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9 The Spin-Charge-Family Theory Offers the Explanation for all the Assumptions of the Standard Model, for the Dark Matter, for the Matter/Anti-matter Asymmetry, Making Several Predictions.

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Abstract. The *spin-charge-family* theory, which is a kind of the Kaluza-Klein theories, but with two kinds of the spin connection fields — the gauge fields of the two kinds of spins [1–5] — is offering the explanation for the appearance and properties of family members (quarks and leptons), of families, of vector gauge fields (weak, hyper, colour), of scalar higgs and Yukawa couplings and gravity. It also explains the appearance of the dark matter and matter/anti-matter asymmetry. In this talk the achievements of this theory, its predictions and also its not yet solved problems are briefly presented and discussed.

Povzetek. Teorija *spinov-nabojev-družin* [1–5] ponuja odgovor na vsa odprta vprašanja standardnega modela fizike osnovnih delcev in polj, pa tudi na marsikatero odprto vprašanje v kozmologiji. Pojasni lastnosti ene družine kvarkov in leptonov, nastanek družin, nastanek barvnega, šibkega in hiper polja, nastanek skalarnih polj, ki pojasnijo pojav Higgsovega polja in Yukavinih sklopitev. Pojasni tudi pojav temne snovi in asimetrijo med snovjo in antisnovjo v vesolju. Teorija, ki ima marsikaj skupnega s Kaluza-kleinovimi teorijami, ponudi dve vrsti spinov. Ena vrsta določa vse naboje osnovnih delcev, druga družinska kvantna števila. V predavanju predstavim dosedanje dosežke te teorije, njene napovedi, pa tudi še nerešena odprta vprašanja.

9.1 Introduction

More than 40 years ago the *standard model* offered the elegant new step in understanding elementary fermion and boson fields. It postulated:

- The existence of the massless family members - coloured quarks and colourless leptons, both left and right handed, the left handed members distinguishing from the right handed ones in the weak and hyper charges and correspondingly mass protected.
- The existence of massless families to each of a family member.
- The existence of the massless gauge fields (colour octet, weak triplet, hyper singlet) to the observed (colour, weak and hyper) charges of the family members. They all are vectors in $d = (3 + 1)$, in the adjoint representations with respect to the weak, colour and hyper charges.

- The existence of a massive self interacting scalar field carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$, respectively, obviously doublets (in the fundamental representation with respect to the weak charge - like fermions), with the "nonzero vacuum expectation values", what breaks the weak and the hyper charge, breaking correspondingly the mass protection of fermions and weak and hyper bosons.
- The existence of the Yukawa couplings, which together with (the gluons and) the scalar higgs take care of the properties of the fermions and heavy bosons, after the break of the weak and the hyper charge.

The *standard model* offers no explanation for the assumptions, suggested by the phenomenology. Its assumptions have been confirmed without offering surprises. The last unobserved field, the higgs scalar, was detected in June 2012 and confirmed in March 2013.

There are several attempts in the literature, offering the extensions of the *standard model*, but do not really offer the explanation for the *standard model* assumptions. The SU(5) and SU(10) *grand unified theories* unify all the charges, but neither they explain why the spin (the handedness) is connected with the (weak and hyper) charges nor why and from where do families appear. *Supersymmetric* theories, assuming the existence of bosons with the charges of quarks and leptons and fermions with the charges of the gauge vector fields, although having several nice properties, do not explain the occurrence of families except by assuming larger groups. Also the theories of *strings* and *membranes*, again having desired features with respect to several requirements, like renormalizability, also do not offer the explanation for the appearance of families, although they do have families, if assuming a large enough group. The*Kaluza-Klein* theories do unify spin and charges, but do not offer the explanation for the appearance of families.

To see the next step beyond the standard model one should be able to answer the following questions:
i. Where do families originate and why there exist families at all? How many families are there?
ii. How are the origin of the scalar field - the higgs - and the Yukawa couplings connected with the origin of families?
iii. How many scalar fields determine properties of the so far (and others possibly be) observed fermions and masses of the heavy bosons?
iv. Why is the higgs, or are all the scalar fields, if there are several, doublets with respect to the weak and the hyper charge, while all the other bosons have charges in the adjoint representations of the group?
v. Why do the left and the right handed family members distinguish so much in charges and why do they - quarks and leptons - manifest so different properties if they all start as massless? **vi.** Are there also scalar bosons with the colour charge in the fundamental representation of the colour group and where, if they are, do they manifest?
vii. Where does the dark matter originate?
viii. Where does the matter/anti-matter asymmetry originate?
ix. Where do the charges and correspondingly the so far (and others possibly be) observed gauge fields originate?

x. Where does the dark energy originate and why is it so small?
xi. And several other questions, like: What is the dimension of space-time?

My statement is: An elegant trustworthy step beyond the *standard model* must offer answers to several of the above open questions, explaining: **o** the origin of the charges of the fermions, **o** the origin of the families of the fermions and their properties, **o** the origin of the vector gauge fields and their properties, **o** the origin of the scalar field, its properties and the Yukawa couplings, **o** the origin of the dark matter, **o** the origin of the "ordinary" matter/anti-matter asymmetry.

Inventing a next step, which covers only one of the open questions, can hardly be the right step.

The *spin-charge-family* theory [1–14] does offer the explanation for all the assumptions of the *standard model*, offering answers to many of the above cited open questions. The more I am working (together with the collaborators) on the *spin-charge-family* theory, the more answers to the open questions of the elementary fermion and boson fields and cosmology the theory is offering. Although still many theoretical proofs, more precise, and first of all the experimentally confirmed, predictions are needed, the theory is becoming more and more trustworthy.

I shall briefly present the achievements of the *spin-charge-family* theory, still open questions and answers to some of the most often posed questions and criticisms.

9.2 *Spin-charge-family* theory, action and assumptions

I present in this section, following a lot the similar one from Refs. [1,5], the *assumptions* of the *spin-charge-family* theory, on which the theory is built.

A i. In the action [1,4,2,5] fermions ψ carry in $d = (13+1)$ as the *internal degrees of freedom only two kinds of spins* (no charges), which are determined by the two kinds of the Clifford algebra objects (there exist no additional Clifford algebra objects) (9.7)) - γ^a and $\tilde{\gamma}^a$ - and *interact correspondingly with the two kinds of the spin connection fields* - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, the *gauge fields* of $S^{ab} = \frac{i}{4}\,(\gamma^a\gamma^b - \gamma^b\gamma^a)$, the generators of $SO(13,1)$ and $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$ (the generators of $\widetilde{SO}(13,1)$) - and the *vielbeins* $f^{\alpha}{}_{a}$.

$$
\begin{aligned}
\mathcal{A} &= \int d^dx\, E\, \mathcal{L}_f + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,,\\
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.},\\
p_{0a} &= f^{\alpha}{}_{a} p_{0\alpha} + \frac{1}{2E}\,\{p_{\alpha}, E f^{\alpha}{}_{a}\}_-, \quad p_{0\alpha} = p_{\alpha} - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2}\,\{f^{\alpha[a} f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + \text{h.c.}\,,\\
\tilde{R} &= \frac{1}{2}\,\{f^{\alpha[a} f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^{c}{}_{b\beta})\} + \text{h.c.}\,.
\end{aligned}
\tag{9.1}
$$

Here [1] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. R and $\tilde{R}$ are the two scalars (R is a curvature).

A ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting as $SO(7,1) \times SU(3) \times U(1)$), affecting both internal degrees of freedom - the one represented by γ^a and the one represented by $\tilde{\gamma}^a$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ massless families (which decouple into twice four families). The rest of families get heavy masses [2].

A iii. The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

A iv. The scalar condensate (Table 9.1) of two right handed neutrinos with the family quantum numbers of one of the two groups of four families, brings masses of the scale of unification ($\propto 10^{16}$ GeV) to all the vector and scalar gauge fields, which interact with the condensate [1].

A v. There are nonzero vacuum expectation values of the scalar fields with the space index $s = (7,8)$, conserving the electromagnetic and colour charge, which cause the electroweak break and bring masses to all the fermions and to the heavy bosons.

Comments on the assumptions:

C i.: This starting action enables to represent the *standard model* as an effective low energy manifestation of the *spin-charge-family* theory [1–13]. It offers the explanation for all the *standard model* assumptions: **a.** One representation of $SO(13,1)$ contains, if analyzed with respect to the *standard model* groups $(SO(3,1) \times SU(2) \times U(1) \times SU(3))$ all the members of one family (Table 9.4), left and right handed, with the quantum numbers required by the *standard model* [3]. **b.** The action explains the appearance of families due to the two kinds of generators

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[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\ e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[2] A toy model [20,21] was studied in $d = (5+1)$ with the same action as in Eq. (9.1). The break from $d = (5+1)$ to $d = (3+1)\times$ an almost S^2 was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, its proof is is in progress, is made in the $d = (13+1)$ case.

[3] It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and the right handed weak chargeless and $SU(2)_{II}$ charged coloured quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless colour anti-triplet anti-quarks and (anti)colourless anti-leptons, and the left handed weak chargeless and $SU(2)_{II}$ charged anti-quarks and anti-leptons. The anti-fermion states are reachable from the fermion states by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$, presented in Ref. [22].

of groups, the infinitesimal generators of one being S^{ab}, of the other $\tilde{S}^{ab}$ [4]. **c.** The action explains the appearance of the gauge fields of the *standard model* [1,5]. (In Ref [5] the proof is presented, that gauge fields can in the Kaluza-Klein theories be equivalently represented with either the vielbeins or spin connection fields.) [5]. **d.** It explains the appearance of the scalar higgs and Yukawa couplings [6]. **e.** The starting action contains also additional $SU(2)_{II}$ (from $SO(4)$) vector gauge fields (one of the components contributes to the hyper charge gauge fields as explained above), as well as the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10,\ldots,14)$. All these fields gain masses of the scale of the condensate (Table 9.1), which they interact with. They all are expressible with the superposition of $f^{\mu}{}_{m}\, \omega_{ab\mu}$ or of $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ [7].

C ii., C iii.: There are many ways of breaking symmetries from $d = (13+1)$ to $d = (3+1)$. The assumed breaks explain the connection between the weak and the hyper charge and the handedness of spinors, manifesting correspondingly the observed properties of the family members - the quarks and the leptons, left and right handed (Table 9.4) - and of the observed vector gauge fields. After the break from $SO(13,1)$ to $SO(3,1) \times SU(2) \times U(1) \times SU(3)$ the anti-particles are accessible from particles by the application of the operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$, as explained in Refs. [22,23] [8].

[4] There are before the electroweak break two massless decoupled groups of four families of quarks and leptons, in the fundamental representations of $\widetilde{SU}(2)_{R,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II,\widetilde{SO}(4)}$ and $\widetilde{SU}(2)_{L,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{I,\widetilde{SO}(4)}$ groups, respectively - the subgroups of $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$ (Table 9.5). These eight families remain massless up to the electroweak break due to the "mass protection mechanism", that is due to the fact that the right handed members have no left handed partners with the same charges.

[5] Before the electroweak break are all observable gauge fields massless: the gravity, the colour octet vector gauge fields (of the group $SU(3)$ from $SO(6)$), the weak triplet vector gauge field (of the group $SU(2)_{I}$ from $SO(4)$), and the hyper singlet vector gauge field (a superposition of $U(1)$ from $SO(6)$ and the third component of $SU(2)_{II}$ triplet).All are the superposition of the $f^{\alpha}{}_{c}\, \omega_{ab\alpha}$ spinor gauge fields

[6] There are scalar fields with the space index $(7,8)$ and with respect to the space index with the weak and the hyper charge of the Higgs's scalar. They belong with respect to additional quantum numbers either to one of the two groups of two triplets, (either to one of the two triplets of the groups $\widetilde{SU}(2)_{R\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$, or to one of the two triplets of the groups $\widetilde{SU}(2)_{L\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{I\,\widetilde{SO}(4)}$, respectively), which couple through the family quantum numbers to one (the first two triplets) or to another (the second two triplets) group of four families - all are the superposition of $f^{\sigma}{}_{s}\ \tilde{\omega}_{ab\sigma}$, or they belong to three singlets, the scalar gauge fields of (Q, Q', Y'), which couple to the family members of both groups of families - they are the superposition of $f^{\sigma}{}_{s}\ \omega_{ab\sigma}$. Both kinds of scalar fields determine the fermion masses (Eq. (9.6)), offering the explanation for the higgs, the Yukawa couplings and the heavy bosons masses.

[7] In the case of free fields (if no spinor source, carrying their quantum numbers, is present) both $f^{\mu}{}_{m}\, \omega_{ab\mu}$ and $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ are expressible with vielbeins, correspondingly only one kind of the three gauge fields are the propagating fields.

[8] The discrete symmetry operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$, Refs. [22,23], does not contain $\tilde{\gamma}^{a}$'s degrees of freedom. To each family member there corresponds the anti-member, with the same family quantum number.

C iv.: It is the condensate (Table 9.1) of two right handed neutrinos with the quantum numbers of one group of four families, which makes massive all the scalar gauge fields (with the index $(5,6,7,8)$, as well as those with the index $(9,\ldots,14)$) and the vector gauge fields, manifesting nonzero τ^4, τ^{23}, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ [1,5]. Only the vector gauge fields of Y, $SU(3)$ and $SU(2)$ remain massless, since they do not interact with the condensate.

C v.: At the electroweak break the scalar fields with the space index $s = (7,8)$ - originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{s's"s}$ with the quantum numbers (Q, Q', Y'), conserving the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to mass matrices of twice the four families, as well as to the masses of the heavy vector bosons.

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

The fourth family to the observed three ones is predicted to be observed at the LHC. Its properties are under consideration [13,14], the baryons of the stable family of the upper four families is offering the explanation for the dark matter [12].

Let us (formally) rewrite that part of the action of Eq.(9.1), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does in the low energy regime manifest by the *standard model* required degrees of freedom of the fermions, vector and scalar gauge fields [2–13].

$$
\begin{aligned}
\mathcal{L}_f = {} & \bar{\psi}\gamma^m (p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
& \{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\
& \{\sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^t p_{0t}\, \psi\},
\end{aligned} \tag{9.2}
$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t} = p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$, with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within either $(0,1,2,3)$ or $(5,6,7,8)$, t runs $\in (5,\ldots,14)$, $(t', t")$ run either $\in (5,6,7,8)$ or $\in (9,10,\ldots,14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families.

The first line of Eq. (9.2) determines (in $d = (3+1)$) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients ${c^{Ai}}_{ab}$ [9].

$$
\tau^{Ai} = \sum_{a,b} {c^{Ai}}_{ab}\, S^{ab}\,, \tag{9.3}
$$

[9] $\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})$, $\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$, $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$. After the electroweak break the charges $Y := \tau^4 + \tau^{23}$, $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, $Q := \tau^{13} + Y$, $Q' := -Y\tan^2\vartheta_1 + \tau^{13}$ manifest. θ_1 is the electroweak angle, breaking $SU(2)_I$, θ_2 is the angle of the break of the $SU(2)_{II}$ from $SU(2)_I \times SU(2)_{II}$.

fulfilling the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak} \,. \tag{9.4}$$

They represent the colour, the weak and the hyper charge. The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}, with (s,t) either $\in (5,6,7,8)$ or $\in (9,\ldots,14)$, in agreement with the assumptions **A ii.** and **A iii.**. I demonstrate in Ref. [5] the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections proposed by the *spin-charge-family* theory.

All vector gauge fields, appearing in the first line of Eq. (9.2), except $A^{2\pm}_m$ and $A^{Y'}_m$ $(= \cos\vartheta_2\, A^{23}_m - \sin\vartheta_2\, A^4_m$, Y' and τ^4 are defined in [10], are massless before the electroweak break. $\vec{A}^3_m$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^1_m$ carries the weak charge $SU(2)_I$ ($SU(2)_I$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and A^Y_m $(= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m)$ carries the corresponding $U(1)$ charge, $Y = \tau^{23} + \tau^4$, τ^4 originates in $SO(6)$ and τ^{23} is the third component of the second $SU(2)_{II}$ group, A^4_m and $\vec{A}^2_m$ are the corresponding vector gauge fields). The fields $A^{2\pm}_m$ and $A^{Y'}_m$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one of the group of four families (the assumption **iv.**, Table 9.1). (See Ref. [5].)

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, there are correspondingly $2^{(7+1)/2-1} = 8$ families [5], which split into two groups of $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$ families.

If there are no fermions present then the vector gauge fields of the family members and family charges - ω_{abm} and $\tilde{\omega}_{abm}$ - are all expressible with the vielbeins [1,5], which are then the only propagating fields.

The scalar fields, the gauge fields with the space index ≥ 5, which are either the superposition of $\tilde{\omega}_{abs}$ or the superposition of $\omega_{s'ts}$, determine, when gaining nonzero vacuum expectation values (the assumption **v.**), masses of fermions (belonging to two groups of four families of family members of spinors) and weak bosons.

The condensate (the assumption **iv.**), Table 9.1, gives masses of the order of the scale of its appearance to all the scalar gauge fields, presented in the second and the third line of Eq. (9.2).

The vector gauge fields of the (before the electroweak break) conserved charges ($\vec{\tau}^3$, $\vec{\tau}^1$, Y) do not interact with the condensate and stay correspondingly massless. After the electroweak break - when the scalar fields (those with the family quantum numbers and those with the family members quantum numbers (Q, Q', Y')) with the space index $s = (7,8)$ start to self interact and gain nonzero vacuum expectation values - only the charges $\vec{\tau}^3$ and $Q = Y + \tau^{13}$ are the conserved charges. No family quantum numbers are conserved, since all scalar fields with the family quantum numbers and the space index $s = (7,8)$ gain nonzero vacuum expectation values.

[10] $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}_L^3$	$\tilde{N}_R^3$
$(\vert\nu_{\mathbf{1R}}^{\mathbf{VIII}}>_{\mathbf{1}}\ \vert\nu_{\mathbf{2R}}^{\mathbf{VIII}}>_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu_{1R}^{VIII}>_1\ \vert e_{2R}^{VIII}>_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e_{1R}^{VIII}>_1\ \vert e_{2R}^{VIII}>_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 9.1. This table is taken from [1]. The condensate of the two right handed neutrinos ν_R, with the VIII^{th} family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{\tau}^4 = -1$, $\tilde{N}_R^3 = 1$, $\tilde{N}_L^3 = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in Table 9.5.

Quarks and leptons have the "spinor" quantum number (τ^4, originating in $SO(6)$ presented in Table 9.4) equal to $\frac{1}{6}$ and $-\frac{1}{2}$, respectively. In the Pati-Salam model [24] twice this "spinor" quantum number is named $\frac{B-L}{2}$ quantum number, for quarks equal to $\frac{1}{3}$ and for leptons to -1.

Let me introduce a common notation A_s^{Ai} for all the scalar fields, independently of whether they originate in ω_{abs} or $\tilde{\omega}_{abs}$, $s \geq 5$. In the case that we are interested in the scalar fields which contribute to masses of fermions and weak bosons, then $s = (7, 8)$. If A_s^{Ai} represent ω_{abs}, $Ai = (Q, Q', Y')$, while if A_s^{Ai} represent $\tilde{\omega}_{\tilde{a}\tilde{b}s}$, all the family quantum numbers of all eight families contribute to Ai.

$$
\begin{aligned}
A_s^{Ai} &\in (\, A_s^{Q}\,, A_s^{Q'}\,, A_s^{Y'}\,, \vec{\tilde{A}}_s^{\tilde{1}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{L}}}\,, \vec{\tilde{A}}_s^{\tilde{2}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{R}}}\,)\,, \\
\tau^{Ai} &\supset (Q,\, Q',\, Y',\, \vec{\tilde{\tau}}^1,\, \vec{\tilde{N}}_L,\, \vec{\tilde{\tau}}^2,\, \vec{\tilde{N}}_R)\,.
\end{aligned} \tag{9.5}
$$

Here τ^{Ai} represent all the operators, which apply on the spinor states. These scalars, the gauge scalar fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$, are expressible in terms of the spin connection fields.

9.3 Achievements of the *spin-charge-family* theory and its predictions

The achievements of the *spin-charge-family* theory.

I. The *spin-charge-family* theory does offer the explanation for all the assumptions of the *standard model*:

I A. It explains all the properties of family members of one family - their spins and all the charges - clarifying the relationship between the spins and charges, Table 9.4 [11].

I B. It explains the properties of the vector fields, the gauge fields of the corresponding charges. They are in the *spin-charge-family* theory represented by the superposition of the spin connection fields ω_{stm}. It is proven in Sect. II of Ref. [5]

[11] The *spin-charge-family* theory explains, why the left handed and the right handed quarks and leptons differ in the weak and the hyper charge. It also explains, why quarks and leptons differ in the colour charge.

that the spin connection fields representation is equivalent to the usual Kaluza-Klein representation with the vielbeins $f^{\sigma}{}_{m} = \vec{\tau}^{A\sigma}\vec{A}^{A}_{m}$, where $\vec{\tau}^{A} = \vec{\tau}^{A\sigma}p_{\sigma}$, $\vec{\tau}^{A}$ determine symmetry properties of the space with $s \geq 5$ and $\vec{A}^{A}_{m}$ are the corresponding gauge fields.

I C. The scalar fields with the space index $s \in (7,8)$ belong to two doublets with respect to the space index s, while they belong with respect to additional quantum numbers either to three singlets with one of the family members charges (Y, Y', Q') or to twice two triplets of the family charges belonging to the groups $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$. These scalar fields explain the appearance of the higgs and Yukawa couplings.

I D. The theory explains why these scalar fields, and consequently the higgs, which is the superposition of several scalar fields [4,5], have the weak and the hyper charge equal to $(\pm\frac{1}{2}, \mp\frac{1}{2})$, respectively, although they are bosons. They do transform as bosons with respect to $\mathcal{S}^{ab}$ [12], but due to the fact that they belong with respect to the space index $s = (5,6,7,8)$ to two $SU(2)$ groups with $\tau^{13} = \frac{1}{2}(S^{56} - S^{78})$ and $\tau^{23} = \frac{1}{2}(S^{56} + S^{78})$, respectively, their weak (τ^{13}) and hyper charge ($\tau^{23} + \tau^{4}$, where $\tau^{4} = -\frac{1}{3}\,(S^{9\,10} + S^{11\,12} + S^{13\,14})$ does not influence $s = (7,8)$) are the ones required by the *standard model*. Table 9.2 presents these two doublets and their quantum numbers.

I E. There are the nonzero vacuum expectation values of the scalar gauge fields

	state	τ^{13}	τ^{23}	spin	τ^{4}	Q
$A^{Ai}_{\substack{78\\(-)}}$	$A^{Ai}_{7} + iA^{Ai}_{8}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(-)}}$	$A^{Ai}_{5} + iA^{Ai}_{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$A^{Ai}_{\substack{78\\(+)}}$	$A^{Ai}_{7} - iA^{Ai}_{8}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(+)}}$	$A^{Ai}_{5} - iA^{Ai}_{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	+1

Table 9.2. The two scalar weak doublets, one with $\tau^{23} = -\frac{1}{2}$ and the other with $\tau^{23} = +\frac{1}{2}$, both with the "spinor" quantum number $\tau^{4} = 0$, are presented. In this table all the scalar fields carry besides the quantum numbers determined by the space index also the quantum numbers $\mathcal{A}i$, which represent either the family members quantum numbers (Q, Q', Y') or the family quantum numbers (twice two triplets), $A^{Ai}_{\substack{78\\(\pm)}} = A^{Ai}_{7} \pm iA^{Ai}_{8}$, Eq. (9.5)

.

with the space index $s = (7,8)$, (with the weak charge equal to $\pm\frac{1}{2}$ and the hyper charge correspondingly equal to $\mp\frac{1}{2}$, both with respect to the space index), and with the family (twice two triplets) and family member quantum numbers (three singlets) in adjoint representations, which start to interact among themselves, gain nonzero vacuum expectation values, causing the break of the weak and the hyper

[12] $\mathcal{S}^{ab}$, which applies on the spin connections ω_{bde} $(= f^{\alpha}{}_{e}\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_{e}\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, is equal to $\mathcal{S}^{ab}\,A^{d\ldots e\ldots g} = i\,(\eta^{ae}\,A^{d\ldots b\ldots g} - \eta^{be}\,A^{d\ldots a\ldots g})$, or equivalently, in the matrix notation, $(\mathcal{S}^{ab})^{c}{}_{e}A^{d\ldots e\ldots g} = i(\eta^{ac}\delta^{b}_{e} - \eta^{bc}\delta^{a}_{e})\,A^{d\ldots e\ldots g}$.

charge symmetry.

II. The *spin-charge-family* theory does offer the explanation for the dark matter and for matter/anti-matter asymmetry:

II A. Neutral clusters of the members of the stable among the upper four families explain the appearance of the dark matter [12].
II B. The scalar fields with the space index $s \in (9, \ldots, 14)$ belong with respect to the space index s to a triplet or an anti-triplet, Table 9.3. They cause transitions of anti-leptons into quarks and anti-quarks into quarks and back, transforming matter into anti-matter and back. The condensate breaks CP symmetry. In the expanding universe, fulfilling the Sakharov request for appropriate non thermal equilibrium, these colour triplet and anti-triplet scalars have a chance to explain the matter/anti-matter asymmetry in the universe [1], as well as the proton decay.
II C. It is the *scalar condensate* of two right handed neutrinos (Table 9.1), which

	state	τ^{33}	τ^{38}	spin	τ^4	Q
$A^{Ai}_{\overset{9\,10}{(+)}}$	$A^{Ai}_{9} - iA^{Ai}_{10}$	$+\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{11\,12}{(+)}}$	$A^{Ai}_{11} - iA^{Ai}_{12}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{13\,14}{(+)}}$	$A^{Ai}_{13} - iA^{Ai}_{14}$	0	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{9\,10}{(-)}}$	$A^{Ai}_{9} + iA^{Ai}_{10}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{\overset{11\,12}{(-)}}$	$A^{Ai}_{11} + iA^{Ai}_{12}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{\overset{13\,14}{(-)}}$	$A^{Ai}_{13} + iA^{Ai}_{14}$	0	$\frac{1}{\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$

Table 9.3. The triplet and the anti-triplet scalar gauge fields, the triplet with the "spinor" quantum number equal to $\mathcal{S}^4 = -\frac{1}{3}$, $\mathcal{S}^4 = -\frac{1}{3}\,(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14})$ and the anti-triplet with the "spinor" quantum number equal to $\mathcal{S}^4 = +\frac{1}{3}$. In this table all the scalar fields carry, besides the quantum numbers determined by the space index, (only) the family quantum numbers, not pointed out in this table. The table is taken from Ref. [1].

gives masses to all the vector and scalar gauge fields appearing in the *spin-charge-family* theory, except to the gravity, colour vector gauge fields, weak vector gauge fields and hyper $U(1)$ gauge field, since they do not interact with the condensate.
II D. The scalar fields, the members of the weak doublets (Table 9.2) with the space index $s = (5, 6)$, and the colour triplets and anti-triplets with the space index $t = (9, \ldots, 14)$ [1], which contribute to transitions of anti-particles into particles and to proton decay, keep masses of the condensate scale, as also do $A^{2\pm}_{m}$ and $A^{Y'}_{M} = \cos\theta_2 A^{2m}_{m} - \sin\theta A^{4}_{m}$.

III. The theory might have a chance to explain the hierarchy of the fermion and boson masses.
III A. By the theory predicted existence of the fourth family to the observed three families with the masses of the fourth family members at 1 TeV or even above [13,14] makes the mass matrices of the family members very close to the

democratic matrix, which suggests that the lower four families masses expand in the interval from less than eV to 10^{12} eV. Correspondingly would the interval of the higher four families be within the interval from ≈ 100 TeV [12] to $\approx 100\times 10^{12}$ TeV, which is above the unification scale 10^{16} GeV) (10^{13} TeV), explaining why are the masses of fermions spreading from few orders of magnitude below eV to TeV and above up to the unification scale.

Predictions of the *spin-charge-family* theory.

I. The *spin-charge-family* theory predicts in the low energy regime two decoupled groups of four families. The scalar fields with the space index $s = (7, 8)$, which are the gauge fields of the family charges, the superposition of $\tilde{S}^{ab}$ belonging to the subgroups $\widetilde{SU}(2)_{II\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II\widetilde{SO}(4)}$, determine the symmetry of each of the two groups of families.

I A. The symmetry of mass matrices

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha},$$

enables to tell what are the masses and matrix elements of the fourth family quarks and leptons within the interval of the accuracy of the experimental data. Any $(n-1) \times (n-1)$ submatrix of the $n \times n$ unitary matrix, $n \geq 4$ determines uniquely the $n \times n$ unitary matrix.

Present experimental data for the mixing matrices are not accurate enough even for quarks to tell, what are the fourth family masses. The estimation: most probably they are above 1 TeV. We can, however, for the chosen fourth family masses, predict the mixing matrix elements of quarks [13,14]. It comes out that the fourth family matrix elements are not very sensitive either to the lower three or to the fourth family quark masses. Our calculations [14] show that the new experimental data are in better agreement with the *spin-charge-family* theory predictions than the old ones.

For leptons the experimental data are less accurate and correspondingly the estimated mixing matrix elements for the fourth family leptons are less predictable.

The higher are the fourth family members masses, the closer are the mass matrices to the democratic matrices for either quarks or leptons - which is expected.
The fourth of the lower four families will be measured at the LHC.

I B. Scalar fields, which cause electroweak phase transition and are responsible for masses of the lower four families of quarks and leptons and weak bosons, determine the higss and the Yukawa couplings.
Besides the higgs, additional superposition of scalar fields are predicted to be measured at the LHC.

I C. The properties of the upper four family members, (almost) decoupled from the lower four families (their mass matrices still manifest the $\widetilde{SU}(2)_{II\widetilde{SO}(3,1)}$ $\times\widetilde{SU}(2)_{II\widetilde{SO}(4)}$ symmetry, provided that the condensate respect this symmetry, and are influenced by the family scalar fields of the upper four families, by the

family members scalar field with the quantum numbers (Q, Q', Y') and by the interaction with the condensate), can be evaluated within this theory by following the evolution of the universe [12].
The masses of the lowest of the upper four families are estimated [12] to be in the interval of several 10 *TeV to several* 10^4 *TeV.*
I D. Very heavy dark matter baryons are opening an interesting new "fifth family nuclear" dynamics.

III. There are besides the scalar fields, which are, like higgs, $SU(2)$ doublets, also the scalar fields, which are $SU(3)$ triplets, involved and responsible for the matter/anti-matter asymmetry of our universe.

9.4 Most common questions about the *spin-charge-family* theory

Let me present and offer a brief answers to the most common questions and complains about the validity and the ability that the*spin-charge-family* theory might be the right answer to the open questions of the *standard model* by attentive participants of the conferences, readers or referees. To most of such questions the answers can be found by carefully reading papers [5,4,1–3,6–14], some of them are discussed in special sections of these papers or in contributions to the Discussion section of the Bled 2015 workshop.

There are also the assumptions in this theory, represented in this talk, chosen in order that the theory manifests in the low energy regime the *standard model* properties, which also need, and want for better answer than the one, that obviously our universe has chosen among many other possibilities, those required by the assumptions.

The most needed are, of course, the experimental data confirmation of the predictions of this theory, making it trustworthy as the right next step beyond the *standard model*. But what does speak for this theory is that the simple starting action (Eq. (9.1)) and only a few assumptions explain all the assumptions of the *standard model*, offering the explanation also for the existence of the dark matter and the matter/anti-matter asymmetry, and might be for more open questions in the elementary particle physics and cosmology.

The order of questions presented below have no special meaning.

1. Can the fourth family (to the observed three ones) with the masses close to or larger than 1 TeV exist at all, since the masses of the higgs, top quark and heavy bosons are all below 200 GeV?
2. If there are so many scalar fields carrying the weak and the hyper charges of the higss (three singlets with the quantum numbers (Q, Q', Y') and two times two triplets carrying the family quantum numbers), how can the masses of the heavy bosons, to which all the scalars contribute, be so low, ≈ 100 GeV?
3. If there are two kinds of charges, the family and the family members ones, why after the electroweak break the colour and the electromagnetic charges are the only conserved charges ?
4. Can there be at all two kinds of the spin connection fields and only one kind of the vielbeins?

5. How can the vector gauge fields at all be represented by spin connection fields and not, like in the Kaluza-Klein ordinary procedure, by vielbeins [5,1]?
6. The two $SO(d-1,1)$ groups - $SO(13,1)$ and $\widetilde{SO}(13,1)$ - have so many representations that there is not difficult to make a choice of the needed ones, but there are many more left.
7. Can the higher loops contributions, making all the off diagonal matrix elements of the mass matrices depending on the scalar singlets with the quantum numbers (Q, Q', Y') keep the symmetry of the three level (Eq. 9.6)?
8. And several others.

Let me try to answer the above questions.

1. Due to not accurate enough experimental data the prediction for the fourth family masses is, that they might be at around one TeV or above. Since for the masses of the fourth families the theory predicts the mass matrices which are very close to the democratic ones, although still keeping the symmetry of Eq. (9.6), the matrix elements of the mixing matrices for the fourth family members are very small. Correspondingly the predictions can hardly be inconsistent with the so far made measurements. I expect that the new experiments on the LHC will confirm the existence of the fourth family of quarks and leptons.
2. The question, which remains to be answered, is, whether the scalar fields belonging to either the three singlets with the quantum numbers $(Q, Q', Y)'$ or to the two times two triplets with respect to the family charges, all carrying the weak and the hyper charge of the higgs, do all together contribute only ≈ 100 GeV to the masses of heavy bosons after the electroweak break (Ref. [5], Eq. (14)). Although it looks like that under certain conditions (the masses and nonzero vacuum expectation values of these scalars) this is possible, the study is not yet finished and the answer is not yet convincing.
3. The answer to the third questions is that all the scalar fields with the space index $s = (7, 8)$ - all having the weak and the hyper charges of the higgs - with the family quantum numbers gain nonzero vacuum expectation values, causing correspondingly the breaking of all the family charges, while their weak and hyper charges cause the breaking of the weak and hyper charge. Correspondingly the only conserved charges after the electroweak break are the electromagnetic and colour charges.
4. The answer to the question number 4. is explained in details in Ref. [5], Sect. IV., and in App. A., Sect. 2.. A short answer to this question is that either γ^a's or $\tilde{\gamma}^a$'s transform in the flat space under the Lorentz transformations as vectors. The curved coordinate space is only one, while both kinds of spin connection fields are expressible in terms of the vielbeins, if there are no spinors (fermion) sources present, while spin connections of both kinds differ among themselves and are not expressible by vielbeins, if there are spinor sources present (Ref. [5], App. C., Eq. (C9)).
5. The relation of the vector gauge fields when they are expressed with the spin connection fields (as it is done in the *spin-charge-family* theory) and the vector gauge fields when they are expressed with the vielbeins (as it is usually in the

Kaluza-Klein theories) is explained in Refs. [5,1]. The vector (as well as the scalar) gauge fields - $A^{Ai}_{m} = \sum_{st} c^{Aist}\omega_{stm}$ - are (Ref. [5], Eq. (C9)) expressible with vielbeins. In Sect. II. of this Ref. the proof is presented that the vielbein $f^{\sigma}{}_{m} = i x^{\tau}\, \vec{\tau}^{A\sigma}_{\tau}\, \vec{A}^{A}_{m}$, where $A^{Ai}_{m} = \sum_{st} c^{Aist}\omega_{stm}$ and $\vec{\tau}^{A} = \vec{\tau}^{A\sigma}\, p_{\sigma} = \vec{\tau}^{A\sigma}{}_{\tau}$ $x^{\tau}\, p_{\sigma}$ (Eqs. (5-13) of Ref [5]). This is true when the space with $d \geq 5$ has the rotational symmetry, $x'^{\mu} = x^{\mu}$, $x'^{\sigma} = x^{\sigma} - i\vec{\alpha}^{1}(x^{\mu})\,\vec{\tau}^{A}(x^{\tau})\,x^{\sigma}$. This symmetry manifests in $f^{\sigma}{}_{s} = \delta^{\sigma}_{s}\, f$, for **any** f, which is the *scalar function of the coordinates* x^{σ} in $d \geq 5$.
For $f = (1 + \frac{\rho^2}{2\rho_0^2})$ the space is an almost S^{d-4} sphere, with one point missing, and the curvature R is equal to

$$R = \frac{d(d-1)}{(\rho_0)^2}\,. \tag{9.6}$$

6. One representation of $SO(13,1)$ contains just all the members of one family of quarks and leptons, left and right handed with respect to $d = (3+1)$, with the quantum numbers required by the *standard model*. Although it contains also anti-quarks and anti-leptons, after the break of the symmetry of space from $SO(13,1)$ (and simultaneously of $\widetilde{SO}(13,1)$) to $SO(7,1) \times SU(3) \times U(1)$ the transformations of quarks into leptons as well as those, which transform spins to charges, are at low energies not possible. All the scalar fields, which would cause such transformations, become too massive.
   All the scalar fields with the space index $s \geq 5$ have phenomenological meaning, either as scalars causing the electroweak break ($s = (7,8)$) or as scalars which contribute to the matter/anti-matter asymmetry of our universe. All the scalar, as well as the vector gauge fields, with the quantum numbers of the condensate, gain masses through the interaction with the condensate as discussed in Sect. II. of this talk and in Ref. [1,5].
7. In Ref. [30] the authors discussed this problem. Although in this paper the proof is not yet done, later studies show that the $U(1) \times SU(2) \times SU(2)$ symmetry remains in all orders of loop corrections.

9.5 Conclusions

I represent in this talk very briefly the so far obtained achievements of the *spin-charge-family* theory, which offers the explanation for all the assumptions of the *standard model*, with the families included, as well as some answers to the open questions in cosmology. Answering so far to so many open questions of the elementary particles and fields physics, this theory might be the right next step beyond the *standard model*.

The theory predicts that there are two triplet (with respect to the family quantum numbers) and three singlet (with respect to the family members quantum numbers) scalar fields, all with the weak and hyper charges of the higgs ($\mp\frac{1}{2}$, $\pm\frac{1}{2}$, respectively, with respect to the space index $s = (7,8)$), which explain the appearance of the scalar higgs and the Yukawa couplings. Some superposition of these scalar fields will be observed at the LH. The LHC will measure also the fourth family to the observed three ones.

I present in this talk also the most often asked questions about the validity of this theory, replying briefly to these questions and discuss the not yet solved problems of this theory.

9.6 Appendix: Short presentation of spinor technique [4,8,17,18]

This appendix is a short review (taken from [4]) of the technique [8,19,17,18], initiated and developed in Ref. [8], while proposing the *spin-charge-family* theory [2–4,6–13,1,29]. All the internal degrees of freedom of spinors, with family quantum numbers included, are describable in the space of d-anticommuting (Grassmann) coordinates [8], if the dimension of ordinary space is also d. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra and anticommuting with one another.The technique was further developed in the present shape together with H.B. Nielsen [19,17,18].

In this last stage we rewrite a spinor basis, written in Ref. [8] as products of polynomials of Grassmann coordinates of odd and even Grassmann character, chosen to be eigenstates of the Cartan subalgebra defined by the two kinds of the Clifford algebra objects, as products of nilpotents and projections, formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

App. B of Ref. [5]briefly represents the starting point [8] of this technique in order to better understand the Lorentz transformation properties of both Clifford algebra objects, γ^a's and $\tilde{\gamma}^a$'s, as well as of spinor, vector, tensor and scalar fields, appearing in the *spin-charge-family* theory, that is of the vielbeins and spin connections of both kinds, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and of spinor fields, family members and families.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{9.7}$$

If B is a Clifford algebra object, let say a polynomial of γ^a, $B = a_0 + a_a\,\gamma^a + a_{ab}\,\gamma^a\gamma^b + \cdots + a_{a_1 a_2 \dots a_d}\,\gamma^{a_1}\gamma^{a_2}\dots\gamma^{a_d}$, one finds

$$\begin{aligned}
&(\tilde{\gamma}^a B := i(-)^{n_B}\, B\gamma^a\,)\;|\psi_0\;>,\\
&\qquad B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,,
\end{aligned} \tag{9.8}$$

where $|\psi_0>$ is a vacuum state, defined in Eq. (9.22) and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's, for any d, even or odd, and I is the unit element in the Clifford algebra.

It follows from Eq. (9.8) that the two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects B.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$
\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,
\end{aligned}
\tag{9.9}
$$

$\{S^{ab},\tilde{S}^{cd}\}_- = 0\,, \{S^{ab},S^{cd}\}_- = i(\eta^{ad}S^{bc}+\eta^{bc}S^{ad}-\eta^{ac}S^{bd}-\eta^{bd}S^{ac})\,, \{\tilde{S}^{ab},\tilde{S}^{cd}\}_- = i(\eta^{ad}\tilde{S}^{bc}+\eta^{bc}\tilde{S}^{ad}-\eta^{ac}\tilde{S}^{bd}-\eta^{bd}\tilde{S}^{ac})\,.$

We assume the "Hermiticity" property for γ^a's

$$
\gamma^{a\dagger} = \eta^{aa}\gamma^a\,,
\tag{9.10}
$$

in order that γ^a are compatible with (9.7) and formally unitary, i.e. $\gamma^{a\,\dagger}\gamma^a = I$.

One finds from Eq. (9.10) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(9.9) that the two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$
\begin{aligned}
S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4,\\
S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned}
\tag{9.11}
$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$
\begin{aligned}
\Gamma^{(d)} &:= (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,\\
\Gamma^{(d)} &:= (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,.
\end{aligned}
\tag{9.12}
$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, subtituting γ^a's by $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(9.10) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$
\stackrel{ab}{(k)}:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,
\tag{9.13}
$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then

$$
\begin{aligned}
\gamma^a &= \stackrel{ab}{(k)} + \stackrel{ab}{(-k)}\,, \qquad \gamma^b = ik\eta^{aa}\,(\stackrel{ab}{(k)} - \stackrel{ab}{(-k)})\,,\\
S^{ab} &= \frac{k}{2}(\stackrel{ab}{[k]} - \stackrel{ab}{[-k]})
\end{aligned}
\tag{9.14}
$$

One can easily check by taking into account the Clifford algebra relation (Eq. (9.7)) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq. (9.9)) that the nilpotent $\stackrel{ab}{(k)}$ and the projector $\stackrel{ab}{[k]}$ are "eigenstates" of S^{ab} and $\tilde{S}^{ab}$

$$
\begin{aligned}
S^{ab}\stackrel{ab}{(k)} &= \frac{1}{2}\,k\stackrel{ab}{(k)}\,, & S^{ab}\stackrel{ab}{[k]} &= \frac{1}{2}\,k\stackrel{ab}{[k]}\,,\\
\tilde{S}^{ab}\stackrel{ab}{(k)} &= \frac{1}{2}\,k\stackrel{ab}{(k)}\,, & \tilde{S}^{ab}\stackrel{ab}{[k]} &= -\frac{1}{2}\,k\stackrel{ab}{[k]}\,,
\end{aligned}
\tag{9.15}
$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$
\begin{aligned}
&\gamma^a\stackrel{ab}{(k)} = \eta^{aa}\stackrel{ab}{[-k]},\;\gamma^b\stackrel{ab}{(k)} = -ik\stackrel{ab}{[-k]},\;\gamma^a\stackrel{ab}{[k]} = \stackrel{ab}{(-k)},\;\gamma^b\stackrel{ab}{[k]} = -ik\eta^{aa}\stackrel{ab}{(-k)}\,,\\
&\tilde{\gamma}^a\stackrel{ab}{(k)} = -i\eta^{aa}\stackrel{ab}{[k]},\;\tilde{\gamma}^b\stackrel{ab}{(k)} = -k\stackrel{ab}{[k]},\;\tilde{\gamma}^a\stackrel{ab}{[k]} = \;i\stackrel{ab}{(k)},\;\tilde{\gamma}^b\stackrel{ab}{[k]} = -k\eta^{aa}\stackrel{ab}{(k)}\;.
\end{aligned}
\tag{9.16}
$$

From Eq.(9.16) it follows

$$
\begin{aligned}
S^{ac}\stackrel{ab}{(k)}\stackrel{cd}{(k)} &= -\frac{i}{2}\eta^{aa}\eta^{cc}\stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, & \tilde{S}^{ac}\stackrel{ab}{(k)}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{aa}\eta^{cc}\stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
S^{ac}\stackrel{ab}{[k]}\stackrel{cd}{[k]} &= \frac{i}{2}\stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, & \tilde{S}^{ac}\stackrel{ab}{[k]}\stackrel{cd}{[k]} &= -\frac{i}{2}\stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
S^{ac}\stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa}\stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, & \tilde{S}^{ac}\stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa}\stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
S^{ac}\stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc}\stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, & \tilde{S}^{ac}\stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc}\stackrel{ab}{(k)}\stackrel{cd}{[k]}\;.
\end{aligned}
\tag{9.17}
$$

From Eq. (9.17) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$
\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, & &\stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\stackrel{ab}{[k]}\,, & &\stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\stackrel{ab}{[-k]}\,, & &\stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, & &\stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, & &\stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, & &\stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, & &\stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, & &\stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, & &\stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, & &\stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, & &\stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, & &\stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\;.
\end{aligned}
\tag{9.18}
$$

We recognize in Eq. (9.18) the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\overset{ab}{(k)}$ and $\overset{ab}{[k]}$, respectively. Defining

$$\overset{ab}{(\tilde{\pm i})} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \overset{ab}{(\tilde{\pm 1})} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{9.19}$$

one recognizes that

$$\begin{aligned}
&\overset{ab}{(\tilde{k})}\,\overset{ab}{(k)} = 0\,, \quad \overset{ab}{(\tilde{-k})}\,\overset{ab}{(k)} = -i\eta^{aa}\;\overset{ab}{[k]}\,,\\
&\overset{ab}{(\tilde{k})}\,\overset{ab}{[k]} = i\;\overset{ab}{(k)}\,, \quad \overset{ab}{(\tilde{k})}\,\overset{ab}{[-k]} = 0\,.
\end{aligned} \tag{9.20}$$

Recognizing that

$$\overset{ab}{(k)}^{\dagger} = \eta^{aa}\;\overset{ab}{(-k)}\,, \quad \overset{ab}{[k]}^{\dagger} = \overset{ab}{[k]}\,, \tag{9.21}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned}
&< \overset{ab}{(k)}^{\dagger}\;\overset{ab}{(k)} > = 1\,,\\
&< \overset{ab}{[k]}^{\dagger}\;\overset{ab}{[k]} > = 1\,.
\end{aligned} \tag{9.22}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\overset{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(9.11)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{aligned}
&\overset{0d}{(k_{0d})}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
&\overset{0d}{[-k_{0d}]}\overset{12}{[-k_{12}]}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
&\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
&\qquad\vdots\\
&\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{[-k_{d-1\;d-2}]}\;|\psi_0>\\
&\overset{od}{(k_{0d})}\overset{12}{[-k_{12}]}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
&\qquad\vdots
\end{aligned} \tag{9.23}$$

All the states have the same handedness Γ, since $\{\Gamma, S^{ab}\}_- = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (9.11)) generate all the states of one representation, $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra (Eq. (9.11)) generate the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq. (9.11)) of the algebra S^{ab} and $\tilde{S}^{ab}$

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}\,,
\end{aligned}
\tag{9.24}
$$

a left handed ($\Gamma^{(13,1)} = -1$) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$
\begin{aligned}
&\stackrel{03}{(+i)}\stackrel{12}{(+)}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\;|\psi_0\rangle = \\
&\qquad\frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)|| \\
&\qquad\qquad(\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi_0\rangle\,.
\end{aligned}
\tag{9.25}
$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (9.11)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (9.11)), generate families from the starting u_R quark, transforming the u_R quark from Eq. (9.25) to the u_R of another family, keeping all of the properties with respect to S^{ab} unchanged. In particular, $\tilde{S}^{01}$ applied on a right handed u_R-quark from Eq. (9.25) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$
\begin{aligned}
\tilde{S}^{01}\;&\stackrel{03}{(+i)}\stackrel{12}{(+)}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(-)}\,\stackrel{13\,14}{(-)} = \\
&\qquad -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(-)}\,\stackrel{13\,14}{(-)}\;.
\end{aligned}
\tag{9.26}
$$

Below some useful relations [6] are presented

$$
\begin{aligned}
&N^{\pm}_{+} = N^{1}_{+} \pm i\,N^{2}_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\,N^{2}_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\stackrel{03}{(\tilde{\mp} i)}\stackrel{12}{(\tilde{\pm})}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{(\tilde{\pm} i)}\stackrel{12}{(\tilde{\pm})}\,,\\
&\tau^{1\pm} = (\mp)\;\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\;\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\;\stackrel{56}{(\tilde{\pm})}\stackrel{78}{(\tilde{\mp})}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\;\stackrel{56}{(\tilde{\mp})}\stackrel{78}{(\tilde{\mp})}\;. \qquad (9.27)
\end{aligned}
$$

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
		(Anti)octet, $\Gamma^{(1,7)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
17	u_R^{c3}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
25	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

Continued on next page

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
		(Anti)octet, $\Gamma^{(1,7)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons										
41	$\bar{d}^{\bar{c}2}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
...												
49	$\bar{d}^{\bar{c}3}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
...												
57	$\bar{e}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 9.4. The left handed ($\Gamma^{(13,1)} = -1$) ($= \Gamma^{(7,1)} \times \Gamma^{(6)}$) multiplet of spinors - the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, is presented in the massless basis using the technique presented in App. 9.6. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^{13} = \pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks of three colours ($c^i = (\tau^{33}, \tau^{38})$) with the "spinor" charge ($\tau^4 = \frac{1}{6}$) and the colourless left handed weak charged leptons and the right handed weak chargeless leptons with the "spinor" charge ($\tau^4 = -\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. It contains also the states of opposite charges, reachable from particle states by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$, presented in Refs. [22,23]. The vacuum state, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [1,29,4].

I present at the end one Weyl representation of $SO(13+1)$ and the family quantum numbers of the two groups of four families.

One Weyl representation of $SO(13+1)$ contains left handed weak charged and the second $SU(2)$ chargeless coloured quarks and colourless leptons and right handed weak chargeless and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from Ref. [22].

The eight families of the first member of the eight-plet of quarks from Table 9.4, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 9.5 [4]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,2}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$u^{c1}_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\mid\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 9.5. Eight families of the right handed u^{c1}_R (9.4) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$), and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (9.27). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (9.27)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

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