Topological Informational Spaces

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Keywords: basis, closed system, connectedness, covering, discrete space, exterior, indiscrete space, informational space: operand formula vector, vector distributiveness (orthogonality), metrics (meaning); interior, linkage, neighborhood, open system, subbasis; system of informational formulas, operands and basic transition formulas; topology of systems

Edited by: Vladimir A. Fomichov

Received: April 28, 1998 **Revised:** August 18, 1998 **Accepted:** August 25, 1998

A system, Φ, of different informational formulas, φ, can possess various topological structures, $\mathfrak{O}$. By this, topological informational spaces of the form $\langle \Phi, \mathfrak{O} \rangle$ can be constructed and the question arises: How can the topological structures be introduced reasonably for concrete systems of informational formulas?

A topology causes certain other concepts, e.g., those concerning closed topology, connetedness, continuum, interior, exterior, neighborhood, basis, subbasis, metric, space, etc. of systems, and especially the concept of meaning as a kind of the informational accumulation point. The paper treats topologies of three types of informational formula systems: Φ_φ, $\Phi_{\xi \models \eta}$, and Φ_ξ. An example of bidirectional consciousness shell is presented enabling a complex engine modeling.

1 Introduction

A basic problem of topology[1] is to define a general space. By topology a mathematical concept (structure, branch) is meant[2] giving sense to various intuitive notions. Topological notions can be innovatively extended into realm of the informational, realizing one of the significant features of the so-called *informational space*. Such a space can be determined also from other points of view concerning, for instance, the distributivity of informational entities (operands)—which can proceed into different concepts of a vector space. In general, a more complete theory of informational space would need concepts of informational subtheories, such of concerning informational topological space, informational vector space, and informational graph theory.

Another, mathematically grounded view to the problem of graph is the so-called topological graph theory [14]. The primitive objective of this theory is to draw a graph on a surface, so that no two edges (graph arrows representing informational operators) cross, an intuitive geometric problem that can be solved by specifying symmetries or combinatorial side-conditions (surface graph-imbedding). Although potentially interesting for the informational graph theory, this kind of problem is not in the focus of informational graph investigation. Informationally, graph is merely a presentation of formula or formula system potentiality concerning the setting of the parenthesis pairs (parenthesizing[3]) in a formula or formula system.

Introducing the topology on (over) a system of informational formulas means a challenge of logic—both the informational and the philosophical one—which comes close to some known metamathematical problems [30]. Just imagine a topology in the realm of mathematical axiomatism where for a set of axioms a topology of axiomatic statements (e.g., true formulas) is constructed. Although topology is a general mathematical principle in the realm of the set and space theory, it seems nonobviuously to take it as a set with topology on the set of formulas.[4]

In mathematics, topology may be considered as an

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[2] Otherwise, topology is a science of position and relation of bodies in space. This paper concerns at least the following topological topics: point system (set) topology (general topology), metric space (e.g., meaning topology), and graph topology.

[3] Parenthesizing (in German, Einklammerung) has also the philosophical meaning in phenomenology, for instance, in Husserl [17].

[4] The author believes too that such an idea reaches beyond the conventional horizon of a mathematician. However, he believes that the following discussion will show the appropriateness of such a trait.

abstract study of the limit-point concept [16]. Which factors could dictate the introduction of a topology for a given system of informational formulas? In informational cases, different kinds of reasonable topologies, corresponding intuitive ideas what an understanding, interpretation, conception, perception and meaning should be, are coming to the consciousness, that is, into the modeling foreground.

In set theory, the concept of a set (collection, class, family, system, aggregate) itself is undefined. Similar holds for an element x of the set X. The phrases like is in, belongs to, lies in, etc. are used. In informational theory, topology may be considered as an abstract study of the concept of meaning [32, 35] (concerning interpretation, understanding, conceptualism, consciousness, etc. of the informational). Here, meaning of something, of some formula or formula system, functions as an informational limit point, to which it is possible to proceed as near as possible by the additional meaning decomposition of something. The concept of a set is replaced by the concept of a system of informational formulas or/and informational formula systems. In this respect, similar notions to those in mathematics can be used, however, considering the informational character of entities (operands) and their relations (operators).

Introducing topological concepts in informational theory, the reader will get the opportunity to experience what happens if the informational concepts, priory described by the author (e.g., [31, 32, 33, 34, 35, 36, 37, 38], to mention some of the available sources) are thrown into the realm of a topological informational space. In this view, informational serialism, parallelism, circularism, spontaneism, gestaltism, transitism, organization, graphism, understanding, interpretation, meaning, and consciousness will appear under various topological possibilities, complementing the already previously presented informational properties, structure, and organization.

Mathematical topology, as presented for example in [7, 8, 16, 18, 19, 21, 24, 25], roots firmly in the mathematical set theory [5, 6, 20]. In informational theory, the set is replaced by the concept concerning a *system of informational formulas* (system, informational system or IS, in short). A system is—said roughly—a set of *informationally (operandly, through or by operands) connected informational formulas.* The question is, which are the substantial differences occurring between the mathematical and the informational conceptualism in concern to topological structure?

Elements of a mathematical set are elements determined by a logical expression (defining formula, relation, statement) and, for example, by notation of the form

$$X = \{x_1, x_2, \dots, x_m\}$$

which presents a concrete structure of the set by its elements.

In informational theory, instead of a set, there is a system of informational formulas being elements of the system. Formulas are active, emerging, changing, vanishing informational entities (by themselves) which can inform in a spontaneous and circular manner. What does not change is their informational markers distinguishing the entities. Notation of the form

$$\Phi \rightleftharpoons \begin{pmatrix} \varphi_1; \\ \varphi_2; \\ \vdots \\ \varphi_n \end{pmatrix}, \quad \text{where}$$

$$\varphi_1, \varphi_2, \dots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \dots, \varphi_m$$

presents,[5] in fact, only an instantaneous description of the parallel system of markers φ_i, by a vertical presentation, denoting concrete formulas (or formula systems), and being separated by semicolons. These are nothing else as a special sort of informational operators, e.g. $\Vdash$, meaning the parallel informing of formulas of the system Φ. Also, there is a substantial difference between the symbols $=$ and $\rightleftharpoons$; the second one is read as 'mean(s)' and denotes meaning and not the usual equality.

Another notions to be determined informationally are informational union and informational intersection of systems. It has to be stressed that formulas in a system "behave" in the similar manner as the elements in a set in respect to the union and intersection operation. Thus, the same operators can be used as in mathematics, without a substantial conceptual difference.

2 A Mathematical vs. Informational Dictionary

The presented dictionary should bring the mathematical feeling into the domain of informational theory. It certainly concerns the topological terms priory. The correspondence between set-theoretical and system-informational terms yields the following comparative table[6]:

MATHEMATICAL VS. INFORMATIONAL TOPOLOGY	
set X	system Φ: general formula system Φ_φ; transition formula system $\Phi_{\xi \models \eta}$; and operand formula system Φ_ξ
set braces: $\{,\}$	system parentheses: $(,)$

[5] For the system-conditional formula, $\varphi_1, \varphi_2, \dots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \dots, \varphi_m$ see the discussion in Sect. 3.1.

[6] In this dictionary, the 'formula φ of Φ' ($\varphi \in \Phi$) is to distinguish from the informational speech convention where the meaning of 'of' represents an informational function, that is, the informational Being-of [29], e.g., $\varphi(\alpha)$ for 'φ of α.

vertical snake-form operand-occurrence braces [5, 6]	formula operand occurrence floor brackets: $\lfloor, \rfloor$
empty set $\emptyset$	empty system $\emptyset$
set element x	system formula φ
x is an element of X, x belongs to X, x is in X, $x \in X$; negation: $x \notin X$	φ is a formula of Φ, φ belongs to Φ, φ is in Φ, $\varphi \in \Phi$; negation: $\varphi \notin \Phi$
subset A	subsystem Ψ
A is a subset of X, A is included in X, $A \subset X$; negation: $A \not\subset X$	Ψ is a subsystem of Φ, Ψ is included in Φ, $\Psi \subset \Phi$; negation: $\Psi \not\subset \Phi$
powerset of set X, $\mathfrak{P}(X)$ or 2^X	powersystem concerning system Φ, $\mathfrak{P}\lfloor\Phi\rfloor$ or $\mathcal{P}\lfloor\Phi\rfloor$
union of sets, $A \cup B$	union of systems, $\Phi \cup \Psi$; or $(\Phi; \Psi)$, informing of both systems
intersection of sets, $A \cap B$	intersection of systems, $\Phi \cap \Psi$ means, e.g., $(\Phi; \Psi; \Phi \models \Psi; \Psi \models \Phi)$, parallel informing of all the systems' components
difference of sets Y and X, $Y \setminus X$	difference of systems Ψ and Φ, $\Psi \setminus \Phi$
complement of set X, $\complement X$	complement of system Φ, $\complement\Phi$
complement of set X, regarding set Y, $\complement_Y X$	complement of system Φ, regarding system Ψ, $\complement_\Psi \Phi$
open set O	open system $\mathfrak{O}$
topology O	informational topology $\mathfrak{O}$
topological structure O	informational topological structure $\mathfrak{O}$
topological space (X, O), simply, X	topological informational space $\langle\Phi, \mathfrak{O}\rangle$, simply, Φ
carrier X	informational carrier Φ
point $x \in X$	informational point, formula, formula system $\varphi \in \Phi$

MATHEMATICAL VS. INFORMATIONAL GRAPH THEORY [4, 35, 39]

vertex (apex, in Russ., вершина [39]), v	operand, operand point, ξ
set of vertices, V	system of operands, Φ_ξ
vertex connection (rib, ребро) u: arc, loop, link [39]	operator, operator arrow, marked by $\models$ or by an operator particularization
set of ribs, U	list of operator markers
edge, unordered pair $e = \{v_1, v_2\}$, or ordered pair (v_1, v_2)	basic transition, $\xi \models \eta$, with binary operator
set of edges, E (incidence function)	system of basic transitions $\Phi_{\xi\models\eta}$
path (route, цепь) $v_1 u_1 v_2 u_2 \cdots u_{n-1} v_n$	informational route, path, formula scheme
graph $G = (V, E)$ [2]	$\xi_1 \models \xi_2 \models \cdots \models \xi_n$ informational graph, presented by $\Phi_{\xi\models\eta}$, derived from an actual system Φ_φ

Informational space shall mean a non-empty formula system which possesses some type of informational structure (and organization), e.g. metaphysicalism, meaning, informational vector, informational metric (in the form of informational distance) and/or informational topology. Within such a possible structure, the elements in an informational space will be called *formulas* or *points* $(\Phi_\varphi, \Phi_{\xi\models\eta})$ and, in a special case (Φ_ξ), *operands.*

3 Systems and Subsystems of Informational Formulas

3.1 Informationally Linked Formulas in a System

Informational linkage of formulas in a formula system deserves a special attention and theoretical treatment. The consequence of formula linkages via common operands makes the difference between formulas of a system on one side, and between the elements of a set on the other one.

DEFINITION 1 *If in informational formulas φ_1 and φ_2 a common operand α appears, that is, $\varphi_1\lfloor\ldots,\alpha,\ldots\rfloor$ and $\varphi_2\lfloor\ldots,\alpha,\ldots\rfloor$,*[7] *respectively, notation*

$$\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2 \quad \textit{or, simply,} \quad \varphi_1 \leftrightsquigarrow \varphi_2$$

will be used and read as formula φ_1 informs formula φ_2 *via* operand α *or, simply,* formula φ_1 is informationally linked to formula φ_2. *This operation is* informationally symmetric. *Thus,*

$$(\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2) \rightleftharpoons (\varphi_2 \overset{\alpha}{\leftrightsquigarrow} \varphi_1)$$

Transitivity of operator $\leftrightsquigarrow$ can exist in the following way: if α is common to φ_1 and φ_2, and β is common to φ_2 and φ_3, that is,

$\varphi_1\lfloor\ldots,\alpha,\ldots\rfloor$, $\varphi_2\lfloor\ldots,\alpha,\beta,\ldots\rfloor$ and $\varphi_3\lfloor\ldots,\beta,\ldots\rfloor$,

then φ_1 is linked informationally with φ_3. Formally,

$$((\varphi_1 \overset{\alpha}{\leftrightsquigarrow} \varphi_2) \wedge (\varphi_2 \overset{\beta}{\leftrightsquigarrow} \varphi_3)) \Longrightarrow (\varphi_1 \overset{\alpha,\beta}{\leftrightsquigarrow} \varphi_3)$$

Operator $\wedge$ denotes informational conjunction (in fact, the operator of parallel informing, $\|\!\models$, or, usually, semicolon ';' [30]) *and operator $\Longrightarrow$ informational implication* [30]. □

[7] It is to stress that a notation $\varphi\lfloor\alpha_1, \alpha_2, \ldots, \alpha_n\rfloor$ means informational operands $\alpha_1, \alpha_2, \ldots, \alpha_n$ occurring in formula φ, and does not represent the so-called functional form, that is, informational Being-of [29] in the form $\varphi(\alpha_1, \alpha_2, \ldots, \alpha_n)$. Evidently, $\varphi(\alpha_1, \alpha_2, \ldots, \alpha_n) \Longrightarrow \varphi\lfloor\alpha_1, \alpha_2, \ldots, \alpha_n\rfloor$.

Further, there can exist more than one common operand, e.g., $\alpha_i, \alpha_j, \alpha_k \ldots, \alpha_m$ in φ_1 and φ_2 in a transitive manner. In this case,

$$\left(\varphi_1 \stackrel{\alpha_i, \alpha_j, \alpha_k, \ldots, \alpha_m}{\leftrightsquigarrow} \varphi_2\right) \rightleftharpoons \begin{pmatrix} \varphi_1 \stackrel{\alpha_i}{\leftrightsquigarrow} \varphi_{i'}; \\ \varphi_{i'} \stackrel{\alpha_j}{\leftrightsquigarrow} \varphi_{j'}; \\ \varphi_{j'} \stackrel{\alpha_k}{\leftrightsquigarrow} \varphi_{k'}; \\ \vdots \\ \varphi_{\ell'} \stackrel{\alpha_m}{\leftrightsquigarrow} \varphi_2 \end{pmatrix}$$

Transitivity of operator $\leftrightsquigarrow$ applies also to the case of more than one common operator through several formulas, and it can be defined from case to case.

Because common operands concern informing between formulas, the implication in the last definition can be expressed by means of a parallel system

$$\begin{pmatrix} \varphi_1 \stackrel{\alpha}{\leftrightsquigarrow} \varphi_2; \\ \varphi_2 \stackrel{\beta}{\leftrightsquigarrow} \varphi_3 \end{pmatrix} \Longrightarrow (\varphi_1 \stackrel{\alpha,\beta}{\leftrightsquigarrow} \varphi_3)$$

Another significant feature follows from the last definition:

THEOREM 1 *Let the linkages in a circular manner*

$$\begin{gathered} \varphi_1 \leftrightsquigarrow \varphi_2, \\ \varphi_2 \leftrightsquigarrow \varphi_3, \\ \vdots \\ \varphi_{m-1} \leftrightsquigarrow \varphi_m, \\ \varphi_m \leftrightsquigarrow \varphi_1 \end{gathered}$$

be given. Then,

$$\varphi_1, \varphi_2, \ldots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m$$

This feature is called the reflexivity *of a circular informational linkage of formulas within (in the framework of) a formula system.* □

PROOF 1 We have to prove that

$$(\varphi_1, \varphi_2, \ldots, \varphi_m \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m) \Longrightarrow (\varphi_i \leftrightsquigarrow \varphi_j); \quad i, j \in \{1, 2, \ldots, m\}$$

Within this conditionality also

$$\varphi_i \leftrightsquigarrow \varphi_i; \quad i \in \{1, 2, \ldots, m\}$$

holds in a transitive (consequently multiple-linkage) manner.

Another evident meaning of the theorem result is

$$\varphi_i \leftrightsquigarrow \varphi_1, \varphi_2, \ldots, \varphi_m \text{ for all } i \in \{1, 2, \ldots, m\}$$

It means that φ_i is informationally linked to each of $\varphi_1, \varphi_2, \ldots, \varphi_m$, including to itself (informational circularity). This proves the theorem. □

3.2 Formula Systems

In informational theory, a system of informational formulas corresponds to the notion of a set of elements in mathematics. A fundamental concept of informational theory is that of the system (short for the system of informational formulas).

DEFINITION 2 *Intuitively, a system is a well-defined list of informationally well-formed formulas (separated by semicolons).*

Formulas consist of operands, (binary) operators, and parenthesis pairs.

In a system, formulas inform in parallel to each other. In a proper system, formulas are informationally linked via common operands, directly or indirectly (transitively), in such a way that each formula is, to some extent, informationally linked with each other formula of the system.

In an improper system, some formulas are informationally isolated.

Informationally, only proper systems appear (inform) to be reasonable. Isolated formulas inform per se, *beyond the informational context of other formulas or subsystems of formulas in a system. For the rest of the system, such formulas are unobservable and do not observe informationally other formulas or formula subsystems.* □

The *union* of two systems Φ_1 and Φ_2, denoted by $\Phi_1 \cup \Phi_2$, means the system

$$(\Phi_1 \cup \Phi_2) \rightleftharpoons \left(\varphi \,\middle|\, \underbrace{(\varphi \in \Phi_1) \vee (\varphi \in \Phi_2)}_{\text{system classifier}}\right)$$

The union classifier can be expressed also as

$$((\varphi \in \Phi_1) \vee (\varphi \in \Phi_2)) \rightleftharpoons \begin{pmatrix} \varphi \in \Phi_1, \\ \varphi \in \Phi_2 \end{pmatrix}$$

which represents the so-called *alternative* system, using comma instead of semicolon between formulas [35].

The *intersection* of two systems Φ_1 and Φ_2, denoted by $\Phi_1 \cap \Phi_2$, means the system

$$(\Phi_1 \cap \Phi_2) \rightleftharpoons \left(\varphi \,\middle|\, \begin{matrix} \varphi \in \Phi_1; \\ \varphi \in \Phi_2 \end{matrix}\right)$$

If systems Φ_1 and Φ_2 do not have any formulas in common, $\Phi_1 \cap \Phi_2 \rightleftharpoons \emptyset$; they are said to be *disjoint* systems.

The *relative complement* of a system Φ_1 with respect to a system Φ, denoted by $\complement_\Phi \Phi_1$, or the *difference* of Φ and Φ_1, denoted by $\Phi \setminus \Phi_1$, is the system

$$\Phi \setminus \Phi_1 \rightleftharpoons (\varphi \,|\, (\varphi \in \Phi) \vee (\varphi \notin \Phi_1))$$

The *complement* of a system Φ_1, denoted by $\complement\Phi_1$, is the system

$$\complement\Phi_1 \rightleftharpoons (\varphi \,|\, (\varphi \in \Upsilon) \vee (\varphi \notin \Phi_1))$$

where Υ functions as a universal system. Evidently, $\complement\Phi_1 \rightleftharpoons (\Upsilon \setminus \Phi_1)$. Usually, in a complex case, the formula system Φ has the role of the currently universal system to which its subsystems can be compared.[8]

4 Topological Informational Spaces

4.1 Definitions

4.1.1 Open Systems

DEFINITION 3 *Let Φ mark a reasonable[9] non-empty system of informational formulas. A class[10] (short for informational class) $\mathfrak{O}$ of subsystems of Φ, $\mathfrak{O} \subset \mathfrak{P}\lfloor\Phi\rfloor$, is a* topology *on Φ iff $\mathfrak{O}$ satisfies the following axioms:*

(T_I) *The union of any number of systems in $\mathfrak{O}$ belongs to $\mathfrak{O}$.*

(T_{II}) *The intersection of any two systems in $\mathfrak{O}$ belongs to $\mathfrak{O}$.*

(T_{III}) *Systems Φ and $\emptyset$ belong to $\mathfrak{O}$.*

*The systems of $\mathfrak{O}$ are then called $\mathfrak{O}$-*open systems, *or simply* open systems, *and Φ together with $\mathfrak{O}$, i.e. the informational pair $\langle\Phi,\mathfrak{O}\rangle$, is called the* topological informational space. □

As we see, a topological space is defined as an ordered pair between the carrier Φ and its topology $\mathfrak{O}$.

Let us formulate Def. 3 in another way to get a different experience of the meaning of an informational topology. Topology can be determined by the following four steps too:

1^0 *A basic system Φ of formulas $\varphi_1, \varphi_2, \cdots, \varphi_m$ exists.*

2^0 *There exists a type characteristics[11] $\mathfrak{O} \in \mathfrak{P}\lfloor\mathfrak{P}\lfloor\Phi\rfloor\rfloor$.*

3^0 *The first axiom is:*
For each system $\mathfrak{O}'$, the informational implication
$$(\mathfrak{O}' \in \mathfrak{O}) \Longrightarrow \left(\left(\bigcup_{\Xi\in\mathfrak{O}'} \Xi\right) \in \mathfrak{O}\right) \text{ and } \Phi \in \mathfrak{O} \textit{ hold.}$$

4^0 *The second axiom is:*
For each Ξ_1 and each Ξ_2, the informational implication $(\Xi_1 \in \mathfrak{O}; \Xi_2 \in \mathfrak{O}) \Longrightarrow ((\Xi_1 \cap \Xi_2) \in \mathfrak{O})$ holds.

Such a structure family is called the topological structure, and the relation $\Xi \in \mathfrak{O}$ can be expressed verbally as: system Ξ is open in topology $\mathfrak{O}$.[12]

[8] A complex system is, for example, that of informational consciousness, in which several complex subsystems are imbedded. However, it does not mean that, in a specific case, a subsystem appears as a kind of universal to which its subsystems can be system-complementally compared.

[9] A reasonable system of informational formulas usually concerns a concrete, cyclically structured informational graph [32, 35].

[10] A class (family, collection) of subsystems means a system of subsystems.

[12] For more details see [6], p. 246. Evidently, the openness of a system concerns its topology.

EXAMPLE 1 Topologies Deducible from Standardized Metaphysicalism. Let the following classes of subsystems of the standardized metaphysicalistic system [32, 35] $\mathfrak{M} \rightleftharpoons (\varphi_1; \varphi_2; \dots; \varphi_6)$ of circular formulas[13] be given:

$$\begin{aligned}
\mathfrak{O}_1 &\rightleftharpoons (\mathfrak{M}; \emptyset; (\varphi_6));\\
\mathfrak{O}_2 &\rightleftharpoons (\mathfrak{M}; \emptyset; (\varphi_5; \varphi_6); (\varphi_6));\\
\mathfrak{O}_3 &\rightleftharpoons (\mathfrak{M}; \emptyset; (\varphi_4; \varphi_5); (\varphi_6));\\
\mathfrak{O}_4 &\rightleftharpoons (\mathfrak{M}; \emptyset; (\varphi_4; \varphi_5; \varphi_6); (\varphi_6));\\
\mathfrak{O}_5 &\rightleftharpoons (\mathfrak{M}; \emptyset; (\varphi_1; \varphi_2; \varphi_3; \varphi_6); (\varphi_4; \varphi_5; \varphi_6); (\varphi_6))
\end{aligned}$$

For $\mathfrak{O}_3$, the union

$$((\varphi_4; \varphi_5) \cup (\varphi_6)) \rightleftharpoons (\varphi_4; \varphi_5; \varphi_6)$$

does not belong to $\mathfrak{O}_3$. Evidently, $\mathfrak{O}_3$ is not a topology of $\mathfrak{M}$. □

Formulas of a topological informational space are often called *points*. System Φ, in which a topology is defined, is said to be the *carrier* of the topological space $\langle\Phi,\mathfrak{O}\rangle$. System Φ can be the carrier of more than one topological space. Thus, a system of different topological spaces for Φ is, for instance,

$$\langle\Phi, (\mathfrak{O}_1; \mathfrak{O}_2; \dots; \mathfrak{O}_n)\rangle \rightleftharpoons \begin{pmatrix} \langle\Phi,\mathfrak{O}_1\rangle; \\ \langle\Phi,\mathfrak{O}_2\rangle; \\ \vdots \\ \langle\Phi,\mathfrak{O}_n\rangle \end{pmatrix}$$

EXAMPLE 2 The carrier of a topological informational space can be expressed in different ways, with different structural and organizational[14] consequences. Conceptualizing an IS, at the first glance, usually, a system of serial and serially circular informational formulas is determined. According to [35], p. 114, such a system includes serial formulas of the type[15]

$${}^{n}_{i}\varphi_{\rightarrow}\lfloor\alpha, \alpha_1, \cdots, \alpha_n\rfloor;\; 1 \le i \le N_{\rightarrow};\; N_{\rightarrow} = \tfrac{1}{n+1}\binom{2n}{n}$$

[13] In this case, the following is meant:
- φ_1, φ_2, and φ_3 mark the short loops ($\ell = 2$) of informing, counterinforming and embedding, respectively;
- φ_4 and φ_5 denote the midle-size loops ($\ell = 4$) for informing-counterinforming and counterinforming-embedding, respectively; and
- φ_4 represents the long loop ($\ell = 7$).

[14] Informational organization always concerns the substantial, semantic, meaning-like structure, the choice of entities and their decomposition, the reasonable setting of parenthesis pairs (parenthesizing), operational connections and informational loops, particularization of informational operators, system parallelization (introduction of parallel interpretive, detailing, explaining, complementary formulas and systems), etc. It concerns the significance, distinguishability, and specificity of a concrete structure representing a contentional informational case.

[15] Here, the delimiters $\lfloor, \rfloor$ are introduced to replace the parentheses (,) when for $\varphi\lfloor\alpha\rfloor$ the occurrence (and not the functional dependence—the informational Being-of [29]) of α in formula φ is meant. Expression $\varphi\lfloor\alpha_1, \cdots, \alpha_n\rfloor$ is read as formula φ with operands $\alpha_1, \cdots, \alpha_n$, or also, formula φ concerning operands $\alpha_1, \cdots, \alpha_n$. The idea to introduce this sort of parentheses (floor parentheses) comes from Bourbaki [5, 6] where the vertical snake-form parentheses are used (not available in LaTeX2ε).

and circular serial formulas of the type

$${}^{n+1}_{\;\;i}\varphi^{\circlearrowleft}_{\rightarrow}\lfloor\alpha,\alpha_1,\cdots,\alpha_n\rfloor;\; 1 \le i \le N^{\circlearrowleft}_{\rightarrow};\; N^{\circlearrowleft}_{\rightarrow} = \tfrac{1}{n+2}\tbinom{2n+2}{n+1}$$

where n applies for different lengths of formulas entering into the system. Thus, this kind of the system includes the last two types of formulas.

Another possibility is to express the system in the form of primitive informational parallelism, corresponding to the expression of an informational graph (for example, in [32, 35]). Such a system is obtained by the parallelization (in fact, reduction of formulas into primitive informational transitions [33]) of serial and circular serial formulas given previously. Parallelization is denoted by

$$\begin{aligned} &{}^{n}\varphi'_{\|}\lfloor\alpha,\alpha_1,\cdots,\alpha_{n-1},\alpha_n\rfloor \rightleftharpoons \\ &\qquad\qquad \Pi'\left\lfloor {}^{n}_{i}\varphi_{\rightarrow}\lfloor\alpha,\alpha_1,\cdots,\alpha_{n-1},\alpha_n\rfloor\right\rfloor; \\ &{}^{n+1}\varphi^{\circlearrowleft\prime}_{\|}\lfloor\alpha,\alpha_1,\cdots,\alpha_{n-1},\alpha_n\rfloor \rightleftharpoons \\ &\qquad\qquad \Pi'\left\lfloor {}^{n+1}_{\;\;i}\varphi^{\circlearrowleft}_{\rightarrow}\lfloor\alpha,\alpha_1,\cdots,\alpha_{n-1},\alpha_n\rfloor\right\rfloor \end{aligned}$$

for the serial and circular serial case, respectively. The parallel system is obtained by setting all the occurring primitive transitions into the system. As we know, such a primitive formula system represents the graph, describing all the possible informational situations of a system.

Different topologies can be constructed and studied for both circular serial and circular primitive parallel systems. We shall learn the essential differences and possibilities for both types of systems. □

4.1.2 Further Examples and Definitions

(1) Discrete spaces. Let Φ be any formula system. The system of all possible subsystems of system Φ, $\mathfrak{P}\lfloor\Phi\rfloor$, satisfies the axioms for the open systems and is, in this sense, the topology in Φ, called the *discrete* topology on Φ. If Φ is topologized by its discrete topology, it is called the *discrete* space.

(2) Indiscrete spaces. Let Φ be any formula system. A system $(\emptyset, \Phi)$ satisfies the axioms for the open systems and is, in this sense, the topology on Φ, called the *indiscrete* (or *trivial*) topology in Φ. If Φ is topologized by its indiscrete topology, it is called the *indiscrete* space.

(3) Basis of a topology. Basis $\mathfrak{B}$ of a topology $\mathfrak{O}$ in Φ is such a subsystem of family $\mathfrak{O}$ ($\mathfrak{B} \subset \mathfrak{O}$) that each open system $\mathfrak{U}$ in $\mathfrak{O}$ ($\mathfrak{U} \in \mathfrak{O}$) is a union of some open systems $\mathfrak{A}_i$ in $\mathfrak{B}$, that is, $\bigcup_{\mathfrak{A}_i \in \mathfrak{B}} \mathfrak{A}_i$. Said in another way: for each $\mathfrak{U} \in \mathfrak{O}$ and each point (formula) $\varphi \in \Phi, \mathfrak{U}$, there exists such a system $\mathfrak{V} \in \mathfrak{B}$ that

$$\varphi \in \mathfrak{V};\; \mathfrak{V} \subset \mathfrak{U}$$

Open systems of the given basis $\mathfrak{B}$ are called *basic open systems* of space $\langle\Phi, \mathfrak{O}\rangle$.

(4) Subbasis of a topology. Subbasis of a topology $\mathfrak{O}$ in Φ consists of finite intersections (intersections of finite families) of open systems in $\mathfrak{S}$ in such a way that these intersections constitute the basis of topology $\mathfrak{O}$. Then, for each $\mathfrak{U} \in \mathfrak{O}$ and each $\varphi \in \mathfrak{U}$ there exists a finite number of systems in $\mathfrak{S}$, for instance, $\mathfrak{W}_1, \ldots, \mathfrak{W}_n$, such that

$$\varphi \in (\mathfrak{W}_1 \cap \ldots \cap \mathfrak{W}_n);\;\; (\mathfrak{W}_1 \cap \ldots \cap \mathfrak{W}_n) \subset \mathfrak{U}$$

Open systems in the given subbasis $\mathfrak{S}$ are called the *subbasic open systems* of space Φ.

It is evident that a topology $\mathfrak{O}$ of space Φ is completely determined by the basis or subbasis of $\mathfrak{O}$.

(5) Topological cover. In general, we say that a family of systems $(\Xi_\iota)_{\iota \in I}$ is a cover of system Ψ, if $\Psi \subset \left(\bigcup_{\iota \in I} \Xi_\iota\right)$.

Cover $(\Xi_\iota)_{\iota \in I}$ of subsystem Ψ of topological informational space Φ is called *open,* if all Ξ_ι are open systems in Φ.

4.1.3 Interior, Exterior, and Boundary Formulas (Points)

According to [13, 20], some further definitions could be useful also for the purposes of informational topology.

DEFINITION 4 *Let Φ be a subsystem of system σ in topological space $\langle\sigma, \mathfrak{O}\rangle$.*

(1) *A formula $\varphi \in \Phi$ is called* interior formula *(point) of Φ if φ belongs to an open system $\Psi \in \mathfrak{O}$ contained in Φ, that is $\varphi \in \Psi$ and $\Psi \subset \Phi$, where Ψ is open. The system of interior formulas of Φ, denoted by $\underline{\Phi}$, is called the interior of Φ.*

(2) *A formula $\varphi \in \Phi$ is called* exterior formula *(point) of Φ if φ belongs to an open system $\Xi \in \mathfrak{O}$ contained in the complement $\complement\Phi$, that is $\varphi \in \Xi$; $\Xi \subset \complement\Phi$, where Ξ is open. The system of exterior formulas of Φ, denoted by $\overline{\Phi}$, is called the exterior of Φ.*

(3) *A formula $\varphi \in \sigma$ is called* boundary formula *(point) of Φ if in each neighborhood of φ formulas of Φ and $\complement\Phi$ occur. The system of all boundary formulas is called the* boundary *and denoted by $\beta\lfloor\Phi\rfloor$.* □

The situation is presented diagrammatically in Fig. 1. For the boundary, there is, evidently,

$$\beta\lfloor\Phi\rfloor \rightleftharpoons ((\overline{\Phi} \setminus \Phi) \cup (\Phi \setminus \underline{\Phi}))$$

A SIGNIFICANT COMMENT A formula system Φ is open iff each of its formula is an interior formula (point). A formula system Ψ is closed iff its complement $\complement\Psi$ is an open formula system. □

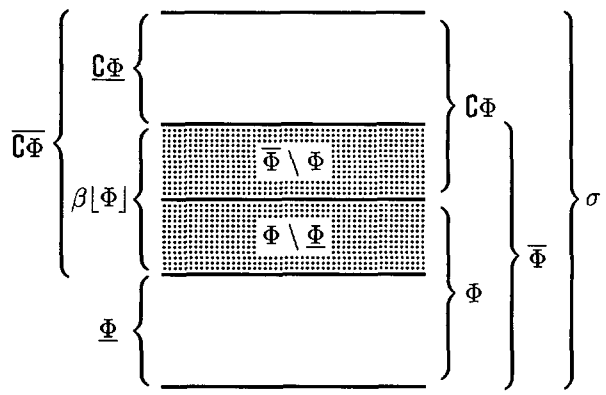

Figure 1: *Diagram presentation of formula system Φ, its interior $\underline{\Phi}$, exterior $\overline{\Phi}$, and boundary $\beta\lfloor\Phi\rfloor$, within a system σ, where the complement of Φ, denoted by $\mathbf{C}\Phi$, appears together with the complement interior $\mathbf{C}\underline{\Phi}$, exterior $\overline{\mathbf{C}\Phi}$, and boundary $\beta\lfloor\mathbf{C}\Phi\rfloor \rightleftharpoons \beta\lfloor\Phi\rfloor$.*

EXAMPLE 3 The connection of an informational loop and to it belonging system, that is, circular formula system, with the environment, can be realized by a special formula, usually a simple transition formula, e.g., $\alpha_j \models \xi_{ij}$ in Fig. 4. This formula belongs to Φ_φ, but is not an interior formula of Φ_φ, that is, of $\underline{\Phi_\varphi}$. □

EXAMPLE 4 Let systems Φ, B, Ψ form a topological space $\langle((\Phi \cup B) \cup \Psi), \mathfrak{D}\rangle$, as presented in Fig. 2. How

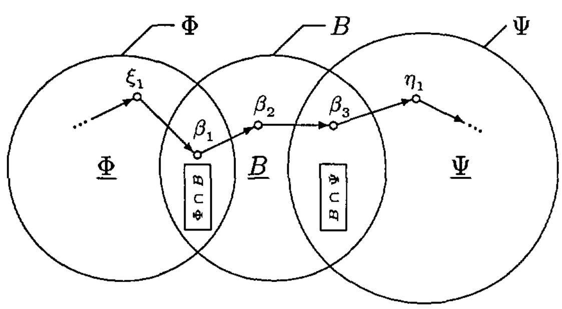

Figure 2: *Graphical presentation of formula system $\sigma \rightleftharpoons (\Phi; B; \Psi)$, where each of the subsystems has its interior $\underline{\Phi}, \underline{B}, \underline{\Psi}$, respectively.*

can this system informational graph be interpreted in different ways?

We must clarify more precisely what subsystems, marked by Φ, B, Ψ, might represent. The aim of the graph is to explicate the so-called interior and neighborhood regions of subsystems in respect to informational formulas in general, their operands, and the so-called primitive transitions, represented in the form of the graph route[16]

$$\cdots \models \xi_1 \models \beta_1 \models \beta_2 \models \beta_3 \models \eta_1 \models \cdots$$

The route drawn in Fig. 2 runs through the operands

$$\xi_1, \beta_1 \in \Phi;\ \beta_1, \beta_2, \beta_3 \in B;\ \beta_3, \eta_1 \in \Psi$$

where

$$\beta_1 \in \Phi, B \quad \text{and} \quad \beta_3 \in B, \Psi$$

According to Fig. 2, parts of system B function as neighborhoods of Φ and Ψ, respectively. System $\underline{B}$ as the interior of B is represented by

$$\underline{B} \rightleftharpoons (B \setminus ((\Phi \cap B) \cup (B \cap \Psi)))$$

that is, considering both systems Φ and Ψ. On the other side, the exterior of Φ, $\overline{\Phi}$, could roughly be understood as the union $\Phi \cup \underline{B}$ and, adequately, $\overline{\Psi}$ as $\Psi \cup \underline{B}$. In general, B is the neighborhood system for both Φ and Ψ. Further, evidently,

$$\mathbf{C}\Phi \rightleftharpoons \underline{B} \cup \Psi;\ \mathbf{C}\Psi \rightleftharpoons \underline{B} \cup \Phi;$$
$$\mathbf{C}\underline{\Phi} \rightleftharpoons B \cup \Psi;\ \mathbf{C}\underline{\Psi} \rightleftharpoons \Phi \cup B;$$
$$\overline{\Phi} \rightleftharpoons \Phi \cup \underline{B};\ \overline{\Psi} \rightleftharpoons \Psi \cup \underline{B}$$

etc. This example shows the importance of distinguishing the three possible types of topological spaces: (1) a space of circular/serial informational formulas of arbitrary lengths, Φ_φ; (2) a space of basic transition formulas of length 1, $\Phi_{\xi\models\eta}$; and (3) a space of simple informational operands (formula length 0), Φ_ξ. □

Another example shows the questionableness of a unique determination of topological structures dealing with different types of informational formula systems (Φ_φ, $\Phi_{\xi\models\eta}$, and Φ_ξ).

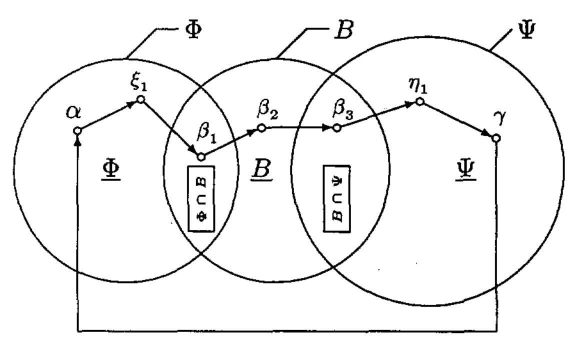

Figure 3: *Graphical presentation of formula system $(\Phi \cup B) \cup \Psi$, including a circular path.*

EXAMPLE 5 According to Fig. 3, let formula systems

[16] In literature, different names are given to the route. Informationally, the name informational scheme or, in short, scheme, is used. The name *edge* denotes the edge (representing an operator) of a graph polyhedron, to which a graph can be transformed. Path instead of the graph route sounds also adequately.

$$((\Phi \cup B) \cup \Psi)_\varphi \rightleftharpoons \begin{pmatrix} \alpha \models (\xi_1 \models (\beta_1 \models (\beta_2 \models (\beta_3 \\ \models (\eta_1 \models (\gamma \models \alpha)))))); \\ \vdots \end{pmatrix};$$

$$((\Phi \cup B) \cup \Psi)_{\xi \models \eta} \rightleftharpoons \begin{pmatrix} \alpha \models \xi_1; \xi_1 \models \beta_1; \beta_1 \models \beta_2; \\ \beta_2 \models \beta_3; \beta_3 \models \eta_1; \\ \eta_1 \models \gamma; \gamma \models \alpha; \ \ldots \end{pmatrix};$$

$$((\Phi \cup B) \cup \Psi)_\xi \rightleftharpoons (\alpha; \xi_1; \beta_1; \beta_2; \beta_3; \eta_1; \gamma; \ \ldots)$$

be given, where $\alpha \in \underline{\Phi}_\xi$ and $\gamma \in \underline{\Psi}_\xi$. What is now evident (or not quite evident) from the topological graph in Fig. 3? Can basic transition formula systems be determined uniquely, and in which way?

The problem occurs at basic transitions crossing the boundaries of systems Φ, B and Ψ. In Fig. 3 such transitions are $\xi_1 \models \beta_1$, $\beta_3 \models \eta_1$, and $\gamma \models \alpha$. Evidently, in a strict situation, it would be not possible to express the basic transition systems rigorously. Thus, the compromise notation[17]

$$\begin{array}{l} \Phi_{\xi \models \eta} \rightleftharpoons (\models \alpha;\ \alpha \models \xi_1;\ \xi_1 \models \beta_1;\ \beta_1 \models); \\ B_{\xi \models \eta} \rightleftharpoons (\models \beta_1;\ \beta_1 \models \beta_2;\ \beta_2 \models \beta_3;\ \beta_3 \models); \\ \Psi_{\xi \models \eta} \rightleftharpoons (\models \beta_3;\ \beta_3 \models \eta_1;\ \eta_1 \models \gamma;\ \gamma \models) \end{array}$$

can be accepted, where the transition $\gamma \models \alpha$ comes additionally. Further,

$$\Phi_\xi \rightleftharpoons (\alpha; \xi; \beta_1);\ B_\xi \rightleftharpoons (\beta_1; \beta_2; \beta_3);\ \Psi_\xi \rightleftharpoons (\beta_3; \eta; \gamma)$$

In this situation, Fig. 3 shows $(\Phi \cap \Psi) \rightleftharpoons \emptyset$. □

The last example presents how basic transition formula systems and operand systems derived from general formula systems can offer various informational interpretations.

4.1.4 Informational Neighborhood

Informational neighborhood (neighborhood, for short) is both a metaphor and a formalistic structure concerning various possibilities of informational relationships between formulas and formula systems.

DEFINITION 5 *Neighborhood of system Ξ in a topological space $\langle \Phi, \mathfrak{O} \rangle$ is called each system which includes an open system including Ξ. Neighborhoods of one-formula system (φ) are said to be also the neighborhoods of formula φ.* □

Let us present the last definition by other words.

(1) Let φ be a formula (point) of topological space $\langle \Phi, \mathfrak{O} \rangle$, that is, $\varphi \in \Phi$. A subsystem Ξ of Φ ($\Xi \subset \Phi$) is a neighborhood of φ iff Ξ is a supersystem of an open system Ψ ($\Xi \supset \Psi$) containing φ. Thus, $\boxed{\varphi \in \Phi, \Psi;\ \Psi \subset \Xi;\ \Xi \subset \Phi}$.

Other possible interpretations of the neighborhood definition are the following:

(2) A system Ξ in a topological space $\langle \Phi, \mathfrak{O} \rangle$ is a neighborhood ($\mathfrak{O}$-neighborhood) of a point (formula) φ iff Ξ contains an open system Ψ to which φ belongs.

(3) In a topological space $\langle \Phi, \mathfrak{O} \rangle$, the neighborhood of the point $\varphi \in \Phi$ is called each subsystem $\Xi \subset \Phi$, including an open system Ψ such that $\varphi \in \Psi$ and $\Psi \subset \Xi$. Then, the neighborhood of subsystem $\Omega \subset \Phi$ is each subsystem $\Xi \subset \Phi$ which includes an open system Ψ, that is, satisfies $\Omega \subset \Psi$ and $\Psi \subset \Xi$. Thus, $\boxed{\varphi \in \Phi, \Psi;\ \Omega, \Xi \subset \Phi;\ \Psi \subset \Xi;\ \Omega \subset \Psi}$.

Evidently, each neighborhood of system Ξ in Φ is also a neighborhood of each system $\Psi \subset \Xi$ and, in particular, of each formula in Ξ. In turn, let Ξ be the neighborhood of each formula of system Ψ and Υ be the union of all open systems included in Ξ; then $\Upsilon \subset \Xi$, as well as each formula of Ψ, belong to an open system, included in Ξ, that is, to $\Psi \subset \Upsilon$; but Υ is open according to ($\mathrm{T_I}$); consequently, Ξ is the neighborhood of system Ψ. In particular, the following comes into the foreground:

SUPPOSITION 1 *That a system is the neighborhood of each its formula, it is necessary and sufficient for it to be open.*[18] □

Let us mark by $\mathfrak{N}\lfloor\varphi\rfloor$ the system of all neighborhoods of formula φ. Systems in $\mathfrak{N}\lfloor\varphi\rfloor$ possess the following properties:

($\mathrm{N_I}$) *Each subsystem of system Φ, including a system of $\mathfrak{N}\lfloor\varphi\rfloor$, belongs to $\mathfrak{N}\lfloor\varphi\rfloor$.*
($\mathrm{N_{II}}$) *Intersection of a finite number of systems of $\mathfrak{N}\lfloor\varphi\rfloor$ belongs to $\mathfrak{N}\lfloor\varphi\rfloor$.*
($\mathrm{N_{III}}$) *Formula φ belongs to each system in $\mathfrak{N}\lfloor\varphi\rfloor$.*

In fact, these three properties are a direct consequence of Def. 5 and Ax. ($\mathrm{T_{II}}$).

($\mathrm{N_{IV}}$) *For each system N belonging to $\mathfrak{N}\lfloor\varphi\rfloor$, there exists a system W belonging to $\mathfrak{N}\lfloor\varphi\rfloor$, such*

[17] The compromise notation is, for example, $\models \alpha$ and $\gamma \models$. Each informational operator $\models$ is a binary operator, dependent on both operands. If one side of the operator is open, it is meant, that the missing operand is not fixed yet.

[18] In the informational sense, the word "neighborhood" has a meaning which points to the informational relationship, proximity, similarity, closeness and the like, coming to the surface as a senseful informational formula of intuitively clear situations, facts and properties. In this way, the choice of this term has the advantage to make the speech figurative. For instance, Sup. 1 can be expressed in the following manner: for a system Ξ to be open, it is necessary and sufficient that for an arbitrary $\varphi \in \Xi$, all the formulas being *sufficiently informationally close to* φ, belong to Ξ. And, in general, if a property is true for all the formulas of a neighborhood of formula φ, it is said that this property holds for all the formulas, being *sufficiently informationally close to* φ. As we shall learn, informational closeness will concern, for instance, formulas of an entirely circularly connected system, with several loop formulas.

that N belongs to $\mathfrak{N}\lfloor\psi\rfloor$ for an arbitrary $\psi \in W$.

Evidently, considering Sup. 1, it suffices to take for W an arbitrary system, including formula φ, and being included in N.[19]

These four properties of system $\mathfrak{N}\lfloor\varphi\rfloor$ are said to be the *neighborhood characteristics*. Thus:

SUPPOSITION 2 *If to each formula φ of system Φ, a system $\mathfrak{N}\lfloor\varphi\rfloor$ of subsystems of Φ corresponds, and properties (N_{I}), (N_{II}), (N_{III}) and (N_{IV}) hold, then in Φ there exists a unique topological structure, for which $\mathfrak{N}\lfloor\varphi\rfloor$ serves as a system of all neighborhoods φ at an arbitrary $\varphi \in \Phi$.* □

If there exists the required topological structure, then, by Sup. 1, as the system of all open systems of this topology necessarily serves the system $\mathfrak{O}$ of all those systems Ξ in Φ, for which $\Xi \in \mathfrak{N}\lfloor\varphi\rfloor$ holds for each $\varphi \in \Xi$; this is the reason of a unique topology, if it exists.

4.1.5 Informational Bases and Subbases

The base $\mathfrak{B}$ of a topological space $\langle\Phi, \mathfrak{O}\rangle$ is a system of formula systems B such that any formula φ of Φ belonging to subsystem B, B is a subsystem of $\mathfrak{G}$ in $\mathfrak{O}$.

DEFINITION 6 *A class (system of subsystems) $\mathfrak{B}$ of open subsystems of Φ, in topological space $\langle\Phi, \mathfrak{O}\rangle$, is a base for the topology $\mathfrak{O}$ iff (i) every open system $\mathfrak{G} \in \mathfrak{O}$ is the union of members of $\mathfrak{B}$. Equivalently, $\mathfrak{B} \subset \mathfrak{O}$ is a base for $\mathfrak{O}$ iff (ii) for any formula φ belonging to an open system $\mathfrak{G}$, there exists $B \in \mathfrak{B}$ with $\varphi \in B$ and $B \subset \mathfrak{G}$.* □

Let us examine the base $\mathfrak{B}$ as a system of singleton systems.

EXAMPLE 6 Let a topological formula space $\langle\Phi, \mathfrak{F}\rangle$ be given. Then the class $\mathfrak{B} \rightleftharpoons ((\varphi) \mid \varphi \in \Phi)$ of all singleton subsystems of Φ is a base for formula topology $\mathfrak{F}$ on Φ. Because each system (φ) is $\mathfrak{F}$-open, each $\mathfrak{A} \subset \Phi$ is $\mathfrak{F}$-open too. Furthermore, every system is the union of singleton systems. Thus, any other class $\mathfrak{B}^*$ of subsystems of Φ is a base for $\mathfrak{F}$ iff it is a superclass of $\mathfrak{B}$, that is, $\mathfrak{B}^* \supset \mathfrak{B}$. □

Which are the necessary and sufficient conditions for a class of systems to be a base for some informational topology?

THEOREM 2 *Let $\mathfrak{B}$ be a system of subsystems of a non-empty system Φ. Then $\mathfrak{B}$ is a base for topology $\mathfrak{O}$ on Φ iff*

(i) $\Phi \rightleftharpoons \bigcup_{B \in \mathfrak{B}} B$ *and*

(ii) *for any $B, B^* \in \mathfrak{B}$, $B \cap B^*$ is the union of members of systems of $\mathfrak{B}$, or, equivalently, if $\varphi \in (B \cap B^*)$ then there exists $B_\varphi \in \mathfrak{B}$ such that $\varphi \in B_\varphi$ and $B_\varphi \subset (B \cap B^*)$.* □

Informational subbasis is another notion which could become relevant in the topological investigation of informational formula systems.

DEFINITION 7 *In a topological space $\langle\Phi, \mathfrak{O}\rangle$, a class $\mathfrak{S}$ of open formula subsystems of Φ, that is, $\mathfrak{S} \subset \Phi$, is a subbasis for the topology $\mathfrak{O}$ on Φ iff finite intersections of members of $\mathfrak{S}$ form a base $\mathfrak{B}$ for $\mathfrak{O}$.* □

Any class $\mathfrak{A}$ of formula subsystems of a non-empty formula system Φ is the subbasis for a unique topology $\mathfrak{O}$ on Φ. Intersections of members of $\mathfrak{A}$ form a base for the topology $\mathfrak{O}$ on Φ.

EXAMPLE 7 Let $\Phi \rightleftharpoons (\varphi_0; \varphi_1; \ldots; \varphi_6)$, according to the formula system, belonging to the graph in Fig. 4, and $\mathfrak{A} \rightleftharpoons ((\varphi_3; \varphi_4); (\varphi_4; \varphi_5); (\varphi_6))$. Finite intersections of members of $\mathfrak{A}$ gives the base

$$\mathfrak{B} \rightleftharpoons ((\varphi_3; \varphi_4); (\varphi_4; \varphi_5); (\varphi_6); (\varphi_4); \emptyset; \Phi)$$

By definition, $\Phi \in \mathfrak{B}$ follows, since it is the empty intersection of members of $\mathfrak{A}$-system. Considering unions of members of $\mathfrak{B}$ gives the family

$$\begin{aligned} \mathfrak{O} \rightleftharpoons \; & ((\varphi_3; \varphi_4); (\varphi_4; \varphi_5); (\varphi_6); (\varphi_4); \emptyset; \Phi; \\ & (\varphi_3; \varphi_4; \varphi_6); (\varphi_4; \varphi_5; \varphi_6); (\varphi_3; \varphi_4; \varphi_5)) \end{aligned}$$

Formula system $\mathfrak{O}$ is the topology on Φ generated by formula system $\mathfrak{A}$. □

4.1.6 Informational Accumulation Point

Accumulation point (also, limit point) is a well-known term in mathematical topology. We need the notion of informational accumulation point, for example, as a formula or formula system approaching as close as possible to the meaning of something. This means that the final meaning of something can never be reached, although the meaning of something can be expressed by a formula system as close as required.

DEFINITION 8 *If $\mathfrak{A}$ is a subsystem of a formula system Φ, formula $\varphi \in \Phi$ is an* accumulation point *of $\mathfrak{A}$, iff every open system $\mathfrak{G}$ containing φ contains a point (formula) of $\mathfrak{A}$ different from φ. There is,*

$$(\mathfrak{G} \text{ is open}; \; p \in \mathfrak{G}) \Longrightarrow ((\mathfrak{A} \cap (\mathfrak{G} \setminus (\varphi))) \neq \emptyset)$$

The system of accumulation points of $\mathfrak{A}$, marked by $\mathfrak{A}'$, is called the derived system *of system $\mathfrak{A}$.* □

It usually happens that an accumulation point is informationally inexpressible, although a formula system as a point comes close and closest to the accumulation point.

[19] This property can be additionally expressed by saying that a neighborhood of formula φ is, besides, a neighborhood of all formulas being sufficiently close to φ.

4.1.7 Informational Connectedness and Compactedness

Most of topological investigation concerns certain topological properties as *connectedness* and *compactedness*. Intuitively, the connectedness of an informational space is a consequence of an adequate operand distribution in informational formulas.

Two formula subsystems $\mathfrak{A}$ and $\mathfrak{B}$ of a topological space $\langle \Phi, \mathfrak{O} \rangle$ are separated if $\mathfrak{A}$ and $\mathfrak{B}$ are operand disjoint and neither contains an accumulation point of the other. This means $\mathfrak{A}_\xi \cap \mathfrak{B}_\xi \rightleftharpoons \emptyset$.

A topological informational space $\langle \Phi, \mathfrak{O} \rangle$ is *disconnected* iff Φ is the union of two open, non-empty, disjoint subsystems of formulas, i.e.,

$$\Phi \rightleftharpoons (\mathfrak{A} \cup \mathfrak{B});\ \mathfrak{A}, B \in \mathfrak{O};\ \mathfrak{A} \cap \mathfrak{B} \rightleftharpoons \emptyset;\ \mathfrak{A}, \mathfrak{B} \not\rightleftharpoons \emptyset$$

DEFINITION 9 *An informational graph is said to be* connected graph *if there is a path (informational scheme) between every pair of operands in the graph. An informational graph is said to be* circularly connected graph *if there is a circular path (circular informational scheme) between every pair of operands in the graph.* □

A connected graph represents an operand connected formula system. How the connected formula systems can be recognized topologically?

Because informational topology deals with formula systems, the connectedness can be recognized by the properties among formulas of a formula system. Visually, connectedness can be inspected by a graph, using Def. 9. Because a graph is uniquely described by the formula system $\Phi_{\xi \models \eta}$ deduced from the original formula system Φ_φ, connectedness is determined by the adequate transitivity of formulas in the sense

$$((\xi_1 \models \xi_2) \wedge (\xi_2 \models \xi_3)) \rightleftharpoons$$
$$((\xi_1 \models \xi_2) \models \xi_3) \vee (\xi_1 \models (\xi_2 \models \xi_3))$$

If $\varphi_i, \varphi_j \in \Phi_\varphi$, formulas φ_i and φ_j are connected iff there exists an operand ζ such that

$$\zeta \in \left(\Phi_\xi^{\varphi_i} \cap \Phi_\xi^{\varphi_j}\right)$$

This means that operand systems $\Phi_\xi^{\varphi_i}$ and $\Phi_\xi^{\varphi_j}$, derived from formulas φ_i and φ_j, respectively, have the common operand ζ. This property coincides with the concept of informationally linked formulas in a system, discussed in Sect. 3.1.

The concept of cover [Sect. 4.1.2/(5)] is needed in the definition of compactedness in the following sense.

DEFINITION 10 *A subsystem Ψ of informational topological space $\langle \Phi, \mathfrak{O} \rangle$ is* compact *if every open cover of Ψ is reducible to a finite cover.* □

In another topological interpretation, if Ψ is compact and $\Psi \subset \left(\bigcup_{\iota \in I} \Xi_\iota\right)$, where Ξ_ι are open formula systems, then it is possible to select a finite number of the open systems, say $\Xi_{\iota_1}, \ldots, \Xi_{\iota_m}$, so that $\Psi \subset (\Xi_{\iota_1} \cup \ldots \cup \Xi_{\iota_m})$.

4.2 Topologies Concerning Meaning

Let us present a direct generalization of the informational topology used in concern with the phenomenon of meaning[20] μ, for instance, as it can emerge within the various forms of understanding, interpretation, conceptualization, perception, consciousness, and the like.

DEFINITION 11 *Let Φ be a system of formulas, and assume that there exists a meaning formula $\mu\lfloor \varphi, \psi \rfloor$ on pairs of formulas $\varphi, \psi \in \Phi$ satisfying the following conditions:*

1. $\mu\lfloor \varphi, \psi \rfloor \not\models \emptyset$;
2. $\mu\lfloor \varphi, \psi \rfloor \models \mu\lfloor \varphi \rfloor$ if and only if $\varphi \rightleftharpoons \psi$;
3. $\mu\lfloor \varphi, \psi \rfloor \models \mu\lfloor \psi, \varphi \rfloor$;
4. $(\mu\lfloor \varphi, \psi \rfloor \models \mu\lfloor \psi, \omega \rfloor) \models \mu\lfloor \varphi, \omega \rfloor$
(the informational transitional consequence)

We say that Φ is a meaning space with meaning μ, *or with* meaning difference μ. □

Condition 1 says that system $\mu\lfloor \varphi, \psi \rfloor$ does not inform to be empty. Thus, this condition has also the meaning $\mu\lfloor \varphi, \psi \rfloor \not\rightleftharpoons \emptyset$. Condition 2 says that meaning concerning operands φ and ψ informs to be $\mu\lfloor \varphi \rfloor$ only and only if φ is the same operand as ψ. In this case, also $\mu\lfloor \varphi, \psi \rfloor \rightleftharpoons \mu\lfloor \varphi \rfloor$. By Condition 3, since meaning concerning two operands, $\mu\lfloor \varphi, \psi \rfloor$, is a meaning difference between φ and ψ, such a difference informs to be $\mu\lfloor \psi, \varphi \rfloor$. Thus, $\mu\lfloor \varphi, \psi \rfloor \rightleftharpoons \mu\lfloor \psi, \varphi \rfloor$. It can certainly be introduced a difference between the symmetrical cases by the distinguishing $\mu\lfloor \varphi, \psi \rfloor \not\rightleftharpoons \mu\lfloor \psi, \varphi \rfloor$. Such a condition would ruin the traditional convenience of space metrication.

Meaning of something as an informational phenomenon follows the possibilities of informational decomposition and, in this sense, offers various possibilities for the play with meaning topologies in informational spaces. The diversity of decomposition is pointed out, for instance, by Haney [15], using the term deconstruction[21]. Meaning as informational decomposition makes an informational space a continuum, in which topological notions of open systems,

[20] Informationally, meaning replaces the so-called metric in mathematics. The metric spaces (introduced by Fréchet in 1906 [12]) are based on the concept of distance [25]. Usually, a metric can be introduced in real numbers and in other kinds of mathematical spaces (e.g., in Hilbert space). A metric provides an easy way to define a topology in a metric space. To the topologist, the particular metric used on a space is merely a convenient way to define open sets [16].

[21] In Haney [15], the following is stated: ... *it would be an overgeneralization to say, as the tendency is in 'American deconstruction', that all meaning is indeterminate, all presence illusory, all*

interior, exterior, boundary, neighborhood, accumulation point, etc. become reasonable for a formalistic and artificial construction (composition) of formula systems.

4.3 Topologies Concerning Distributed (Parallel) Systems

DEFINITION 12 (Distributed System) *A parallel informational space $\mathfrak{D}$ with operands (points, a kind of vectors) δ being distributed by their components (point coordinates) $\xi_1^\delta, \ldots, \xi_{n_\delta}^\delta$ is called the distributed informational space or $\mathfrak{D}$-space if:*

1. *A rule is given by which to each pair of points δ, ε of space $\mathfrak{D}$ the parallelism $(\delta; \varepsilon)$ corresponds.*
2. *This rule satisfies the following conditions:*
   a) $(\varepsilon; \delta) \rightleftharpoons (\delta; \varepsilon)$ (displacement law);
   b) $(\delta; \varepsilon \models \zeta) \rightleftharpoons ((\delta; \varepsilon) \models (\delta; \zeta))$ (distributive law);
   c) $(\mu\delta; \varepsilon) \rightleftharpoons \mu(\delta; \varepsilon)$ for an arbitrary functional operand μ;
   d) $(\delta, \delta) \not\models \emptyset$ for $\delta \not\rightleftharpoons \emptyset$ and $(\delta, \delta) \rightleftharpoons \emptyset$ for $\delta \rightleftharpoons \emptyset$.

By axioms b) and c) the general formula

$$\begin{aligned}(\varphi\lfloor \mu_1\delta_1, \ldots, \mu_k\delta_k \rfloor; \psi\lfloor \nu_1\varepsilon_1, \ldots, \nu_m\varepsilon_m \rfloor) \rightleftharpoons \\ \omega\lfloor \mu_1\nu_1(\delta_1; \varepsilon_1), \mu_1\nu_2(\delta_1; \varepsilon_2), \ldots, \mu_1\nu_m(\delta_1; \varepsilon_m), \\ \mu_2\nu_1(\delta_2; \varepsilon_1), \mu_2\nu_2(\delta_2; \varepsilon_2), \ldots, \mu_2\nu_m(\delta_2; \varepsilon_m), \\ \ldots, \\ \mu_k\nu_1(\delta_k; \varepsilon_1), \mu_k\nu_2(\delta_k; \varepsilon_2), \ldots, \mu_k\nu_m(\delta_k; \varepsilon_m) \rfloor\end{aligned}$$

is obtained which holds for arbitrary vectors $\delta_1, \ldots, \delta_k, \varepsilon_1, \ldots, \varepsilon_m$ and arbitrary meanings $\mu_1, \ldots, \mu_k, \nu_1, \ldots, \nu_m$. □

To explicate the vector nature of points (formulas and/or formula systems) in a distributed informational space $\mathfrak{D}$, let us introduce the vector notation (of the basic degree) of points in the form $\|\delta\rangle$ [32, 38]. The question is which formula components constitute the informational vector $\|\delta\rangle$? Evidently, the structure of an informational vector is not as simple as in a mathematical vector space. Let us discuss several formula notations constituting a vector.

The basic constituent is obviously the system of all simple operands appearing in a formula system δ and being denoted by

$$^{\rightarrow}\delta_{\|}^{0}\left\lfloor \xi_1^\delta, \ldots, \xi_{n_\delta}^\delta \right\rfloor \rightleftharpoons \left(\xi_1^\delta, \ldots, \xi_{n_\delta}^\delta\right)$$

theme of intention irrelevant, all reference a fiction, etc. (see Attridge [1] p. 12). That a text for Derrida, especially a literary text, is always situated, read and re-read in a specific place and times makes it 'iterable' or repeatable, the same but always different, and therefore never reducible to an abstraction by theoretical contemplation (Derrida [11] pp. 172–97). A text is unique and repeatable, concrete and abstract simultaneously. This coexistence lies in the heart of deconstruction and reflects the connectedness of the subject and object in the experience of the self as pure consciousness.

Besides, parallel components $\xi_1^\delta, \ldots, \xi_{n_\delta}^\delta$ can appear and be distributed within different kinds of formulas, or even form a serial or circular serial formula as a whole, that is

$$^{\rightarrow}\delta_{\|}^{\ell_\rightarrow}\left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\rightarrow}^\delta \right\rfloor \rightleftharpoons {}^{\ell_\rightarrow}_{\;;}\delta_\rightarrow \left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\rightarrow}^\delta \right\rfloor \quad \text{and/or}$$
$$^{\circ}\delta_{\|}^{\ell_\circ}\left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\circ - 1}^\delta \right\rfloor \rightleftharpoons {}^{\ell_\circ}_{\;;}\delta_\rightarrow^{\circ} \left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\circ - 1}^\delta \right\rfloor$$

respectively. Vector $\|\delta\rangle$ corresponding to system δ is determined by

$$\|\delta\rangle \rightleftharpoons \begin{pmatrix} ^{\rightarrow}\delta_{\|}^{0}\left\lfloor \xi_1^\delta, \ldots, \xi_{n_\delta}^\delta \right\rfloor; \\ ^{\rightarrow}\delta_{\|}^{\ell_\rightarrow}\left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\rightarrow}^\delta \right\rfloor; \; \ell_\rightarrow \in \{1, 2, \ldots, \ell_\rightarrow^{\max}\}; \\ ^{\circ}\delta_{\|}^{\ell_\circ}\left\lfloor \xi_1^\delta, \ldots, \xi_{\ell_\circ - 1}^\delta \right\rfloor; \; \ell_\circ \in \{2, 3, \ldots, \ell_\circ^{\max}\} \end{pmatrix}$$

The structure of vector $\|\delta\rangle$ needs to be additionally explained. What does such a vector include and in which sense the difference between the mathematical and informational vector comes to the surface?

First, let us list all the components of vector $\|\delta\rangle$ in concern to the origin system δ. System δ is simply a parallel system of serial and/or circular serial formulas. But, in fact, this list is in no way a complete one in regard to the complex parallelism hidden in particular formulas of δ. The reader should remind the axiomatic approach of the informational where the fundamental axiom is expressed by the implication

$$(\alpha \models \beta) \Longrightarrow \begin{pmatrix} \alpha; \\ \beta; \\ \alpha \models \beta \end{pmatrix}$$

If this rule is recursively applied to a serial or circular serial formula ${}^{n}_{;}\varphi_\rightarrow \lfloor \alpha, \alpha_1, \cdots, \alpha_n \rfloor$ or ${}^{n+1}_{\;\;;}\varphi_\rightarrow^{\circ} \lfloor \alpha, \alpha_1, \cdots, \alpha_n \rfloor$, respectively, then, evidently, the application of the last axiom delivers all the subformulas appearing in a serial and/or circular serial formula, that is, in the serial case,

$${}^{n}_{;}\varphi_\rightarrow \lfloor \xi, \xi_1, \cdots, \xi_n \rfloor \Longrightarrow \begin{pmatrix} ^{\rightarrow}\delta_{\|}^{0}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ ^{\rightarrow}_{;}\delta_{\|}^{1^*}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ \vdots \\ ^{\rightarrow}_{;}\delta_{\|}^{(n-1)^*}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ {}^{n}_{;}\varphi_\rightarrow \lfloor \xi, \xi_1, \cdots, \xi_n \rfloor \end{pmatrix}$$

and in the circular serial case,

$${}^{n+1}_{\;\;;}\varphi_\rightarrow^{\circ} \lfloor \xi, \xi_1, \cdots, \xi_n \rfloor \Longrightarrow \begin{pmatrix} ^{\rightarrow}\delta_{\|}^{0}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ ^{\rightarrow}_{;}\delta_{\|}^{1^*}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ \vdots \\ ^{\rightarrow}_{;}\delta_{\|}^{n^*}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor; \\ {}^{n+1}_{\;\;;}\varphi_\rightarrow^{\circ} \lfloor \xi, \xi_1, \cdots, \xi_n \rfloor \end{pmatrix}$$

where the asterisked markers ${}_{i}\overset{\rightarrow}{\delta}{}_{\|}^{1^*}, \ldots, {}_{i}\overset{\rightarrow}{\delta}{}_{\|}^{(n-1)^*}, {}_{i}\overset{\rightarrow}{\delta}{}_{\|}^{n^*}$ denote the systems of serial subformulas of lengths $1, \ldots, n-1, n$ *conditionally* in respect to operands $\xi, \xi_1, \cdots, \xi_n$ in floor parentheses. Namely, a system ${}_{i}\overset{\rightarrow}{\delta}{}_{\|}^{1^*}\lfloor \xi, \xi_1, \ldots, \xi_n \rfloor$ includes only and only such basic transitions of the form $\alpha_i \models \alpha_j$ $(\ell = 1)$ which appear in formula ${}^{n}_{i}\varphi_{\rightarrow}\lfloor \xi, \xi_1, \cdots, \xi_n \rfloor$ or formula ${}^{n+1}_{\;\;i}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi, \xi_1, \cdots, \xi_n \rfloor$ (as a whole), respectively. Similar concerns lengths ℓ up to value n or $n+1$, respectively.

A short analysis shows that in the serial and circular serial case the number of all possible subformulas of a given length can be evaluated by simple formulas. Let ℓ_{sub} mark the length of a subformula in a serial formula with the length $\ell_{\rightarrow}$. Then, evidently, the number of such subformulas in a formula is

$$\vec{n}_{\text{sub}} \rightleftharpoons \begin{cases} \dfrac{\ell_{\rightarrow}}{\ell_{\text{sub}}}, & \text{if } \ell_{\rightarrow} \text{ is even} \\ \dfrac{\ell_{\rightarrow}+1}{\ell_{\text{sub}}}, & \text{if } \ell_{\rightarrow} \text{ is odd} \end{cases}$$

In a circular case there is

$$\overset{\circlearrowleft}{n}_{\text{sub}} = \begin{cases} \dfrac{\ell_{\circlearrowleft}}{\ell_{\text{sub}}}, & \text{if } \ell_{\circlearrowleft} \text{ is even} \\ \dfrac{\ell_{\circlearrowleft}-1}{\ell_{\text{sub}}}, & \text{if } \ell_{\circlearrowleft} \text{ is odd} \end{cases}$$

5 Variants of Informational Topologies

A topology $\mathfrak{O}$ depends on the carrier system Φ, that is, on the characteristic forms of its formulas. Which kinds of formulas in Φ can be distinguished?

The most usual system of formulas is composed of different serial and circular-serial formulas. These formulas emerge during the analysis of an informational case, usually in a kind of top-down and bottom-up decomposition of an initial (top) marker or an end (bottom) marker, carrying implicitly a yet-not-determined concept, proceeding stepwise into a more detail of the case—a progressive case decomposition from different points of view. This approach seems to be the most natural one, seen from the human point of consciousness. Just after of such a case identification more abstract and convenient approach with possibilities can be considered.

DEFINITION 13 *The constructed system of formulas, Φ, can take the following characteristic forms:*

$$\begin{aligned}
\Phi_\varphi &\rightleftharpoons (\varphi\lfloor \cdots \xi_1 \cdots \rfloor;\ \varphi\lfloor \cdots \xi_2 \cdots \rfloor;\ \ldots;\ \varphi\lfloor \cdots \xi_n \cdots \rfloor); \\
\Phi_\xi &\rightleftharpoons (\xi_1;\ \xi_2;\ \ldots;\ \xi_n;) \cup \Phi\lfloor \xi_i | \text{implicit operands} \rfloor; \\
\Phi_{\xi\models\eta} &\rightleftharpoons (\xi_1 \models \xi_2;\ \xi_2 \models \xi_3;\ \ldots;\ \xi_{n-1} \models \xi_n) \cup \\
&\qquad \Phi\lfloor \xi_i \models \xi_j | \text{implicit basic transitions} \rfloor
\end{aligned}$$

The first system, Φ_φ, is an authentic, intuitively constructed representation of a real case. The second system, Φ_ξ, is strictly expressed by all the occurring system operands as the title operands of a circular formula system, each operand in at least one circular formula. The third system, $\Phi_{\xi\models\eta}$, is the representative of all possible situations occurring by all possible parenthesis pairs displacements within the constructed (analyzed and synthesized) system. □

As said, the originally conceptualized system (obtained by the top-down or bottom-up approach or from both of them) is Φ_φ. Thus, the remaining two systems, Φ_ξ and $\Phi_{\xi\models\eta}$, evidently emerge from Φ_φ, that is,

$$\Phi_\varphi \longrightarrow \Phi_\xi \quad \text{and} \quad \Phi_\varphi \longrightarrow \Phi_{\xi\models\eta}$$

where $\longrightarrow$ denotes the corresponding derivation approach. On this basis, three different topologies can be determined, as formula, operand and basic-transition topology, respectively. $\Phi_\varphi \longrightarrow \Phi_{\xi\models\eta}$ is the formal representative of the corresponding informational graph [35].

Now, let us show, how different topologies can be defined on Φ_φ, Φ_ξ and $\Phi_{\xi\models\eta}$ in a concrete case, and how all they mirror one and the same informational graph, with different possibilities in regard to various parenthesis displacements in formulas of the system. As an example we choose the metaphysicalistic case.

5.1 Topologies of a Simple Metaphysicalism

Simple metaphysicalism is a basic scheme of informational invariance which can be further decomposed in greater details during identification of the involved entities, that is, a formula expressed in the metaphysicalistic form. Thus, the graph in Fig. 4 can be understood as a consequence of the circular metaphysicalistic formula system $\boxed{{}_{k_j}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor}$. Subscript j concerns the formula system component ${}_{k_j}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$ of system Φ_φ, e.g., $j = 1, 2, \ldots, n$. Subscript i concerns the operand component ξ_i of metaphysicalistic formula system ${}_{k_j}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor \in \Phi_\varphi$. Subscript k_j concerns the parenthesis-pair combination $1 \leq k_j \leq \overset{\circlearrowleft}{n}_{\text{sub},j}$ of the formula subsystem system ${}_{k_j}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$, with altogether $\boxed{\prod_{\forall i,j} \frac{\ell_{ij}}{\ell_{ij}+1}\binom{2\ell_{ij}}{\ell_{ij}}}$ possibilities, considering serial (input) and circular serial formulas of a system of formula systems, where ℓ_{ij} denotes the length of the formula in a formula subsystem.

5.1.1 Topologies on the circular formula system ${}_{k_j}\overset{\circlearrowleft}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$

According to the graph in Fig. 4, one of the possible formula systems can be constructed (reconstructed). Let it be the consequent observing type of metaphysicalism for which the extreme left-parenthesis heaping is characteristic, that is, $k_j = 1$. In this case, the graph

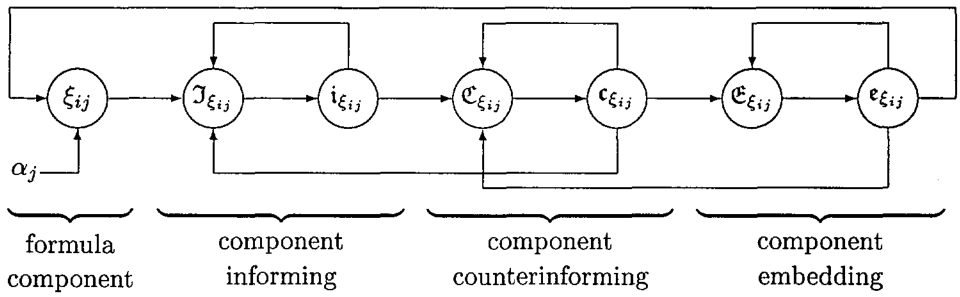

Figure 4: *The graph representing the basic metaphysicalism of a formula system* ${}_{k_j}\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor \in \Phi_\varphi$ *component* ξ_{ij}*, impacted by something (interior and/or exterior)* α_j*.*

is interpreted by the one of possible formula systems, that is,

$$
{}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor \rightleftharpoons
\left(\begin{array}{ll}
\alpha_j \models \xi_{ij}; & [\varphi_0] \\
((((((\xi_{ij} \models \mathfrak{I}_{\xi_{ij}}) \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models & [\varphi_1] \\
\quad \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}) \models \xi_{ij}; & \\
(((\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_2] \\
(((\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_3] \\
(\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_4] \\
(\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_5] \\
(\mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}} & [\varphi_6]
\end{array}\right)
$$

(0) According to the preceding notation, there is $\Phi_\varphi \rightleftharpoons {}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$. To be more transparent, let us replace this system by the abbreviated one, in the form

$$
\Phi_\varphi \rightleftharpoons \begin{pmatrix} \varphi_0; \\ \varphi_1; \\ \varphi_2; \\ \varphi_3; \\ \varphi_4; \\ \varphi_5; \\ \varphi_6 \end{pmatrix}
$$

representing the input transition formula and the six circular formulas, respectively. Which kind of topologies on Φ_φ can then be defined in a meaningful way?

(1) Let topology $\mathfrak{D}_{\varphi,1}$, if possible, maintain the meaning of the original formula system Φ_φ in respect to the graph in Fig. 4. Let the meaningful condition be

$$(\varphi_1), (\varphi_2), (\varphi_3), (\varphi_4), (\varphi_5), (\varphi_6) \in \mathfrak{D}_{\varphi,1}$$

by which all of the loops and the input transition enter the topology. What are the consequences of such a choice? First, the mutual intersections of formulas are empty systems, that is,

$$(\varphi_p \cap \varphi_q) \rightleftharpoons \emptyset;\ p \neq q;\ p, q = 0, 1, \ldots, 6$$

However, all the possible unions of formulas φ_0, φ_1, φ_2, φ_3, φ_4, φ_5, φ_6 must enter $\mathfrak{D}_{\varphi,1}$. This condition delivers together with $\emptyset$ and Φ_φ a topology which is the power system of Φ_φ, $\mathfrak{P}\lfloor\Phi_\varphi\rfloor$, called the discrete topology $\mathfrak{D}_{\varphi,1}$ (see Sect. 4.1.2). The precept of this example is that for all formulas $\varphi \in \Phi_\varphi$, $(\varphi) \in \mathfrak{D}_\varphi$ implies that $\mathfrak{D}_\varphi$ is $\mathfrak{P}\lfloor\Phi_\varphi\rfloor$.

(2) A look to the graph in Fig. 4 brings to the surface another logic of topological understanding. Let us take the main loop φ_1 and one of the subloops, say φ_2. In this case, further subsystems of topology $\mathfrak{D}_{\varphi,2}$ are (φ_1), (φ_2), and $(\varphi_1; \varphi_2)$. Thus,

$$
\mathfrak{D}_{\varphi,2} \rightleftharpoons \begin{pmatrix} \emptyset; \\ (\varphi_1); \\ (\varphi_2); \\ (\varphi_1; \varphi_2); \\ \Phi_\varphi \end{pmatrix}
$$

One sees that this type of topology emerges independently on the chosen subloop. In case of two chosen subloops, say φ_2 and φ_3, the topology becomes

$$
\mathfrak{D}_{\varphi,3} \rightleftharpoons \begin{pmatrix} \emptyset;\ (\varphi_1);\ (\varphi_2);\ (\varphi_3); \\ (\varphi_1; \varphi_2);\ (\varphi_1; \varphi_3);\ (\varphi_2; \varphi_3); \\ (\varphi_1; \varphi_2; \varphi_3;);\ \Phi_\varphi \end{pmatrix}
$$

Let Φ_φ include n_φ formulas, $\varphi_1, \ldots, \varphi_{n_\varphi}$. Then, for a topology $\mathfrak{D}_{\varphi,i}$, implication

$$
(((\varphi_1), (\varphi_2), \ldots, (\varphi_i) \in \mathfrak{D}_{\varphi,i}) \wedge (i < n)) \Longrightarrow \\ (n_{\mathfrak{D}_{\varphi,i}}) = 2^i + 1)
$$

holds. Here, $n_{\mathfrak{D}_{\varphi,i}}$ marks the number (cardinality) of subsystems in $\mathfrak{D}_{\varphi,i}$.

(3) Other senseful topologies could consider specific situations in respect to the graph in Fig. 4. In constructing a topology, one can proceed from the other topological side, taking subsystems with more than one formula. Certainly, any other topology $\mathfrak{D}_\varphi$ on system Φ_φ is merely a subsystem of $\mathfrak{P}\lfloor\Phi_\varphi\rfloor$.

If the formula subsystems $(\varphi_0; \varphi_1; \varphi_2; \varphi_3)$ and $(\varphi_4; \varphi_5; \varphi_6)$ (the split of system Φ_φ) are joined to $(\emptyset; \Phi_\varphi)$, no further subsystems are necessary for the topology ($\emptyset$; $(\varphi_0; \varphi_1; \varphi_2; \varphi_3)$; $(\varphi_4; \varphi_5; \varphi_6)$; Φ_φ). Further, a characteristic implication is, for instance,

$$((\varphi_2;\varphi_4);(\varphi_3;\varphi_5);(\varphi_6) \in \mathfrak{D}_\varphi) \Longrightarrow \\ \left(\begin{pmatrix} (\varphi_2;\varphi_3;\varphi_4;\varphi_5;\varphi_6); \\ (\varphi_2;\varphi_3;\varphi_4;\varphi_5); \\ (\varphi_2;\varphi_4;\varphi_6); \\ (\varphi_3;\varphi_5;\varphi_6) \end{pmatrix} \in \mathfrak{D}_\varphi\right)$$

The initial intention (premise of the implication) is to cover explicitly both subloops for $\mathfrak{I}_{\xi_{ij}}$ by φ_2 and φ_4, respectively, and both subloops for $\mathfrak{C}_{\xi_{ij}}$ by φ_3 and φ_5, respectively, including the covering of the main loop ξ by φ_1. All these loops are implicitly covered by Φ_φ.

5.1.2 Topologies on the system of operands of the circular formula system ${}_{k_j}\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$

Any system operand as such (as an entity) can explicitly be extracted (expressed) by means of the other operands (including itself) of the system. Therefore, one can imagine the operand system as the one in which operands are representatives of their formula systems, that is, specific formula system markers. For instance, instead of the circular formula $\varphi_1 \in {}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$, we can put explicitly,

$$\xi_{ij} \rightleftharpoons \begin{pmatrix} ((((((\xi_{ij} \models \mathfrak{I}_{\xi_{ij}}) \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models \\ \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}) \models \xi_{ij} \end{pmatrix}$$

etc. Thus, ξ_{ij} marks a formula of the kind

$${}_1\overset{\circ}{\xi}_{ij\rightarrow}\lfloor \alpha_j, \xi_{ij}, \mathfrak{I}_{\xi_{ij}}, \mathfrak{i}_{\xi_{ij}}, \mathfrak{C}_{\xi_{ij}}, \mathfrak{c}_{\xi_{ij}}, \mathfrak{E}_{\xi_{ij}}, \mathfrak{e}_{\xi_{ij}} \rfloor$$

In this sense, an operand system does not differ substantially from the formula system discussed in Sect. 5.1.1. The difference is that instead of formulas, the operands structuring them, come into the foreground. Thus, the basic system for the case in Fig. 4 is

$$\Phi_{\xi_{ij}} \rightleftharpoons \begin{pmatrix} \xi_{ij}; \\ \mathfrak{I}_{\xi_{ij}}; \\ \mathfrak{i}_{\xi_{ij}}; \\ \mathfrak{C}_{\xi_{ij}}; \\ \mathfrak{c}_{\xi_{ij}}; \\ \mathfrak{E}_{\xi_{ij}}; \\ \mathfrak{e}_{\xi_{ij}} \end{pmatrix}$$

In this situation, additionally, different operand roles can be explicated by means of different topologies. According to Fig. 4, some particularities can be stressed which do not proceed directly from the graph. For instance, topologically, a certain informational impact can be expressed between operands being not directly connected, for instance between $\mathfrak{I}_{\xi_{ij}}$ and $\mathfrak{C}_{\xi_{ij}}$, that is, the impacting of informing onto embedding. This means, $(\mathfrak{I}_{\xi_{ij}}, \mathfrak{C}_{\xi_{ij}}) \in \mathfrak{D}_{\xi_{ij}}$. Such a topological condition could lead to the request that $\mathfrak{I}_{\xi_{ij}}$ and $\mathfrak{C}_{\xi_{ij}}$ must be explicitly expressed, for instance, applying the rotation principle of operands in the main loop, and then searching, how $\mathfrak{I}_{\xi_{ij}}$ depends informationally on $\mathfrak{C}_{\xi_{ij}}$, and vice versa. Namely, in a loop, cause and its consequence depend on each other.

In case of an operand topology, operands must be expressed explicitly, anyhow. The principle of operand rotation must be applied for operands which do not function as the main operands, that is, for $\mathfrak{I}_{\xi_{ij}}$, $\mathfrak{i}_{\xi_{ij}}$, $\mathfrak{C}_{\xi_{ij}}$, $\mathfrak{c}_{\xi_{ij}}$, $\mathfrak{E}_{\xi_{ij}}$, and $\mathfrak{e}_{\xi_{ij}}$.

Besides, all the loops in the graph must be covered consequently. The rotation principle is one of the possibilities on this way. Let us rotate the embedding operand $\mathfrak{e}_{\xi_{ij}}$, considering system ${}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$, and looking into the graph in Fig. 4. The result is, from the consequent observational point of view,

$$\mathfrak{e}_{\xi_{ij}} \rightleftharpoons \\ \begin{pmatrix} ((((((\mathfrak{e}_{\xi_{ij}} \models \xi_{ij}) \models \mathfrak{I}_{\xi_{ij}}) \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models & [\varphi_1]^\mathfrak{e} \\ \quad \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}; & \\ (((\mathfrak{e}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}; & [\varphi_3]^\mathfrak{e} \\ (\mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}; & [\varphi_6]^\mathfrak{e} \end{pmatrix}$$

One can recognize how the rotation principle brings a new understanding of the metaphysicalistic system when new formula systems for metaphysicalistically interior operands come into consideration, complexing the system as a whole by detailing the before hidden additional possibilities of the interior operands. The last formula system is in its first part ($[\varphi_1]^\circ$, $[\varphi_3]^\circ$, and $[\varphi_6]^\circ$) informationally different to the adequate part of ${}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$.

The topological concept concerning operand system $\Phi_{\xi_{ij}}$ requests a more complex system in regard to the initial ${}_1\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$. Substantially, the graph in Fig. 4 must be covered systematically irrespective of the operator rotation to the title (the leftmost) position of a circular formula.

5.1.3 Topologies on system of basic transitions of the circular formula system ${}_{k_j}\overset{\circ}{\varphi}_{\rightarrow}\lfloor \xi_{ij}, \alpha_j \rfloor$

Finally we come to the most significant and attractive form of topology determined on the system of basic transitions, that is, on basic serial formulas with the length $\ell = 1$ (e.g., of the form $\alpha \models \beta$). Why such a formula system could be of the primary interest, and why topologies on this formula system are informationally significant to a substantial extent?

To come into the course of the relevant discussion we have to remind on the informational equivalence existing between the so-called informational graph [35] and the corresponding parallel system of basic transitions. For instance, parallelizing a serial or circular

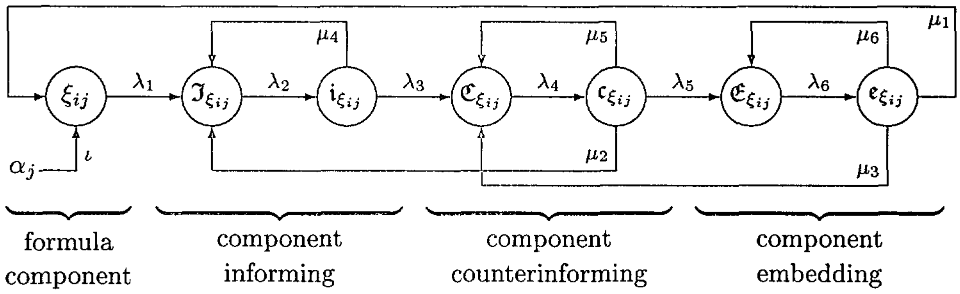

Figure 5: *The graph representing the basic metaphysicalism of Fig. 4 in respect to primitive transitions* ι, λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, μ_1, μ_2, μ_3, μ_4, μ_5, μ_6.

serial formula means to decompose it into elementary transitions. Parallelization of a lengthy formula occurs by the following steps:

1. within a formula φ, all parenthesis pairs are omitted; what remains is called the *route*, sometimes *scheme*, or also *framed scheme* [37] (in Russian, *маршрут*, in [39], p. 85);

2. from the route, the system of basic transitions is constructed, moving from the left to the right along the route, building basic transition formulas by taking the two operands, connected by the operator between them in the route.

We see how the obtained paralleled system differs informationally from the original formula. The substantial difference occurs in the domain of informational operators. In a lengthy formula, an operator can connect two arbitrary subformulas which are not simple operands. However, an informational operator (a binary operator in any case) is always a product of its left and its right operand. Therefore the route must be understood as a frame scheme which only fixes the position of an operator (in the formula and then in the route), but does not definitely determine the subject of a concrete operator. This is the price which must be paid in any case of a formula reduction (in this case, in fact, generalization) where a concrete formula serves only as a sort of syntactical (structural, organizational) template.

A route $\xi_1 \models \xi_2 \models \ldots \xi_{n-1} \models \xi_n$ is called *informational chain* (in Russian *цепь*,[39], p. 90) if operators $\models$ in the route are mutually different. In an informational formula, informational operators are axiomatically mutually different. In mathematics, on contrary, equally denoted operators in the context of a mathematical formula (with a unique meaning, unique and firm definition) always represent equal operations. Routes or chains, respectively, are usually framed, to distinguish them clearly from formulas. For instance, $\boxed{\xi_1 \models \xi_2 \models \ldots \xi_{n-1} \models \xi_n}$. Such a notation can be useful in cases where informational formulas and chains are combined, to enable the expression of parts where parenthesizing is let open. A formula with framed routes is called framed informational formula.

The system from which the parallelization proceeds is ${}_{k_j}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor$, where k_j can be an arbitrary subscript in the interval concerning the formula system ${}_{k_j}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor$. Thus, one can take ${}_{1}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor$ and parallelize it according to the rules discussed in the previous text. Sometimes, the parallelization of ${}_{1}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor$ into the system of primitive transition formulas is marked by $\Pi'\left({}_{1}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor\right)$. The result is

$$\Pi'\left({}_{1}\varphi_{\rightarrow}^{\circ}\lfloor \xi_{ij}, \alpha_j \rfloor\right) \rightleftharpoons \varphi_{\parallel}^{\circ\prime}\lfloor \xi_{ij}, \alpha_j \rfloor$$

and, accordingly to Fig. 4, evidently,

$$\varphi_{\parallel}^{\circ\prime}\lfloor \xi_{ij}, \alpha_j \rfloor \rightleftharpoons \left(\begin{array}{ll} \alpha_j \models \xi_{ij}; & [\varphi_0'] \\ \xi_{ij} \models \mathfrak{I}_{\xi_{ij}}; \mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}}; \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_1'] \\ \quad \mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}}; \mathfrak{c}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}}; \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}}; & \\ \quad \mathfrak{e}_{\xi_{ij}} \models \xi_{ij}; & \\ \mathfrak{c}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_2''] \\ \mathfrak{e}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_3''] \\ \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_4''] \\ \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_5''] \\ \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} & [\varphi_6''] \end{array}\right)$$

An informational transition formula can appear in the system only once. In this way, the last five rows include only the remaining feedback transitions.

(0) Let us introduce the notation $\Phi_{\xi\models\eta} \rightleftharpoons \varphi_{\parallel}^{\circ\prime}\lfloor \xi_{ij}, \alpha_j \rfloor$. For a better transparency, we replace the upper system $\varphi_{\parallel}^{\circ\prime}\lfloor \xi_{ij}, \alpha_j \rfloor$ by the abbreviated

$$\Phi_{\xi\models\eta} \rightleftharpoons \left(\begin{array}{ll} \iota; & [\varphi_0'] \\ \lambda_1; \lambda_2; \lambda_3; \lambda_4; \lambda_5; \lambda_6; \mu_1; & [\varphi_1'] \\ \mu_2; & [\varphi_2''] \\ \mu_3; & [\varphi_3''] \\ \mu_4; & [\varphi_4''] \\ \mu_5; & [\varphi_5''] \\ \mu_6 & [\varphi_6''] \end{array}\right)$$

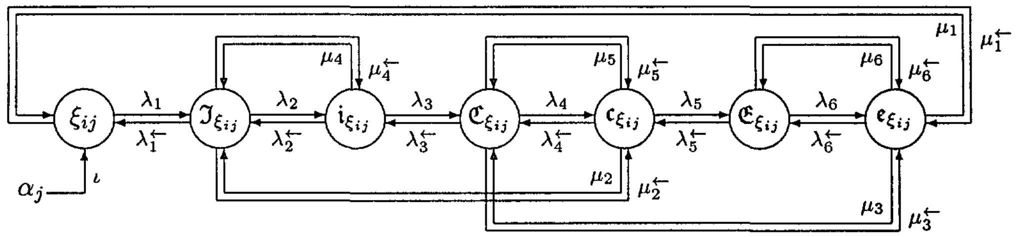

Figure 6: *The bidirectional graph representing the metaphysicalism of Fig. 5 by considering the primitive onedirectional and counterdirectional transition pairs* $(\lambda_1, \lambda_1^{\leftarrow})$, $(\lambda_2, \lambda_2^{\leftarrow})$, $(\lambda_3, \lambda_3^{\leftarrow})$, $(\lambda_4, \lambda_4^{\leftarrow})$, $(\lambda_5, \lambda_5^{\leftarrow})$, $(\lambda_6, \lambda_6^{\leftarrow})$, $(\mu_1, \mu_1^{\leftarrow})$, $(\mu_2, \mu_2^{\leftarrow})$, $(\mu_3, \mu_3^{\leftarrow})$, $(\mu_4, \mu_4^{\leftarrow})$, $(\mu_5, \mu_5^{\leftarrow})$, $(\mu_6, \mu_6^{\leftarrow})$.

respectively. The correspondence between transition formulas in $\varphi_{\parallel}^{\circ'}\lfloor \xi_{ij}, \alpha_j \rfloor$ and their abbreviated notations in $\Phi_{\xi\models\eta}$ is evident. Notation ι marks the input transition, λ_p $(p = 1, \ldots, 6)$ the forward transition of the main loop, and μ_q $(q = 1, \ldots, 6)$ the feedback transition, corresponding to the graph in Fig. 4. The transitional situation is presented in Fig. 5. Systems $[\varphi_2''], \ldots, [\varphi_6'']$ are already reduced by the common transitions within the main loop $[\varphi_1']$. Which kinds of senseful topologies can now be defined on $\Phi_{\xi\models\eta}$?

(1) The basic topological question could concern the main loop in Fig. 5. The circular route of this loop is $\boxed{\xi_{ij} \models \mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \xi_{ij}}$.

To this route, evidently, the subsystem $[\varphi_1']$, that is,

$$(\lambda_1; \lambda_2; \lambda_3; \lambda_4; \lambda_5; \lambda_6; \mu_1) \in \mathfrak{O}_{\xi\models\eta,1}$$

corresponds. If $[\varphi_1'] \subset \Phi_{\xi\models\eta}$ is the only element besides $\emptyset$ and $\Phi_{\xi\models\eta}$ which has to enter in $\mathfrak{O}_{\xi\models\eta,1}$, topology $\mathfrak{O}_{\xi\models\eta,1}$ already satisfies the axioms $(\mathrm{T_I})$, $(\mathrm{T_{II}})$, and $(\mathrm{T_{III}})$. Thus, $\mathfrak{O}_{\xi\models\eta,1} \rightleftharpoons (\emptyset; \varphi_1''; \Phi_{\xi\models\eta})$.

(2) Let us study topology $\mathfrak{O}_{\xi\models\eta,2}$ in which the basic transition systems, covering the loops in Fig. 4, are included. Thus,

$$\begin{array}{l} (\lambda_1; \lambda_2; \lambda_3; \lambda_4; \lambda_5; \lambda_6; \mu_1), \\ (\lambda_2; \lambda_3; \lambda_4; \mu_2),\ (\lambda_4; \lambda_5; \lambda_6; \mu_3), \\ (\lambda_2; \mu_4);\ (\lambda_4; \mu_5);\ (\lambda_6; \mu_6) \in \mathfrak{O}_{\xi\models\eta,2} \end{array}$$

This choice of topological subsystems causes the inclusion of further subsystems. By the intersection axiom $(\mathrm{T_{II}})$, there is

$$(\lambda_2; \lambda_3; \lambda_4),\ (\lambda_4;\ \lambda_5;\ \lambda_6),\ (\lambda_2),\ (\lambda_4),\ (\lambda_6) \in \mathfrak{O}_{\xi\models\eta,2}$$

Evidently, these subsystems represent the common parts of the loops. By the union axiom $(\mathrm{T_I})$, elements like

$$\begin{array}{l} (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2; \mu_3; \mu_4; \mu_5; \mu_6), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2; \mu_3; \mu_4; \mu_5), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2; \mu_4; \mu_5; \mu_6), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_3; \mu_4; \mu_5; \mu_6), \\ (\lambda_1; \ldots; \lambda_6; \mu_2; \mu_3; \mu_4; \mu_5; \mu_6), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2; \mu_3; \mu_4), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2; \mu_3), \\ (\lambda_1; \ldots; \lambda_6; \mu_1; \mu_2), \\ \vdots \\ (\lambda_1; \ldots; \lambda_6; \mu_5; \mu_6) \end{array}$$

etc. must additionally enter topology $\mathfrak{O}_{\xi\models\eta,2}$. Now, again axiom $(\mathrm{T_{II}})$ has to be applied, etc. The number of elements in $\mathfrak{O}_{\xi\models\eta,2}$ becomes enormous.

5.2 Topologies of a Bidirectional Metaphysicalism

Bidirectionality in informational sense means introducing a strict counterdirectional path (reverse serial or circular serial formula) in regard to the existing path (initial formula). In a graph, this situation is evidently visible by the occurrence of counterarrows or, in some cases, by the operand connection lines with arrows on both sides of the line.

The graph in Fig. 7 represents a conceptually invariant shell of the possible bidirectional artificial consciousness. Bidirectionality is ensured in every point of the informational structure. Further, the graph can be used as a template for any formula system development on one side, and as a individual semantic approach to the choice of vertical components in several specific domains of the informational, that is, of the conscious individualism, its structure and organization on the other side. A suggestion for the choice of vertical components is given in [36, 38]. Thus, vertical components can fit best the specific field of research

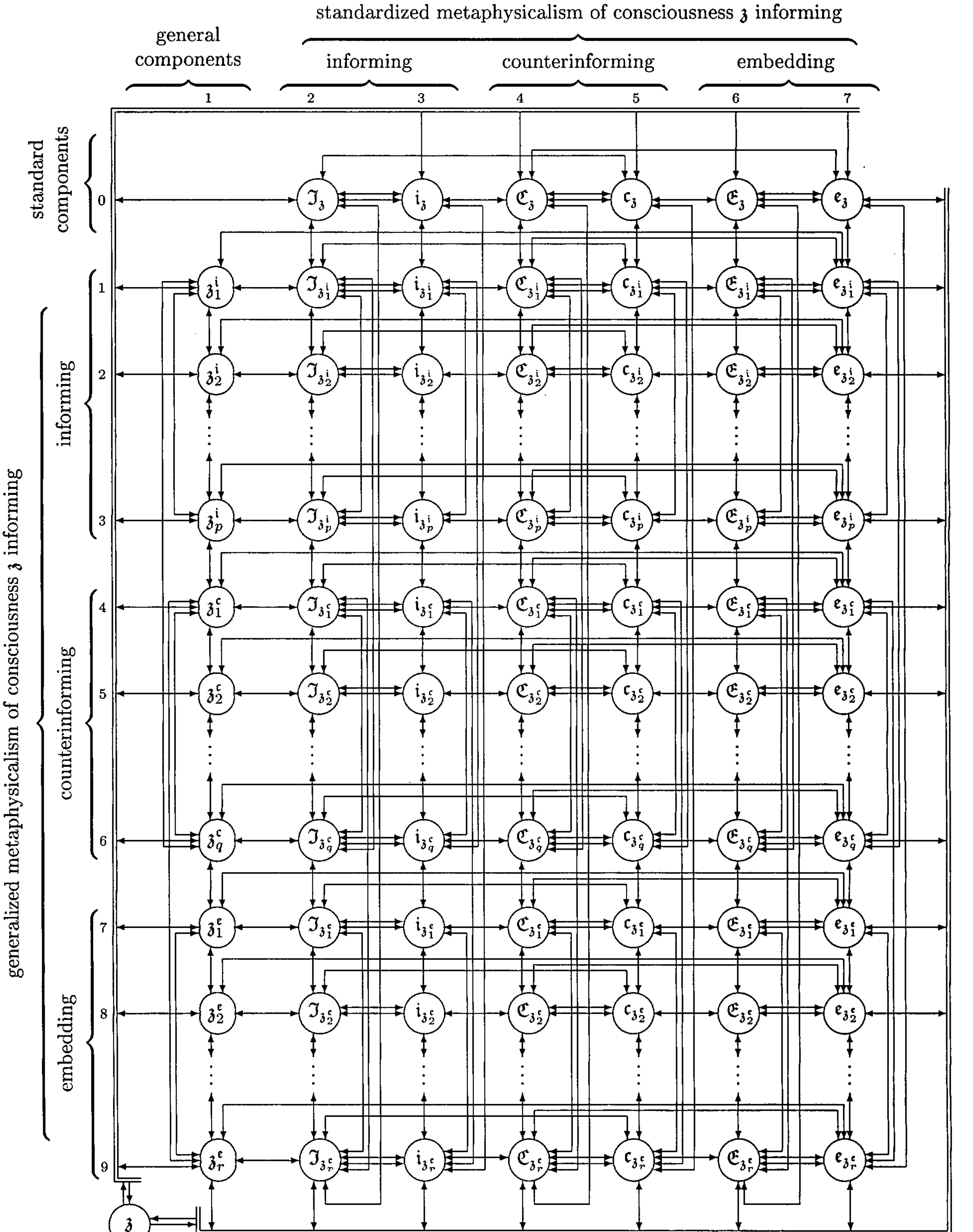

Figure 7: *An initial informational shell of the generalized and standardized metaphysicalism of consciousness system* $\mathfrak{z}$ *(a kind of pure consciousness), exploring the bidirectional metaphysicalism.*

in respect to the function in the vertical metaphysicalistic scheme. On the other side, chosen vertical components can be again metaphysically decomposed in the horizontal direction.

In the framework of consciousness circumstances, the stream of consciousness can be forced consciously into the opposite direction within informational cycles as shown in Fig. 6. A critical conscious informing must investigate its own conscious stream (of informing, counterinforming, and informational embedding) in one and the other direction, changing the causal conditions circularly in the opposite direction. In an unidirectional graph, each arrow, representing an operator, is replaced by the bidirectional arrow, representing two operators, the direct and the reverse one. Thus, in Fig. 7, a bidirectional arrow $\longleftrightarrow$, marks $\rightleftharpoons$, meaning two separate and functionally (essentially) different operands.

Topologically, each concrete case concerning the graph in Fig. 6 can be informationally distinguished, foe example, by a definite setting of the parenthesis pairs in formulas. As mentioned frequently before, the formula system Φ_φ is the originally conceptualized model of a real informational situation. In this sense, bidirectionality offers the possibility to investigate a loop in one and the opposite direction. For a loop, for instance, the principles of the pure observing and the pure informing can be applied in one and the opposite direction, simultaneously. For the case in Fig. 6, there is, for example,

$$
{}_{1,1}\overset{\circ}{\varphi}_{=}\lfloor \xi_{ij}, \alpha_j \rfloor \rightleftharpoons
$$

$$
\left(\begin{array}{ll}
\alpha_j \models \xi_{ij}; & [\varphi_0] \\
((((((\xi_{ij} \models \mathfrak{I}_{\xi_{ij}}) \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models & [\varphi_1] \\
\quad \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}) \models \xi_{ij}; & \\
\xi_{ij} \models (\mathfrak{e}_{\xi_{ij}} \models (\mathfrak{E}_{\xi_{ij}} \models (\mathfrak{c}_{\xi_{ij}} \models (\mathfrak{C}_{\xi_{ij}} \models & [\varphi_1^{\leftarrow}] \\
\quad (\mathfrak{i}_{\xi_{ij}} \models (\mathfrak{I}_{\xi_{ij}}) \models \xi_{ij})))))); & \\
(((\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}) \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_2] \\
\mathfrak{I}_{\xi_{ij}} \models (\mathfrak{c}_{\xi_{ij}} \models (\mathfrak{C}_{\xi_{ij}} \models (\mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}))); & [\varphi_2^{\leftarrow}] \\
(((\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}}) \models \mathfrak{e}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_3] \\
\mathfrak{C}_{\xi_{ij}} \models (\mathfrak{e}_{\xi_{ij}} \models (\mathfrak{E}_{\xi_{ij}} \models (\mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}))); & [\varphi_3^{\leftarrow}] \\
(\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}}) \models \mathfrak{I}_{\xi_{ij}}; & [\varphi_4] \\
\mathfrak{I}_{\xi_{ij}} \models (\mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}); & [\varphi_4^{\leftarrow}] \\
(\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}}) \models \mathfrak{C}_{\xi_{ij}}; & [\varphi_5] \\
\mathfrak{C}_{\xi_{ij}} \models (\mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}); & [\varphi_5^{\leftarrow}] \\
(\mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}}) \models \mathfrak{E}_{\xi_{ij}} & [\varphi_6] \\
\mathfrak{E}_{\xi_{ij}} \models (\mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}}) & [\varphi_6^{\leftarrow}]
\end{array}\right)
$$

This is the original (initial) formula system, a consequently observing case in each system formula, from which the graph in Fig. 6 was drawn, consistently following the rule of an arrow and its counterarrow. It is clear that according to a specific informational case, the parenthesis pairs can be set adequately (and differently), following the realistic circumstances for each of the system formula. However, any other setting of the parenthesis pairs in the system formulas does not change the informational graph in Fig. 6. Maybe, in a specific case, some direct and/or reverse paths can even be omitted or left simply void for a later final decision.

(0) Let us denote $\Phi_\varphi^{\rightleftarrows} \rightleftharpoons_{1,1} \overset{\circ}{\varphi}_{=}$. Now, for the sake of transparency, let be

$$
\Phi_\varphi^{\rightleftarrows} \rightleftharpoons \begin{pmatrix}
\varphi_0; \\
\varphi_1;\ \varphi_1^{\leftarrow}; \\
\varphi_2;\ \varphi_2^{\leftarrow}; \\
\varphi_3;\ \varphi_3^{\leftarrow}; \\
\varphi_4;\ \varphi_4^{\leftarrow}; \\
\varphi_5;\ \varphi_5^{\leftarrow}; \\
\varphi_6;\ \varphi_6^{\leftarrow}
\end{pmatrix}
$$

representing the input transition formula φ_0 (bringing into the system the exterior object α at point ξ_{ij}) and the twelve circular formulas.

(1) To represent the variability of a formula system rooting in the possibility of arbitrary parenthesis pairs displacements in formulas, we can use the formal expression of informational schemes (the so-called graph routes of graph paths) for Fig. 6, and write the graph equivalent scheme in regard to the initial system $\Phi_\varphi^{\rightleftarrows}$ in the form

$$
\boxed{\Phi_\varphi^{\rightleftarrows}} \rightleftharpoons
$$

$$
\left(\begin{array}{ll}
\alpha_j \models \xi_{ij}; & \boxed{[\varphi_0]} \\
\xi_{ij} \models \mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models & \boxed{[\varphi_1]} \\
\quad \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \xi_{ij}; & \\
\xi_{ij} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} \models & \boxed{[\varphi_1^{\leftarrow}]} \\
\quad \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}} \models \xi_{ij}; & \\
\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & \boxed{[\varphi_2]} \\
\mathfrak{I}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & \boxed{[\varphi_2^{\leftarrow}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_3]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_3^{\leftarrow}]} \\
\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & \boxed{[\varphi_4]} \\
\mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}}; & \boxed{[\varphi_4^{\leftarrow}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_5]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_5^{\leftarrow}]} \\
\mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} & \boxed{[\varphi_6]} \\
\mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} & \boxed{[\varphi_6^{\leftarrow}]}
\end{array}\right)
$$

In this formula system scheme, some directed and counterdirected paths obtain equal formal expression, e.g., $\boxed{[\varphi_4]}$ and $\boxed{[\varphi_4^{\leftarrow}]}$, $\boxed{[\varphi_5]}$ and $\boxed{[\varphi_5^{\leftarrow}]}$, and $\boxed{[\varphi_6]}$

and $\boxed{[\varphi_6^{\leftarrow}]}$ seem to be equal. However, it is to understand that they originate from different informational situations and, according to the original circumstances, they have different operators between the equal operands[22].

(2) An interesting case occurs in dealing with the operand system

$$\Phi^{\rightleftarrows}_{\xi_{ij}} \rightleftharpoons \left(\alpha;\ \xi^{\rightleftarrows}_{ij};\ \mathfrak{I}^{\rightleftarrows}_{\xi_{ij}};\ \mathfrak{i}^{\rightleftarrows}_{\xi_{ij}};\ \mathfrak{C}^{\rightleftarrows}_{\xi_{ij}};\ \mathfrak{c}^{\rightleftarrows}_{\xi_{ij}};\ \mathfrak{E}^{\rightleftarrows}_{\xi_{ij}};\ \mathfrak{e}^{\rightleftarrows}_{\xi_{ij}}\right)$$

concerning the graph in Fig. 6 and with possible topologies on this system.

First, let us explain in which way the bidirectional operands $\xi^{\rightleftarrows}_{ij}$; $\mathfrak{I}^{\rightleftarrows}_{\xi_{ij}}$; $\mathfrak{i}^{\rightleftarrows}_{\xi_{ij}}$; $\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}$; $\mathfrak{c}^{\rightleftarrows}_{\xi_{ij}}$; $\mathfrak{E}^{\rightleftarrows}_{\xi_{ij}}$; $\mathfrak{e}^{\rightleftarrows}_{\xi_{ij}}$ are formally and explicitly represented. Fig. 6 shows how many causal circular paths (loops) pass a certain operand. The following correspondence is evident:

$$\xi^{\rightleftarrows}_{ij} \blacktriangleright 2;\ \mathfrak{I}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 6;\ \mathfrak{i}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 6;\ \mathfrak{C}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 8;$$
$$\mathfrak{c}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 8;\ \mathfrak{E}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 6;\ \mathfrak{e}^{\rightleftarrows}_{\xi_{ij}} \blacktriangleright 6$$

Operator $\blacktriangleright$ reads *directly informs the number of loops.*

How, for instance, operand $\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}$ is expressed explicitly by means of the loops passing it, using the so-called operand rotation principle for each of particular loop, and the informational path (scheme) form? The advantage of the path formula is that the setting of the parenthesis pairs remains open, and in this case various possibilities of the final setting of parenthesis pairs can be considered. Evidently, the following comes out from the graph in Fig. 6:

$$\boxed{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}} \rightleftharpoons$$
$$\left(\begin{array}{ll}
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \xi_{ij} \models & \\
\quad \mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_1^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}} \models \xi_{ij} \models \mathfrak{e}_{\xi_{ij}} \models & \\
\quad \mathfrak{E}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_2^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_3^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{i}_{\xi_{ij}} \models \mathfrak{I}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_4^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_5^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{e}_{\xi_{ij}} \models \mathfrak{E}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_6^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}}; & \boxed{[\varphi_7^{\mathfrak{C}}]} \\
\mathfrak{C}_{\xi_{ij}} \models \mathfrak{c}_{\xi_{ij}} \models \mathfrak{C}_{\xi_{ij}} & \boxed{[\varphi_8^{\mathfrak{C}}]}
\end{array}\right)$$

[22] Operator $\models$ denotes a general informational joker. In two cases, the equal transition formulas $\alpha \models \beta$ and $\alpha \models \beta$ can represent different transitions. For instance, between two substantives different verb forms can be set. It means that in virtually equal formal cases, the intention of α's informing follows the first and then the second verbal form. Finally, the cases are resolved as being different by the particularization of operators.

The last two paths are virtually equivalent (see the footnote [22]). Similar schemata can be obtained from the graph in Fig. 6 for the remaining operand systems $\boxed{\xi^{\rightleftarrows}_{ij}}$, $\boxed{\mathfrak{I}^{\rightleftarrows}_{\xi_{ij}}}$, $\boxed{\mathfrak{i}^{\rightleftarrows}_{\xi_{ij}}}$, $\boxed{\mathfrak{c}^{\rightleftarrows}_{\xi_{ij}}}$, $\boxed{\mathfrak{E}^{\rightleftarrows}_{\xi_{ij}}}$, and $\boxed{\mathfrak{e}^{\rightleftarrows}_{\xi_{ij}}}$. The rule for an explicit expression of an operand out of given formula system is to collect all the formulas in which the operand occurs and then express these circular formulas, according to the principle of an operand rotation, in a way by which the operand comes to the title position (the most left and the most right position in a circular formula).

What can then be said to the topological outlook of the obtained framed operand (in fact, a system of informational paths) representing formula a system by each of the system path? It is to stress that the graph for the formula system scheme $\boxed{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}$ (with 8 formula paths) is a subgraph of the graph in Fig. 6 (merely the local informing and embedding loops are missing).

In this sense we introduce a new concept of topology consisting of informational paths (routes, marked by ρ) instead of informational formulas φ, representing $\rho \rightleftharpoons \boxed{\varphi}$. Thus, instead of Φ_φ we introduce Φ_ρ or $\Phi_{\boxed{\varphi}}$, respectively. Each path (graph route) ρ represents potentially $\frac{1}{\ell_\rho}\binom{2\ell_\rho}{\ell_\rho}$ formulas if ℓ_ρ is the length of the path corresponding formula (number of the adequate formula binary operators). In this way a new sort of topological space is introduced, for instance pertaining to $\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}$,

$$\left\langle \Phi^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1}, \mathfrak{O}^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1} \right\rangle, \text{ where}$$

$$\Phi^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1} \rightleftharpoons \left(\boxed{[\varphi_1^{\mathfrak{C}}]};\ \boxed{[\varphi_2^{\mathfrak{C}}]};\ldots;\ \boxed{[\varphi_8^{\mathfrak{C}}]}\right), \text{ and, e.g.,}$$

$$\mathfrak{O}^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1} \rightleftharpoons \left(\begin{array}{l}
\emptyset; \\
\left(\boxed{[\varphi_1^{\mathfrak{C}}]}\right);\ \left(\boxed{[\varphi_2^{\mathfrak{C}}]}\right); \\
\left(\boxed{[\varphi_1^{\mathfrak{C}}]};\ \boxed{[\varphi_2^{\mathfrak{C}}]}\right); \\
\Phi^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1}\ ()
\end{array}\right)$$

Evidently, $\Phi^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1} \rightleftharpoons \mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}$.

(3) One could construct other reasonable topologies being subsystems of $\mathfrak{P}\left\lfloor \Phi^{\mathfrak{C}^{\rightleftarrows}_{\xi_{ij}}}_{\rho,1} \right\rfloor$. But, the next provoking question concerns a topology of formula systems Φ_φ (not just formulas φ) and topologies of topological spaces of the form $\langle \Phi, \mathfrak{O} \rangle$.

Let Φ_{Φ_φ} mark a system of formula systems Φ_φ and $\Phi_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle}$ a system of topological spaces $\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle$, in general. Let

$$\Phi_\varphi \in \Phi_{\Phi_\varphi} \text{ and } (\Psi \in \mathfrak{O}_{\Phi_\varphi}) \Longrightarrow (\Psi \subset \Phi_{\Phi_\varphi})$$

This structure delivers a system topological space $\langle \Phi_{\Phi_\varphi}, \mathfrak{O}_{\Phi_\varphi} \rangle$.

Another concept of topology of topological spaces follows the condition

$$\begin{array}{l} \langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle \in \Phi_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle} \text{ and} \\ (\Psi \in \mathfrak{O}_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle}) \Longrightarrow (\Psi \subset \Phi_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle}) \end{array}$$

delivering a topology topological space of the form $\langle \Phi_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle}, \mathfrak{O}_{\langle \Phi_\varphi, \mathfrak{O}_\varphi \rangle} \rangle$.

5.3 Topological Informational Spaces Possessing Informational Metrics

What kinds of informational metrics could come to the surface, could be considered, and finally theoretically (constructively) applied in artificial systems of consciousness and other cognitive models? Which are the possibilities of introducing various kinds of metrics concepts—the informationally static[23] and informationally dynamic[24] ones—into a topologically structured informational space?

Several candidates come into consideration as measures of the informational metrics. The properties of such measures could be, for instance, meaningness, understandingness, interpretativeness, perceptiveness, conceptiveness, determinativeness, and several others. If so, the corresponding decomposition and expressiveness of informational measures as entities must be available.

Where could these measures reside within a metaphysicalistic model? The answer is, anywhere. By the principle of operand rotation in a circular formula, any loop operand can be rotated to the initial (main) position of the loop and, by this, expressed by an adequate informational formula in respect to the parenthesis-pair setting in the formula. Meaning of something as an informational measure can usually appear in the embedding part of a metaphysicalistic loop. As a meaning of something it could represent the informational value (informational length) of something. In a similar manner, the informational distance between two informational operands could be determined, implicitly and explicitly, by a functionally inner and outer informational difference, respectively.

Various concepts of understanding, conception, perception, etc. can serve as special measures of meaning (metrics). They can be placed constructively in any part of the metaphysicalistic loop and, then, rotated to the main position of a formula and expressed explicitly [32]. This kind of constructive approach must remain within the reasonable limits, preserving the common logical principles or direction.

[23] By an informationally static metrics, the most common concepts of informing are meant, for instance, that of something's meaning. Typical, purely static metrics concerns numerical or any other value, distance, or any other geometrical measure.

[24] By informationally dynamic metrics, the individually organized informational phenomena are meant, for instance that of an individual understanding structure, which has something in common with the individual structures of others, and which is to some extent structured invariantly (standardized) in concern to the meaning or understanding.

6 Possible Geometry and Topology of the Informational

That what will be stressed in this section concerns the interpretation possibilities of informational topologies by means of geometric bodies—their surfaces, intersections, volumes, and arbitrary substructures occurring interiorly, on the surface, and/or exteriorly of these bodies. Interpretation ideas can be found in several sources dealing with geometry [9, 22, 23, 26, 27]. Mathematica [9] seems to be the tool for an adequate graphical presentation.

By such an interpretation of systems of informational formulas, geometrical bodies become also a means for informationally semantic presentation of modeled entities. For instance, a sphere—its interior, surface and exterior—can be taken as a body of consciousness (or a body of any other informational entity). The surface of the sphere can represent topologically that which is potentially possible to become conscious, and a circle on the sphere surface can represent the currently conscious. Such circles can expand as parts of different toruses which intersect with the sphere. They can represent different intentional informings within the consciousness activity.

Further, the interior of the sphere can represent the subconscious which can come to the surface. On contrary, the exterior of the sphere can be grasped as the non-conscious and non-subconscious yet. Thus, a system of spheres and toruses intersecting each other can built a complex and to some degree globally transparent model of interacting consciousness systems.

Such a complex geometrical model can be particularly, that is, additionally, characterized with specific topologies, bringing into the modeling system an interaction of different topological spaces. In this context, both informational topologies and geometrical bodies can become a reasonable unit for complex informational investigation and experiments in the domain of the informational, and particularly in the domain of the conscious in an informational sense.

Geometrically interpreted, informational topological spaces of informational topological spaces could get a transparent view to an arbitrary (recurrent) depth. Further, such interpreting geometrical structures can behave variable in any possible aspect, for instance, in moving of geometrical body intersections together with bodies which can change also dimensions (volumes, radii, sides, surfaces) to follow the dynamic picture of informational circumstances emerging, changing, and vanishing. Such problems of informational and consciousness geometry interpretation deserve a special attention and will be treated somewhere else.

7 Conclusion

We see how the concept of mathematical topology comes intuitively close to the informational topology. However, the substantial differences occurring between them, e.g. the nature of emergence of operands, operators, and formula systems, have to be stressed over and over again. Some of the differences are already recognized from the mathematical-informational dictionary in Sect. 2, and other follow from the discussion and examples in this paper. It is worth to refresh these differences by the following list:

1. A formula system is obviously a set of interdependent formulas, irrespective, how it is expressed; e.g., by (1) serial circular formulas of different lengths, (2) primitive transition formulas, or (3) informational operands that are in some way, by some specific formula systems given on some other places.

2. Formulas (elements) of an informational formula system are directly dependent on each other through the common operands. Thus, the change of an operand in a given formula changes the same operand in an other formula and, thus, changing the informing of the other formula. As said, the interdependence of formulas as system elements is a rule, that is, a consequence of their formal linkage through common formula operands. In this respect, informational formulas as system elements behave differently in respect to the elements of a mathematical set.

3. A consequence of the preceding item is that elements of a set are meant as a sort of constant determined entities, and are in this way represented as (fixed) set elements. On the other hand, formulas as system elements possess their emerging nature in any respect: in emerging operands and operators, in setting of parenthesis pairs in a formula, and, most significantly, in expanding or contracting a formula by the number of occurring operands and operators, that is, in spreading and narrowing the meaning power of a formula.

4. A concrete formula system can also emerge according to the circumstances of its informing, for instance, by adding the interpretational formulas concerning the occurring operands, expressing the operand properties by additional (new) formulas. On the other side, a concrete mathematical set is defined constantly, even its cardinality is infinite. The elements of a set are determined by an unchangeable rule (e.g., predicate) or by a sort of concrete or recursive enumeration.

By informational topology, a complex meaningly structured grouping and coupling of formulas concerning substantial informational spaces can formally be expressed (implemented), keeping the entire, that is, a non-reductional informational nature of involved entities as they perform in their reality. In this respect, a topologized formula system is not a simplified model for real informational situations, for instance in the domain of cognitive science[25].

Tangled webs of causal influences are target phenomena in recent biology and cognitive science [10]. Such twisted influences include both internal and external factors as well as patterns of reciprocal (also bidirectional) interaction. The shell graph in Fig. 7 is a general scheme for the most pretentious informational modeling and experimenting, where the so-called reductionist approach can be entirely circumvented. Such an initial informational shell can be used as an informing model for any other problems beside consciousness (e.g., in philosophy, cognitive science, biology, psychology, psychiatry, language, on-line economic simulation, etc., as shown in [32, 37] where additional references are listed.). This points evidently to the applicability of informational topology with its deep intuitive background being appropriate for natural and artificial modeling of interactive philosophical and scientific problems.

An evident example of the informational metaphysicalism could be the so-called inner speech (talking to oneself) [3]. Such a speech is constituted by the experienced meaning (informing), emergence of speech (counterinforming), and logical articulation (informational embedding), respectively. But, all components of this sort can emerge in a distributed form across the inner speech informing. They can be treated (grasped, understood) topologically as certain informational or semantical unity through topological grouping by subsystems $\sigma_\varphi \in \mathfrak{O}_{\Phi_\varphi}$, where $\sigma_\varphi \subset \Phi$ and $\langle \Phi, \mathfrak{O}_{\Phi_\varphi} \rangle$ is the corresponding topological informational space.

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[3] BLACHOWICZ, J. 1997. The dialoge of the soul with itself. Journal of Consciousness Studies 4:485–508.

[25] In contrast to artificial neural networks (ANNs) being *simplified mathematical models for neural systems formed by massively interconnected computational units running in parallel* [28], informational topological systems can always fit adequately and arbitrarily precisely the point of meaning in a topological continuum.

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