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AN OVERVIEW OF MENTAL CALCULATION STRATEGIES AND THE FREQUENCY OF THEIR APPLICATION

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Abstract/Izvleček

The paper aims to explore mental calculation strategies and the frequency of their application in tasks. In the paper, we categorized mental calculation strategies related to four basic mathematical operations, looked at several different strategies used by students in a test containing mental calculation tasks, and presented an overview of the interview results. We also analyzed age-related differences in the number of strategies applied by participants. It was found that Mathematics in school does not always contribute to the development of and flexibility in the use of mental calculation strategies. One apparent reason is student preference for previously acquired written algorithms.

Pregled miselnih računskih strategij in pogostost njihove uporabe

V prispevku predstavljamo raziskavo strategij miselnega računanja in pogostost njihove uporabe pri nalogah. Kategorizirali smo strategije miselnega računanja pri štirih osnovnih matematičnih operacijah in ugotavljali koliko različnih strategij so učenci uporabili pri testu, ki je vseboval miselne računske naloge ter pripravili pregled odgovorov v intervjujih. Analizirali smo tudi razlike v številu uporabljenih strategij glede na starost. Ugotovili smo, da uporaba matematike pri pouku, ne prispeva vedno k razvoju in fleksibilnosti uporabe miselnih strategij računanja. Eden od ugotovljenih razlogov je ta, da študenti raje uporabljajo že prej usvojene pisne algoritme.

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Introduction

The twenty-first century requires problem solvers, people who can apply their arithmetical knowledge to unknown problems in new situations. Today's society is looking for a mathematically literate individual who has the capacity "to identify and understand the role that mathematics plays in the real world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2003, p. 24). Although we live in a world of rapid technology development, Mead's (2014) research on technology dependence in everyday calculation has shown that most people still solve real-world Math problems with mental calculation.

According to the *Cambridge Dictionary* (2018), *calculation* is the process of using the information you already have and adding, taking away, multiplying, or dividing numbers to judge the number or amount of something. *Collins English Dictionary* (Forsyth and Mangan, 2014) defines calculation as something that you think about and work out mathematically; something that you think carefully about and arrive at a conclusion on after having considered all the relevant factors. A *method* (Greek *μέθοδος:* research path, mode, procedure) is a planned or premeditated procedure for achieving a certain theoretical or practical goal (*Hrvatska enciklopedija*, 2013). As with calculation methods, numerical tasks can be solved with three methods: using mental calculation, or written calculation (paper and pencil), or using a calculator (McIntosh et al., 1995; Selter, 2000). In this paper, we will focus on mental calculation (known also as mental computation).

"*Mental calculation* means solving arithmetic problems mentally without using a standard written procedure" (Rathgeb-Schnierer and Green, 2019, p. 2). It is integrated into the child's knowledge of numbers and refers to calculating the exact result "in the head," without the use of aids such as a calculator or paper and pen (Sowder, 1988). Mental calculation is based on understanding the structure of numbers; it increases the comprehension of the number system and is closely related to the concept of number sense (Bruinsma, 1961; Lemonidis, 2016; Sowder 1992). Number sense refers to a person's general understanding of numbers and operations and the ability to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations (McIntosh et al., 1997b).

When children learn to manipulate numbers in their heads, they develop better number sense and increase confidence in their mathematical abilities that will encourage them to consider mental calculation as an option when solving an arithmetic problem (McIntosh et al., 1997a). Real life demands the ability to perform simple mental calculations quickly and flexibly; the child must therefore be able to promptly understand relationships and know which calculation to perform (Bruinsma, 1969). "Flexible mental calculating is a situation-dependent and individual response to specific number and task characteristics and the corresponding construction of a solution process using strategic tools" (Korten, 2017, p. 362). It includes two aspects: the knowledge of various *strategies* and the ability to adapt these appropriately when solving a problem (Rathgeb-Schnierer and Green, 2013; Threlfall, 2009; Verschaffel et al., 2009).

Mental Calculation Strategies

In this section, we will present a definition of and recommendations for teaching mental calculation strategies, the results of certain research on this topic, and an overview of the strategies of mental addition, subtraction, multiplication, and division.

Mental calculation strategies are "the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known" (Thompson, 1999, p. 2).

Teaching Mental Calculation Strategies

Formal education has the greatest impact on one's arithmetic development, and thus their mental calculation skills. Mental calculation can be developed through carefully planned teaching and practice, to which primary school Mathematics makes a decisive contribution. "The term of mental calculation describes best the objective to be achieved by learning calculating in mathematics classes. Mental operations, knowledge linking, applying strategies, efficiently, accurately and quickly are the benefits we get from calculating" (Cindrić et al., 2019, p. 80).

For rapid, accurate mental calculations, students need to apply learned or invented strategies (Baranyai et al., 2019). "Using mental computation strategies flexibly requires sound number sense and by using a strategies approach to computation, rather than a focus on procedural algorithms, students have opportunities to work with numbers in flexible ways" (Hartnett, 2007, p. 345). Systematic work on mental calculation promotes the development of students' own strategies by exploring, discussing, and justifying their thinking and solutions based on the interrelationships of numbers and properties of operations, allowing students to develop logical thinking that will help them learn algebra (Carvalho and da Ponte, 2013; Heirdsfield, 2011). Murphy (2004, p. 16) points out that teaching mental calculation strategies supports children in moving towards more flexible deductive strategies by making links to their existing knowledge. Schröder (2007) noted problems in the usage of strategies where students cannot determine which are the best options for solving a given calculation task: even if students do know the strategies, they often cannot adapt and use these. During Math class, talking about strategies can help students choose the best one: "Through class discussions, students can compare the ease of use and ease of explanation of various strategies. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and they can often be used efficiently and accurately" (NCTM, 2000, p. 84). In addition to the calculation strategies recommended in the curriculum, students can demonstrate their own strategies during class discussions, which has the following benefits: the student who explains crystallizes his/her thoughts; students who listen become familiar with the idea that there are various strategies; and some strategies may be more effective than those currently used (QCA, 1999). Analyzing task-solving strategies is not only beneficial for students; it also gives the teacher information about the students' cognitive development level, individual learning style, and readiness to adopt a new concept (Sharma, 2001).

Teacher actions that influence students' mental calculation performance include carefully selected tasks to highlight coherence and encouragement in strategic thinking (Heirdsfield, 2005). Therefore, during preparation for teaching mental calculation and related strategies, connected tasks should be chosen, and during lessons, the teacher must actively participate in the discussion by asking carefully chosen questions to guide the students and clarify potential misconceptions. This approach supports the development of strategies and enables students to become more competent and effective. Furthermore, to increase student motivation for learning Mathematics and to internalize success, it is necessary to create learning environments that will enable students to experience success in Mathematics, support their self-confidence, and develop positive attitudes towards Mathematics

(Suren and Kandemir, 2020). "Teacher support has proven to be a statistically significant predictor of students' self-confidence and mathematics anxiety, whereas enthusiasm makes an independent contribution to explaining student self-confidence" (Vidić et. al., 2022, p. 63). Activities that contribute to developing a different view of number patterns and numerical relationships do not emphasize solving the problem in the first place, but instead focus on problem characteristics, patterns, and numerical relationships. "Mental procedures that a person applies trying to determine the solutions are more important than the results themselves, especially in today's availability of computer technology" (Cindrić et al., 2019, p. 80). This approach supports the development of flexibility in mental calculation and conceptual knowledge (Rechtsteiner and Rathgeb-Schnierer, 2017). Moreover, Korten (2017) argues that during mutual learning (the combination of an individual and an interactive way of learning), flexible mental calculation competences were fostered on different cognitive levels in this inclusive situation.

Categorization of Mental Calculation Strategies

Several mental calculation strategies have been formally categorized using different names and varying numbers of categories. The application of proficient number facts (speedy recall and efficient number fact strategies) is not separately listed in every table, but in the data analysis we will observe it as a mental calculation strategy. Table 1 shows strategies for mental addition and subtraction in the set of natural numbers, where each strategy is illustrated by an example (Beishuizen et al., 1997; Beishuizen and Anghileri, 1998; Fuson et al., 1997; Heirdsfield, 2011; QCA, 1999; Rezat, 2011; Threlfall, 2002; Thompson, 2000; Van den Heuvel-Panhuizen, 2000).

Table 1. Strategies for mental addition and subtraction

Name of strategy	Example
partition	65+77 → (60+70)+(5+ 7)=130 + 12 → 65+77=142; 85−32 → (80–30)+(5–2) → 85–32=53
advanced version of partition	87–49 → 87–49=(70+17)–(40+9)=(70–40)+(17–9)=30+8 → 87–49=38
sequencing	56+32 → 56+30=86 and 86+2 → 56+32=88; 53–28 → 53–20=33 and 33–8=33 –3–5 → 53–28=25
modified sequencing	45+27 → 45+10=55, 55+10=65, 65+7 → 45+27=72; 34–25 → 34–10=24, 24–10=14, 14–5 → 34–25=9
combination of partition and sequencing	57+35 → (50+30)+5+7=85+7 → 57+35=92; 68–31 → (60–30)+8–1= 98–1 → 68–31= 37; 84–27 → (80–20)+4–7 → 64–7=64–4–3 → 84–27=57
compensation	45+29 → 45+30–1 → 45+29=74;

	65–39 → 65–40+1 → 26; 47–18 → 47–20+2 → 47–18=29
complementary addition	83–78 → 78+2+3 → 83–78=5
counting by adding or subtracting tens then ones	48+25 → 48, 58, 68, 69, 70, 71, 72, 73 → 48+25=73; 74–26 → 74, 64, 54, 53, 52, 51, 50, 49, 48 → 74–26=48
counting by adding (subtracting) tens to tens and then ones	56+25 → 50, 60, 70, 76, 77, 78, 79, 80, 81 → 56+25=81; 63–24 → 60, 50, 40, 43, 42, 41, 40, 39 → 63–24=39
balancing	45+27 → 45+27=50+22 → 45+27=72
permanence of the difference for supplementing the subtrahend to ten	73–25 → 73–25=78–30 → 73–25=48; 73–24 → 73–24=69–20 → 73–24=49
near double	37+35 → 35+2+35 → 35+35+2 → 37+35=72

Table 2 shows strategies for mental multiplication in the set of natural numbers (Baranyai et al., 2019; Caney and Watson, 2003; Heirdsfield et al., 1999; Hope, 1987; Mulligan and Mitchelmore, 1997; QCA, 1999).

Table 2. Strategies for mental multiplication

Name of strategy	**Example**
counting by	7·5 → 5, 10, 15, 20, 25, 30, 35 → 5·7=35; 3·4 → 4, 8, 12 → 4·3=12
multiplication as addition	6·5 → 5+5=10, 10+5=15, 15+5=20, 20+5=25, 25+5=30 → 6·5=30
using multiplication table facts	5·22 → 10·22=220, and 10 is twice as 5 → 5·22=110
double and half	25·36 → 50·18=100·9 → 25·36=900
separation from the right	4·26 → 4·6=24, 4·20=80, 80+24=104 → 4·26=104
separation from the left	5·17 → 5·10=50, 5·7=35, 50+35=85 → 5·17=85
compensation	6·29 → 6·30–6=180–6 → 6·29=174; 7·21 → 7·20+7=140+7 → 7·21=147
factorization	25·6 → 25·2·3=50·3 → 25·6=150; 75·16 → 3·25·4·4=3·100·4=12·100 → 75·16=1200
zero exclusion	5100·20 → 5100·20=51·2=102 → 5100·20=102000
using distributive property	12·35 → 12·(30+5)=12·30+12·5=360+60 → 12·35=420

Table 3 shows strategies for mental division (Caney and Watson, 2003; Heirdsfield et al., 1999; Mulligan and Mitchelmore, 1997; QCA, 1999).

Table 3. Strategies for mental division

Name of strategy	**Example**
halving	40:8 → 40:2, 20:2, 10:2 → 40:8=5
counting by	20:4 → 20, 16, 12, 8, 4, 0 → 20:4=5
division as subtraction	10:5 → 10–5=5, 5–5=0 → 10:5=2
division as addition	27:9 → 0+9=9, 9+9=18, 18+9=27 → 27:9=3
using multiplication table	8·6=48 → 48:8=6

separation from the left	84:4 → 8:4=2, 4:4=1 → 84:4=21; 120:6 → 12:6=2, 0:6=0 → 120:6=20
separation from the right	84:4 → 4:4=1, 8:4=2 → 84:4=21; 120:6 → 0:6=0, 12:6=2 → 120:6=20
holistic	154:22 → 22·5=110, 154–110=44, 22·2=44, 5+2=7 → 154:22=7
compensation	84:7 → 77:7=11, 84–77=7, 11+1=12 → 84:7=12; 108:12 → 120:12=10, 120–108=12, 10–1=9 → 108:12=9
zero exclusion	400:20 → 400:20=40:2 → 400:20=20
dividing a sum	76:4 → 76:4=(36+40):4=36:4+40:4=9+10 → 56:4=19; 125:5 → 125:5=100:5+20:5+5:5=20+4+1 → 125:5=25

Methodology

Research Objective

Having reviewed the literature on mental calculation and the variety of existing strategies, we were curious to see how much students apply the strategies. In this paper, we seek to answer the following questions:

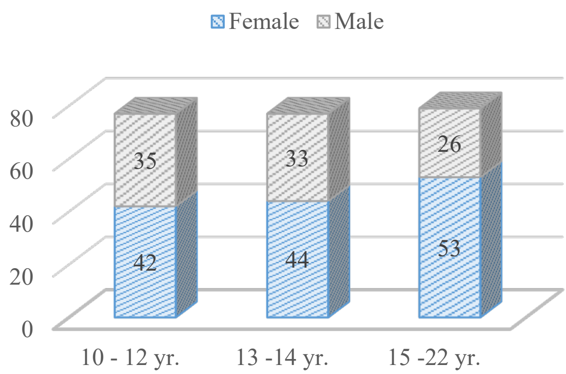
- What is the number of different strategies used in mental calculation tasks that include four basic arithmetic operations (addition, subtraction, multiplication, division)?
- Do students use procedural algorithms of written calculation in tasks that require mental calculation, i.e., do they use procedural algorithms as a "method" of mental calculation?
- Are there age-related differences among students in terms of the number of different strategies used in tasks that require mental calculation?

Research Instrument, Sample, Data Collection, and Analysis Techniques

For the purpose of the research, a test and an interview were conducted with each respondent, with mental calculation tasks modeled on a study about mental calculation by McIntosh et. al. (1995). In the analysis of the data obtained in the empirical part of the research, descriptive statistics were used in combination with parametric methods to check the differences between age groups in the number of different strategies used. The resulting data were supplemented by analyzing the students' answers from the in-depth interviews. Since it includes both qualitative and quantitative data collection procedures, this is a multi-method study.

It was conducted in Croatia during the 2020-2021 academic year. The research sample was composed of 233 students (139 females and ninety-four males) aged 10 to 22.

The mental calculation test consisted of twenty mental arithmetic tasks, five tasks for each of the four basic arithmetic operations. When compiling the assignments, care was taken to ensure that the selected numbers in the assignments were appropriate for students at a certain level of education according to the Croatian curriculum. Therefore, the test was made for three levels: students 10-12 years old attending upper elementary and lower middle school (grades 4-6); students 13-14 years old attending upper middle school (grades 7-8), and students 15-22 years old attending high school or university. The following graph shows the number of students taking the age-appropriate levels of the test.



Graph 1. Number of students taking different test levels

The tasks were chosen in various ways so that students could use several different mental calculation strategies. The mental calculation tasks were presented via mobile phone with the help of a wireless speaker, and students were given 20 seconds per task. The same test conditions were ensured for all participants including the volume and duration of reading the tasks, the number of readings, and estimated time needed for the calculation. Tasks were presented orally, not visually, and with a time limit to avoid written calculation. The mental calculation lasted approximately 20 minutes, including giving instructions and distributing worksheets. This was followed by an interview lasting approximately 10 minutes per participant. Student responses were recorded, and based on the theoretical part of the research, a coding system was constructed and a calculation strategy was identified for each completed task.

Those who solved tasks correctly were asked this question during the interview: *"What strategy did you use? Describe the calculation procedure in words."* Students who did not calculate correctly were asked to try to recalculate the result and describe the strategy they would use. This part of the research certainly required an interview, not a questionnaire with multiple-choice questions or open-ended questions. The skilled examiner thus can better explain the question to the respondent and understand what the respondent wants to say and describe because the students in the school were not taught most of the strategies and had not heard their names. In contrast, multiple-choice questions would suggest the strategy to be used. Open-ended questions could produce many unanswered questions because the respondent does not know how to express, write, or describe the applied strategy. The interview resulted in the identification of a range of different mental calculation strategies used for each individual arithmetic operation. The prevalence of written calculation was also determined, i.e., how often in mental calculation tasks a written algorithm was used. Namely, although students were asked to solve tasks using mental calculation, some still applied a written arithmetic algorithm by imagining it in their heads. A pilot study with seventeen students aged 12, and 14 students aged twenty-one was conducted before creating the final version of the test. Based on the results of the pilot study, the number of mental calculation tasks was reduced from 28 to 20.

Data collection lasted about two months during the school year. The whole procedure was carried out independently by the first author of the paper, because the research results, especially the interviews, are the most credible.

The data analysis first identified elements of descriptive statistics: arithmetic mean ± standard deviation, median, minimum, maximum of the number of mental calculation strategies used for each age group. The Kolmogorov-Smirnov test verified the normality of the data, and the homogeneity of variance was confirmed using Levene's test for equality of variances. One-way analysis of variance (ANOVA) was then applied. The statistical significance of the main effect was calculated, and the existence of significant differences between individual subgroups of respondents was examined by Bonferroni's post hoc correction.

Results and Discussion

We will first look at the number of mental calculation strategies used in each of the subgroups. Descriptive data on the number of strategies used in each of the subgroups are shown in Tables 4 and 5.

Table 4. Descriptive statistics for the number of addition and subtraction strategies

		mental addition strategies				**mental subtraction strategies**			
age	N	AM ± SD	MED	MIN	MAX	AM ± SD	MED	MIN	MAX
10	19	3.84±0.83	4	2	5	2.21±0.97	2	1	4
11	16	3.44±0.81	2	0	4	2.38±1.09	2	0	4
12	42	3.33±1.03	3	1	5	2.41±0.94	2	1	4
13	46	3.35±0.85	3	1	5	2.46±1.19	2	0	4
14	31	3.59±1.12	4	1	5	2.55±0.81	2	1	4
15	26	3.42±0.95	3.5	2	5	2.92±1.02	3	1	5
16	23	3.61±0.84	4	2	5	2.44±0.73	2	1	4
17-22	30	3.57±0.94	4	2	5	2.40±0.93	2	1	4

This shows a surprisingly low minimum for addition tasks, i.e., it ranges from 0 to 2 for the strategy used in five tasks. Zero strategies used means that the respondent used written calculation strategies instead of mental calculation strategies by imagining the procedure in their head. The median ranges from 3 to 4 in all age groups except the 11-year-old group, which used two or fewer mental addition strategies. This group also has the lowest maximum number. The minimum number of subtraction strategies for all respondents is low: one strategy or no strategies being used in all five tasks. Half the students in all age groups, except the group of 15-year-olds, used two or fewer than two mental subtraction strategies. Only in the group of 15-year-olds were all five mental subtraction strategies used. In general, respondents in all age groups used more different mental addition strategies compared to subtraction strategies.

The number of mental multiplication and division strategies ranges from zero to a maximum of five. Half the number of students younger than 15 years use a maximum of two strategies for mental multiplication. Surprisingly, the median of the number of mental division strategies used by students in elementary and middle school is greater than the median number of mental multiplication strategies used.

Table 5. Descriptive statistics for mental multiplication and division strategies

		mental multiplication strategies				mental division strategies			
age	N	AM ± SD	MED	MIN	MAX	AM ± SD	MED	MIN	MAX
10	19	1.95±1.35	2	0	4	2.16±0.90	2	1	3
11	16	1.69±1.30	1	0	5	2.31±0.88	3	1	3
12	42	2.14±1.26	2	0	4	2.62±0.80	3	1	4
13	46	2.20±1.05	2	1	4	2.04±1.05	2	1	4
14	31	2.71±1.01	2	1	4	2.65±1.05	3	0	5
15	26	2.81±1.06	3	1	4	2.73±1.15	3	1	5
16	23	2.74±0.86	3	2	5	2.96±0.88	3	2	5
17-22	30	2.40±0.86	2.5	1	4	2.13±0.14	2	0	4

The following are the results of examining the statistical significance of differences in arithmetic means in the number of strategies for mental calculation among the observed age groups, using one-way analysis of variance for independent samples (ANOVA).

Table 6. ANOVA test results

	F	**p**	η^2
Number of mental addition strategies	0.84	0.56	0.03
Number of mental subtraction strategies	1.08	0.38	0.03
Number of mental multiplication strategies	3.11	<0.001	0.09
Number of mental division strategies	3.10	<0.001	0.09

The results in Table 6 show the main effect of addition and subtraction is not statistically significant (0.56>0.05, 0.38>0.05) and that increasing the years of education does not increase the base and application of mental addition and subtraction strategies, which leads us to conclude that formal education neither offers nor improves mental calculation skills. This contrasts with the previously conducted research (Caney and Watson, 2003; Carpenter et al., 1997, Heirdsfiel et al., 1999). Regarding the number of mental multiplication and division strategies, the main effect proved to be statistically significant ($p<0.001$). To note exactly where the differences occur, we also conducted the Bonferroni post hoc test, the results of which are shown in Table 7 and Table 8.

Table 7. Results of the Bonferroni post hoc test for the number of mental multiplication strategies

	11	12	13	14	15	16	17-22
10	0.49	0.52	0.42	0.02	0.01	0.02	0.16
11		0.16	0.11	0.01	0.00	0.00	0.04
12			0.82	0.03	0.02	0.04	0.33
13				0.04	0.02	0.05	0.43
14					0.74	0.92	0.27
15						0.83	0.17
16							0.26

Table 7 shows that the greatest differences are present among respondents aged 10 to 13, compared to those aged 14 to 16. With an accuracy of 95%, we can say that students attending the first and second years of secondary school use more mental multiplication strategies than elementary and middle school students.

Table 8. Results of the Bonferroni post hoc test for the number of mental division strategies

	11	12	13	14	15	16	17-22
10	0.66	0.11	0.69	0.10	0.07	0.01	0.94
11		0.31	0.37	0.30	0.20	0.06	0.57
12			0.01	0.91	0.67	0.21	0.04
13				0.01	0.01	0.00	0.71
14					0.75	0.27	0.05
15						0.00	0.03
16							0.26

Older students use more mental division strategies compared to younger students. Although the main effect is statistically significant, a more detailed analysis of the data in Table 8 shows that in only 29% of couples is the difference statistically significant. In contrast to the significant differences in the use of mental multiplication strategies between groups of students attending primary school and those in the first half of secondary education, no clear pattern can be seen in significant differences between age groups in terms of the number of mental division strategies.

Since for each arithmetic operation in at least one of the age groups, the application of zero mental calculation strategies was noted, we will pay special attention to this phenomenon. When examining the students about the methods of calculation, it was found that a certain number of students imagined the procedure of written calculation in their heads.

According to Baranyai et al. (2019), most students do not use various mental calculation strategies: more than a quarter of respondents did not use mental calculation strategies, but only followed the written arithmetic algorithms in their heads, and more than one-third used only one or, at most, two different strategies. The most common answers received during the interview and indicating written calculation include: "*It's like school ... on the board, so I imagine writing,*" "*I imagine paper and write one below the other,*" "*I imagine in my head one below the other and subtract,*" "*I divide just like in school on the board.*" Students who used 12 or fewer mental calculation strategies, out of a total of 20, resorted to imagining written calculation in their heads. On the other hand, students who used 13, 14, or 15 mental arithmetic strategies made only sporadic use of written arithmetic by imagining it in their heads. This is noticeable in all age groups. In the following table, we look at some answers and explanations given when interviewing students.

Table 9. Students' verbal descriptions of mental calculation strategies used.

Mental addition tasks	47+4 or 147+4 → "*I break apart 4 into 3 and 1; add 3 to 47 to obtain the next ten and add 1 to 50*" or "*I know that 4+7 is 11, so the sum is 51*"
	70+80 → "*It's like 7+8; I know by heart that it's 15 and then I add zero*"
	54+99 → "*I add 100 to 54; then subtract 1 from 154*" or "*It's the same as 53+100*"
	57+36 → "*It's 50+30 and 7+6; then I add 13 to 80*" or "*I take 3 from 36, then 57 and 3 is 60; then add the remaining 33*"
	26+25 → "*That is twice 25 and then 1 more*" or "*That is 25 and 25 and 1 more*" or "*20 and 20 and 11*"
Mental subtraction tasks	153–99 → "*That's the same as 154–100*" or "*I take 100 from 153, then add 1*"
	44–26 → "*That's the same as 50–32*" or "*I take 24 from 44 and get 20 and take 2 more*" or "*I see how many I have from 26 to 30 and then from 30 to 44, which is a total of 18*"
	200–54 → "*I know that 100–54 is 46 and then add 100 more*" or "*200–50 is 150 and then I take 4 from 150*"
	85–78 → "*From 78 to 80 is 2 and another 5 to 85 which is all together 7*" or "*That is the same as 15–8*"
	63–21 or 142–21 or 263–21 → "*60–20 and 3–1*" or "*63–20 and take 1 more*" or "*I know that 42 is twice as 21, then 142–21=121*" or "*263–20 is 243 and then I take 1 more*" or "*63 is three times 21 so 63–21=42 and then I add 200 more*"
Mental multiplication tasks	23·6 → "*20·6 and 3·6*" or "*23 and 23 equals 46, and 46 and 46 equals 92, plus 46 equals 138*"
	14·4 or 34·4 → "*14·2 equals 28, and 28·2 equals 56*" or "*10·4 and add 4·4" or "30·4=120 and 16 more*" or "*34·5 and then take 34" or "34·2 and then 68·2*"
	19·5 or 39·5 or 69·5 → "*10·5 and 9·5 more*" or "*20·5 equals 100, then take 5 and get 95*" or "*69·10=690, then 690:2*" or "*40·5 and then take 5*"
	25·3 → "*25 and 25 equals 50, and 25 more is 75*" or "*I know it by heart, that's 75*" or "*20·3 equals 60 and 15 more*" or "*That is 100–25*"
	40·20 or 300·20 or 800·70 → "*That's 4·2 and write two zeros*" or "*8·7 with three zeros*" or "*300·10 equals 3000, and this is twice as much, 6000*"

Mental division tasks	143:13 → "*I know 13·10 equals 130 and 13 one more time, so answer is 11" or "That is 130+13, so the answer is 11*" or "*Using Vedha math, 4 equals 1+3, this is 11*"
	56:4 or 136:4 → "*I know that 14·4=56*" or "*That is 120+16, so the result is 30 and 4, i.e. 34*" or "*140:4 equals 35, so this is 34*"
	63:9 or 153:17 → "*7·10 equals 70, then this is 7·9*" or "*I know it by heart–multiplication table*" or "*That's 170–17, so the answer is 9*" or "*I know that result is a one-digit number, so I look for a number that multiplied by 7 gives last digit 3, and that is 9*
	60:5 → "*I know it by heart*" or "*That is 50+10, so I get 10 and 2 and result is 12*" or "*11·5 equals 55, so this is 12*"
	400:20 or 300:20 or 2400:80 → "*4:2 and put 0*" or "*20 fits five times in 100 and 3 times more, which is 15 altogether*" or "*That is the same as 240:8, which is 30" or "24:8 and put zero*"

Finally, let us look at the use of the "school" strategy for mental calculation. The test follows the official Mathematics curriculum in the Republic of Croatia, which recommends several mental calculation strategies: sequencing and addition to the next ten (using permanence of sum and difference) in addition and subtraction; separation from left to right and zero exclusion in multiplication and division; dividing a sum in division. Since these are recommended strategies, they are widespread in teaching Mathematics in Croatian classrooms. However, students in this study used other strategies as well. This could be a result of their teachers' instruction or of students' own "invention." It should also be noted that the least used strategy in mental addition is the one learned in school. We can explain this by the fact that addition is the easiest operation for students, so they feel confident in their own knowledge and free to apply their own creations and modifications in calculating.

Conclusion and Implications

Mathematical competences are among the key competences for personal development, active citizenship, social inclusion, and employability in the knowledge society of the 21st century. An indispensable segment of mathematical competences is mental calculation, which is the most common method in the calculations we perform during everyday activities (paying bills, calculating time, estimating, etc.). The results of our research showed that school Mathematics does not contribute to students' progress in terms of mental calculation addition and subtraction strategies. Furthermore, according to our results, the use of mental calculation strategies learned in school decreases with increasing years of education.

This is in contrast to previously conducted research; the study shows that "calculation ability of the primary school children increases with age; scores of calculation fluency and accuracy increased in higher school grades" (Zhou et. al., 2021, p.289). Older students apply a wider range of mental calculation strategies for all four arithmetic operations, and at the same time independently modify them or combine several known strategies to suit them in order to be as efficient as possible in a given task (Anghileri, 1989; Caney and Watson, 2003; Carpenter et al., 1997; Heirdsfiel et al., 1999). Because of the lack of time and the wide scope of the mathematics teaching content, teachers often stick to the curriculum requirements, which assign a lot of time to teaching and practising written calculation. Once the written calculation procedure is demonstrated, it is given priority in the calculation. After students learn a standard calculation algorithm, they tend to stop using previously learned strategies, even when those are more advantageous and appropriate (Selter, 2009). When students learn by example, they acquire specific procedures rather than general rules, and those procedures tend to have a negative impact on the development of flexibility (Schütte, 2004). The results of our research showed the use of written calculation, even though the tasks required only mental skills. We believe that the rigid imposition of written arithmetic rules that are taught in school affects students in their choice and ability to use effective mental calculation strategies. Students are forced to develop and discover various strategies based on their own knowledge, insights, and experience independently and outside the mathematics classroom. Most students are not motivated to do this, and others find it impossible without the teacher's guidance. However, it would be good to further investigate whether preferences for written calculation limit the use of mental calculation strategies or, conversely, underdeveloped mental calculation skills push students towards the use of written calculation.

The results of this research provide insight into the number of different mental calculation strategies used in mental calculation tasks. Of course, we must say that the results are not satisfactory; some students did not use mental calculation strategies in the tasks at all, and the arithmetic mean is also low, considering that these tasks rely on four basic calculation operations used daily. Future research should focus on studying the possible correlation between student confidence and motivation to learn Mathematics with the ability to create their own mental calculation strategies.

We also recommend organizing training for teachers to show them the variety of mental calculation strategies and to help them improve their teaching of it, and to choose and prepare tasks and heuristic questions for class discussion.

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