New and Old Technologies: a Suitable Combination for Obtaining Efficient Educational Results

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The daily use of sophisticated technologies and devices could hide the physical meaning of everyday observations on natural phenomena. The technological solutions continuously proposed by the market, also for the teaching applications, are more and more sophisticated. The result is that more time is actually spent on the learning of the tools than in exploiting the didactic content that may be contained in the programs themselves. The aim of this paper is to show how a comparative analysis of "old" and "current" techniques, applied to the same problem, could allow to study in depth its scientific contents and to highlight both the development and the methods of scientific thought. Two different cases are considered here: a simple analogue mechanic system for introducing students to non-linear systems and the use of the dark room meridian. Both papers were presented at the Second and Third International Workshops "Developing Creativity and Broad Mental Outlook in the Computer Age" (CBMO-2002 and CBMO-2006) in conjunction with the 8th and10th Conference of the International Society for the Study of European Ideas – ISSEI 2002 (Aberystwyth, University of Wales, 22-27 July 2002) and ISSEI 2006 (University of Malta, July 24 - 29, 2006).

Povzetek: Ali moderna orodja dejansko izboljšujejo učenje?

1 A Simple System to Introduce High School Students to Nonlinearity

1.1 Introduction

As a consequence of the use and diffusion of the automatic calculus, the solutions to non-linear systems become daily present: see, for example, the weather forecast. Nonlinear systems are quite complex to handle with simple mathematics. Nevertheless one of the simplest non-linear systems, an oscillator, could be used to introduce these systems in secondary schools. In this paper it is shown how it is possible to realise a simple nonlinear oscillator and observe the behaviour of the main nonlinear characteristics from a mechanical (and mathematical) point of view. To emphasise the differences, you can observe a gradual transition "linear–nonlinear" and detect the threshold value for this transition of the control parameter.

The aim of the mechanical devices presented is to make non–linear systems affordable by non-specialised people, and in the same time to keep strictly the correctness of observations and conclusions.

Before the introduction of computers, at the middle of the past century, analytical predictions of physical systems were limited to the solution of linear (differential) equations. But linear equations describe very simple and often non real systems. Real systems are more complex and need non linear differential equations, generally not easy to solve.

Numerical methods for the solution of non-linear differential equations running on a computer enable us to solve many kinds of equation describing real systems without the need for linearization that means simplification or abstraction and the consequent loss of information and generality. This has brought about, starting from the sixties, the growth of a broad field of studies concerning the numerical modelling of systems involving several science fields as physics, chemistry, biology, and especially climatology. From the development of the above activities, a new approach to the phenomena whose name is enlightening was born: *experimental mathematics* [1]. This approach is the study of phenomena by means of their computer simulation. The parameters representing quantities or real entities are simulated by numerical variables. Laws are expressed by equations or equation sets (generally non-linear). In this way "simulated experiments" are carried out and the results or previsions obtained may be compared, when possible, with the results of real experiments.

Theoretical attempts to extend these changes to the school environment are made using mathematical non-linear systems[2, 3], electronic circuits[4, 5], mechanical systems like a chaotic pendulum[6]. Some mathematical systems were published in textbooks prepared with the assistance of the National Council of Teachers of Mathematics in USA [7] .

Here is a mechanical apparatus is presented which allows us to visualise a linear system in its gradual conversion toward non-linearity. This apparatus allows a full control of its dynamical state as the visualisation of the corresponding potential energy. The study of this non-linear system is carried out without the use of differential calculus or other mathematical tools not usually treated at high school. This study consists in the observation of the system's behaviour and characteristics and in the comparison with the corresponding linear system.

This presentation is only a trace, any possible application could be enlarged, completed and adapted to the pupils. The language, the tools, and the kind of representations used here are chosen to stimulate and to come into play spatial ability rather than logical and formal ones[8].

1.2 Harmonic oscillator

The harmonic oscillator is a traditional argument in the study of the mechanics. Our attention will be devoted to the system and to its potential. Differential equation's solution will not be directly studied.

To build a harmonic oscillator is simple: any stable system which tends to return to its equilibrium position, if set out of it (of a small quantity), can be regarded as a harmonic oscillator. In our case the harmonic oscillator is a steel plate fixed at its ends. The system realised allows us to have such a linear system and to "switch" it, when needed, to non-linear condition.

1.3 Description of the Apparatus

The system is shown in figure 1. A steel strip **P** is fixed on a wooden support just like a bucksaw: whereas the blade saw is in traction, in our case the strip P is working in compression through the spring **S** adjusted by nut **N**. When a force F is applied perpendicular to the steel strip, as shown in figure 1, the system tends to return to its equilibrium position. The reaction of the strip is proportional to its displacement x:

$$F = -k\,x \qquad (1)$$

k is the elastic constant.

The corresponding potential can be obtained from the work spent by an external force to give a displacement x.

$$W = 1/2\ k{*}x^2 \qquad (2)$$

this way, we find that the shape of the potential is a parabola.

To understand the meaning of this curve, we may observe that as x increases, potential energy increases as x square. This happens on the Earth when a body moves in a well whose section is parabolic. Then if we let a ball roll on the bottom of a similar well, we are simulating a harmonic potential. Figure 2 shows the mechanical simulation of this potential. This apparatus also consists of a wooden support and two flexible iron wires hanging by their ends from the wells of a wooden support. Under the wires, there is a support that can be lifted in such a way to hold up and lift the centre of the wires, as to the function of this support.

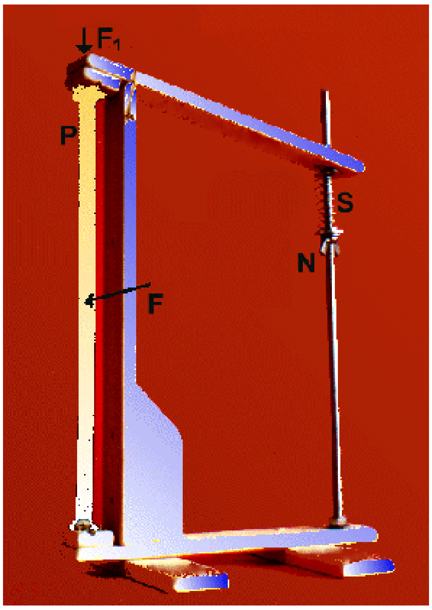

Figure 1. A harmonic oscillator realised with a steel strip **P** fixed at its ends. When a force **F** is applied perpendicular to the strip, the system tends to return to its equilibrium position. The spring **S** can apply a force $\mathbf{F_1}$ regulated by the nut **N**.

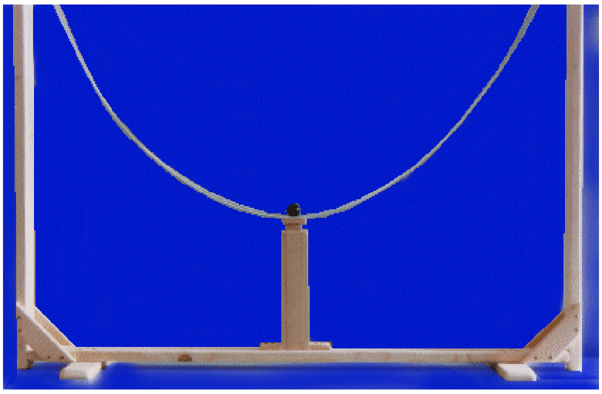

Figure 2. Mechanical simulation of a harmonic potential. The curve of the metallic guides is actually closer to a catenary than to a parabola, but for our purposes the approximation is good enough. If we move the ball along the guide, it tends to return to its equilibrium position.

1.4 Towards Non-linear Behaviour

The device shown in figure 1 allows to apply a longitudinal force ($\mathbf{F_1}$) on the strip. As this longitudinal force is due to the spring **S**, its intensity is given by the contraction of the spring itself.

We can observe that when increasing the longitudinal force $\mathbf{F_1}$, as we turn the nut **N**, the stable equilibrium point remains x = 0. (figure 3) until a critical value $\mathbf{F_c}$ is reached.

As we overcome this value $\mathbf{F_c}$ and try to put the strip in the zero position, the strip quickly start to bend in one direction or the other.

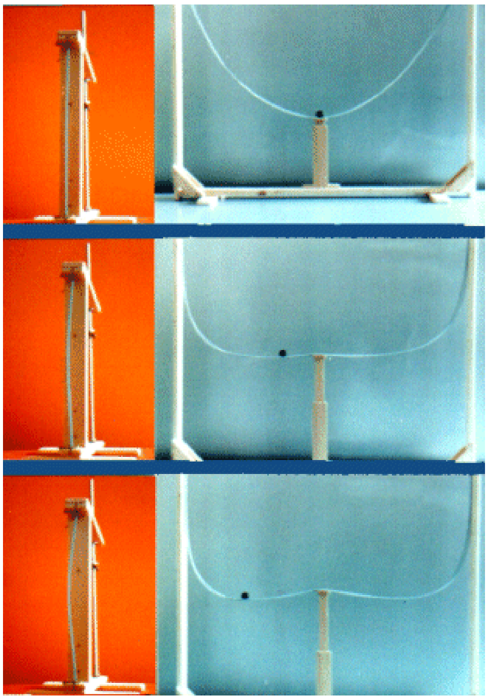

Figure 3. Pictures show the real physical system on the left-hand side, and its mechanical simulation potential on the right-hand one. Beyond the critical value **Fc**, the strip starts to bend towards either one of the two possible directions. The corresponding simulating potential changes from a single well to a double well shaped function

In such a situation the system is said to have a bifurcation. For $x = 0$, we have an unstable equilibrium point and two new symmetrical stable equilibrium points. In this way the bifurcation is directly visualised and the relative potential shape can be obtained: the situation is well represented by a double well potential (figure 4).

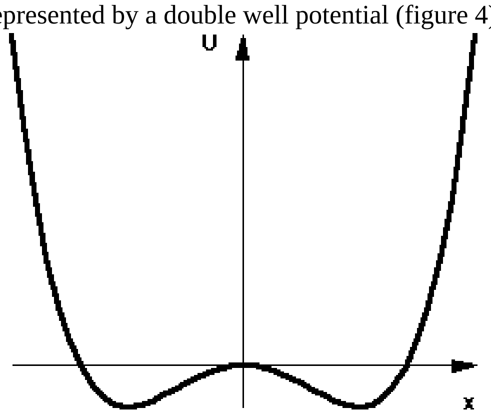

Figure 4. Plot of a potential function of the oscillator when $\mathbf{F_1}$ > **Fc**. The stable equilibrium point is moved from $x = 0$ to one of the two possible directions. This shape is known as double-well potential and is the representation of the equation (3). The value of the parameter r controls the deepness of the two lateral wells.

The changing between the potential of the system when $F_1 < F_c$ and the potential $F_1 > F_c$ is simulated by the rise of the central part of the well and the consequent progressive displacement of the ball, representing our reference position, towards one of the two lateral wells. At this point the system "makes a choice": the symmetry is broken (figure 5).

To emphasise the differences, it is possible to observe a gradual transition “linear–nonlinear” and detect the threshold value for this transition of the control parameter.

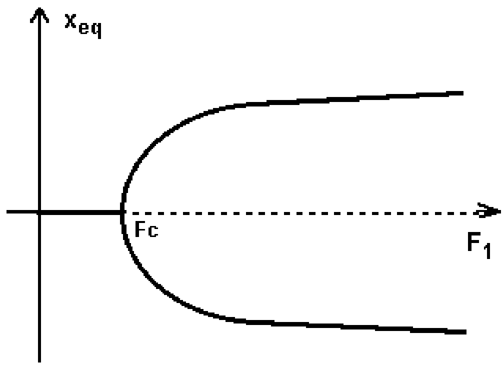

Figure 5. Plot of the strip equilibrium position with respect to the longitudinal force $\mathbf{F_1}$ applied to the same strip. The solid line represents the stable equilibrium points, while the dashed line represents the unstable equilibrium points. In this kind of diagrams the equilibrium position represents the **order parameter**, while the independent variable $\mathbf{F_1}$ is the **control parameter**. The point corresponding to $\mathbf{F_1}$ = **Fc** is a **bifurcation point**.

1.5 A simple computer simulation

By using basic language tools it is possible to write a simple code with elementary graphics to visualize the curve depicted in figure 4 represented by the function

$$U(x) = 1/2\,(1 - r)\,x^2 + 1/4\,x^4 \quad . \qquad (3)$$

By changing the value of r, you can see how this parameter is related to the difference between the central maximum and the two minima. If $r < 1$, the curve looks like (but is not actually) a parabola since it has one minimum only, and the oscillator behaves as it were a harmonic one, in the sense that if moved from its equilibrium position, it tends to return there.

The potential well grows from small starting values of r, until the critical value $r = 1$ is reached. Above this threshold, two lateral minima appear, and the central value becomes a maximum (unstable equilibrium point). When r further increases, the minima get deeper and farther.

From the observations described above, it is easy to relate the parameter r to the longitudinal force $\mathbf{F_1}$ applied to the strip.

In order to see an actual change of the system (and of its potential) due to the parameter r, we can increase the longitudinal force acting on the spring (of the device shown in figure 1), lifting at the same time the central part of the device in figure 2. This process is shown in figure 3.

The physical system represented by this potential is called "nonlinear oscillator".

We can try to plot a diagram of the system's equilibrium position $\mathbf{x}_{\mathbf{eq}}$ (the equilibrium position of the strip centre or equilibrium position of the little ball of the simulated potential) varying the longitudinal force $\mathbf{F}_1$ as done in figure 5.

In a broad class of non linear systems, the parameter $\mathbf{x}_{\mathbf{eq}}$, that in our case is the stable equilibrium position, is the order parameter;

$\mathbf{F}_1$, on which $\mathbf{x}_{\mathbf{eq}}$ depends, is called order parameter.

In the point where two stable equilibrium positions stem from one, we have a bifurcation.

2 Dark Room Meridian

2.1 Introduction

This section describes a project devoted to the application of the above expressed approach in the field of the time observation, measurement and regulation through the comparison between dark room meridians and modern instruments as GPS or telescopes. The dark room meridian is taken in the paper as a case study of an ancient instrument of observation. The knowledge and use of this old instrument and the comparison with current techniques can broaden the mental outlook and stimulate pupils to discover the deep characteristics of the scientific culture evolution. Due to the location of the meridians in historical buildings, this project could emphasize also the connections of science, arts and history.

The achievements of science and development of technologies made easy and in some way “natural” to obtain results coming from years or centuries of research work and attempts. The determination of our position in space and in time is made easy by the use of new technologies as having a look on a calendar or on a watch.

2.2 Position in Space

The knowledge of our position on the earth (i.e. geographic coordinates) can be done by pushing a button on a portable low-cost GPS (Global Positioning System) receiver. The position in a 3-dimensional space is done: latitude, longitude & altitude. In addition an electronic compass gives the north direction.

In spite of the seeming simplicity of the receiver (dimensions and appearance are similar to a cell phone, an antenna of an inch across only is enough to receive the signals), the GPS consists in a fleet of 27 satellites and 10 stations over the world. Each satellite has atomic clocks on board the precision of wich is ±1 second in 1 million years and is necessary for the functioning of the system.

The system is controlled by the US Army with a master central station and 10 more monitor stations over the world.

Actually determining the position on the Earth (latitude and longitude coordinates) has a long and meaningful history and has been discussed both in scientific and popular literature[9].

2.3 Position in Time

To obtain the time with an accuracy of a second is enough to connect with the site: http://wwp.greenwichmeantime.com/ , or to have a look on a radio-controlled watch. In spite of the simplicity of this action, our time position is due to the duration of a tropical year watched by the IERS (International Earth Rotation Service) that updates the national institutes and sets the variations of the Universal Time Coordinates (UTC) to keep the earth’s rotation linked with the day. On 31st December 2005 one leap second has been added, and in the recent past several leap seconds have been added (eight seconds from 1989).

2.4 This Work

Here we focalize our attention on an instrument used for the determination of the tropical year (the time between two spring equinoxes) that belongs to the history of the astronomy and the evolution of the calendar. The evolution of the calendar has been broadly discussed in literature. The study of this instrument involves geometrical, physical properties and astronomical knowledge.

The difficulty in the determination of the date is due to the fact that the ratio between the duration of the year and the duration of a day is not an integer number. This brings the date of the equinoxes to shift of several days over some hundred years.

One of the first instruments designed to resolve the question was the dark room meridian.

2.5 The Instrument

A dark room meridian consists in an indoor space where the light is penetrating through a small hole (the pinhole). So an image of the sky is projected on the floor of this space. Below the hole a line oriented to the North is traced on the same floor. In spite of the simple structure of the device, the results obtained allow high accuracy and reliability. In fact, the accuracy depends on the hole height and dimensions and on the accuracy of line orientation. The reliability depends on the stability of the building hosting the room

This instrument works as time regulator on two different scales: daily and annual.

The passage of the sun on the meridian line is a time reckoning at daily scale. This passage was setting the noon for each city or village until the end of the XVIII century.

Seasonal variation of the sun inclination and hence the longitudinal position on the meridian line sets the time over a year, in particular the extremities of the line, summer and winter solstices, and the equinoxes, spring and autumn.

The inclination of the sun changes of about 0.36 degrees over a day, across the equinoxes. If the gnomon is 20 m high (at 45° of latitude), the difference on the line is about 25 cm, this means that it is possible to know the moment of the equinox within half an our if the edge of the image is about 1 cm. With a sundial it is impossible to have an image so sharp because the sun is a disk with an angular aperture of half degree. In a dark room the edge of the image is half of the diameter of the hole, that means 1 cm in this case.

The reliability depends on the stability of the building where the room is located, for this reason these instruments are located in old and solids monuments.

These ancient devices were used since XV until the end of the XVIII century, today most of them are abandoned even if they are inside historically and artistically important buildings.

2.6 History

In the second half of the XVI century Cosimo dei Medici, pursuing his project of transformation of the Duke of Florence in the Duke of Tuskany, commissioned embellishment of many civil and religious buildings. One of the artists employed in these activities was the astronomer and cartographer Ignazio Danti.

Cosimo discussed with Danti the problem of the calendar reform (at that time the equinox fell 10 days before the 21st of march). Danti needed to make accurate measurements of the tropical year. He mounted two instruments on the face of S.M.Novella in Florence[10].

The accuracy of these instruments was not enough to give a close estimation of the year length. Danti made a hole in the face of S.M.Novella about 21 m high and traced a line on the pavement of the church. In this way the church works as a “camera obscura”.

But when Cosimo I died in 1574. this work was not finished.

Danti was reassigned in Bologna, and here he built another meridian in S.Petronio church.

Seventy-five years later the wall where Danti placed the hole was removed to enlarge the church. G. D. Cassini Jesuit mathematician and astronomer, suggested to put another hole and to trace a new line in S. Petronio. The line was traced during a public demonstration on the 21st and 22nd of June 1665.

Cassini works on this “heliometer” gave information on several astronomical and physical values included sun eccentricity, refraction of the light in the atmosphere, parallax error, variation of the obliquity, etc.

Dozens of meridians have been made in Europe, most of them in important historical buildings, see for example St. Maria del Fiore in Florence, St. Petronio in Bologna, St. Sulpice in Paris[11], El Escorial in Madrid[12] or the Grand Master Palace in Malta[13, 14].

2.7 The Beauty of the Meridian

The charm of the meridian is due to the antiquity of its origins, the essentiality of its construction, and perhaps to its substantial immateriality: it is a light beam passing through a hole. The meridian realizes the aim of the Geography: to represent a spherical body (the sun) on a plane surface (a circle). In addition the meridian transforms a circular movement into a (locally) linear movement, it makes visible an object (the sun) by naked eye, hard to observe otherwise, and it makes visible also a movement not sensibly visible[15].

2.8 The Project

The described project is devoted to the pupils of primary and secondary schools with different educational objectives and methods.

The program includes both learning (as geometrical error theory, optics, astronomy, etc.) as well as design and manual pupils' activities (as the construction of a “camera obscura” to observe the sun and the pinhole characteristics, the drawing of the meridian line, the search for disappeared meridians and/or the “adoption” of an existing one). These activities are divided by school level: primary and secondary.

Let’s consider the examples of learning activities.

For primary and secondary school the subjects are: pinhole and dark room general properties, historical overview of the mean dark room meridians, visiting, if possible, some important dark room meridians.

For secondary school only: geometrical optics, diffraction limits of the pinhole, error theory.

The examples of practical activities for primary and secondary school are: the construction of pinholes with different diameters and shapes to observe that the image of the sun has same dimensions and shape, the drawing of the meridian line using different methods:

1) Through the knowledge of the official time, latitude and time equation it is possible to mark (several times along the year, for example, September, October, etc.) the image of the sun at the local noon. The line passing on these points is the meridian line.
2) Tracing the line using a compass and taking account of the magnetic declination (secondary school only).
3) Tracing a circle with the centre on the perpendicular to the hole. The image of the sun intercepts the circle in two points. The axe of the segment done by these two points is the line (secondary school only).

As a secondary effect this project involves the participation of public institutions. In fact, due to the loose of interest as scientific instrument, many of such devices are abandoned, damaged or cancelled by restoration works of the buildings hosting them.

These activities can contribute to valorising cultural and artistic goods of a local territory.

3 Conclusion

Considering both examples of comparison between old and new technologies, we can underline the following:

- the approach to the problem using old techniques is a door to enter the phenomena comprehension;

- the light use of computer technologies and programming tools makes the complex and abstract concepts more affordable for students;
- the student scientific culture could be improved by using, building or remaking a simple instrument that involves mechanical and physical or geographical and astronomical knowledge;
- comparing "old" and "current" techniques for the same purposes emphasizes the scientific progress and contributes to reducing barriers between scientific and humanistic culture.

All these activities can stimulate students' involvement in and awareness of both natural and cultural environments where we all live.

References

[1] D.K. Campbell, 1987, *Los Alamos Science* **15**, 217

[2] G.J. Rodgers, 1992, *Physics Education* **27**, 14

[3] V. Detcheva, V. Spassov, 1993, *Physics Education* **28**, 39

[4] P.R. Hobson, A.N. Lansbury, 1996, *Physics Education* **31**, 39

[5] P.H. Borchards, 1995, *Physics Education* **6**, 372

[6] S.J. Martin, P.J. Ford, 2001, *Physics Education* **36**, 108

[7] D.R: Camp, 1995, *Zenterblatt für Didaktik der Matematik* **5**, 143

[8] E. Eisenstein, 1979, *The Printing Press as an Agent of Change*, Cambridge Univ. Press

[9] Dava Sobel "*Longitude*", Walker & Company, 1995.

[10] J. L. Heilbron "*The Sun in the Church*", Harvard University Press, 1999.

[11] G. Camus, P. Divonne, A. Gotteland, B. Tailliez "Les meridiennes de l'eglise Saint-Sulpice à Paris", in *L'astronomie*, mai 1990, p. 195-214.

[12] M.M. Valdés, "Meridianas en los Palacios reales" in *Analema* n.15 sept.-dic. 1995.

[13] Frank Ventura, "Maltese sundials" in *Antique Maltese Clocks*, Progress Press Co. Ltd. 1992, p. 134-139

[14] William Soler, "*Palace of Grand Master in Valletta*", Patrimonju Publishing, 2001, p.109-124

[15] F. Farinelli, "*Geografia*" Einaudi, Torino 2003