}^{\oplus i} = 0$; $i \in \{1,2,3\}$. Eq. (12.24) tells us that the strengths $F_{56}, \tilde{F}_{56}$ of the spin connection fields ($\omega_{56\sigma}$ and $\tilde{\omega}_{56\sigma}$) can make a choice between the massless solutions ($\mathcal{A}_n^{I,II}$, $\mathcal{A}_n^{III,IV}$) and ($\mathcal{B}_n^{I,II}, \mathcal{B}_n^{III,IV}$):

For

$$
0 < 2(F_{56}+\tilde{F}_{56}) \le 1, \quad \tilde{F}_{56} < F_{56}
\tag{12.25}
$$

there exist four massless left handed solutions with respect to $(3+1)$. For

$$
0 < 2(F_{56}+\tilde{F}_{56}) \le 1, \quad \tilde{F}_{56} = F_{56}
\tag{12.26}
$$

the only massless solution are the two left handed spinors with respect to $(3+1)$

$$
\psi^{(6\ I,II) m=0}_{\frac{1}{2}} = \mathcal{N}_0\, f^{-F_{56}-\tilde{F}_{56}+1/2}\, \overset{56}{(+)}\, \psi^{(4\ I,II)}_{(+)}\,.
\tag{12.27}
$$

The solutions (Eq.12.27) are the eigen functions of M^{56} with the eigen value $1/2$. Since no right handed massless solutions are allowed, the left handed ones are mass protected. For the particular choice $2(F_{56}+\tilde{F}_{56}) = 1$ the spin connection fields $-S^{56}\omega_{56\sigma} - \tilde{S}^{56}\tilde{\omega}_{56\sigma}$ compensate the term $\frac{1}{2Ef}\{p_\sigma, Ef\}_-$ and the left handed spinor with respect to $d = (1+3)$ becomes a constant with respect to ρ and ϕ. To make one of these two states massive, one can try to include terms like $\tilde{F}^{\oplus i}$.

Let us keep $\tilde{F}^{\oplus i} = 0 \;\; i \in \{1,2,3\}$ and $F_{56} = \tilde{F}_{56}$, while we take $\tilde{F}^{\ominus 3}, \tilde{F}^{\ominus \boxplus}$ non zero. Now it is still true that due to the conditions in Eq. (12.24) there are no massless solutions determined by $\mathcal{A}^{III,IV}$ and $\mathcal{B}^{III,IV}$. There is now only one massless and mass protected family for $F_{56} = \tilde{F}_{56}$. In this case the solutions $\mathcal{A}^{I-}_0$ and $\mathcal{A}^{II-}_0$ are related

$$
\begin{aligned}
\mathcal{A}^{I-}_0 &= \mathcal{N}^-_0 \; f^{\frac{1}{2}[1-2F_{56}-2\tilde{F}_{56}-2\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}]}\,,\\
\mathcal{A}^{II-}_0 &= -\frac{(\sqrt{(\tilde{F}^{\ominus 3})^2+\tilde{F}^{\ominus\boxplus}\tilde{F}^{\ominus\boxminus}}+\tilde{F}^{\ominus 3})}{\tilde{F}^{\ominus\boxplus}}\,\mathcal{A}^{I-}_0\,.
\end{aligned}
\tag{12.28}
$$

There exists, however, one additional massless state, with $\mathcal{A}^{I+}_0$ related to $\mathcal{A}^{II+}_0$ and $\mathcal{B}^{I+}_0$ related to $\mathcal{B}^{II+}_0$, which fulfil Eq. (12.24). But since we have left and right handed massless solution present, it is not mass protected any longer.

One can make a choice as well that none of solutions would be massless.

According to Eq. (12.38) from sect. 12.3 the equation of motion presented in Eqs. (12.5, 12.3) are covariant with respect to the discrete symmetry operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ (Eq. (12.35)), what means that the antiparticle feels the transformed gauge fields and carry the opposite charge with respect to the starting particle.

Let us conclude this section by recognizing that for $\tilde{F}^{\oplus\boxplus} = 0$ and $\tilde{F}^{\ominus\boxplus} = 0$ all the families decouple. There is then the choice of the parameters (F_{56} , $\tilde{F}_{56}$, $\tilde{F}^{\oplus 3}$, $\tilde{F}^{\ominus 3}$) which determine how many massless and mass protected families exist, if any.

12.2.1 Solutions after the scalar gauge fields gain nonzero vacuum expectation values

Let us now assume that the spin connection fields gain nonzero vacuum expectation values

$$
\begin{aligned}
m^{(56)}_{\pm} &: = < \omega_{56\pm} >, \quad \tilde{m}^{(56)}_{\pm} :=< \tilde{\omega}_{56\pm} >,\\
\tilde{m}^{(\tilde{N}_R i)}_{\pm} &: = (< \tilde{\omega}_{23\pm} - i\tilde{\omega}_{01\pm} >, < \tilde{\omega}_{31\pm} - i\tilde{\omega}_{02\pm} >, < \tilde{\omega}_{12\pm} - i\tilde{\omega}_{03\pm} >)\,,\\
\tilde{m}^{(\tilde{N}_L i)}_{\pm} &: = (< \tilde{\omega}_{23\pm} + i\tilde{\omega}_{01\pm} >, < \tilde{\omega}_{31\pm} + i\tilde{\omega}_{02\pm} >, < \tilde{\omega}_{12\pm} + i\tilde{\omega}_{03\pm} >)\,,
\end{aligned}
\tag{12.29}
$$

breaking the charge S^{56} symmetry, as well as all the "tilde charges" ($\tilde{S}^{56}$, $\vec{\tilde{N}}^{(R,L)}$).

Then the equation of motion (12.10) can be rewritten as

$$
\begin{aligned}
&\{\gamma^0\gamma^m p_{0m} + \gamma^0 \sum_{+,-}^{56} (\pm) \;\; p_{0\pm}\}\psi = 0\,,\\
&p_{0m} = p_m - S^{56}\,\omega_{56m}\,,\\
&p_{0\pm} = p_{\pm} - S^{56}\,m^{(56)}_{\pm} - \tilde{S}^{56}\,\tilde{m}^{(56)}_{\pm} - \sum_{i=1}^{3} \tilde{N}^i_R\,\tilde{m}^{(\tilde{N}_R i)}_{\pm} - \sum_{i}^{3} \tilde{N}^i_L\,\tilde{m}^{(\tilde{N}_R i)}_{\pm}
\end{aligned}
\tag{12.30}
$$

One finds that requiring the hermiticity of the equations of motion (Eq. (12.30)) leads to the relations

$$-m^{(56)}_{+} = m^{(56)}_{-}\,,\; \tilde{m}^{(56)}_{+} = \tilde{m}^{(56)}_{-}\,,\; \tilde{m}^{(\tilde{N}_R i)}_{+} = \tilde{m}^{(\tilde{N}_R i)}_{-}\,,\; \tilde{m}^{(\tilde{N}_L i)}_{+} = \tilde{m}^{(\tilde{N}_L i)}_{-} \tag{12.31}$$

We also must require, to be consistent with the definition and the Eqs. (12.35, 12.36, 12.37, 12.38) and Eq. (12.30), that

$$\begin{aligned}
\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\, m^{(56)}_{\pm}\, (\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}})^{-1} &= -m^{(56)}_{\mp}\,,\\
\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\, \tilde{m}^{(56)}_{\pm}\, (\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}})^{-1} &= \tilde{m}^{(56)}_{\mp}\,,\\
\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\, \tilde{m}^{(\tilde{N}_R i)}_{\pm}\, (\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}})^{-1} &= \tilde{m}^{(\tilde{N}_R i)}_{\mp}\,,\\
\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}\, \tilde{m}^{(\tilde{N}_L i)}_{\pm}\, (\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}})^{-1} &= \tilde{m}^{(\tilde{N}_L i)}_{\mp}\,.
\end{aligned} \tag{12.32}$$

Eq. (12.30) has then the solutions

$$\begin{aligned}
m^{\tilde{N}_L}_{1,2} &= \frac{1}{2}\,(m^{(56)}_{-} - \tilde{m}^{(56)}_{-}) \pm \sqrt{\sum_i (\tilde{m}^{(\tilde{N}_L i)}_{-})^2}\,,\\
m^{\tilde{N}_R}_{1,2} &= \frac{1}{2}\,(m^{(56)}_{-} + \tilde{m}^{(56)}_{-}) \pm \sqrt{\sum_i (\tilde{m}^{(\tilde{N}_R i)}_{-})^2}\,,
\end{aligned} \tag{12.33}$$

with the spinor states with no conserved charge S^{56} any longer

$$\begin{aligned}
\psi^{(6)\tilde{N}_L}_{m(1,2)} &= N^{\tilde{N}_L}\,\{(\tilde{m}^{(\tilde{N}_L 3)}_{-} \pm \sqrt{\sum_i (\tilde{m}^{(\tilde{N}_L i)}_{-})^2}\,)\,(\,\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{[+]} - \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(-)})\\
&+ (\tilde{m}^{(\tilde{N}_L 1)}_{-} + i\tilde{m}^{(\tilde{N}_L 2)}_{-})\,(\,\stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} - \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)})\}e^{-imx^0}\,,\\
\psi^{(6)\tilde{N}_R}_{m(1,2)} &= N^{\tilde{N}_R}\,\{(\tilde{m}^{(\tilde{N}_R 3)}_{-} \mp \sqrt{\sum_i (\tilde{m}^{(\tilde{N}_R i)}_{-})^2}\,)\,(\,\stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{(+)} - \stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{[-]})\\
&+ (\tilde{m}^{(\tilde{N}_R 1)}_{-} - i\tilde{m}^{(\tilde{N}_R 2)}_{-})\,(\,\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{(+)} - \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{[-]})\}e^{-imx^0}\,,
\end{aligned} \tag{12.34}$$

while handedness in the "tilde" sector is conserved.

12.3 Discrete symmetries of spinors and gauge fields of the toy model

In the subsection of this section 12.3.1 the discrete symmetry operators for particles and antiparticles in the second quantized picture are presented, as well as for the gauge fields. This definition for the discrete symmetry operators, as they manifest from the point of view of $d = (3 + 1)$, is designed for all the Kaluza-Klein like theories. At least this way of looking for the appropriate discrete symmetry operators from the point of view of $d = (3 + 1)$ can be helpful in all the Kaluza-Klein cases to find the appropriate discrete symmetry operators in the observable dimensions.

One sees that the operators of discrete symmetries, presented in Eqs. (12.39, 12.41, 12.42), do not depend on the family quantum numbers $\tilde{\gamma}^a$, which means that every particle, described as a member of one family, transforms under the product of the two discrete symmetry operators $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$, presented in Eq. (12.39) and Eq. (12.42), into the corresponding antiparticle state, which belongs to the same family (carrying the same family quantum numbers).

The discrete symmetry operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ (Eqs. (12.39, 12.42)) is in our case with $d = (5+1)$ equal to

$$\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} = \gamma^0 \gamma^5 I_{\vec{x}_3} I_{x_6} . \tag{12.35}$$

It has an even number of γ^a's, which guarantees that the operation does not cause the transformation into another Weyl representation in $d = (5+1)$, which means that we stay within the Weyl representation from which we started.

Let us check what does this discrete symmetry operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ do when being applied on several operators.

One easily finds

$$\begin{aligned}
\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} (\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5, \gamma^6) (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} &= (-\gamma^0, \gamma^1, \gamma^2, \gamma^3, -\gamma^5, \gamma^6) , \\
\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} (p^0, p^1, p^2, p^3, p^5, p^6) (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} &= (p^0, -p^1, -p^2, -p^3, p^5, -p^6) , \\
\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \stackrel{56}{(\pm)} (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} &= \stackrel{56}{(\mp)} , \\
\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \tilde{\gamma}^a (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} &= \tilde{\gamma}^a , \text{ for each } a ,
\end{aligned}$$

$$\begin{aligned}
&\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} (\omega_{565}(x^0, \vec{x}_3, x^5, x^6), \omega_{566}(x^0, \vec{x}_3, x^5, x^6)) (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} = \\
&\qquad (-\omega_{565}(x^0, -\vec{x}_3, x^5, -x^6), \omega_{566}(x^0, -\vec{x}_3, x^5, -x^6)) , \\
&\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} (\tilde{\omega}_{\tilde{5}\tilde{6}5}(x^0, \vec{x}_3, x^5, x^6), \tilde{\omega}_{\tilde{5}\tilde{6}6}(x^0, \vec{x}_3, x^5, x^6)) (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} = \\
&\qquad (\tilde{\omega}_{\tilde{5}\tilde{6}5}(x^0, -\vec{x}_3, x^5, -x^6), -\tilde{\omega}_{\tilde{5}\tilde{6}6}(x^0, -\vec{x}_3, x^5, -x^6)) ,
\end{aligned} \tag{12.36}$$

where we write $\tilde{\omega}_{\tilde{5}\tilde{6}s}, s = (5, 6)$ to point out that the first two indices belong to the $\widetilde{SO}(5, 1)$ group. We also use the notation $\stackrel{56}{(\pm)} = \frac{1}{2} (\gamma^5 \pm i\gamma^6)$.

One correspondingly finds, taking into account Eqs. (12.7, 12.9)

$$\begin{aligned}
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; S^{56} \; (\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}})^{-1} &= -S^{56} , \\
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; \tilde{S}^{56} \; (\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}})^{-1} &= \tilde{S}^{56} , \\
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; \omega_{56m}(x^0, \vec{x}_3) \; (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} &= -\omega_{56m}(x^0, -\vec{x}_3) , \\
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; (\vec{\tilde{A}}^{\oplus}_5(x^5, x^6), \vec{\tilde{A}}^{\oplus}_6(x^5, x^6)) \; (\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}})^{-1} &= (\vec{\tilde{A}}^{\oplus}_5(x^5, -x^6), -\vec{\tilde{A}}^{\oplus}_6(x^5, -x^6)) , \\
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; \vec{\tilde{A}}^{\oplus}_{\pm}(x^5, x^6) \; (\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}})^{-1} &= \vec{\tilde{A}}^{\oplus}_{\mp}(x^5, -x^6) , \\
\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \; A^{56}_{\pm}(x^5, x^6) \; (\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}})^{-1} &= -A^{56}_{\mp}(x^5, -x^6) ,
\end{aligned} \tag{12.37}$$

with $A^{56}_{\pm} = (\omega_{565} \mp i\omega_{566})$.

From Eqs. (12.36, 12.37) it follows

$$\begin{aligned}
&\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \; \{\gamma^0 \gamma^m (p_m - S^{56} \omega_{56m}(x^0, \vec{x}_3))\} \; (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1} \\
&\quad = \{(-\gamma^0)(-\gamma_m) \; (p^m - (-S^{56}) (-\omega_{56}{}^m)(x^0, -\vec{x}_3))\} \\
&\quad = \{\gamma^0 \gamma^m (p_m - S^{56} \omega_{56m}(x^0, -\vec{x}_3))\}
\end{aligned}$$

$$\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \, \{\gamma^0 \stackrel{56}{(\pm)} \, (p_{\pm} - S^{56}\, \omega_{56\pm}(x^5, x^6) - \frac{1}{2}\, \tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}\pm}(x^5, x^6))\} \, (\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}})^{-1}$$
$$= \{\gamma^0 \stackrel{56}{(\mp)} \, (p_{\mp} - S^{56}\, \omega_{56\mp}(x^5, -x^6) - \frac{1}{2}\, \tilde{S}^{ab}\, \tilde{\omega}_{\tilde{a}\tilde{b}\mp}(x^5, -x^6))\}\,. \tag{12.38}$$

Taking into account Eq. (12.4) and the equations of (12.38), we see that the equations of motion are covariant with respect to a particle and its antiparticle: A particle and its antiparticle carry the same mass, while the antiparticle carries the opposite charge S^{56} than the particle and moves in the transformed $U(1)$ field $-\omega_{56}{}^{m}(x^0, -\vec{x}^3)$ [15].

The equations of motion for our toy model (Eqs. (12.10,12.4), and correspondingly the solutions (Eq. (12.16)) manifest the discrete symmetries $\mathbb{C}_{\mathcal{N}}\cdot \mathcal{P}_{\mathcal{N}}$, $\mathcal{T}_{\mathcal{N}}$ and $\mathbb{C}_{\mathcal{N}}\cdot \mathcal{P}_{\mathcal{N}} \cdot \mathcal{T}_{\mathcal{N}}$, with the operators presented in Eqs. (12.39, 12.42). Both, $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \cdot \Psi^{(6)}$ and $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}} \Psi^{(6)}$ (12.42) solve the equations of motion, provided that $\omega_{56m}(x^0, \vec{x}_3)$ is a real field. The field $\omega_{56m}(x^0, \vec{x}_3)$ transforms under $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ and $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ to $-\omega_{56}{}^{m}(x^0, -\vec{x}_3)$, like the $U(1)$ field must [15].

The starting action (12.5) and the corresponding Weyl equation (12.10) manifest discrete symmetries $\mathbb{C}_{\mathcal{N}}\cdot \mathcal{P}_{\mathcal{N}}$, $\mathcal{T}_{\mathcal{N}}$ and $\mathbb{C}_{\mathcal{N}}\cdot \mathcal{P}_{\mathcal{N}} \cdot \mathcal{T}_{\mathcal{N}}$ from Eqs. (12.39,12.42). Correspondingly all the states with the conserved charges M^{56} respect this symmetry, transforming particle states into the antiparticle states.

12.3.1 Discrete symmetry operators

To discuss properties of the representations of particle and antiparticle states and of the gauge fields with which spinors interact let us first define the discrete symmetry operators as seen from the point of view of $d = (3+1)$ in the second quantized picture as proposed in the ref. [9], where the definition of the discrete symmetries operators for the Kaluza-Klein kind of theories, for the first and the second quantized picture was defined, so that the total angular moments in higher dimensions manifest as charges in $d = (3+1)$. The ref. [9] uses the Dirac sea second quantized picture to make presentation transparent.

The ref. [9] proposes the following discrete symmetry operators

$$\mathcal{C}_{\mathcal{N}} = \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^6, x^8, \ldots, x^d}\,,$$
$$\mathcal{T}_{\mathcal{N}} = \prod_{\Re\gamma^m, m=1}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^0}\, I_{x^5, x^7, \ldots, x^{d-1}}\,,$$
$$\mathcal{P}^{(d-1)}_{\mathcal{N}} = \gamma^0\, \Gamma^{(3+1)}\, \Gamma^{(d)}\, I_{\vec{x}_3}\,. \tag{12.39}$$

The operator of handedness in even d dimensional spaces is defined as

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\, \gamma^a)\,, \tag{12.40}$$

with products of γ^a in ascending order. We choose γ^0, γ^1 real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, alternating imaginary and real up to γ^d real. Operators

I operate as follows:

$$
\begin{aligned}
I_{x^0} x^0 &= -x^0; \\
I_x x^a &= -x^a; \\
I_{x^0} x^a &= (-x^0, \vec{x}); \\
I_{\vec{x}} \vec{x} &= -\vec{x}; \\
I_{\vec{x}_3} x^a &= (x^0, -x^1, -x^2, -x^3, x^5, x^6, \dots, x^d); \\
I_{x^5, x^7, \dots, x^{d-1}} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &= \\
(x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \dots, -x^{d-1}, x^d); \\
I_{x^6, x^8, \dots, x^d} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &= \\
(x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \dots, x^{d-1}, -x^d), \; d = 2n.
\end{aligned}
$$

$\mathcal{C}_{\mathcal{N}}$ transforms the state, put on the top of the Dirac sea, into the corresponding negative energy state in the Dirac sea.

We need the operator, we name [11,10,9] it $\mathbb{C}_{\mathcal{N}}$, which transforms the starting single particle state on the top of the Dirac sea into the negative energy state and then empties this negative energy state. This hole in the Dirac sea is the antiparticle state put on the top of the Dirac sea. Both, a particle and its antiparticle state (both put on the top of the Dirac sea), must solve the Weyl equations of motion.

This $\mathbb{C}_{\mathcal{N}}$ is defined as a product of the operator [11,10] "$emptying$", (which is really an useful operator, although it is somewhat difficult to imagine it, since it is making transformations into a completely different Fock space)

$$
\text{"}emptying\text{"} = \prod_{\Re\gamma^a} \gamma^a \, K = (-)^{\frac{d}{2}+1} \prod_{\Im\gamma^a} \gamma^a \, \Gamma^{(d)} K \,, \tag{12.41}
$$

and $\mathcal{C}_{\mathcal{N}}$

$$
\begin{aligned}
\mathbb{C}_{\mathcal{N}} &= \prod_{\Re\gamma^a, a=0}^{d} \gamma^a \;\; K \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \;\; \Gamma^{(3+1)} \, K \, I_{x^6, x^8, \dots, x^d} \\
&= \prod_{\Re\gamma^s, s=5}^{d} \gamma^s \;\; I_{x^6, x^8, \dots, x^d} \,.
\end{aligned} \tag{12.42}
$$

Let us present also the second quantized notation, following the notation in the ref. [9]. Let $\underline{\Psi}^{\dagger}_p[\Psi_p]$ be the creation operator creating a fermion in the state Ψ_p and let $\Psi^{\dagger}_p(\vec{x})$ be the second quantized field creating a fermion at position $\vec{x}$. Then

$$
\{\underline{\Psi}^{\dagger}_p[\Psi_p] = \int \Psi^{\dagger}_p(\vec{x}) \, \Psi_p(\vec{x}) d^{(d-1)}x \} |vac>
$$

so that the antiparticle state becomes

$$
\{\underline{\mathbb{C}_{\mathcal{N}}} \, \underline{\Psi}^{\dagger}_p[\Psi_p] = \int \Psi_p(\vec{x}) \, (\mathcal{C}_{\mathcal{N}} \, \Psi_p(\vec{x})) d^{(d-1)}x\} |vac> \;.
$$

The antiparticle operator $\underline{\Psi}^{\dagger}_{a}[\Psi_p]$, to the corresponding particle creation operator, can also be written as

$$\underline{\Psi}^{\dagger}_{a}[\Psi_p]\,|vac> = \underline{\mathbb{C}}_{\mathcal{N}}\,\underline{\Psi}^{\dagger}_{p}[\Psi_p]\,|vac> = \int \Psi^{\dagger}_{a}(\vec{x})\,(\mathbb{C}_{\mathcal{N}}\,\Psi_p(\vec{x}))\,d^{(d-1)}x\;|vac>,$$
$$\mathbb{C}_{\mathcal{H}} = \text{"emptying"}\cdot\mathcal{C}_{\mathcal{H}}\,. \tag{12.43}$$

While the discrete symmetry operator $\mathbb{C}_{\mathcal{N}}$ has an odd number of γ^a operators and correspondingly transforms one Weyl representation in $d = (5+1)$ into another Weyl representation in $d = (5+1)$, changing the handedness of the representation, stays the operator $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}$ within the same Weyl. The same is true for $\mathcal{T}_{\mathcal{N}}$ and also for the product $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}\cdot\mathcal{T}_{\mathcal{N}}$.

12.4 Conclusions and discussions

We make in this contribution a small step further with respect to the refs. [3,10] in understanding the existence of massless and mass protected spinors as well as the massive states in non compact spaces in the presence of families of spinors after breaking symmetries. We take a toy model in $\mathcal{M}^{5+1}$, which breaks into $\mathcal{M}^{3+1}\times$ an infinite disc curled into an almost S^2 under the influence of the zweibein. Following the *spin-charge-family* theory we have in this toy model four families. We study properties of families when allowing that besides the spin connection field, which are the gauge field of $S^{st} = \frac{i}{4}(\gamma^s\gamma^t - \gamma^t\gamma^s)$, also the gauge fields of $\tilde{S}^{st} = \frac{i}{4}(\tilde{\gamma}^s\tilde{\gamma}^t - \tilde{\gamma}^t\tilde{\gamma}^s)$, determining families, affect the behaviour of spinors.

We simplify our study by assuming the same radial dependence of all the spin connection fields (Eq. (12.4)), while the strengths of the fields (F_{56}, $\tilde{F}_{56}$, $\tilde{F}_{mn}$) are allowed to vary within some intervals.

We found that the choices of the parameters allow within some intervals of parameters (F_{56}, $\tilde{F}_{56}$, $\tilde{F}_{mn}$) four, two or none massless and mass protected spinors.

We allowed the nonzero vacuum expectation values of all the spin connection fields, $f^{\sigma}{}_{s'}\ \omega_{56\sigma}$, $f^{\sigma}{}_{s'}\ \tilde{\omega}_{\tilde{5}\tilde{6}\sigma}$ and $f^{\sigma}{}_{s'}\ \tilde{\omega}_{\tilde{m}\tilde{n}\sigma}$, where $\sigma = ((5),(6))$, $s = (5,6)$, $\tilde{m} = (\tilde{0},\tilde{1},\tilde{2},\tilde{3})$. All indices $\tilde{a}$ belong to the $\widetilde{SO}(5,1)$ group, while indices a belong to the $SO(5,1)$ group. The nonzero vacuum expectation values of all the gauge fields causes that the $U(1)$ charge (S^{56}) breaks, as well as also all the family quantum numbers, while the handedness in the "tilde" degrees of freedom keep two groups of families non coupled.

We studied also the discrete symmetries of equations of motion and of solutions, for massless and massive states.

We found: **a.** Almost S^2 or any other shape with the symmetry around the axis, perpendicular to the infinite disc, has the rotational symmetry around this axis. But almost S^2 has not the rotational symmetry around the axis which goes through the centre of almost sphere because of the singular point on the southern pole unless we make the translation of the axis. Equivalently the almost torus - infinite disc curled into an almost torus - has no symmetry. **b.** Even number of families stay massless and mass protected for the intervals of parameters. **c.** Non zero vacuum expectation values of the scalar gauge fields break all the charges, while the two handedness in the "tilde" sector keeps the two groups of families separated. **d.** Let us add that while the weak charge and the hyper charge have fractional values in the *spin-cahreg-family* theory in $d = (13 + 1)$, have the scalar fields in this case of $d = (5 + 1)$ integer valued charges.

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13 Influence from Future, Arguments

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Abstract. It is the purpose of the present article to collect arguments for, that there should exist in fact -although not necessarily yet found - some law, which imply an adjustment to special features to occur in the future. In our own "complex action model" we suggest a version in which the "goal" according to which the future is being arranged is to diminish the integral over time and space of the numerical square of the Higgs field. We end by suggesting that optimistically calculated the collected evidences by coincidences runs to that the chance for getting so good agreement by accident would be of the order of only 1 in 30000. In addition we review that the cosmological constant being so small can be considered evidence for some influence backward in time. Anthropic principle may be considered a way of simulating influence backward in time.

Povzetek. Namen tega prispevka je zbrati argumente za trditev, da obstaja nek (najbrž še ne odkrit) naravni zakon, ki dopušča vpliv prihodnosti na dogodke v sedanjosti. Avtor je v "modelu kompleksne akcije", skupaj s sodelavci, predlagal, da je "cilj", ki določa prihodnost, povezan z zmanjšanjem integrala vrednosti kvadrata Higgsovega polja po prostor-času. Avtor meni, da je antropsko načelo lahko način za simulacijo vpliva nazaj v času. Zbere nekaj primerov, ki jih uporabi za argument, da smo vpliv prihodnosti na sedanjost in preteklost že opazili. Predlaga poenoteno sliko enačb gibanja in začetnih pogojev. Predstavi model s kompleksno akcijo, ter povzame napovedi tega modela. Povzame tudi, kako drugi avtorjevi modeli potrjujejo vpliv prihodnjosti na sedanjost. Tudi majhnost kozmološke konstante se da pojasniti s vpivom preteklosti na sedanjost.

13.1 Introduction

Since long the present author and various collaborators[1–4] have speculated on possibilities for a physical theory having in it some preorganization in the sense, that there is some law that adjust initial condition and/or coupling constants so as to arrange for special "goals" to occur in the future. In works with K. Nagao[6,8,7,9–11] we sought to calculate, if effects of an imaginary part of the action of the type of the works with M. Nimomiya[3,4] could be so well hidden, that such a model would be viable. One could even say, that it is speculations about, that future could somehow act back on the present and the past. Usually - since Darwin and Wallace - it is considered (essentially) a fundamental law of nature, that this kind of back action does *not* exist. But is that trustable? In the present article we shall

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collect arguments for the opposite, namely that there truly IS such a back action in time.

I would like to seek to make a kind of review of the evidence for such an influence from the future, and use it as an excuse for talking about some relatively recent works[12], some of which may not immediately seem to be relevant, such as my work with Masao[15,13,14] Ninomiya on "A Novel String Field Theory"[13–15]. My real motivation is to look for what the fine tuning problems for the various coupling constants may tell us about the fundamental laws of physics, which we seek to find [12].

In other words we could say, that I want to investigate if retro causation is possible and plan to argue for that there are indications that it is possible.

Although the idea of having retro causation is generally believed not to be true there were at least one proposal for a theory of that kind proposed, namely the theory about electromagnetic radiation being radiated *in equal amounts backward and forward in time* by Wheeler and Feynman [16].

The work of Feynman and Wheler avoids influence from the future by a discussion of the absorbers of the light emitted backward and forward, a mechanism a priori rather different form the one we use in our complex action model already mentioned; but the quantum mechanics interpretation inspired from the Feynman-Wheeler theory, which is called transactional interpretation and is due to Cramer[17], is the same one as the one supported by our complex action theory.

The plan of this talk about the influence from future will be like this :

- 1)Introduction
- 2) Listing of arguments for influence from future.
- 3) Discussion of Time reversal
- 4)Why should we NOT unite initial state information with equations of motion?
- 5) The finiteness of String Theory may hide in mine and Ninomiyas Novel String Field Theory [13–15] - an influence from the future, and that might be the reason for it being string theory.
- 6) Some fine-tunings as if "God hated the Higgs squared field
- 7) Bennetts and mine argument that at the time the Cosmological Constant must already have hat its value, when densities of energy so low as the present were unknown/did not yet occur.
- 8) The Multiple Point Principle being successful means influence from future.
- 9) If we count optimistically do we have sufficient evidence for a planned universe development?
- 10) At the end we conclude that one must take the possibility seriously.

13.2 Listing of Arguments

Here I should like to list a series of arguments for that there *is* indeed some adjustment going on to achieve some "goals" we may hope to guess some time:

- A) Funny that many religious people imagine, that there is a Governor of the world, if the principle preventing such government were truly valid.

- B) Strange that the laws about the initial conditions and equations of motion behave differently under the CPT-like symmetry (or under time-reversal)
- C) Cosmological constant were very small compared to the energy density in the beginning; how could it then be selected so small, when it had no significance at that time (argument with D. Bennett).
- D) Several evidences for anthropic principle, but mostly physicists do not like it. (Personally I would say: The anthropic principle is much like putting in the experimental fact that we humans exist into the theory; putting in experimental results can always help avoiding finetuning problems, so a good theory should be more ambitious than have to include such an input.)
- E) Multiple Point Principle (almost) successful: Higgs mass, top Yukawa coupling, and Weak scale relative to Planck scale.
- F) Our Complex Action model with Higgs field square taken to dominate gives[12]:
  - 1) n and p+e+antineutrino suppress Higgs field equally much (within errors).
  - 2) The "knee" cut the cosmic ray spectrum down close to the effective Higgs threshold.
  - 3) Nuclear matter has low binding energy.
  - 4) Higgs field in vacuum at lowest Higgs field square.
  - 5) Smallness of weak scale/Higgs field relative to fundamental/Planck scale.
- G)It may be very hard to make an ultraviolet cut off, that does not violate locally in time a little bit. So an ultraviolet meaningful theory may imply influence from future?
- H) General Relativity allows closed time-like loops...(well known to lead to time machines by worm holes etc.)
- I) Horowich and Maldacenas influence backward inside the black hole.
- J) The bad luck of SSC and the - though too little - bad luck of LHC would follow from Higgs machines getting bad luck.
- K)With large extra dimensions there appear in principle a frame dependence of which moments are earlier than which due to the frame motion in the extra dimension directions.
- L) Wheeler space time foam and baby universes imply almost unavoidably influence from future, at least small influences from near future. Baby universes make effective coupling constant depending on very far away influences in e.g. Time.
- M) In String theory in the formulation of Ninomiyas and mine (Novel SFT) the hanging together of "objects to strings, or chains giving strings better, is put in as an initial condition AND IT LOOKS ALSO AS A FINAL STATE CONDITION!

The following arguments are even more theoretical speculation arguments for influence from future:

- N) When we e.g. Astri Kleppes and mine derivation of space time and locality etc. - seek to derive in Random Dynamics e.g. Feynman path integral we get

the complex action and thus future influence from it. And seeking to derive locality we get left with effective couplings, which much like in baby universe theory depends on, what goes on averaged over all space and time.
- O) Were the many e-foldings in inflationary organized in order to get a big universe (a miracle) ?

Somewhat aesthetically arguments form the time reversal symmetry should also be mentioned:

- P) The usual picture: The laws concerning the time development the equations of motion are perfectly invariant under the CPT-symmetry. Nevertheless the initial conditions determining the actual solution to these equations of motion is chosen in a way that makes it look more and more complicated as one progresses forward in time! (This is the law of increasing entropy) Really the mystery is not why finally the world ends up in a state in which one can say almost nothing in a simple way; but we rather should take it that a huge number of states have same probability/ the heat death state. Rather it is the mystery why it ever were in a state that could be described rather simply, the state in early big bang times, with high Hubble expansion rate.
- Q) And even more mysterious we could claim: Why were the Universe in such a special state in the beginning, but do not also end up in such special and simple state? Initial State Versus Development Laws (equations of motion) seem not to have the same symmetry under time reversal (or say instead CPT) Since Newton we have distinguished between initial state information and the laws for the time development. Seeking the great theory beyond the Standard Models our best hope to progress is to unite some of the information about Nature, which we already have in our literature. One lacking unification is the unification of initial state information and the equations of motion. One little may be indicative trouble is that time reversal or better CPT symmetry is valid for equations of motion but NOT for the initial state information!

13.3 Discussion of Time Reversal-like Symmetry

Let us look now a bit on the problem for the usual point of view and thus the argument for influence from the future Q). What are the possibilities?:

- 1 Possibility) CPT symmetry could be the more fundamental and the asymmetry w.r.t. time direction of the initial state information (we know a lot about the start, but the future gets more and more chaotic) could be due to some sort of spontaneous break down, as e.g. in mine and Ninomiyas complex action model:
  In principle the "initial state information" could be put in at any time, but due to some special conditions in a certain time early compared to our era "the actual solution to the equations of motion chosen to be realized (by Nature)" became mainly determined by this certain era early compared our era. This should mean that in that special era the realized solution is arranged to obey some relatively simply rules, e.g. some strongly expanding universe being the rule.

- 2. possibility) The time direction asymmetry might be the more fundamental and the CPT symmetry just some effective result coming out of an a priori time and even CPT noninvariant theory. So the initial state CPT noninvariance were the more fundamental feature, and the CPT symmetry for laws of nature is only some sort of effective or “accidental” symmetry [5]. It is well known that CPT largely follows from Lorentz invariance, so that if it were correct as I have claimed for years, that Lorentz invariance could be a low energy approximation (only for the “poor physicists”), then also CPT would be a low energy limit symmetry.

Taking the first possibility means that you have in principle also the possibility of having some influence from the future, so that our question as to, whether such influence is at all possible, gets answered by yes; but of course the effect may be essentially zero such as the situation of the “spontaneous break down ” is realized, since otherwise we should already have observed it so safely, that we would have had to believe it.

If the time dominating the fixation of the solution as in mine and Ninomiyas model becomes a certain time which is earlier than our time - but not necessarily the very first moment (if such one should exist) - there would be an opposite axis for the entropy running on the other side of this special time era(that dominantly fixes the solution being realized). In other words before the solution-determining time-era the entropy would decrease! So in that “before solution dominating era ” there would formally be influence from the future. Of course, if we lived in such an era, we would invert our time axis and still say, that entropy grows, except if we get contact theoretically or truly to an era with another entropy development axis.

If the second possibility were realized, we should expect Lorentz non-invariant effects in principle. We should namely expect CPT not to be fundamentally true, but then Lorentz invariance could only with violation of other presumably good assumptions be exact.

If we fundamentally did not have Lorentz invariance it could mean that there were in the “fundamental terminology” beyond the Lorentz invariance appearance perhaps some fundamental frame in which the physics would develop strictly causally, in the sense that it would develop more and more chaotic (i.e. increasing entropy) and without any influence from the future. But logically it could nevertheless be so that in some Lorentz frames moving relative to the fundamental frame there could be influence from future.

13.4 Why Not Unite Initial Conditions and Equations of Motion

In looking for a unified theory of all physics, one often finds the idea of seeking to unify the various simple gauge subgroups of the Standard Model gauge group into some simple gauge group such e.g. $SU(5)$ or groups containing $SU(5)$ as a subgroup, such as SO(10). But since making progress towards finding the “theory of everything” is expected to go via successive unifications, one should also possibly imagine other types of unifications. Here we then ask: Should we not unify initial-state conditions with equations of motion? This is actually what our already in this article suggested complex action model (see subsection 13.6) would do. It predicts both (something about) the initial conditions and of course the equations of motion from the form of the action (as usual). In this sense one should really guess the form of the complex action so that we can obtain relations between features of the initial state conditions and the equations of motion. We can say that with a Standard Model real part of the action taken as phenomenologically suggested the dimensional arguments used to predict that the most important part of the imaginary part of the action determining (or at least providing some information concerning initial conditions) and ending with that the mass square term for the Higgs field, are results of of such a unification. So in this sense our results from this Higgs-dominated imaginary part can be considered results of a unification of initial state conditions and equations of motion.

Also the Hawking-Hartle no-boundary assumption for their (and others) quantum gravity gives information about initial state conditions, and thus it should be considered a unification of initial conditions and equations of motion.

But now one may have general worries about - this kind of - unifications of initial conditions and equations of motion, unless one allows for the influence from the future:

In fact the time reversal or the CPT-like symmetry leads to that the unified theory presumably should have such a symmetry, at least both in our complex action theory and in the non-boundary theory there IS cpt-like symmetry, except that the whole theory is on the manifold. Therefore it gets very hard not to have also a final state condition. In fact it seems only to be a spontaneous breaking of the symmetry of this type that is likely to solve the phenomenological problem. But then there appears indeed easily some remaining effect of influence form the future.

13.5 String Theory, Regularization Problem, and Our Novel SFT

Only String Theory Seems to Cope with the Cut Off problem in Nice Way!

Presumably the best argument for believing, that (super)String Theory should be the theory of everything(T.O.E.), is that it does NOT HAVE THE USUAL DIVERGENCE PROBLEM. One might wonder how string theory manages to avoid the problem of divergent loops. It is well know that by summing up the infinitely many loops from the various string states the integrand for the loop 26-momentum obtain a damping factor going with an exponential of the square of the loop momentum. Thus the divergence of the usual type got effectively cut off. A related property of the lowest order scattering amplitudes is, that they for large transverse momenta fall off even with an exponential in the square of the transverse momentum. Since String theory has gravity (almost unavoidably) having such wonderful cut off of loops behavior is remarkable good!

13.5.1 Cut off in the Light of Mine and Ninomiyas Novel String Field Theory

Let us now consider the for the success of the string theory in coping with the divergences plaguing the usual quantum field theories so important Gaussian cut off of the large momenta.

As an orientation let us look at the transverse momentum cut off from the point of view of mine and Ninomiyas novel string field theory:

The momentum of an open string say in our formalism is given by a sum over the "contained "objects, each of which has the variables (J, Π), i.e. 24 momenta J and their conjugates Π, and the total momentum of the open string is proportional to the sum of the even "objects, because the momentum contribution from the odd ones become zero due to their construction as *difference* of conjugate momenta of the two even neighbors. The scattering is in our SFT-model is simply exchanges of "even objects, while no true interaction takes place, only strings are divided and recollected, so that the "even objects in the initial strings get distributed into various final strings.

So how does the limiting/the strong cutting off of the transverse momenta come about in the optic of our model?

Although there is a divergent number "objects in any string in our novel string field theory, these "objects are sitting in chains with strong negative correlation between the momenta of neighbors (in the chains). So any connected piece of such a chain never reaches momenta much bigger than of the order of one over square root of alpha prime $\sqrt{\alpha'}$, except for the momentum assigned the total strings. So if we only split the chains of objects into a few connected pieces we cannot get any combination of the pieces, when recombined to final state strings, to contain big amounts of momenta compared to the alpha prime order of magnitude value$\sqrt{\alpha'}$. It is this restriction that means, that we get in Veneziano model the exponential of the squared momentum falling off amplitudes.

The limitation actually exponentially with the square of the momentum in the exponent, i.e. Gaussianly of the amplitude of scattering for large transverse momenta of strings coming out of collisions of strings in our novel string field theory (SFT) is due to the very strong anti-correlation of the momenta of the "objects - crudely functioning as constituents of the strings so that only very limited momenta are statistically found on connected pieces of object-chains. Since this so important - for the momentum cut off (anti)correlation of the "objects on the chains used for strings is put in as INITIAL and even as FINAL STATE conditions in order to describe the strings by means of "object-chains, one can say that in mine and Ninomiyas SFT we have arranged the transverse momentum cut off effectively by the initial or final states having been assumed to have the appropriate (anti)correlations!

13.5.2 The Limitation of Momenta in Loops

For each limited loop order corresponding in our novel SFT to splitting the "cyclically ordered chains of "objects into a limited number of subchains before being recollected into new "cyclically ordered chains forming the final state strings (depending on the order (of loops)) the amount of momentum, that can be sent out as transverse momentum in a scattering is limited due to the correlations among the "objects (neighboring on the chains). The higher the order though the higher is the effectively allowed order of magnitude of the transverse momentum, corresponding to the well known fact that higher and higher loop order in unitarity corrections to the Veneziano model has a slower and slower fall off for large momenta the higher the order (i.e. the larger the number of loops). Roughly this relevant correlation corresponds to the "stringness" in the sense, that it is also this correlation (between neighboring "objects"), that ensures that very small pieces of strings carry only very little momentum. But have in mind, that in OUR theory the hanging together to strings is only put in as initial state (and even final state) conditions. Even the alpha prime α' scale so needed to make a chance of having a cut off effectively is in our model *only put in as an initial and final state condition* (nothing in the completely trivial and basically non existing dynamics talks about alpha prime!)

So one really in mine and Ninomiyas novel string field theory must ask: String theory cut off, from where does it come?

Generally: When one interacts (locally) with the string in our formalism or in other ones you can only transfer little meaning given by apha prime (inverse square root $\frac{1}{\sqrt{\alpha'}}$) momentum into the scattering. Via Heisenberg uncertainty this is turned into an extension of the strings due to quantum fluctuations. But it is crucial for the effective cut off, that the string hangs piecewise together; if e.g. in mine and Ninomiyas novel SFT you could split the "objects in a way, in which no "objects kept attached to their neighbors almost, then the momentum in the scattering could be much larger, and very likely a divergence problem would reappear.

In fact it is well known that the higher loops one consider in string theory (unitarity corrections to Veneziano model) the slower becomes the coefficient in the Gaussian fall off of the amplitude with the exponential of the square of the transverse momenta. This means that the more pieces the string or in our model the to the strings corresponding "cyclically ordered chains" are cut into and recollected under the scattering, the larger can the transverse momentum become.

If one would attempt to split up the string to be actually built form discretized elements, one would be back in quantum field theory and it would be as hard as usual to avoid divergencies. The continuity of the string or in our novel SFT formulation the cyclically ordered chains is crucial for the achievements w.r.t. avoiding divergencies and keep tranverse momenta low.

13.5.3 Looking for a Cut Off Machinery

Let us now look whereto we are led when we look for a way to make a cut off:

Now I would like to speculate as to where we are led to think, if we which to get sense out of a theory, in e.g. too many dimensions, so that ultraviolet cut off is truly a necessity:

First we could think of modifying geometry or we may seek to keep it:

- 1) Cut offs like lattices which have a discretized geometry.
- 2) Keep e.g. flat geometry or at least a manifold.

In this second case where are we led, if we seek a cut off of the ultraviolet divergencies, but cling to continuous manifold or let us for simplicity say simple Minkowskian geometry (but continuous space and time) ?

If we use point particles with interactions we have no chance to get any form factors to rescue us against the ultraviolet divergencies.(we might though use higher order derivative on the fields in the Lagrangian density, but let us leave that as another possibility). So we are let in the direction, that we must take the particles, with which we want to work, to be composite objects / bound states or rather most importantly extended objects, so that interactions with the various components have the chance to cancel out couplings to very high momentum states (which is what cause the divergencies).

Thus let us at least look towards seeking cut off in direction of bound states:

Let us now think along the line, that we replace the particles, we consider phenomenologically, by bound states or composite structures. That is to say, that looking more deep inside they shall turn out to consist of some "smaller parts "partons say. It is fine that we may then get form factors, since they have the chance to cut off the loop integrals and make them converge.

Now we may talk the language of Bjorken x being the fraction of longitudinal momentum carried by a "parton.

If the partons have non-zero Bjorken x, then you get parton parton scatterings, when the bound states collide and the situation is much like, if the partons really existed and we are back to the point particle play: there will finally result divergencies again.

So if we are looking for avoiding divergencies we are driven in the direction of taken all the Bjorken $x = 0$. But that then in succession means that collision of only a few partons from one particle(=bound state) with partons in the colliding particle(=bound state) will hardly give any momentum transfer, hardly mean even a scattering.

Once assuming $x = 0$ for all the partons we will get negligible momentum transfer by just scattering a few partons with each other; that is too much cutting off. The effective way to get some significant scattering to identify with the scattering of the particles(=bound states), we want phenomenologically, is to exchanges from one bound state to another one a large number(infinitely many) partons. This means we are driven towards a picture, in which a scattering is mainly an exchange of some part of one composite particle with part of another one. But none of the constituents (=partons) truly interact. Rather the constituents individually just continue undisturbed as if not interacting at all!

Remark how we got driven towards the picture of String Theory in mine and Ninomiyas novel string field theory: The bound state, we consider should be composed from constituents not interacting at all!

These constituents or partons, we are driven towards, are of course to be identified with the "objects in Ninomiyas and mine novel SFT(= string field theory); precisely these "objects of our theory do not change at all.

So we for the moment think of "Even Objects as Partons:

Does it matter whether we consider our "Objects as constituents or the true string interpretation definition of the "Objects J from discretizing right and left movers in the string? For this true definition of the "objects" we have to refer to the other article in the present Bled Conference proceedings 2014 on what comes beyond the standard models [15] (starting on p. 184).

Very shortly let us though on the definition of the "Objects say:

Since the "objects are defined as the difference between the values of say the right mover component of $X^\mu(\sigma,\tau) = X^\mu_R(\tau-\sigma) + X^\mu_L(\tau+\sigma)$ i.e. as $J^\mu_{RI} = X^\mu_R(\tau_R(I+1/2)) - X^\mu_R(\tau_R(I-1/2))$ (where $\tau_R = \tau-\sigma$, and we imagine a discretization replacing τ_R by an integer number I instead and let $\tau_R(I \pm 1/2)$ denote the neighboring τ_R points around the point corresponding to I in the discretization.) at two near to each other values of the ONE relevant variable, it is in fact proportional to the derivative of the right mover component. To reconstruct the position field we both have to integrate (or sum) up and we need both left and right. On open strings boundary conditions causes the left and right mover to be the same. But for open strings they are different.

After we have identified the right with left mover "objects for the open string (as the boundary condition for open string leads to) the objects describing an open string sits topologically in a circle, called by us "a cyclically ordered chain of objects. So the topology of the structure describing the open string by us is a circle and and not as the open string itself an interval. But the momenta of the open string is written as a sum over contributions from the "objects sitting along the cyclically ordered chain (the circle). So as long as one can consider a distribution of momenta to the various "objects, we can consider the "objects constituents (for that momentum distribution purpose at least).

So we might ask: Can we forget the string and only think on Our "Objects ?

If you go over to considering the "objects of our model as constituents of the composite particle(described as the string), you ignore the string as not being the right way of thinking of the same theory.

Contrary to the string point of view, in which the string moves internally as it moves along, the "objects are stale and just do not change (Well, their position is a bit more tricky to consider, so we may think of them as free partons). The "objects fit with the constituents not interacting but just being exchanged en block from bound state to bound state. Pieces of String Time Track per Pair of "Objects with Lightlike Sides Time Track of String from Pieces per Pair of "Objects Lightlike Sides The Very Scattering Moment, Only Exchange of Pieces

Whatever the string may develop mechanically after a collision it is an almost pure exchange of parts that take place at the very collision. At least if the hit is only at ONE POINT of the hitting strings, then from locality nothing can happen at other places in the very first moment. So in the limit of infinitely many constituents (like continuum string) the first moment of a scattering ONLY an exchange of pieces can matter. So, if indeed no parton withx different from 0 is allowed in order to make a good cut off bound state theory, then when first partons hit we can ONLY have exchange of pieces interaction: So in this first moment there is in this sense no true scattering! (Like in mine and Ninomias model).

But there is a need for exchange of pieces

If we have $x = 0$ bound states, there would without exchange of pieces be no scattering, no essential momentum transfer at all.

Now I say: We are driven in seeking for a cut off to a theory with a system of particles (corresponding to the strings in string theory) being bound states with all partons having Bjorken $x = 0$, and they scatter only by exchange of pieces. So it is essentially only how one thinks the constituents as distributed between the particles, that change in the scattering.

It is well known that the higher dimensions spacetime has the more severe are the ultraviolet divergencies: High dimensions give ultraviolet divergences.

13.5.4 Rescuing the Species Doubler Problem by Pushing Chiral Charge to Central Station in Extra Dimension

In the Standard Model one has a remarkably tricky cancellation of the chiral anomalies associated with the (chirally coupled) gauge fields. Non of the fermions in Standard model have their "species doubler (with opposite handedness, but same charge combination). So it should after mine and Ninomiyas no-go theorem be impossible to put the Standard Model on a lattice, or for that matter regularize it in gauge invariant way at all. I.e. No cut off should exist, which can keep gauge invariance. The way Norma Mankoc Borstnik and I attempted to escape this problem were the following:

The way we attempted to escape the no-go theorem was by having infinitely large extra dimensions allowing superfluous fermions to be pushed out to infinity.

Let me look at the nogo theorem problem by thinking of the anomaly telling that the chiral charge is not conserved, but has a lack of conservation correction proportional $F\tilde{F}$(with some gauge fields put in for the two F's).

13.5.5 Anomaly way of Looking at No-Go Anomaly Requires Pushing out or Fetching in Chiral Fermions

Because of the anomaly we need locally in space-time to be able to obtain extra chiral fermions in spite of them having conservation laws making that impossible in the regularized theory. In Norma Mankoč Borštniks and mine attempt to cope with Wittens no go theorem we propose to have non-compact extra dimensions: Then the superfluous or missing chiral fermions may be pushed out or be brought in from the infinitely far away in the extra dimensions. You almost bring them out to a mysterious central station for pushed out chiral fermions, from where they may reappear in the practical world later or earlier or somewhere else than from where they were pushed out.

With such central station whereto chiral particles are brought in and out to various places or times in the 3 +1 dimensional world is to be imagined in the model needed (say Norma Mankoc's and mine), then one may suspect that one easily get times mixed up having such an exchange station for chiral fermions. There namely has to be somehow a control that the total number of chiral fermions of a certain type is conserved in the regularized model. But then how to get the information of the creation seemingly of one at a certain point in the 3+1 space time transfered and brought together with the uses or further creations around space time without endangering the no influence from future principle(which we attempt to attack in this article)?

If really the chiral fermions are fundamentally conserved in the regularization scheme here thought upon as the true theory but just seem not to be because they are pushed out to an in the extra dimensions infinitely far away place, it may seem difficult to keep truly no influence from future from the practical 3+1 dimensional point of view. Would one really could have the number of chiral fermions being added to the central station for such fermions pushed out be kept to net zero without some influence back from the future?

13.6 Some Potential Killings of Our Complex Action Turned Out Supporting It.

Funnily enough I have found a few cases, where seemingly arguments against the validity of the complex action model with its influence from future, actually get turned around and leads to evidence for the influence from future instead, because they turn out rather to show that nature has just some number just finetuned almost to solve the problem.

13.6.1 Short Review of Complex Action Model

Let me here review a bit the main point of the theory of the complex action. A priori it would seem obvious that if we took the action $S[history]$ to be complex rather than as assumed in the usually believed theory, then one would immediately see that effects of non-unitarity and if one used classical calculation one would also expect that otherwise real variables would run complex. In other words at first it would look as if the idea of taking the action complex is phenomenologically so bad that any hope is out unless the imaginary part is extremely small; and so if real and imaginary were about equal in size as one would guess there seems at first to be no chance. But that is according to the calculations or estimations on which we are still working not true! Most convincingly this is seen in a Hamiltonian formalism, in which not so surprisingly a complex action would lead to a non-Hermiten Hamiltonian. In fact the main point is that as long time has past since the start, almost certainly the universe developing by the now assumed non-Hermitean Hamiltonian gets increasing probability for being in those states, which have the largest (eigen)values for the antiHermitean part (divided by i) H_I of the Hamiltonian, if we think of having split it as $H = H_R + iH_I$ where then $H_I = \frac{1}{2i}(H - H^\dagger)$. If we now have assumed - as we have to assume to avoid that the Wentzel-Dirac-Feynmann-path integral shall not be divergent due to the imaginary part of the action $S_I[history]$ going to plus infinity - that there is an upper bound on the antiHermitean part H_I or almost equivalently a lower bound on the imaginary part of the action S_I, then we argue that the system after long time will arrive to a superposition of states with their (eigen)value for H_I close to the assumed upper bound. Once we have argued the system to be in such a state we have the suggestive approximation of $H_I \approx$ “upper bound” and can consider the antiHermitean part H_I an approximate c-number and by a time dependent normalization we can completely remove the effect of this antiHermitean part. This crude argument thus allows us to suppose that after all the antiHermiteamn part H_I of the Hamiltonian is not important provided we study what happens in a universe, that is already very old compared to some fundamental scale for the theory provided we have just an upper bound on this antiHermitean part. This may not be totally convincing as written, but we have formal formulations and it is essentially correct but in order not to have troubles with the Born rule of quantum mechanics that one shall the probability for measuring a state by using the numerical square of the coefficient to a normalized states one shall a new inner product which we call $|_Q$ (so that we can write $< b|_Q a >$) with the property that w.r.t. this inner product the Hamiltonian H gets *normal*. Normality means that the antiHermitean part commutes with the Hermitean part i.e. $[H_R, H_I] = 0$. (The Q that occurs as an index to the new inner product $|_Q$ to be used instead of the original inner product | is an operator constructed from the Hamiltonian - using it diagonalization - and then we defined $< a|_Q b >=< a|Q|b >$.)

Even though now we have argued, that one will obtain a time development as if there existed a Hermitean Hamiltonian even, when the true Hamiltonian is not Hermitean, provided one uses the modified inner product $|_Q$, there is one very interesting and important effect of the antiHermitean part H_I or of the imaginary part $S_I[history]$ of the action left: These antiHermitean or imaginary parts determine the initial condition effectively seen! We saw already just above that the antiHermitean part of the Hamiltonian were important for the states into which the likelihood of finding the world got larger and larger as time went on. So effectively in a late stage of the development of the universe it becomes most likely to find that this universe is in a state with a high -i.e. close to the upper bound - value for the (eigen)value of the antiHermitean part H_I. This really means that we shall look at the complex action theory as a model *unifying the initial conditions with the equations of motion*.

Such a unification of course is in principle very wellcome, if one can find it. In the Hamiltonian formalism with a non-Hermitean Hamiltonian one can see that unless one puts the system/world in a state that has absolutely zero component after some eigenvectors of the Hamiltonian, it will go so that as time goes on the various eigenstates in an expansion of the actual state will grow up exponentially with coefficients going as $-it\lambda_i$ where λ_i is the for the coefficient relevant eigenvalue of the non-Hermitean Hamiltonian $H = H_R + iH_I$. Have in mind that for non-Hermitean Hamiltonian of course the eigenvalues λ_i are typically complex. It is of course the imaginary part of λ_i which gives rise to the time development of the numerical value of a coefficient $c_i \exp -t\lambda_i$ to some eigen vector $|\lambda_i >$ (even though these eigenvectors are not orthogonal to each other, one could still imagine using them in expansion). Exponentially soon a rather small collection of the eigenstates with the largest - in the sense of most positive - imaginary parts of their λ_i's will soon take over. Thereby a rather specific development of the universe gets selected out and one can understand that the antiHermitean part of the Hamiltonian can have strong influence on which states one at a late stage in time is likely to find such a universe with non-Hermitean Halmiltonian. Thus it is understandable that there can be something in the statement that the theory unites initial condition theory with equation of motion theory.

Our studies have led to that one may distinguish reasonably defendable ways of extracting the information from a quantum theory with a given action - two different ways especially suggestive in the case complex action - namely 1)“with future” and 2)“without future”.

13.6.2 Guessing the Standard Model Imaginary Part of the Action

At the present conditions in the Universe - but not at all applicable perhaps in the early times just after a possible big bang say less than 10^{-12} s say - the Standard Model seems to work perfectly except perhaps in very high energy accelerators and in cosmic radiation. So we should expect that at least the real part $S_R[history]$ of the action $S[history]$ should be given well by the action of Standard Model. Now the very natural guess is, that you get the full complex action by just letting all the coefficients of the various terms in the Standard Model action become complex. You might even as the a priori most promising guess think, that the phases are rather random and of order unity, meaning of the order of 100^0, except though, that the mass term for the Higgs particle deserves special discussion.

Let us remind about the discussion around the hierarchy or the scale problem for the usual real action Standard Model:

If you imagine a cut off at the Planck scale or some new physics at some GUT scale at almost Planck energy scale, then one has the problem that corrections to the bare Higgs mass square as written in the Lagrangian density m^2_{Hbare} in order to obtain from that the measured mass square m^2_{Hren} becomes typically very large, either it is divergent or by means of fixing some unified scale it becomes when renormalized to that scale anyway huge compared to the scale of measured Higgs mass square or the weak scale. So it is a well known finetunig problem how to get the weak scale be small compared to the huge scales involved in the loop calculations even if one renormalizes to some unifying scale. You might keep the corrections smaller by having supersymmetric partners - but the LHC results so far rather show the surprise that such ones are so far not found -. But whatever might be the solution to this problem of how the weak scale became so small say compared to the Planck scale and how to keep it there it might it easily becomes so that the bare mass square m^2_{Hbare} becomes appreciably bigger than the renormalized one m^2_{Hren} numerically.In the case when some supersymmetric particles exist and makes the mass square correction *only* logarithmically (divergent) the size of the bare divided by renormalize will though only be “logarithmic”, which means not so fantastically big after all. But if the supersymmetric partners do not exist or are very heavy then again the bare mass square will typically be much larger than the renormalized/observed Higgs mass square.

When we now want to guess the size of the imaginary part of the Higgs mass square, the suggested guess is that it should be of the same order as the real one; but now should it be as the real renormalized or as the real bare ? Most likely the loop corrections for the real and for the imaginary parts are completely different and huge, so the question becomes: Would the same mysterious fine tuning, which made the real part $m^2_{Hren}|_R$ = observed/effective Higgs mass square of the renormalized mass square for the Higgs also function for the imaginary part, so that in some way - which we may or may not understand - the effective/renormalized (whatever that might exactly mean) imaginary part of the Higgs mass square $m^2_{Hren}|_I$ becomes as small as the real renormalized part order of magnitudewise?

Very likely the solution to the finetuning problem (= the scale problem) of why the weak scale is so low compared to the Planck scale say will be solved in a way that will not make also the "renormalized" scale for the imaginary part of the "Higgs mass square" small compared to say the Planck scale. For instance this is the case for our own "solution" to this problem by means of the multiple point principle: This "solution" means, that, if we make the very strong assumption that there is some finetuning fixing the parameters/coupling constants of the theory working in nature in a way restricted so that there becomes *several different vacua all having very small energy densities(=dark energies = cosmological constants)* (for purposes of the weak scale we just say exactly zero energy densities are assumed in the vacua) we how found a viable picture with strongly bound states of 6 top + 6 anti-top quarks and a set of three different vacua in the Standard Model, in which this requirement leads to an exponentially small value of the weak scale compared the scale of the Higgs field in one of the vacua considered degenerate. In other words with our assumption of vacua with zero energy density (called "multiple point principle" (=MPP)) and some in principle calculable speculation about bound states of quarks and anti quarks the parameters of the standard model need to to take such values that the renormalized Higgs mass square must be very small compared to the scale for the Higgs field in one of the by us assumed vacua. We then add as an extra assumption to our multiple point principle that for one of the vacua the Higgs field present should be of the order of the Planck energy. This latter assumption is already supported by the parameters of the Standard Model if one assumes this Standard Model to be valid up to so high energies (or Higgs fields). It found a support together with the multiple point principle by the Higgs mass found in Nature agreeing with our PREdiction.

But really in our complex action model physics coming out of the real and of the imaginary part of the action are *quite different*, crudely the real part gives equation of motion and the imaginary the initial conditions, so to expect that some mysterious mechanism make the same finetunin on both is not at all likely. Therefore we shall conclude that it is most likely that there is no finetuning going on to make the effectively observed/"renormalized" imaginary part of the Higgs mass square small compared to say the Planck scale value. If so, then we should expect it to be probably of the order of the Planck scale. Putting into Standard Model extended to have complex action this size of the Higgs mass square imaginary part would mean that considering a process of daily life or of LHC the Higgs mass square term would give contribution to the imaginary part of the action, which are larger than the contributions from the other terms by a factor $M^2_{Pl}/(\ TeV^2) \approx 10^{34}$. This means that we from dimensional arguments think we could argue that the most important term in the imaginary part of the action should be the part from the Higgs mass (square) term.

With this we argued that we under present conditions can approximate the imaginary part of the action $S_I[history]$ by only the contribution from the Higgs-mass-square term

$$S_I[\mathrm{history}] \approx \int m^2_{Hbare}|_I|\phi_H(x)|^2\sqrt{g}d^4x \tag{13.1}$$

(the $\sqrt{g}$ is just 4-volume measure inserted to make the formula o.k. in the general relativity case, but really you may use flat space approximation and ignore it). The Higgs field were denoted $\phi_H(x)$ and depends of course on the event coordinate (set)$x = \{x^\mu\}$. The integral is, provided we use the "with future"-interpretation of the complex action theory, to be integrated over all space time including *both future and past*, and then it is this quantity (13.1) which at least in first approximation selects initial conditions or what really happens by letting the true happening $\mathrm{history}$ have the minimal value for the imaginary part of the action $S_I[\mathrm{history}]$ among all the say by equations of motion allowed possible histories. For a crude understanding of our complex action theory one may take it that it predicts roughly that

$$S_I[\mathrm{true\ history}] \leqq S_I[\text{any other } \mathrm{history}]. \tag{13.2}$$

(more detailed calculations of some predictions may be found in [9–11] and in some of the papers with Ninomiya [3]).

One way of putting forward the idea of the universe initial conditions being arranged in a way governed so as to achieve say small (or preferably numerically large negative) contributions to $S_I[\mathrm{history}]$ is to call it a "God" (it is only a god in quotes(thanks to Mette Høst)) governing the world so as to seek to minimize the imaginary part of the action $S_I[\mathrm{history}]$. In this language our expression (13.1) means that this "God" only cares for the integral over space time of the Higgs field; "He"to day mainly care for Higgs particles and modifications in the Higgs field. Oscillations in the Higgs field meaning physical Higgs particles will obviously make the square of the Higgs field integrated over all space time bigger. So producing Higgses should e.g. be hated and avoided by the "God. (Had the sign been so that it corresponded to "God" loving Higgs bosons instead "He" would have filled more up with Higgs bosons, say an expectation value of the Planck order of magnitude at least).

But if "He hates the Higgs "He" should love the particles suppressing in there neighborhood the Higgs field? And fill the whole Universe with the most favoured ones.

It is for instance the quarks and the charged leptons that are surrounded by a Yukawa potential region in which the Higgs field has an additional Higgs field - the Yukawa potential -, and so a more strong field the bigger the mass or the lepton causing this field. One may easily understand that the Higgs field having in vacuum its well known expectation value $< \phi_H(x) >= 246\ GeV$ is a bit diminished numerically in the Yukawa-potential-region around a quark or a (charged) lepton. Now in principle we do not know whether the square of the Higgs field $|\phi_H(x)|^2$ increases or decreases as one enforce a little region in space(-time) to have a given Higgs field diminished say w.r.t. the usual vacuum Higgs field. Intuitively one would think the square would decrease when the Higgs field itself decreases but there could - and indeed there are - be effects causing it to go oppositely(as have argued for below and in the articles[12]). In any case unless there is just an extremum of the square $< |\phi_H(x)|^2 >$ as a function of the Higgs field itself $< \phi_H(x) >$ in the usual vacuum situation there would be an effect positive or negative upon the imaginary action $S_I[history]$ as given by (13.1) from the Yukawa-potential regions around the quarks or (charged)leptons, because the normal Higgs fields a bit suppressed in such Yukawa field neighborhoods.

This means that e.g. the "God" would either love or hate these quarks and charged leptons, and that the more strongly the heavier they and the stronger they therefore couple to the Higgsfield.

This in turn means that e.g. a particle like the neutron with its three valence quarks and further quark pairs inside it will suppress the Higgs field from its usual vacuum value a bit and then depending on the sign of the derivative $\frac{d<|phi_H(x)|^2>}{d<\phi_H(x)>}$ increase or decrease the imaginary part of the action $S_I[history]$, thus the neutron would be respectively hated or loved by "God".

Now in nature one can by weak interactions get a neutron transformed into a proton, an electron and an electron-anti-neutrino. Thus if the "God" loved say the neutron itself more than the proton the electron and the electron-anti-neutrino together we would expect that "He" would have arranged initial conditions - and if "He" were allowed to it also that coupling constants or whatever could help - so as to make there be only neutrons but no protons and electrons etc. We know from astronomy and our own earth neighborhood that there exist both neutrons and protons and electrons (and even neutrinos) in rather large amounts, none of them being truly so much suppressed compared to the other.

At first we may look at this fact there there are both neutrons and protons in the world today as a falsification of the minimization of imaginary part of action ideas!

It becomes in our complex action theory an embarrassing question: Why not only n or only p+e+antineutrino ?

An idea to an attempt to disprove our complex action model with the Higgs field square integrated as the imaginary part of the action: Why do we not have either?:

- 1) Only neutrons n and no protons nor electrons, or
- 2) Only protons with their electrons e and antineutrinos, but no neutrons at all.

Either one or the other would probably be favoured and thus by "God be arranged to be realized!

13.6.3 Solution to: Why both protons and neutrons?

Actually this problem of why not only protons(with their electrons) or only neutrons in the world in our complex action model has the "solution":

If the neutron is exactly equally much "loved" as the the proton the electron and the electron-anti-neutrino together -in the sense of contributing the same to the imaginary part of action $S_I[history]$, *then there would be no reason for "God" to eradicate one of the two types of particles. But this requires a certain relation between the masses of the quarks corrected by their Lorentz contraction factors and the electron mass. But remarkably this* *relation is satisfied within calculational accuracy!* (light quark masses are rather badly known so the accuracy is not so high)

Basically[12] in order that there shall be no reason to either remove from the world the neutrons nor the combinations of protons and electrons (the neutrinos anyhow contribute much less to the imaginary part than the massive quarks or leptons) we should get just same imaginary part of action contribution from a neutron and from such a combination of proton and electron. In an short time the contribution is estimated as an integral over space of the Higgs field suppression. We here just assume by Taylor expansion in the presumably rather small Higgs field around the quarks and leptons, that any effect will in first approximation be linear in the change in the Higgs field. Now we find small Yukawa-potential regions of size given by the inverse Higgs mass and centered around quark or lepton. A crucial little problem making the estimation a bit less trivial and bit less accurate is, that these regions of significant Yukawa-potentials are *Lorentz contracted*, because of the non-zero velocity of say the quark it surrounds. (The electrons most copiously found in our universe have actually very small velocities compared to the light velocity, so for them Lorentz contraction is not important.)

The following the reader should have in mind in order to estimate the contribution to the imaginary part of the action $S_I[history]$ under the assumption of the dominant Higgs mass term for a neutron relative a pair of proton and an electron:

- a Of course - unless a linear term should be lacking - the contribution must go linearly with the Yukawa coupling for the quark or lepton in question. Really the suppression of the Higgs field around a particle - quark or lepton say - must go proportionally to the Higgs Yukawa coupling (for fixed velocity)
- b But it will vary with velocity due to the Lorentz contraction of the Higgs-Yukawa effective extension volume, around the particle.
- c So at the end the effect on the imaginary action $S_I[history]$ becomes proportional to

$$\Delta S_I[history] \propto g_{particle} * \frac{m}{E}|_{averaged} \propto \frac{m^2 < \gamma >< \gamma^{-1} >}{E_{average}} \quad (13.3)$$

  where m is the mass of the quark say (or lepton) and E its actual kinetic energy including the Einstein energy. The average as the quark flies around in the nucleon say is denoted of its $\gamma = E/m$ is denoted $< \gamma >$, while the average of the inverse of this same γ is denoted$< \gamma^{-1} >$. The average kinetic including Einstein energy E is denoted $E_{average}$. The combination $< \gamma >< \gamma^{-1} >$ would in the case of no fluctuations of the actual velocity of the quark be just unity, and thus we may hope that we can estimate this product somewhat more accurately than say its two factors separately.

The various types of quarks have of course the deeper Higgs fields around them the stronger their Higgs Yukawa couplings $g_{particle}$. The Higgs field is effectively extended over a range of size given by the Higgs mass but not dependent on the species of quark or lepton in question. The extend of the Yukawa potential rather is over an elliptic region, that is the Lorentz contraction of the spherical Yukawa potential, which is obtained around a resting particle. So the contribution to the integral of the Higgs field or presumably also over its square over all space from a quark or lepton is proportional to $g_{particle}$ and to the inverse of E/m where E is the energy and m the mass of the quark or lepton. The Lorentz contraction factor is for Yukawa potentials for quarks due to motion inside nucleons, if we have - as is most copiously the case - resting nucleons. Well, really the speed of the nucleons inside the nuclei is not so negligible again but compared to the speed of quarks inside nucleons it is small.

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The bigger integrated Yukawa potentials around the quarks and leptons the more the Higgs field is suppressed. The strength of the suppressions is proportional to the Yukawa coupling for particle making the suppression. The extension is roughly like the Lorentz contracted of a sphere forming an ellipsoid given by the Higgs mass(as inverse radius of the sphere).

The proton is almost identical to the neutron except, that one up-quark has been replaced one down-quark.

To keep Universe chargeless a proton should be accompanied by an electron.

A neutrino typically runs so fast that its Yukawa potential is much less extended in volume than those of quarks and charged leptons.

13.6.4 Contributions to See Whether Neutrons or Non-neutrons Favored My Prediction from Future Influence

To estimate the contributions coming from a neutron to compared it to that coming from what is its decay products a proton and an electron and even a not so significant electron anti neutrino we need the light quark masses which are not so well determined (and that makes our uncertainty rather large), but let us take

$$m_u = 1.7 \text{to} 3.3 MeV \tag{13.4}$$

$$m_d = 4.1 \text{to} 5.8 MeV \tag{13.5}$$

for respectively the up and the down quark masses.

One arrives as also sketched here to the relation

$$\sqrt{m_d^2 - m_u^2} = \sqrt{E_q m_e}/\text{“}ln\text{''} \tag{13.6}$$

where we have denoted

$$\text{“}ln\text{''} = <\gamma><\gamma^{-1}> \tag{13.7}$$

because this quantity for light quarks compared to the energy $E_{average}$ tends to be approximately a logarithm. The relation(13.6) is relatively well satisfied, if we take the quark masses (13.5), $E_q \approx 160\ MeV$ and "ln" $= 2.3_7$.(see my previous article for this crude estimate) In fact then we would get (using $m_e = 0.511\ MeV$)

$$R.H.S. = \sqrt{E_q m_e}/\text{“}ln\text{''} = 3.81\ MeV \tag{13.8}$$

$$L.H.S. = \sqrt{m_d^2 - m_u^2} = \sqrt{13._9} \text{to} \sqrt{22._75}\ MeV \tag{13.9}$$

$$= 3.7_3 \text{to} 4.7_7\ MeV. \tag{13.10}$$

13.7 Fine Tuning Calls for Influence Going Back in Time

One argument, which Don Bennett and myself would give for some influence from the future being called for, is this:

We know the fine tuning problem of why the cosmological constant/dark energy /energy density in the vacuum is so small compared to the *energy density* given by the most fundamental constants G, c, and $\hbar$, i.e. the Planck energy density? The ratio of the actual vacuum energy density to the from the dimensional arguments expected value is enormously small. So it is clear that there must have been some enormous fine tuning arranging this enormously small energy density in the vacuum. Now we expect that the vacuum energy density should be constant as time has gone on. So even in a time of say minutes after the start of the universe or Big bang or whatever the vacuum energy should have had the present extremely small value. But now at these early times there were so big energy densities of radiation or matter that the present small vacuum energy density would be very small and insignificant compared to radiation energy density. But when it were at that time so insignificant, how could *at that time* any physical effect have made a so precisely close to zero as the vacuum energy density to day? So it seems that an influence from the future somehow must have arranged at this early stage already the exceedingly small energy density in vacuum? It is of course because of an argument in the direction of this that is the reason for that, when Weinberg looks through the various explanations for the cosmological constant being so small, then the most promising explanation is to use anthropic principle. The entropic principle, which states that parameters shall be so arranged that humans can come to exist, is namely in reality a method to to arrange a simulated effect of the future influencing the past. By throwing away the scenarios which happen not to allow for humans one has got what functions as a back in time effect.

13.8 Our Multiple Point Principle

There is one very general deduction from such a theory with a principle of minimizing some quantity as we above told that the imaginary part $S_I[history]$ would be minimized for the actually realized history. This deduction would be best achieved if we instead of minimizing over histories of the universe minimized over combinations/sets of coupling constants, but since one could imagine some vacuum being selected among several at least the effective coupling constants relevant for the by a quantity like $S_I[history]$ selected vacuum would effectively have been determined as if they were adjusted to minimize something (S_I) by adjusting the coupling constant combination. The deduction related to is found an article by Ninomiya and myself [18] in the Bled proceedings from 2011. The point is, however, to imagine that the right combination of coupling constants is achieved by asking to obtain the minimum for some quantity - in fact our S_I, which we now imagine to depend also on the coupling constants (with an effective vacuum providing such couplings this imagination would be true in our model) - under the restriction that the energy density of the various (local) ground states the vacua should be positive. This assumption of vacuum energy density being positive may be understandable in our model - as well as phenomenologically supported as a principle - by noting, that if a vacuum gets (appreciably) less energy density than zero, then the usual vacuum becomes unstable against making a transition to this low energy density vacuum. From the point of view of the history being selected such an instability would mean that it would be this vacuum rather than the usual one that got realized and the potential history meant as a history in the "usual" vacuum would no longer be realized; so if this latter history gave a smallest S_I that would be a lost achievement if another vacuum tales over. So one should avoid competing vacuum threatening the stability severely for the realized one, or one should presumably preferably think that there are several vacua getting their realization in a turn adjusted to be the most beneficial for the S_I being as negative as possible. Also in such a scenario of several vacua coming to exist as time passes on, the transition from to the next should not be too quick, they are to exist for of the order of 13 milliard years. Thus they should be approximately stable and we would obtain an approximate multiple point principle in such a scenario. In any case we already earlier argued that once you have the minimization of something like our S_I that just can manage some way to effectively depend on the coupling constants, then the couplings get very likely adjusted to lead to several degenerate vacua, meaning multiple point principle.

Having in mind that this multiple point principle is thus to be considered a deduction from a minimization of some quantity model including future in such a way that it really means influence from the future, we can now look at successes of our multiple point point principle (MPP) as also being evidence for there existing in the laws of nature some influence from the future.

Now I remind the reader that the most impressive confirmation of our multiple point principle were that we - Colin D. Froggatt and myself - PREdicted the Higgs mass[19] many years before the Higgs boson were found to 135 GeV $\pm$ 10 GeV ! With the present calculations and top-mass measured our prediction would rather have been 129.4 GeV with an uncertainty now rather down to about $\pm$ 1 GeV. So although our prediction is now only 3.4 GeV above the experimental Higgs mass 126 GeV, the deviation compared to the uncertainty may have gone slightly up compared to the old day PREdiction, but we should still consider it a great success for the multiple point principle that the Higgs mass is so close to our prediction!

Historically we - Don Bennett and myself and also in some papers with Colin D. Froggatt - we looked for some way of justifying to fit fine structure constants by phase transition couplings in lattice Yang Mills theories. We worked at that time with what we call Anti-GUT (meaning anti-grand-unification) meaning that we rather than as were most popular to look for simple groups like SU(5) or SO(10) etc. we did not unify in the sense that we used the not at all simple group $S(U(2) \times U(3)) \times \cdots \times S(U(2) \times U(3))$ (with N_{gen} cross product factors), rather meaning that we gave every family of fermions in the Standard Model its own family of also gauge bosons, so that our "anti grand unifying group" were the cross product of one Standard Model gauge group, one for each family (the number N_{gen} of families not yet known at that time; we had to fit it to the fine structure constants and PREdict it; luckily we PREdicted $N_{gen} = 3$). But the problem for which we needed the multiple point principle were to give an explanation or at least formulate a principle that could imply that the *phase transition* finestructure constant values were the ones for which Nature should care. But if we somehow had derived that Nature should have a couple (or more) energy density wise degenerate vacua/phases of course if nature really were a lattice Yang Mills theory, then it would mean that Nature should choose the phase transition value of the coupling constant/the finestructure constant.

Once we have suggested to believe in such a multiple point principle in the form of there being many/several energy-wise degenerate vacua, you just have to find one with an appropriate small cosmological constant and you can so to speak transfer that small energy density to other vacua, thus explaining the smallness and even fit *the* cosmological constant (or the dark energy). Roman Nevzorov, Froggatt and me did such an application in several versions, explaining the cosmological constant[20].

We even managed to make a solution of the scale problem (related to the hierarchy problem) in the sense of using the postulation of the multiple point principle to fix the scale of the weak interactions (compared to the Planck scale, taken as the fundamental scale). This we -Colin D. Froggatt, Larisa Laperashvili and myself - did by speculating up the existence of a further vacuum in which there is a Boson condensate of bound states of 6 top and 6 anti-top quarks. In the spirit of the multiple point principle postulating a further vacuum is somewhat natural, and at least each time we postulate a new vacuum, we get the information out of multiple point principle that this vacuum shall have the same energy density as the other vacua. Thus for each new vacuum we postulate - and take to be degenerate with the other ones - we get one more of the say Standard Model (if that is what we use) determined, because one more relation among them is obtained. Luckily it turns out that we essentially may use this new information to fix the weak energy scale and most importantly:

We get the weak scale out as restriction on between which values of the running top-Yukawa coupling $g_t(\mu)$ *shall be taken on at 1) the high field scale of the second Higgs field effective potential minimum (assumed by us to be essentially the Planck scale) and 2) the weak scale.*

Since then the running top Yukawa coupling must "run" between the two predicted values $g_t(\mu = 18^{18}\mathrm{GeV})$ and $g_t(\mu = \text{"weak scale"}) = 1.02$, the ratio of the weak to the supposed more fundamental scale gets predicted to be "exponentially" small! Really the point is that with the rather weak couplings of the Standard Model the 'running" is actually a bit slow as a function of the logarithm of μ. Thus to get a given distance of change in the Yukawa coupling an exponentially big ratio of scales is needed. Actually our prediction of the logarithm of the scale ratio, the scale problem gets very well!

So our multiple point principle is here a great success: both explaining the exponential smallness and giving a good value for its logarithm.

13.9 Do we have Enough Evidence for Influence from Future?

I would like towards the end very optimistically for the hypothesis of there being indeed an influence form the future to give - the relatively optimistic, but still crudely true - numbers for how unlikely it would be that our small coincidences favouring the complex action model with the assumption that the Higgs field square dominates without such a model being true.

Say we look at the coincidence that the "knee" in cosmic radiation spectrum just order of magnitude wise happens to coincide with the threshold for Higgs production. If we say one has studied cosmic rays from some electron volts up to say 10^{20} electron volts, we could say over 19 orders of magnitude. Then if one finds a knee to coincide within one or two orders of magnitude, it represents a coincidence that should happen by accident only in about 1/10 cases. Similarly looking at the agreement of our formula (13.6) as being that we get inside the right interval of length one MeV for quantities - sides of the equation - being of order of 4 MeV, this is something that should only happen in one out of four cases.

Our argument that the Higgs-field vacuum expectation value should just have gotten that value, that minimizes the *squared* Higgs field expectation value - we get agreement up to some factor of one or two orders of magnitude - means that our minimization principle led to the right order of magnitude for the weak/Higgs field scale to say a couple of orders of magnitude out of 17 orders of magnitude (taking the Planck scale as the fundamental one). this means again that our influence from future got the right scale among say $17/2 \approx 10$.

We may even count here the smallness of the binding energy in nuclei compared to the separately bigger kinetic and potential energy of the nucleons, say one out of 2 cases accident.

These "numerical" coincidences together would give us a one out of 800 coincidence, which is a factor 4 more than 3 standard deviations. Taking this optimistic estimate seriously we really have more than 3 standard deviation evidence for the influence from future seeking to minimize the Higgs field square (integrated over space time), so as to use it to tune some couplings or the like.

Further to support this complex action with Higgs mass (square) term dominating model for the development of the world being supported we should collect also the evidence coming from the very bad lick of the S.S.C. machine, that would if it had worked according to plans have produced more Higgs bosons than L.H.C. has so far, and the - for our model though too little - bad luck of an explosion in the tunnel, which though were repaired and mainly so far had the effect of making the physicists choose to postpone the running of the L.H.C. with its planned beam energy of 7 TeV against 7 TeV (meaning $\sqrt{s} = 14$ TeV) till 2015. Although it now looks that finally it will come to run, we may though consider it, that this caused postponing of the full energy could be a result of our complex action model with Higgs mass term dominance. Together we might consider these after all not so terribly miraculous bad lucks for Higgs producing machines as something that would not be at least the very first expectation without theory predicting it like ours. So we might say e.g. that in at most one out of say 5 cases would so much bad luck hit the Higgs producing machines.

If we combine this estimate with the just counted, we would say that now the Higgs mass square term dominated complex action model has scored a success corresponding to one out of 800 *5 = 4000 cases!

If we add to this counting the evidence coming from say the Higgs mass being PREdicted from our multiple point principle, which also would follow from an influence from the future type theory, and take it that the range for Higgs mass were at first up to 600 GeV or just use the actually Higgs mass to set the scale for Higgs masses the deviation $129.4 - 126$ GeV = 3.4 GeV (relative to respectively 600GeV or 126 GeV) means a luck for our multiple point principle as one out of ≈ 200 or one out of ≈ 36 respectively.

If we already have counted the luck of our theory of getting the right weak scale it might no longer be new prediction to use the multiple point principle to predict the top -Yukawa coupling to be $1.02 \pm 14\%$ (oterwise this result should give a one out of 7 good luck for our model).

Also it would probably be too much to seek to include as a result of our influence the very remarkable smallness of the cosmological constant because this influence from future type theory in itself does not predict this smallness, although firstly it is very hard to see how such a small cosmological constant could come without an influence from the future and secondly we have works with Roman Nevzorov et al. [20] in which we actually even fit the cosmological well using the multiple point principle(which indeed is consequence of an influence from the future much like the one we discuss here. If we include this cosmological constant as were it prediction it would increase much our measure of the success since even counted only as a success on the logarithmic we could a priori have expect a "Planck energy density value" about 100 orders of magnitude larger. Counting with natural logarithm say we should then say we succeeded as one ot of 100*2.3 = 230.

But even as presumably most fair leaving out the cosmological constant proper as being a success for our model(s), but only taking in the Higgs mass PREdiction from multiple point principle (after replacing the one prediction of MPP by the Higgs or weak scale gotten by adjusting this scale to minimize the squared Higgs field integrated over space and time) we get that good luck for our model is of the order of getting one out of $4000 * 7 \approx 30000$ cases/possibilities correct!

This of course were optimistically counted, but it sounds that one should take possibility of there being effects from the future. especially we did not even in this number include anything from the arguments related to the need for ultraviolet cut off, which especially for gravity may be very hard without a bit of non-locality, thereby allowing the influence from the future sneak in in principle.

13.10 Conclusion and Outlook

We have in the present article looked at a series of arguments for that there should be in the laws of nature some law that makes e.g. the initial conditions or the coupling constants or both be adjusted as if it were with a special purpose (such as as here suggested to make a certain quantity depending on the history "the imaginary part of the action be minimal).

The main classes of arguments, which I suggested are:

- Numerical or observational successes of assumptions involving such an influence from the future. This includes:
  - The bad luck of SSC, and if we take it seriously the very minute bad luck of the LHC, both machines (potentially) producing relative to human history exceptionally many Higgs bosons.
  - Our relation relating the light quark mass square difference to the electron mass square and the fraction of energy carried by the quarks in the nucleons. This relation just organizes that the contribution from a neutron and from an electron and a proton (and an electron anti neutrino) together to this imaginary action is same. Thus when this relation - which seems to be fulfilled within errors in nature - happens to be fulfilled there would be no gain in minimizing the imaginary part of the action by neither arranging for more neutrons than for more of its decay products electron + proton (+ anti electron neutrino). The world would potentially be able to exist at a minimum for the imaginary part of the action.
  - analogously I argued that including the effects of virtual top quarks in the vacuum it could within errors be so that the Higgs field *square* is in fact at a minimum with just the present Higgs expectation value in the vacuum. So indeed the parameters of the Standard Model could have been arranged just so as to minimize the Higgs field square, and that could have led just to the from hierarchy problem consideration rather difficult to accept compared to the Planck scale or Grand Unification scale point of view exceptionally small value Higgs field expectation value.
  - Even the "knee" in the cosmic ray spectrum is so close to the threshold for the severe production of Higgs bosons that we can claim that it is as if it had been arranged to be just like that to make the production of Higgs bosons by the cosmic rays hiding material or planets etc. in the galaxies so small as possible under some restrictions.

    The "God" did not quite switch off the cosmic rays above the effective Higgs production threshold, but the "knee" looks like a weak attempt to do so.
- We called attention to that cut off methods which are needed to make especially renormalizable gravity theories are very hard if at all possible to conceive of without some non-locality. And then since non-locality really means that influence from future is getting allowed for small distances, also such cut off needs in fact calls strongly for that influence from future cannot be totally avoided. We looked especially as an example on string theory in the recent formulation of Ninomiya and myself. In this model the for the cut off effectivity crucial feature - the "stringiness" one could say - is put in as an initial state - and even as a *final state* - condition! If instead of or in addition to inclusion of gravity you also want to have more than the experimental number of dimensions 3+1, the need for such cut offs that in turn leads to non-locality and thereby formally admits influence from future gets even stronger.

- We also mentioned the old worries about that the usually assumed laws of nature for the initial conditions and those for the equations of motion do not have the same CPT ot say just time reversal invariance: The initial conditions usually assumed are only for the *initial state*, but not for the final state also as a time reversal invariant theory would have to have it. So again some influence from future is called for in order to make the symmetry be a least formally uphold.
- Although I did not go so deeply into it in the present article, it is of course also one of the arguments for influence future coming into the physical theory that one in general relativity has wormholes and baby universes. very easily leading to time machines. Such time machines namely leads to inconsistencies unless the happenings are finetuned to just make things go in a with the time machine consistent manner. This has been discussed by Novikov.

We will at the end stress that with the lists of arguments in the present article one should at least admit that the absolutely safe belief that there is no influence from the future deserves being investigated and confronted with as much knowledge as we can collect concerning this question. If one truly will uphold this absolutely safe belief that nothing from the future can influence us in any way, there is really no government of the universe - at least no government with any interest in the future fundamentally - then one would have to throw away as bad science/misunderstandings or pure (poetic) invention all stories about the government of God or destinies or the like which may be found in mythology in the holy texts or the like.

At least I hope to have put a little doubt on the validity of this by now in first approximation well working law of nature that future cannot influence anything in past or now and that there is no government of the universe whatsoever.

Instead one could look at it that the strong belief in this no influence from future nor government arranging for the future will turn out to be only something humanity believed in a relatively short historical era from Darwin Wallace Lamark to some day may be next year when a truly bad luck for LHC e.g. would convince humanity that there exists a “God” (here in quotation marks) that hate the Higgs sufficiently to stop a Higgs producing machine before it gets produced too many Higgses!

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14 Deriving Diffeomorphism Symmetry

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Abstract. In an earlier article, we have "derived" space, as a part of the Random Dynamics project. In order to get locality we need to obtain reparametrization symmetry, or equivalently, diffeomorphism symmetry.

There we sketched a procedure for how to get locality by first obtaining reparametrization symmetry, or equivalently, diffeomorphism symmetry. This is the object of the present article.

Povzetek. V enem od prejšnjih člankih sta avtorja v okviru projekta Naključne Dinamike "izpeljala" pojavnost prostora. Za izpeljavo lokalnosti postora je potrebno vključiti reparametrizacijsko simetrijo, to je simetrijo na difeomorfizme. V tem prispevku nakažeta avtorja izpeljavo postopka, kako do lokalnosti prostora iz reparametrizacijske simetrije.

14.1 Introduction

In an earlier article [1], we have "derived" space, as a part of the Random Dynamics project [2]. Since we want to have locality, we also need to derive reparametrization symmetry, or more generally, diffeomorphism symmetry [3], essentially ensuring that the choice of coordinates plays no role in the formulation of the physical laws.

We propose that diffeomorphism symmetry comes about as a result of a selection principle, in reality a selection principle for how Nature "chooses" its symmetry groups, a scheme that has been developed by Holger Bech Nielsen and his collaborators [4]. The initial idea was that the small representations of the Standard Model gauge group

$$SMG = S(U(2) \times U(3)) \tag{14.1}$$

is a signature of such a selection principle, singling out groups that have the "smallest" representations.

In the present article we use similar arguments, but instead of taking the Standard Model group $SMG = S(U(2) \times U(3))$ as the selected group, we consider the combined diffemorphism-and-gauge group

$$\mathcal{B} = \{(\lambda, \varphi) \mid \lambda \in \mathcal{G}, \varphi \in \mathcal{D}\} \tag{14.2}$$

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where $\mathcal{G}$ is the group of all gauge transformations that map the four-dimensional spacetime manifold $\mathcal{M}$ on the 12-dimensional manifold of SMG: the Lie group is a manifold,

$$\lambda : \mathcal{M} \to SMG$$

and $\mathcal{D}$ is the group of diffeomorphisms, a diffeomorphism φ given by a bijective differentiable map

$$\varphi : \mathcal{M} \to \mathcal{M}$$

14.1.1 An alternative to grand unified models

A major part of the success of the GUT $SU(5)$ model is that the representations of the $SU(5)$ gauge group automatically represent the $SU(5)$ subgroup $S(U(2) \otimes U(3))$ with the Standard Model Lie algebra. The GUT $SU(5)$ group thus presents the needed restrictions on the allowed represenstations of the Standard Model algebra. Any successful GUT group, like for example $SO(10)$, reproduces the same restrictions as $SU(5)$ on the representations of the Standard Model Lie algebra, restrictions corresponding to $S(U(2) \otimes U(3))$.

Any viable alternative to the GUT scheme must thus supply a prediction not only of the Standard Model Lie algebra, but also of the group structure. There are however many possible scenarios, so unless one has some guiding principle for selection the unificaton group, there isn't much predictive power.

One way to get the Standard Model without a GUT scheme, is by using some selection principle for how Nature selects the Standard Model group. The underlying philosophy is that of Random Dynamics, namely that the fundamental physics is random, and that the observed symmetries are emergent. If only some symmetries emerge, supposedly by accident, but maybe even by some more precise mechanism, then the initially random action could be considered as taking random values for some small region of the value space of the representation of the group, with the transformation properties of the fields or degrees of freedom under the group. The elements of a representation of the group in question then move quite slowly as the group elements themselves vary. (One can vary the group elements much before one varies the fields or matrices of the representation). The slower the representation moves as a function of the variation of the group elements the more likely it is that a symmetry emerges, since displacements inside the group itself corresponding to a small region (over which we assume essentially constancy of the action) become bigger with a slower representation motion rate. A symmetry of the random action is thus more likely to occur when the symmetry is represented by "slowly moving" representation elements (e.g. matrices).

By means of some "goal quantities" we single out the groups that have the largest chance to emerge from a random action model, favouring the experimental gauge group and dimension of spacetime.

14.1.2 Groups and algebras

In Yang Mills theories, only the Lie algebra is important, since two groups $\mathcal{G}_1$ and $\mathcal{G}_2$ that have the same Lie algebra also have the same Yang Mills system. There are however many Lie groups with the same algebra. These groups are locally similar, but globally they can be very different, with different representations. For example, $SU(2) \neq SO(3)$, as for $SU(2)$ we have $j = 0, 1/2, 1, 3/2, \ldots$, while for $SO(3)$ $j = 0, 1, 2, \ldots$, and it is only by studying the representations of Nature like q_L, q_R, the Higgs and so on, that we can establish which groups are at stake. To a group corresponds

- The Lie algebra and thus the structure constants f^l_{km} and the Yang-Mills Lagrangian $\mathcal{L}_{YM}$.
- The system of allowed representations, a given set of representations only being allowed by some Lie groups.

The covering group (for a given Lie algebra) can manage all the representations, so the goal is to find the most choosy group, i.e. the one that allows the fewest representations - which also corresponds to experimental data. Our point of deparure is the Standard Model Lie algebra

$$S(U(2) \times U(3)) \sim R \times SU(2) \times SU(3) \sim U(1) \times SU(2) \times SU(3) \tag{14.3}$$

and since we don't find all its possible representations in Naure, we will concentrate on the Lie group rather than on the Lie algebra. It is so to speak stronger to "predict" the group $S(U(2) \times U(3))$, such that

$$\det \begin{pmatrix} . \ . & 0 & 0 & 0 \\ . \ . & 0 & 0 & 0 \\ 0 \ 0 & . & . & . \\ 0 \ 0 & . & . & . \\ 0 \ 0 & . & . & . \end{pmatrix} = 1,$$

a group which admits all phenomenological representations, which all obey the rule

$$\frac{y}{2} + j_3 + \frac{1}{3}\text{"triality"} = 0(\mathrm{mod}1) \tag{14.4}$$

14.2 Skewness

Small representations as one possible selection principle, but another way of singling out Nature's chosen group, is by studying group *skewness* [5], defined as a lack of symmetry.

Nature seems to select the Lorentz group with the smallest representations; perhaps space moreover prefers those dimensions that give the skewest Lorentz group. The Standard Model group $SMG = S(U(2) \times U(3))$ is very skew, and most probably very "complicated".

There is always the worry that the choice of "goal property" is such that it gets dramatically bigger or smaller with the dimension or some other size parameter of the group. In the case of choosing a skewness measure, this can be dealt with by defining it as

$$\frac{\ln(\text{number of outer automorphisms})}{\text{rank of the group}}$$

14.2.1 Inner and outer automorphisms

The degree of skewness is thus a function of the number of outer automorphisms of the group $\mathcal{G}$. An automorphism is an isomorphism of the group onto itself,

$$\beta : \mathcal{G} \to \mathcal{G} \tag{14.5}$$

i.e. a correspondance ϕ of $\mathcal{G}$ with itself respecting the group multiplication, and such that ϕ is bijective and $\phi(gh) = \phi(g)\phi(h)$, $g, h \in \mathcal{G}$.

The map $\beta(g)$ is an inner automorphism if there is an element $h \in \mathcal{G}$, such that for all $g \in \mathcal{G}$,

$$\beta(g) = \beta_h(g) = hgh^{-1} \tag{14.6}$$

The group of outer automorphisms $\mathcal{O}$ is then defined modulo the inner automorphisms in the sense that in the group of all automorphisms $\mathcal{A}$, we discern the subgroup of inner automorphisms,

$$\mathcal{A}_{inn} = \{\beta_h | h \in \mathcal{G}\}, \tag{14.7}$$

and then define the group of outer automorphisms as

$$\mathcal{A}_{out} = \{\mathcal{O}/\{\beta_h | h \in \mathcal{G}\} \tag{14.8}$$

For the Standard Model group, we have that

- The automorphisms of **R** (~ the $U(1)$ factor) are scalings with a factor $k \neq 0$.
- The $SU(2)$ factor has complex conjugation (in the defining representation) as an automorphism, it is however an inner automorphism.
- For the $SU(3)$, as for all $SU(N)$ algebras with $N \geq 1$, complex conjugation is an outer automorphism.

All outer automorphisms of the Standard Model algebra are combinations of these, since an automorphism maps the three invariant subalgebras into three isomorphic invariant subalgebras. There are infinitely many such automorphisms, but the Standard Model algebra together with the set of Standard Model properties (the rule system) is invariant under only one outer automorphism, namely complex conjugation of the $SU(3)$ combined with the $U(1)$ scaling factor $k = -1$.

Among all algebras of dimensionality up to 12 dimensions, taking quantization rule systems into account, there are four combinations of algebras and rule systems that have no generalized outer automorphisms, namely those with semisimple algebras $\mathfrak{su}(3)$ and $\mathfrak{so}(3)$.

14.3 The size of a representation

The other suggested selection principle, namely the size of a representation, was inspired by the fact that after the trivial representation, the lowest-dimensional non-abelian representations in the Standard Model are the remarkably small representations of $SU(2)$ and $SU(3)$.

A probablility argument for the presence of a selection principle can be formulated as follows: look at $S(U(2) \times U(3))$ and count the Lie groups of similarly low rank [6]. It turns out that there are about $2^8 = 256$ groups with low dimension (up to 12, i.e. the dimension of the SMG). Among these about 256 groups, $S(U(2) \times U(3))$ is singled out - most probably by means of some selection principle like the size of the representations.

In order to obtain a more precise formulation of the selection principle, we need to establish what we mean by the "size" of a representation. For this purpose we define a measure for this size in terms of the quadratic Casimir operators, which 'tag' the representations in the sense that they are not defined for the algebra itself, but only for the representations.

A general Casimir invariant is a function $f(F)$ of the Lie group generators F_j which is invariant under the group and commutes with all the generators,

$$[f(F), F_j] = 0.$$

The generators F_j of the group constitute a basis for the corresponding Lie algebra and satisfy the commutation relation $[F_i, F_k] = f^j_{ik} F_j, \quad i, k, j = 1, 2, \ldots, d_G$, where d_G is the dimension of the group and f^j_{ik} are the structure constants by means of which we can construct a Killing metric tensor $g_{kl} = f^j_{ki} f^i_{jl}$. The quadratic Casimir operator

$$\mathbf{C_2} = g^{kl} F_k F_l \tag{14.9}$$

is used for measuring the "size" of a representation r. This is done by normalizing the quadratic Casimir of the representation by dividing it with the quadratic Casimir for the adjoint representation, which consists of $d_G \times d_G$ matrices $\mathbf{A}_j$, such that $(\mathbf{A}_j)^k_l = -f^k_{jl}$. The metric can thus be written $g_{kl} = \mathrm{Tr}(\mathbf{A}_k \mathbf{A}_l)$, and in the first approximation, the "size" of the representation r is taken to be

$$\mathcal{S} = \left(\frac{C_r}{C_A}\right), \tag{14.10}$$

where C_r and C_A are the Casimirs for the representation r and the adjoint representation, respectively.

In the search for the groups with the smallest representations, we thus examine the quadratic Casimir operators, bearing in mind that the quadratic Casimir is well defined only for irreducible representations. Our goal is to show that the combined group $\mathcal{B} = \{(\lambda, \varphi) | \lambda \in \mathcal{G}, \varphi \in \mathcal{D}\}$ has a measure which is smaller than the Standard Model group measure,

$$\left(\frac{C_r}{C_A}\right)_{\mathcal{B}} < \left(\frac{C_r}{C_A}\right)_{SMG}, \tag{14.11}$$

since this is a way of necessitating the existence of the group of diffeomorphisms. When we talk about SMG, it should be noted that we actually have a SMG in each point of spacetime, corresponding to a product of SMG's: $SMG \times SMG \ldots \times SMG$, this product however has the same size measure as SMG itself, i.e. $(C_r/C_A)_{SMG \times SMG \ldots} = (C_r/C_A)_{SMG}$.

According to Schur's lemma [7], in an irreducible representation, any operator that commutes with all the generators of the Lie algebra must be a multiple of the identity operator. Therefore $C_r = g^{kl}F_kF_l = c_r\mathbf{1}$,where $\mathbf{1}$ is the $d_r \times d_r$ identity matrix, and c_r is a coefficient which only depends on the representation r, so we have

$$\mathcal{S} = \frac{c_r\mathbf{1}}{c_A\mathbf{1}} = \frac{c_r}{c_A} \tag{14.12}$$

The point is to minimize the relation c_ρ/c_A, with some normalization of c_A (the normalization in reality being arbitrary).

We are interested in the SU(N) group, which has the defining representation

$$\begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix}$$

and the group elements are **U** = NxN complex unitary matrices with determinant 1. The matrices $\mathbf{U}$ are $\approx \mathbf{1}$, and can be written as $\mathbf{U} = e^{iF}$, with infinitesimal generators F. These F's constitute a real vector space with the dimension N^2-1, i.e. the dimension of SU(N), and we can choose a basis in the F-space, $F_1, F_2, \ldots, F_{N^2-1}$, which can be normalized.

In an irreducible representation

$$\rho : \mathcal{G} \rightarrow (Matrices)$$

$$\rho(g) = \rho(1) + i\rho(F_j)g^j$$

the quadratic Casimir $g_{kl}\rho(F_k)\rho(F_l)$ is only an eigenvalue, but it represents how intensively $\rho(g)$ varies, in the sense that a small c_ρ corresponds to a "lazy" ρ.

The Casimirs thus function as a crude measure for how much the representation matrix varies as a function of the group element it represents. In a lattice context we take the contribution from one plaquette to be the trace of some representation of the group, the most general action $S_\square$ is then a linear expansion on traces of all the possible representations of the gauge group, and the traces of the smallest representations supposedly dominate. This domination corresponds to the variation of the action as a function of how the combination of the link variables varies over the gauge group, and if the action varies relatively slowly over the group, it's taken as an indication that it also varies relatively slowly when we vary the gauge group.

So with an action which is dominated by the contributions from small representations, the variation along the gauge variation is presumeably quite small, a situation corresponding to small quadratic Casimir values. This increases the chance that an action which was not perceived as invariant under a gauge transformation, would nevertheless appear as gauge invariant.

To get an intuition of this "smallness" of a representation, consider SU(2) with its quadratic Casimir $\vec{J}^2$. On an irreducible representation, $\vec{J}^2$ effectively only takes one value, i.e. it has the same eigenvalue on the whole representation. With

$\vec{J}^2 = g_{ij} J^i J^j$ we have a notion of distance, and we can visualise $\vec{J}^2$ as performing a rotation,

$$\vec{J}^2 |a> = j(j+1)|a>$$

where the "smallness" of the representation means that $|a>$ is just slightly rotated,

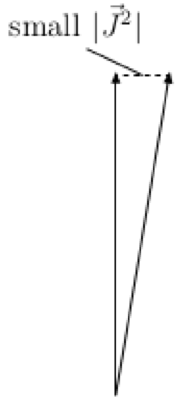

Fig. 14.1.

By means of the Casimir measure we can thus define a size measure, making it meaningful to say that the representations of the non-abelian parts of the Standard Model are "small". In the abelian case, it is however problematic to establish what we mean by the "size" of a representation. We cannot apply a similar reasoning for the U(1) groups as for non-abelian groups, because in the abelian case we cannot use the dimension of the representation as a measure, since abelian groups always have 1-dimensional representations, so dimension doesn't tell anything. There simply is no Casimir element defined, since for an abelian Lie algebra the Killing form is zero.

What we can do is to consider the ratio of the charges of the representation and refer to a "Quantum of charge", for example the Millikan unit quantum. The unique abelian invariant subgroup in the Standard Model gauge group corresponds to the weak hypercharge. That can however not be used as the Quantum, since the quantum for $y/2$ is $1/6$, while right-handed charged leptons have $y/2 = -1$, which is 6 times larger than $1/6$, and 6 is obviously not the smallest integer after zero.

So instead we consider non-invariant abelian subgroups, and define an abelian representation as small if it has relatively many charges (generators of the Lie algebra) with only the values 0, 1 or -1 measured in the Quantum.

It turns out that the Standard Model has a relatively large set of such charges, so in this perspective, even from the abelian point of view the Standard Model is a model with "small" representations.

14.3.1 The size of a composite group

In order to normalize the measure by means of the Casimir of the adjoint representation, we clearly need that there is a well-defined adjoint representation. In the case of a composed, non-simple group like $\mathcal{B} = \{(\lambda, \varphi)|\lambda \in \mathcal{G}, \varphi \in \mathcal{D}, \}$, there is however no straightforward definition of the adjoint representation. For $\mathcal{B}$, we therefore must find a way of varying the two adjoint normalizations relative to each other.

In order to achieve this, we seek to establish a (faithful, 1-1) representation r of $\mathcal{B}$ on which we define a metric, whereby the image of $\mathcal{B}$ becomes a manifold with a metric, allowing us to define a volume.

One way to do this is to establish a (faithful) representation r of $\mathcal{B}$ on which we define a metric. Thus the image of $\mathcal{B}$ becomes a manifold with a metric, which makes it possible to define a volume. The measure $(c_F/c_A)_{\mathcal{B}}$ is then given as the volume ratio of the two representations, taken to the power $2/d_{\mathcal{B}}$,

$$\left(\frac{c_F}{c_A}\right)_{\mathcal{B}} = \left(\frac{V_F}{V_{adj}}\right)^{2/d_{\mathcal{B}}}$$

In the representation picture $c_r \sim g_{ik}$, i.e.

$$g_{ik}^{(\mathcal{G})} = \frac{c_A}{c_r} g_{ik}^{(r)} \tag{14.13}$$

thus

$$\frac{\mathrm{Vol}(\mathcal{G})}{\mathrm{Vol}(\text{representation r})} = \left(\frac{c_A}{c_r}\right)^{d_{G/2}} \tag{14.14}$$

and in the case of

$$\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \ldots \times \mathcal{G}_k \tag{14.15}$$

where the $\mathcal{G}_j$ are simple, the quantity

$$\frac{c_r}{c_A} = \left[\left(\frac{c_{r_1}}{c_{A_1}}\right)^{d_1} \left(\frac{c_{r_2}}{c_{A_2}}\right)^{d_2} \ldots \left(\frac{c_{r_k}}{c_{A_k}}\right)^{d_k}\right]^{\frac{1}{d_1+d_2+\ldots+d_k}} \tag{14.16}$$

is a "good quantity".

14.3.2 Competing groups

For an irreducible representation r, consisting of a set of $r \times r$ matrices $\mathbf{M}_j$, the second-order index $I_2(r)$ of the representation is defined by

$$\mathrm{Tr}(\mathbf{M}_r^i \mathbf{M}_r^j) = I_2(r)\delta^{ij} \tag{14.17}$$

Taking the trace of (14.9), we get for the quadratic Casimir

$$c_2(r) = I_2(r) d_G/d_r \tag{14.18}$$

where d_r is the dimension of the representation r, and d_G is the group dimension. For the defining, fundamental representation N of $SU(N)$ (i.e., in reality the algebra

$\mathfrak{su}(N)$) the second-order index is $I_2(N) = 1/2$, and for the adjoint representation $I_2(A) = N$ and $d_A = N^2 - 1$, which gives $\left(\frac{c_N}{c_A}\right)_{SU(N)} = \frac{N^2-1}{2N^2}$, thus for $SU(2)$ $\left(\frac{c_N}{c_A}\right)_{SU(2)} = 3/8$.

There are presumably other candidates, like $SO(N)$, with the fundamental representation consisting of $N \times N$ real matrices. One can define higher tensor representations from the defining vector representation N, but there are also additional, double-valued spinor representations, similar to $SO(3) \sim SU(2)$, generated by direct products of the fundamental spinor.

In the case of the $SU(N)$ group, the faithful representation with the smallest quadratic Casimir, is the fundamental representation N, while for the $SO(N)$ group the picture is much more complicated, as the faithful representation F with the smallest quadratic Casimir might be either the vector representation, or the spinor representation, the spinor representation being the winner for $N < 8$.

For the vector representation, the second-order index for the $SO(N)$ fundamental and adjoint representations are $I_2(N) = 2$ and $I_2(A) = 2N - 4$, respectively, and the dimension of the adjoint representation $d_A = N(N-1)/2$, thus $\left(\frac{c_N}{c_A}\right)_{SO(N)} = \frac{N-1}{2(N-2)}$, and for the corresponding spinor representation we have $\left(\frac{c_N}{c_A}\right)^{spinor}_{SO(N)} = \frac{N-1}{2(N-2)}\frac{N}{8}$. Another competitor is $\mathfrak{sp}(2N)$, with $\left(\frac{c_N}{c_A}\right)_{sp(2N)} = \frac{2N+1}{4(N+1)}$, thus

$$
\begin{aligned}
\left(\frac{c_N}{c_A}\right)^{vector}_{SO(N)} &= \frac{N-1}{2(N-2)}, & \left(\frac{c_N}{c_A}\right)^{spinor}_{SO(N)} &= \frac{N-1}{2(N-2)}\frac{N}{8} \\
\left(\frac{c_N}{c_A}\right)_{SU(N)} &= \frac{N^2-1}{2N^2}, & \left(\frac{c_N}{c_A}\right)_{sp(2N)} &= \frac{2N+1}{4(N+1)}
\end{aligned}
\tag{14.19}
$$

In the search for the groups chosen by Nature, we examine the (c_N/c_A) for the different groups, but we also worry about possible differences between 3+1 and 4 spacetime dimensions. For example, for dimension d = 3 +1, we have for the Lorentz group $SO(3,1) \sim SL(2,C)$, while for d = 4, $SO(4) \sim SU(2)_R \times SU(2)_L$. For both d = 3 + 1 and d = 4, the Lorentz group however has the same small $\mathcal{S}$,

$$
\mathcal{S} = \left(\frac{c_N}{c_A}\right) = \frac{\frac{1}{2}(1+\frac{1}{2})}{1(1+1)} = \frac{3}{8}
\tag{14.20}
$$

while for d = 2 and d = 5, the value is bigger.

The Lorentz group $SO(d-p, p)$ in reality comes from a symmetric metric $g_{ik} = g_{ki}$. If g_{ki} instead had been antisymmetric, we would have symplectic groups, which are not so competitive, as they have bigger (c_N/c_A). We consider the Lorentz group as a function of the dimension d and of the "geometry", in the sense of the dependence on whether g_{ik} is symmetric, antisymmetric or nonexistent. But an antisymmetric g_{ik} actually does very poorly, while for d = 3,4 it looks good for symmetric g_{ik}; and for the case without metric for d = 2 [8].

Among simple groups, SU(2) has the smallest (c_N/c_A), but in order to allow SU(3) be let in, some cooperation with SU(2) is necessary, since the Spin(5), which is the covering group of SO(5), in reality seems to outdo SU(3).

SU(3) however has a relatively big center Z_3, so if we divide by the group center SU(3) is in good shape, since the SO(5) covering group has a smaller center. For SU(3), the number of elements in the center is 3, while the center of SU(2) merely has 2 elements, and likewise for Spin(5). We thus redefine our measure of representations as

$$\mathcal{S} = (\frac{c_r}{c_A})\frac{1}{[(\text{Number of elements in the center})]^{2/d}} \tag{14.21}$$

We in reality consider volumes:

$$\frac{Vol(SU(3))}{Vol(SU(3)/Z_3)} = 3$$

And with $SU(2)/Z_2 = SO(3)$,

$$\frac{Vol(SU(2))}{Vol(SO(3))} = 2.$$

the quadratic Casimir being a sort of area in the group.

14.4 The group of diffeomorphisms

We define our group $\mathcal{B}$ as the combination of the gauge transformations of SMG and the group of diffeomorphisms.

A diffeomorphism so to say moves a function, by the operation

$$x^\mu \to x^\mu + \eta^\mu$$

The displacement takes place in a given direction, and if we perceive the diffeomorphisms as vectors over a manifold, then for infinitesimal η^μ the set of displacements $\{\eta^\mu\}$ constitutes a tangent field. The group of diffeomorphisms does not have a (usual) Lie algebra, but we take as the Lie algebra a set of fields $\{\epsilon^\mu\}$ corresponding to the tangents

$$f(x) = \sum_\mu \epsilon^\mu \partial_\mu, \tag{14.22}$$

which amounts to substituting a manifold with a space of functions on the manifold,

$$[f_1(x), f_2(x)] = [\sum_\mu \epsilon^\mu \partial_\mu, \sum_\nu \epsilon^\nu \partial_\nu], \tag{14.23}$$

and then we could take

$$C = \int g_{\mu\nu} \epsilon^\mu(x) \epsilon^\nu(x) dx \tag{14.24}$$

as a kind of Casimir. There are scarcely any outer automorphisms for the group of diffeomorphisms, and if all the automorphisms for the group of diffeomorphisms are inner, the group of diffeomorphisms is maximally skew. It should however be noted that the group of diffeomorphisms depends on the topology of the space on which it is operating, for example for $\mathbf{R}^4$, the diffeomorphism group has a trivial center.

Even though the Lie algebra for the group of diffeomorphisms is not a usual Lie algebra, the group is still a Lie group. Consider the mappings of a manifold onto itself, $\mathcal{M}$, $\varphi : x \to x'$, i.e.

$$\varphi : \mathcal{M} \to \mathcal{M} \quad \text{where} \;\; \varphi \;\; \text{is}$$

- bijective,
- sufficiently many times continuously differentiable,
- a group under the group of diffeomorphisms,

then $\varphi : \mathcal{M} \to \mathcal{M}$ is really a "Lie group", which is clear by considering $\varphi + \delta\varphi$ and take the commutators $[\delta\varphi_1, \delta\varphi_2] = [1 + \delta\varphi_1, 1 + \delta\varphi_2] \neq 0$.

One difficulty we meet with respect to the combined group $\mathcal{B}$ is that the group $\mathcal{D}$ of diffeomorphisms is probably simple, while the group $\mathcal{G}$ of gauge transformations is not,

$$g \in \mathcal{G} | g : \mathbf{R}^4 \to SMG,$$

meaning

$$g : \mathbf{R}^4 \to S(U(2) \times U(3)); g(x) \in S(U(2) \times U(3)),$$

and

$$f, g : \mathbf{R}^4 \to S(U(2) \times U(3)); (fg)(x) = f(x) \cdot g(x)$$

and for a non-simple group we cannot define a straightforward measure like (c_r/c_A) for the size of representation. There is however one possibility to define a quadratic Casimir replacement, viz.

$$\ln"c_N" = \int \ln C \sqrt{g} d^4x \tag{14.25}$$

where $g = \det(g_{ik})$. The problem is that we cannot really have a metric, since a metric would not be diffeomorphism-symmetric. On the other hand, we don't quite need the metric g_{ik}, but only $\sqrt{g}$.

14.5 The composite group

Our selected group is $\mathcal{B} | \{(\lambda, \varphi), \lambda \in \mathcal{G}, \varphi \in \mathcal{D}\}$, composed by the group of gauge transformations

$$\mathcal{G} = \{\lambda : \mathcal{M} \to SMG\}$$

and the group of diffeomorphisms

$$\mathcal{D} = \{\varphi : \mathcal{M} \to \mathcal{M}\}$$

In order to investigate the group structure, we determine the action of the group elements.

Let Ψ_l be a fermion state, and let (λ, φ) operate on Ψ_l. With the dfinition

$$(\lambda, \varphi)[\Psi_l](x) = \rho_l^k(\lambda(\varphi(x)))[\Psi_k](\varphi(x)) \tag{14.26}$$

where ρ_l^k is the representation matrix, we want to determine the properties of the group operation $\circ$ of $\mathcal{B}$, i.e. of $(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2)$. First consider

$$(\lambda, 1)[\Psi_k](x) = \rho_k^l(\lambda(x))[\Psi_l](x)$$
$$(1, \varphi)[\Psi_k](x) = [\Psi_k](\varphi(x)) = [\Psi_k \circ \varphi](x),$$

thus

$$\begin{aligned}(\lambda, 1) \circ (1, \varphi)[\Psi_k](x) = (\lambda, 1)[\Psi_k \circ \varphi](x) = \\ = \rho_k^l(\lambda(x))[\Psi_l(\varphi(x))]\end{aligned} \tag{14.27}$$

Then consider

$$\begin{aligned}(1, \varphi) \circ (\lambda, 1)[\Psi_k](x) = (1, \varphi)\rho_k^l(\lambda(x))[\Psi_l(x)] = \\ = \rho_k^l(\lambda(\varphi(x)))[\Psi_l(\varphi(x))]\end{aligned}$$

which leads to the conclusion that

$$\begin{aligned}(\lambda, \varphi) \neq (\lambda, 1) \circ (1, \varphi) \\ (\lambda, \varphi) = (1, \varphi) \circ (\lambda, 1)\end{aligned} \tag{14.28}$$

When investigating $(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2)$ we use that

$$(1, \varphi)[\chi](x) = (\chi \circ \varphi)(x)$$

thus

$$\begin{aligned}(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2)[\Psi_k](x) &= (\lambda_1, \varphi_1)\rho_k^l(\lambda_2(\varphi_2(x)))[\Psi_l(\varphi_2(x))] \\ &= (\lambda_1, \varphi_1)[\rho_k^l(\lambda_2(\varphi_2(x)))(\Psi_l \circ \varphi_2)(x)] \\ &= \rho_l^m(\lambda_1(\varphi_1(x)))\rho_k^l(\lambda_2(\varphi_2 \circ \varphi_1(x)))(\Psi_m \circ \varphi_2 \circ \varphi_1)(x)\end{aligned} \tag{14.29}$$

which we identify with

$$(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2)[\Psi_k](x) = (\lambda_3, \varphi_3)[\Psi_k](x) = \rho_k^m(\lambda_3(\varphi_3(x)))(\Psi_m \circ \varphi_3(x)) \tag{14.30}$$

thus

$$\varphi_3 = \varphi_2 \circ \varphi_1, \tag{14.31}$$

We demand that for all $[\Psi_k](x)$

$$\rho_k^m(\lambda_3(\varphi_3(x))) = \rho_l^m(\lambda_1(\varphi_1(x)))\rho_k^l(\lambda_2(\varphi_2 \circ \varphi_1(x)))$$

which can only be achieved for a faithful representation, and

$$\lambda_1(\varphi_1) \cdot \lambda_2(\varphi_3) = \lambda_3(\varphi_3)$$

(where $\cdot$ is the group operation for $\mathcal{G}$) which applies to all x, a special case being $\varphi_3^{-1}(x)$. We perform the substitution $x \to \varphi_3^{-1}(x)$, thus obtaining

$$\lambda_1(\varphi_1 \circ \varphi_3^{-1}(x)) \cdot \lambda_2(x) = \lambda_3(x)$$

and with $\varphi_3 = \varphi_2 \circ \varphi_1$, we get $\varphi_1 \circ \varphi_3^{-1} = \varphi_2^{-1}$ and

$$\lambda_1(\varphi_2^{-1}(x)) \cdot \lambda_2(x) = \lambda_3(x), \tag{14.32}$$

thus

$$(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2) = (\lambda_3, \varphi_3) = (\lambda_1(\varphi_2^{-1}(\cdot)) \cdot \lambda_2, \varphi_2 \circ \varphi_1) \tag{14.33}$$

Now $\lambda_1(\varphi_2^{-1}(\cdot)) \in \mathcal{G}$, and

$$(\lambda_1(\varphi_2^{-1}(\cdot), 1))\Psi_k(x) = \rho_k^l(\lambda_1(\varphi_2^{-1}(x)))\Psi_l(x),$$

but exchange of argument in λ, i.e. $\lambda(x) \to \lambda(\varphi(x))$, is an automorphism in $\mathcal{G}$, and in this sense,

$$\lambda_1(\varphi_2^{-1}(x)) = [\Phi_{\varphi_2^{-1}}(\lambda_1)](x),$$

is an automorphism in $\mathcal{G}$. The product of two elements of $\mathcal{B}$ finally reads

$$(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2) = (\lambda_3, \varphi_3) = (\Phi_{\varphi_2^{-1}}(\lambda_1) \cdot \lambda_2, \varphi_2 \circ \varphi_1) \tag{14.34}$$

With the alternative definition $(\lambda, \varphi)[\Psi_l](x) = \rho_l^k(\lambda(\varphi^{-1}(x)))[\Psi_k](\varphi^{-1}(x))$, we moreover get that

$$(\lambda_1, \varphi_1) \circ (\lambda_2, \varphi_2) = (\lambda_3, \varphi_3) = (\Phi_{\varphi_2}(\lambda_1) \cdot \lambda_2, \varphi_1 \circ \varphi_2) \tag{14.35}$$

14.5.1 Subgroups of $\mathcal{B}$

Does $\mathcal{B} = \{(\lambda, \varphi)\}$ have any subgroups? The relation (14.32) seems to indicate that the gauge group $\mathcal{G}$ is a normal (invariant) subgroup of $\mathcal{B}$, which means that for $b \in \mathcal{B}$, $b\mathcal{G}b^{-1} \subseteq \mathcal{G}$.

With $(\lambda, \varphi)[\Psi_l](x) = \rho_l^k(\lambda(\varphi(x)))[\Psi_k](\varphi(x))$ and (14.34), i.e.

$$\lambda_3 = \Phi_{\varphi_2^{-1}}(\lambda_1) \cdot \lambda_2, \qquad \varphi_3 = \varphi_2 \circ \varphi_1$$

we take

$$\begin{aligned}(\Lambda, \varphi) \circ (\lambda, 1) \circ (\Lambda, \varphi)^{-1} &= (\Lambda, \varphi) \circ (\lambda, 1) \circ (\Phi_\varphi^{-1}(\Lambda), \varphi^{-1}) = \\ &= (\Phi_\varphi(\Lambda) \cdot \Phi_\varphi(\lambda) \cdot \Phi_\varphi^{-1}(\Lambda), 1) \in \mathcal{G}\end{aligned} \tag{14.36}$$

and specifically for $(1, \varphi)$, we get $(1, \varphi) \circ (\lambda, 1) \circ (1, \varphi)^{-1} = (\lambda, 1)$, so for these specific representatives $(1, \varphi)$ of the cosets of

$$\{(\lambda, 1) | \lambda : \mathcal{M} \to \mathcal{G}\},$$

$(\lambda, 1)$ is similarity transformation invariant, and we conclude that $\mathcal{G}$ is a normal subgroup of $\mathcal{B}$. This implies that $\mathcal{B}$ is not simple, but a semi-direct product group, unless the subgroup $\mathcal{D}$ of diffeomorphisms also is normal, i.e.

$$(\lambda, 1)(1, \varphi)(\lambda, 1)^{-1} \in \mathcal{D}$$

Again using (14.34), we get that

$$(\lambda, \omega) \circ (1, \varphi) \circ (\lambda, \omega)^{-1} = (\lambda, \omega) \circ (1, \varphi) \circ (\Phi_\omega^{-1}(\lambda), \omega^{-1}) = (\Phi_\varphi^{-1}(\lambda) \cdot \lambda^{-1}, \varphi) \notin \mathcal{D} \tag{14.37}$$

thus $\mathcal{D}$ is not an invariant subgroup of $\mathcal{B}$, and $\mathcal{B}$ is a semidirect product of $\mathcal{G}$ and $\mathcal{D}$,

$$\mathcal{B} = \mathcal{G} \rtimes \mathcal{D} \tag{14.38}$$

This means that $\phi_{\varphi^{-1}}$ in (14.34) is a group homomorphism $\phi_{\varphi^{-1}} : \mathcal{D} \to \mathrm{Aut}(\mathcal{G})$, where $\mathrm{Aut}(\mathcal{D})$ denotes the group of automorphisms of $\mathcal{D}$.

14.6 To evaluate the size of $\mathcal{B}$

We started out from the Standard Model group SMG, which in itself is a compact, 12-dimensional manifold. When we go to the bigger group $\mathcal{B}$ encompassing the group of gauge transformations extended with the group of diffeomorphims, we are dealing with an infinite dimensional Lie group. But this does not necessarily have to be so devastating, keeping in mind that the effect of $\mathcal{D}$ in $\mathcal{B}$ in reality is nothing more than to dislocate the different SMG in the product $SMG \times SMG \ldots \times SMG$, $g^{ab}\rho(F_a)\rho(F_b)$. Compared to $\prod SMG$, the group $\mathcal{B}$ (where also $\mathcal{D}$ is included) will still have the same representations.

In deciding on how to measure the size of a representation, we have encountered a set of problems,

- How to establish a viable 'size' for the $U(1)$ group in SMG.
- How do we handle the problem with the adjoint representation in the case of $\mathcal{B}$?
- How do we define c_F/c_A for a semidirect product?

In spite of all these problems, let us make an attempt to evaluate the difference between the measures for SMG and $\mathcal{B}$, respectively. Represent $\mathcal{D}$ by the Lorentz group, taken as $SO(3,1)$ or $SO(4)$, supposing we are in 3+1 or 4 dimensions, and use $(c_F/c_A) = 3/8$ (with the corresponding group dimension 6). In accordance with (14.16) we then define a tentative measure for the composite group $\mathcal{B} = \mathcal{G} \rtimes \mathcal{D}$, as

$$''\mathcal{S}_{\mathcal{B}}'' = \left[\left(\frac{c_F}{c_A}\right)_{SU(2)}^{d(SU(2))} \cdot \left(\frac{c_F}{c_A}\right)_{SU(3)}^{d(SU(3))} \cdot \left(\frac{c_F}{c_A}\right)_{SO(4)}^{d(SO(4))} \right]^{\frac{1}{\sum d_i}} = \\ = \left[\left(\frac{3}{8}\right)^3 \cdot \left(\frac{4}{9}\right)^8 \cdot \left(\frac{3}{8}\right)^6 \right]^{\frac{1}{17}} = 0.406213... \tag{14.39}$$

where we for $\mathcal{S}_{\mathcal{G}}$ have used $\mathcal{S}_{SU(2)\otimes SU(3)}$, keeping in mind that the quadratic Casimir for $\prod SMG$ is the same as for SMG itself, and ignored $U(1)$.

For SMG alone, we get

$$''\mathcal{S}_{\mathcal{G}}'' = \left[\left(\frac{c_F}{c_A}\right)_{SU(2)}^{d(SU(2))} \cdot \left(\frac{c_F}{c_A}\right)_{SU(3)}^{d(SU(3))} \right]^{\frac{1}{\sum d_i}} = \\ = \left[\left(\frac{3}{8}\right)^3 \cdot \left(\frac{4}{9}\right)^8 \right]^{\frac{1}{11}} = 0.424320..., \tag{14.40}$$

so in this crude approach, $''\mathcal{S}_{\mathcal{B}}'' < ''\mathcal{S}_{\mathcal{G}}''$.

$$''\mathcal{S}_{\mathcal{G}}'' = \left[\left(\frac{c_F}{c_A}\right)_{SU(2)}^{d(SU(2))} \cdot \left(\frac{c_F}{c_A}\right)_{SU(3)}^{d(SU(3))} \right]^{\frac{1}{\sum d_i}} = \\ = \left[\left(\frac{3}{8}\right)^3 \cdot \left(\frac{4}{9}\right)^8 \right]^{\frac{1}{11}} = 0.424320..., \tag{14.41}$$

There is however another aspect to this. Let us make no assumption about the dimension N in $SO(N)$, and simply plot the expression (14.39) for $''\mathcal{S}_{\mathcal{B}}''$ as a function of N for $N \leq 8$,

$$
\begin{aligned}
{}''\mathcal{S}_{\mathcal{B}}'' &= \left[\left(\frac{c_F}{c_A}\right)_{SU(2)}^{d(SU(2))} \cdot \left(\frac{c_F}{c_A}\right)_{SU(3)}^{d(SU(3))} \cdot \left(\frac{c_F}{c_A}\right)_{SO(N)}^{d(SO(N))} \right]^{\frac{1}{\sum d_i}} = \\
&= \left[\left(\tfrac{3}{8}\right)^3 \cdot \left(\tfrac{4}{9}\right)^8 \cdot \left(\frac{N(N-1)}{16(N-2)}\right)^{\frac{N(N-1)}{2}} \right]^{\frac{1}{11+\frac{N(N-1)}{2}}},
\end{aligned}
\tag{14.42}
$$

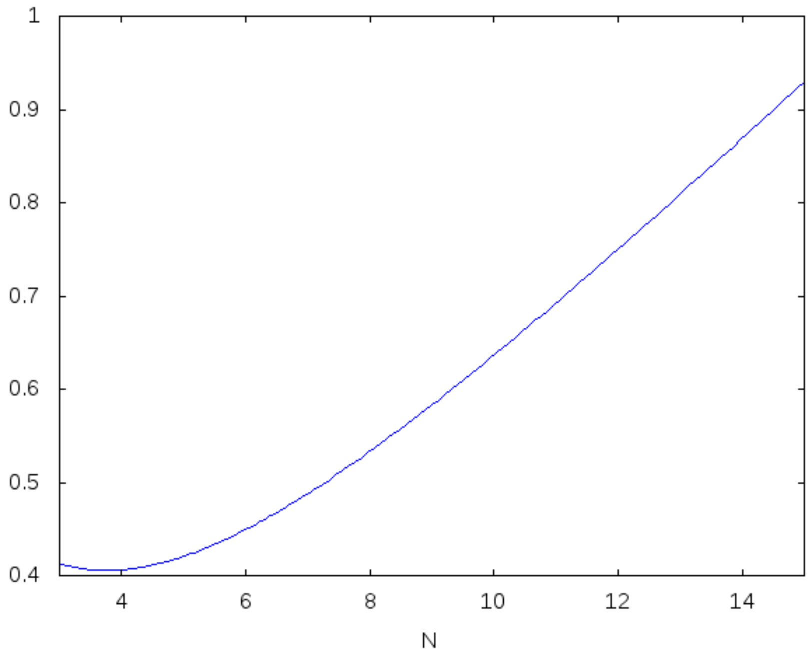

Fig. 14.2.

corresponding to a minimal value for the size $''\mathcal{S}_{\mathcal{B}}''$ at $N = 4$. These encouraging results, both for the relative smallness of $''\mathcal{S}_{\mathcal{B}}''$ compared to $''\mathcal{S}_{\mathcal{G}}''$, and well as the singling out of $N = 4$, are of course based on a coarse evaluation, which is to be refined with a more precise formulation of the quadratic Casimir for the group $\mathcal{B}$, in order to accomplish a fair comparison between the sizes of the two groups.

14.7 Conclusion

In this article we have taken the first steps in "deriving" diffeomorphism symmetry, which is called for within the framework of the derivation of space. We have discussed different "goal quantities", especially the size of a representation of a group, identified as the size of the quadratic Casimir, which is connected with natural metric on the space of unitary matrices in the representations.

With this ”goal quantity” in mind, we argue that diffeomorphism symmetry necessarily comes about, because the size of the bigger group, which is the semidirect product of the Standard Model group and the group of diffeomorphisms, is smaller than the size of th Standard Model group.

The next step will be to calculate the Casimirs for the entire group $\mathcal{B}$, and more precisely evaluate the size of $\mathcal{B}$ as compared with the size of the Standard Model group.

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Virtual Institute of Astroparticle Physics Presentation

BLED WORKSHOPS
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15 Virtual Institute of Astroparticle physics and "What comes beyond the Standard model?" in Bled

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Abstract. Virtual Institute of Astroparticle Physics (VIA) is a unique multi-functional complex of science and education online. VIA website
`http://viavca.in2p3.fr/site.html`
supports participation in conferences and meetings, various forms of collaborative scientific work as well as programs of education at distance, combining online videoconferences with extensive library of records of previous meetings and Discussions on Forum. The VIA facility is regularly effectively used in the programs of Bled Workshops. At XVII Bled Workshop it provided a world-wide discussion of the open questions of physics beyond the standard model.

Povzetek. Virtual Institute of Astroparticle Physics (VIA) predstavlja edinstven večnamenski kompleks znanosti in izobraževanja na spletu. Njegova domača stran
`http://viavca.in2p3.fr/site.html`
podpira udeležbo na konferencah in srečanjih, različne oblike znantsvenega sodelovanja in programe izobraževanja na daljavo. Kombinira video konference z obsežno knjižnico digitalnih zapisov prejšnjih srečanj in diskusije na forumu VIA. Možnosti VIA se redno in obsežno uporabljajo v programih Blejskih delavnic. Na sedemnajsti delavnici je omogočil diskusijo o odprtih problemih fizike onkraj obeh standardnih modelov udeležencem širom sveta.

15.1 Introduction

Studies in astroparticle physics link astrophysics, cosmology, particle and nuclear physics and involve hundreds of scientific groups linked by regional networks (like ASPERA/ApPEC [1,2]) and national centers. The exciting progress in these studies will have impact on the knowledge on the structure of microworld and Universe in their fundamental relationship and on the basic, still unknown, physical laws of Nature (see e.g. [3,4] for review).

Virtual Institute of Astroparticle Physics (VIA) [5] was organized with the aim to play the role of an unifying and coordinating structure for astroparticle physics. Starting from the January of 2008 the activity of the Institute takes place on its website [6] in a form of regular weekly videoconferences with VIA lectures, covering all the theoretical and experimental activities in astroparticle physics and related topics. The library of records of these lectures, talks and their presentations was accomplished by multi-lingual Forum. In 2008 VIA complex was effectively used for the first time for participation at distance in XI Bled Workshop [7]. Since then VIA videoconferences became a natural part of Bled Workshops' programs, opening the virtual room of discussions to the world-wide audience. Its progress was presented in [8–12]. Here the current state-of-art of VIA complex, integrated since the end of 2009 in the structure of APC Laboratory, is presented in order to clarify the way in which VIA discussion of open questions beyond the standard model took place in the framework of XVII Bled Workshop.

15.2 The structure of VIA complex and forms of its activity

15.2.1 The forms of VIA activity

The structure of VIA complex is illustrated on Fig. 15.1. The home page, presented on this figure, contains the information on VIA activity and menu, linking to directories (along the upper line from left to right): with general information on VIA (About VIA), entrance to VIA virtual rooms (Rooms), the library of records and presentations (Previous) of VIA Lectures (Previous → Lectures), records of online transmissions of Conferences(Previous → Conferences), APC Colloquiums (Previous → APC Colloquiums), APC Seminars (Previous → APC Seminars) and Events (Previous → Events), Calender of the past and future VIA events (All events) and VIA Forum (Forum). In the upper right angle there are links to Google search engine (Search in site) and to contact information (Contacts). The announcement of the next VIA lecture and VIA online transmission of APC Colloquium occupy the main part of the homepage with the record of the most recent VIA events below. In the announced time of the event (VIA lecture or transmitted APC Colloquium) it is sufficient to click on "to participate" on the announcement and to Enter as Guest (printing your name) in the corresponding Virtual room. The Calender links to the program of future VIA lectures and events. The right column on the VIA homepage lists the announcements of the regularly up-dated hot news of Astroparticle physics and related areas.

In 2010 special COSMOVIA tours were undertaken in Switzerland (Geneva), Belgium (Brussels, Liege) and Italy (Turin, Pisa, Bari, Lecce) in order to test stability of VIA online transmissions from different parts of Europe. Positive results of these tests have proved the stability of VIA system and stimulated this practice at XIII Bled Workshop. The records of the videoconferences at the XIII Bled Workshop are available on VIA site [13].

Since 2011 VIA facility was used for the tasks of the Paris Center of Cosmological Physics (PCCP), chaired by G. Smoot, for the public programme "The two infinities" conveyed by J.L.Robert and for effective support a participation at distance at meetings of the Double Chooz collaboration. In the latter case, the experimentalists, being at shift, took part in the collaboration meeting in such a virtual way.

The simplicity of VIA facility for ordinary users was demonstrated at XIV Bled Workshop in 2011. Videoconferences at this Workshop had no special technical

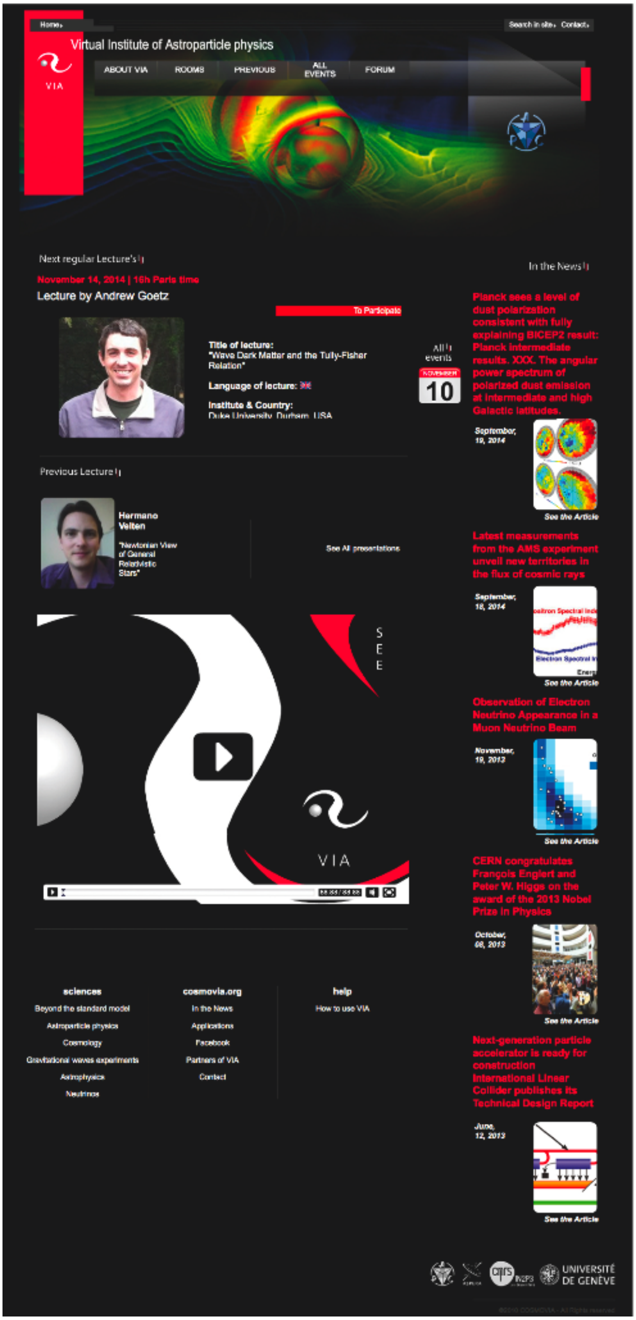

Fig. 15.1. The home page of VIA site

support except for WiFi Internet connection and ordinary laptops with their internal video and audio equipments. This test has proved the ability to use VIA facility at any place with at least decent Internet connection. Of course the quality of records is not as good in this case as with the use of special equipment, but still it is sufficient to support fruitful scientific discussion as can be illustrated by the record of VIA presentation ″New physics and its experimental probes″ given by John Ellis from his office in CERN (see the records in [14]).

In 2012 VIA facility, regularly used for programs of VIA lectures and transmission of APC Colloquiums, has extended its applications to support M.Khlopov's talk at distance at Astrophysics seminar in Moscow, videoconference in PCCP, participation at distance in APC-Hamburg-Oxford network meeting as well as to provide online transmissions from the lectures at Science Festival 2012 in University Paris7. VIA communication has effectively resolved the problem of referee's attendance at the defence of PhD thesis by Mariana Vargas in APC. The referees made their reports and participated in discussion in the regime of VIA videoconference.

In 2013 VIA lecture by Prof. Martin Pohl was one of the first places at which the first hand information on the first results of AMS02 experiment was presented [15].

In 2012 VIA facility was first used for online transmissions from the Science Festival in the University Paris 7. This tradition was continued in 2013, when the transmissions of meetings at Journes nationales du Dveloppement Logiciel (JDEV2013) at Ecole Politechnique (Paris) were organized [16].

In 2014 the 100th anniversary of one of the foundators of Cosmoparticle physics, Ya. B. Zeldovich, was celebrated. With the use of VIA M.Khlopov could contribute the programme of the ″Subatomic particles, Nucleons, Atoms, Universe: Processes and Structure International conference in honor of Ya. B. Zeldovich 100th Anniversary″ (Minsk, Belarus) by his talk ″Cosmoparticle physics: the Universe as a laboratory of elementary particles″ [17] and the programme of ″Conference YaB-100, dedicated to 100 Anniversary of Yakov Borisovich Zeldovich″ (Moscow, Russia) by his talk ″Cosmology and particle physics″ [18].

The discussion of questions that were put forward in the interactive VIA events can be continued and extended on VIA Forum. The Forum is intended to cover the topics: beyond the standard model, astroparticle physics, cosmology, gravitational wave experiments, astrophysics, neutrinos. Presently activated in English,French and Russian with trivial extension to other languages, the Forum represents a first step on the way to multi-lingual character of VIA complex and its activity.

One of the interesting forms of Forum activity is the educational work at distance. For the last five years M.Khlopov's course ″Introduction to cosmoparticle physics″ is given in the form of VIA videoconferences and the records of these lectures and their ppt presentations are put in the corresponding directory of the Forum [19]. Having attended the VIA course of lectures in order to be admitted to exam students should put on Forum a post with their small thesis. Professor's comments and proposed corrections are put in a Post reply so that students should continuously present on Forum improved versions of work until it is accepted as satisfactory. Then they are admitted to pass their exam. The record of videoconference with their oral exam is also put in the corresponding directory of Forum. Such procedure provides completely transparent way of evaluation of students' knowledge. In 2014 the second part of this course was used for a test of VIA system as a possible supplementary tool for Massive Online Open Courses (MOOC) activity [20]. In the context of MOOC VIA facility can be used for individual online work with advanced students.

15.2.2 Organisation of VIA events and meetings

First tests of VIA system, described in [5,7–9], involved various systems of videoconferencing. They included skype, VRVS, EVO, WEBEX, marratech and adobe Connect. In the result of these tests the adobe Connect system was chosen and properly acquired. Its advantages are: relatively easy use for participants, a possibility to make presentation in a video contact between presenter and audience, a possibility to make high quality records, to use a whiteboard facility for discussions, the option to open desktop and to work online with texts in any format.

The normal amount of connections to the virtual room at VIA lectures and discussions usually didn't exceed 20. However, the sensational character of the exciting news on superluminal propagation of neutrinos acquired the number of participants, exceeding this allowed upper limit at the talk "OPERA versus Maxwell and Einstein" given by John Ellis from CERN. The complete record of this talk and is available on VIA website [21]. For the first time the problem of necessity in extension of this limit was put forward and it was resolved by creation of a virtual "infinity room", which can host any reasonable amount of participants. Starting from 2013 this room became the only main virtual VIA room, but for specific events, like Collaboration meetings or transmissions from science festivals, special virtual rooms can be created. This solution strongly reduces the price of the licence for the use of the adobeConnect videoconferencing, retaining a possibility for creation of new rooms with the only limit to one administrating Host for all of them.

The ppt or pdf file of presentation is uploaded in the system in advance and then demonstrated in the central window. Video images of presenter and participants appear in the right window, while in the lower left window the list of all the attendees is given. To protect the quality of sound and record, the participants are required to switch out their microphones during presentation and to use the upper left Chat window for immediate comments and urgent questions. The Chat window can be also used by participants, having no microphone, for questions and comments during Discussion. The interactive form of VIA lectures provides oral discussion, comments and questions during the lecture. Participant should use in this case a "raise hand" option, so that presenter gets signal to switch our his microphone and let the participant to speak. In the end of presentation the central window can be used for a whiteboard utility as well as the whole structure of windows can be changed, e.g. by making full screen the window with the images of participants of discussion.

Regular activity of VIA as a part of APC includes online transmissions of all the APC Colloquiums and of some topical APC Seminars, which may be of interest for a wide audience. Online transmissions are arranged in the manner, most convenient for presenters, prepared to give their talk in the conference room in a normal way, projecting slides from their laptop on the screen. Having uploaded in advance these slides in the VIA system, VIA operator, sitting in the conference room, changes them following presenter, directing simultaneously webcam on the presenter and the audience.

15.3 VIA Sessions at XVII Bled Workshop

VIA sessions of XVII Bled Workshop have developed from the first experience at XI Bled Workshop [7] and their more regular practice at XII, XIII, XIV, XV and XVI Bled Workshops [8–12]. They became a regular part of the Bled Workshop's programme.

In the course of XVII Bled Workshop meeting the list of open questions was stipulated, which was proposed for wide discussion with the use of VIA facility. The list of these questions was put on VIA Forum (see [22]) and all the participants of VIA sessions were invited to address them during VIA discussions. During the XVII Bled Workshop the test of not only minimal necessary equipment, but either of the use of VIA facility by ordinary users was undertaken. VIA Sessions were supported by personal laptop with WiFi Internet connection only, as well as in 2014 the members of VIA team were physically absent in Bled and all the videoconferences were directed by M.Khlopov at distance. It proved the possibility to provide effective interactive online VIA videoconferences even in the absence of any special equipment and qualified personnel at place. Only laptop with microphone and webcam together with WiFi Internet connection was proved to support not only attendance, but also VIA presentations and discussions. In the absence of WiFi connection, the 3G connection of iPhone was sufficient for VIA management and presentations.

In the framework of the program of XVII Bled Workshop, M. Khlopov, gave his Introduction at distance to the M.Laletin's talk "Dark Atoms and Their decaying Constituents" (Fig. 15.2). It provided an additional demonstration of the ability of VIA to support the creative non-formal atmosphere of Bled Workshops (see records in [23]).

VIA sessions also included the talks "Novel string field theory solving string theory liberating left and right movers" by Holger Bech Nielsen, "The spin charge family theory offers the explanation for the assumption of the Standard model, for the Dark matter, for the Matter-antimatter asymmetry..., making several predictions" by Norma Mankoc-Borstnik (Fig. 15.3) and"Dark matter particles in the galactic halo" by Rita Bernabei (Fig. 15.4), followed by discussions with distant participants. The records of these lectures and discussions can be found in VIA library [23].

15.4 Conclusions

The Scientific-Educational complex of Virtual Institute of Astroparticle physics provides regular communication between different groups and scientists, working

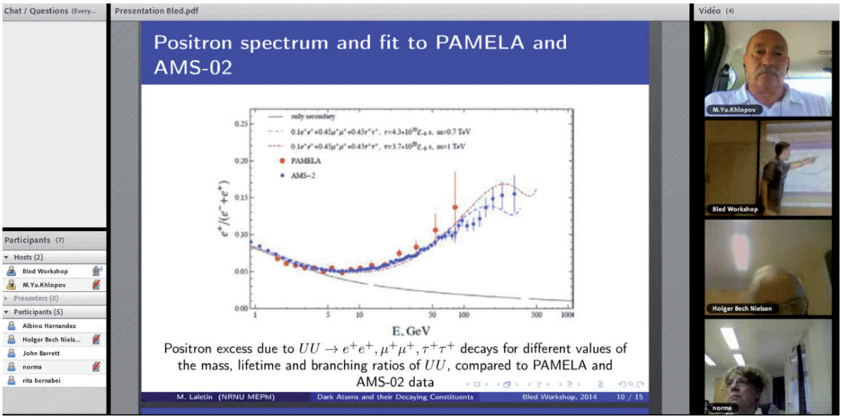

Fig. 15.2. VIA talk by M.Khlopov served as Introduction to M.Laletin's talk at XVII Bled Workshop

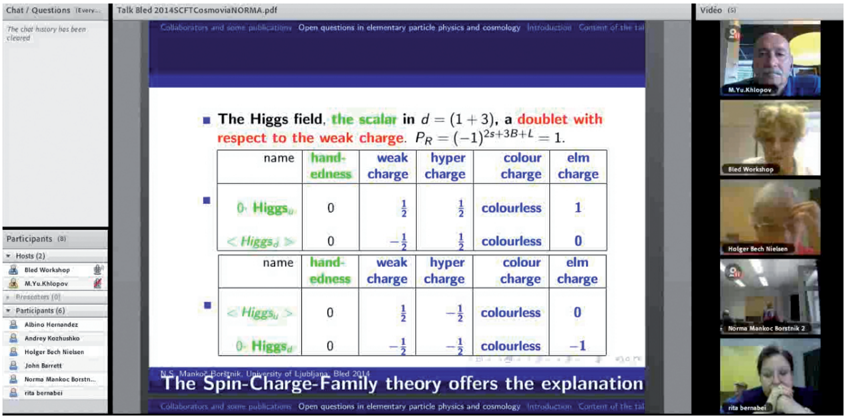

Fig. 15.3. VIA talk by N. Mankoc-Borstnik at XVII Bled Workshop

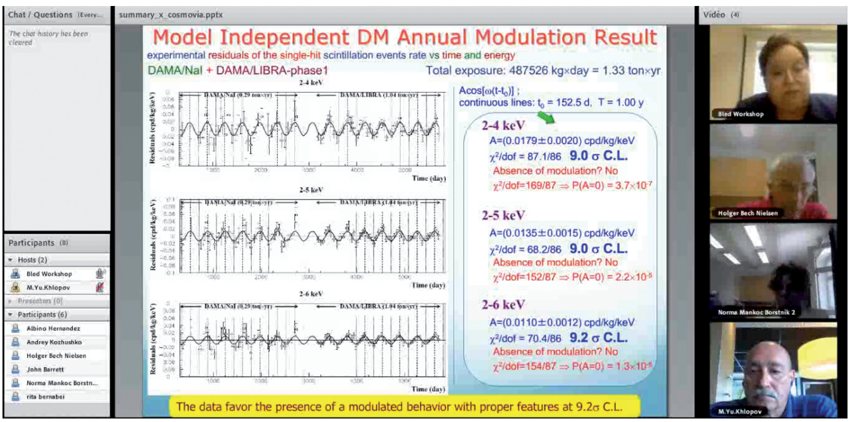

Fig. 15.4. VIA talk by R.Bernabei at XVII Bled Workshop

in different scientific fields and parts of the world, the first-hand information on the newest scientific results, as well as support for various educational programs at distance. This activity would easily allow finding mutual interest and organizing task forces for different scientific topics of astroparticle physics and related topics. It can help in the elaboration of strategy of experimental particle, nuclear, astrophysical and cosmological studies as well as in proper analysis of experimental data. It can provide young talented people from all over the world to get the highest level education, come in direct interactive contact with the world known scientists and to find their place in the fundamental research. These educational aspects of VIA activity can provide nontrivial supplementary tool for MOOC. VIA applications can go far beyond the particular tasks of astroparticle physics and give rise to an interactive system of mass media communications.

VIA sessions became a natural part of a program of Bled Workshops, maintaining the platform of discussions of physics beyond the Standard Model for distant participants from all the world. The experience of VIA applications at Bled Workshops plays important role in the development of VIA facility as an effective tool of science and education online.

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