

ON THE EXPLOITATION OF MECHANICAL ADVANTAGE NEAR ROBOT SINGULARITIES

Jon Kieffer

Engineering Department, The Faculties, Australian National University
Canberra ACT 2601 Australia

Jadran Lenarčič

Jozef Stefan Institute, University of Ljubljana
Jamova 39, 61111, Ljubljana, Slovenia
E-mail: jadran.lenarcic@ijs.si

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Since the earliest days of robotics research when it was recognized that kinematic singularities physically hamper the free manipulation of objects in task space, there has been a popular consensus that singular configurations are unsuitable for practical use and should be avoided. At best they may be included with the expectation of gracefully-degraded performance in their vicinity [1,2]. In this article we question the validity of such conclusions, citing examples that show how humans use singularity configurations of their limbs to gain mechanical advantage, and investigate the possibility of obtaining similar benefits in robotic systems. It is shown that minimization of joint torques in redundant systems leads to human-like behavior that favors singularities, but that stable implementation of such behavior requires a strategy which gives the robot more autonomy with respect to timing task execution. Application of such a strategy to a 2R robot performing static lifting is considered in detail.

1 Introduction

A fundamental objective of robot design and control is to mask the characteristics of the underlying machine to provide an abstraction that is easily understood and programmed. To ease task specification we would like a manipulator arm (walking machine leg) to behave as a disembodied hand (foot) whose motion, force, or impedance [3] can be arbitrarily programmed to fit a task. For convenience it is sensible to specify tasks in a cartesian space that we and our sensors can readily identify with. To this end, robot mechanisms, actuators, and control systems have been designed and implemented with varying degrees of success.

Unfortunately, all real robots have limitations in speed, workspace, and force capability that compromise the ideal abstraction of

an arbitrarily-programmable disembodied hand (foot). To further complicate matters, these limitations are interrelated in a complex way for all robots, except gantry robots. To a large extent these problems can be ameliorated through design and control. Mechanical design methodologies have been developed to optimize workspace size and shape [4], minimize the directional distortion of motion and force capability [5] and even to move kinematic singularities, which severely distort directional force and motion characteristics, out of the intended workspace [6]. In addition, researchers have turned to redundant manipulators whose extra degrees of freedom offer new possibilities for avoiding singularities and improving the directional uniformity of motion and force characteristics, now called dexterity [7], through control. Nevertheless, by their vary nature, articulated robots distort cartesian force and motion

characteristics and include configurations of ultimate distortion: kinematic singularities.

Our objective in this paper is to demonstrate that kinematic singularities and ill-conditioned configurations offer untapped potentials that may be exploited through an alternative model (abstraction) for robot behavior that is better suited to taking advantage of these potentials. In doing so we hope to provoke increased interest in kinematic singularities as well as in alternative strategies for programming useful robot behavior.

2 Background

Kinematic singularities can be conceptualized as configurations of local folding in the mapping of a toroidal joint space onto an dissimilar manifold of spatial end-effector positions/orientations [8]. For the case of a 2R planar robot, two joint positions (elbow-left and elbow-right) map onto the same end-effector position and folding of the joint space manifold with respect to the mapping accounts for the transition from inside the workspace, where two joint solutions generally exist, to outside the workspace, where there are no solutions. For the special case of equal link length geometry, the inside workspace boundary (normally a circle) collapses to a point which is the image of a one parameter locus of singular configurations in joint space. Physically, the end point will be in the workspace center whenever the two links fold over each other.

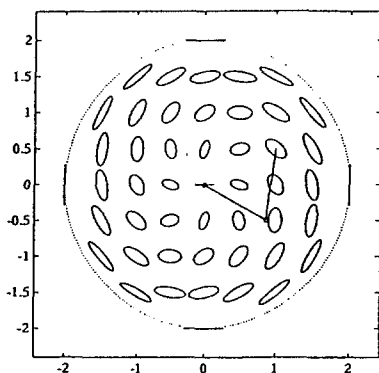


Figure 1: Velocity ellipses for 2R robot with equal link lengths.

Although singularity configurations are sparse compared to regular configurations, their influence extends to significant portions of the sur-

rounding workspace, distorting the directional properties of nearby regular configurations in proportion to their proximity. As illustrated in Fig.1, uniform joint velocity capability, represented by circles in the joint space, become ellipses in the task space and collapse in at least one direction near singularity configurations. This is true of both force and motion characteristics and extends to dynamic properties, such as effective inertia, as well [9, 10]. It is an exaggeration to say that singularities cause this behavior, they are only extreme manifestations of it.

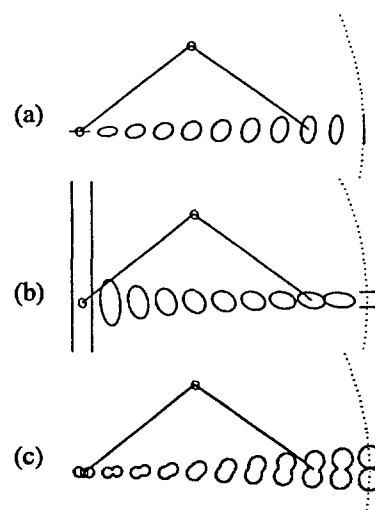


Figure 2: (a) Velocity ellipses, (b) force ellipses, and (c) torque "peanuts" along radial workspace section (2R robot with equal link lengths).

A more complete understanding can be obtained by considering the relation between the force and motion distortions. By comparing force and velocity ellipses along a radial section of the 2R manipulator's workspace (Fig.2(a-b)) we see that the distortions are complementary. At the workspace boundary, joint actuators can support infinite radial loads, but radial velocities become impossible. Conversely, tangential velocities at the workspace boundary are enhanced and tangential force capability is diminished. This complementary property persists, with diminished distortion, at regular configurations and extends to general spatial manipulators as well. An alternate way to represent directional force characteristics is to plot the norm of joint torques needed to support a unit tip force in each direction. As shown in Fig.2(c) this results in "peanut"-shaped figures that, in contrast to force ellipses, remain

bounded at singularities.

It can be shown that force and velocity ellipses share the same principle directions, but have reciprocal principle axes lengths [11]. For this reason it is convenient to consider only velocity ellipses, henceforth called manipulability ellipses, with the understanding that both force and velocity characteristics can be inferred from them.

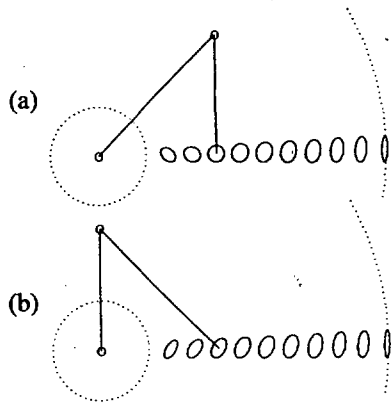


Figure 3: Manipulability ellipses: (a) the only 2R robot with isotropic configurations $L_1 = L_2\sqrt{2}$, (b) 2R robot without isotropic configurations $L_2 = L_1\sqrt{2}$.

Truly uniform properties only occur at so-called isotropic configurations [12], which only exist for special robot geometries. Figure 3 compares the manipulability ellipses for two robots with identical workspaces. The first robot is the only 2R geometry that has isotropic configurations.

3 Potentials for Exploitation

The idea that singularities may be useful was first suggested by K.H. Hunt [13] who recognized that, at singularity configurations, the space of instantaneous joint screws fails to span the entire screw space. This enables any reciprocal wrench applied to the end effector to be transmitted through the structure without loading any actuators. This means that there is a potential for applying or withstanding extremely high loads in certain directions at singularities. Although this argument has been often repeated, and suggested as useful for practical application, e.g. drilling [14], to the authors' knowledge it has yet to be applied in practice.

Nevertheless, it is easy to verify that humans take advantage of this principle when walking or

standing, for example. To see this consider the 2R planar manipulator as a approximation of the human leg. When outstretched to a singularity configuration, radial loads can be transmitted through the structure without loading the joint actuators. This is how humans avoid muscle fatigue while supporting their weight when walking or standing.

Although effective for walking, this example also points out the specialization of motion required to take full advantage of singularities which are local and directionally-oriented within the workspace. In walking, the endpoint (foot) only moves tangentially around the workspace boundary where singularities support the radially-directed load. If the 2R manipulator were applied to drilling for example, it would be necessary to locate the end-effector and workpiece near either the center or the boundary of the workspace and to align them appropriately with respect to the direction of heavy loading. Furthermore, once this is done, we should expect to apply only small motions in the direction of heavy loading (drilling) because large ones would move robot too far from the region of maximum mechanical advantage. Obviously, this approach would be awkward to apply in practice.

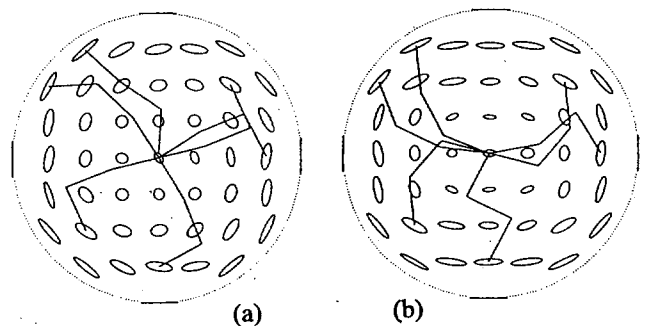


Figure 4: Manipulability ellipses for a 3R robot: (a) configurations optimized for maximum dexterity (minimum condition number), (b) configurations optimized to minimize the norm of joint torques for a vertical tip load.

Ideally, we would like to achieve maximum mechanical advantage in any chosen direction at any end-effector position in the workspace (arbitrary placement and alignment of singularities). However, because this seems impossible, we might settle for extending the influence of singularities to larger portions of the workspace. In principle,

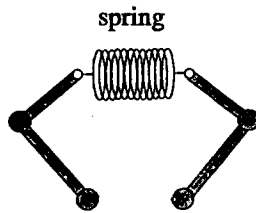


Figure 5: Archer modeled by planar dual arms with a constant force spring.

this can be achieved through redundancy: analogous to the way redundancy can be exploited to increase dexterity (uniformity in properties) throughout the workspace [9], it can also be exploited to propagate nonuniformity of properties (i.e., ill-conditioned configurations and singularities) throughout the workspace. This principle is confirmed in Fig. 4 which compares manipulability ellipses for a 3R planar robots whose configuration at each workspace position has been optimized to (a) maximize uniformity (minimize condition number), and (b) minimize the joint torques needed to balance a vertical tip load.

4 Human Exploitation of Singular Configurations

Further insight into potentials for robotic applications can be gained by considering how humans use singularities.

In performing the so-called "clean and jerk", weight lifters take advantage of the singularity configurations of their arms. From the floor, the weight is thrown upward using forces generated by leg muscles and transmitted through their arms which are stretched downward in a singular configuration. The weight is then caught with arms folded in a second singularity configuration that is similar to the center point singularity of the 2R manipulator. After a pause, the weight is again thrown upward using leg muscles and caught with arms stretched overhead in a third singularity configuration. Obviously, the weight is too heavy for the arms to manipulate except near these singularities and the lifting process is one of ballistically transferring the weight between them.

These examples show that in walking and

weightlifting humans use singularity configurations mainly to support heavy loads, rather than to apply forces. In this sense, singularities transform the mechanism (limbs) into a structure with respect to certain directions of applied loads.

In drawing a bow, archers also make use of ill-conditioned configurations and singularities to minimize muscle effort in their arms. As an explanation, consider the planar dual arm system shown in Fig.5 as approximation of an archer. The bow has been modeled by a constant force spring and the system has three degrees of redundancy with respect to extending the spring. Fig 6(a) shows the pseudoinverse solution which minimizes joint motion. Figure 6(b) shows an alternate solution which minimizes joint torques (section 7) and is obviously similar to human behavior. Figure 7(a) shows plots that compare joint torques versus spring length for the two solutions, confirming that the second solution substantially reduces effort.

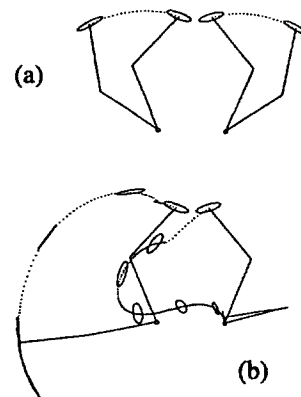


Figure 6: Archer solutions: (a) minimal joint motion (b) minimal joint torques.

The manipulability ellipsoids in Fig.6(b) show that singularity configurations play an important role in minimizing effort. Because the initial configuration is not well suited to the spring load, the flattened ellipses are first reoriented by self-motion. The resulting increase in mechanical advantage provides a dramatic decrease in torque (Fig.7(a)). The spring is then extended with the left arm outstretched and the right arm drawing backward toward its center point. The final posture can be sustained without effort due to singularity configurations of both arms. Although this model does not consider joint limits or in-

terference with the body, it is clear that human archers use essentially the same strategy, taking advantage of singularity configurations to minimize effort and to reduce fatigue while aiming.

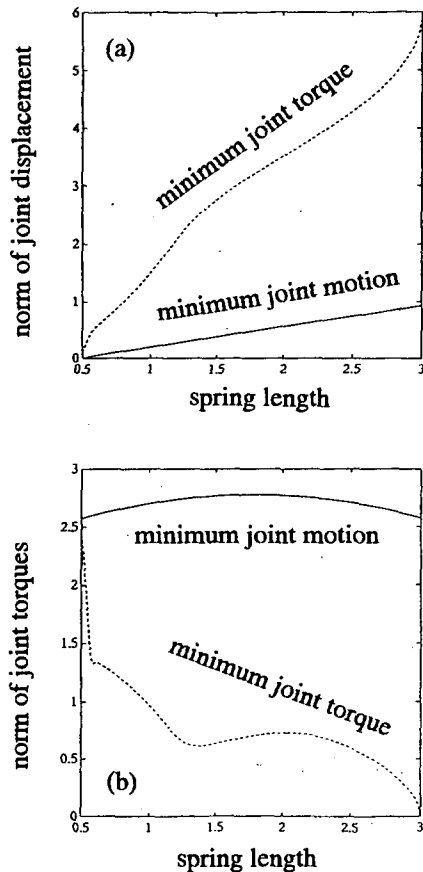


Figure 7: Comparison of archer solutions: (a) norm of joint torques, (b) joint displacement versus spring length for archer solutions.

In addition to confirming the importance of redundancy, this example shows that ill-conditioned configurations near singularities are well suited to applying forces.

5 The Need for Temporal Autonomy

As stressed in the introduction, kinematic singularities are usually regarded as nuisances that should be avoided because they confound the planning and execution of timed-end effector trajectories.

Exactly at a singularity, the Jacobian matrix relating joint velocities to the cartesian end-effector velocity loses rank and the usual joint rate

solution becomes indeterminate. Although this mathematical problem can be overcome by an untimed parametric formulation [15] which considers higher-order kinematics if necessary [16], there still remains the underlying physical problem of joint rates becoming unbounded for certain directions of finite end-effector velocity. More importantly, joint rates also become unreasonably large for substantial regions of ill-conditioned regular configurations close to singularities.

Physically, the situation can be understood from the flattening of the manipulability ellipses near singularities: small end-effector velocities (displacements) in the flattened direction require large joint rates (displacements). Although, this is generally viewed as a problem, it is actually the definition of mechanical advantage. Large end-effector velocities in the flattened direction are simply not possible, but large forces are. From this perspective, redundancy can be interpreted as a mechanism for changing transmission ratio: ill-conditioned configurations provide low transmission ratios with respect to motions in flattened directions and high transmission ratios with respect to motions in the lengthened directions.

The main obstacle to making use of ill-conditioned configurations is the complexity of velocity limitations which make it very difficult to plan the timing of end-effector motions. This is illustrated in Fig.7(b) which plots joint displacement versus spring length for the two archer solutions of Fig 6. The minimum torque solution is far less uniform and requires far more joint motion, especially near singularities. This makes it almost impossible to prescribe the rate of spring extension without exceeding joint rate limits.

This means that the ideal abstraction of a disembodied hand that can be arbitrarily programmed within simple velocity bounds must be compromised and replaced with a more sophisticated view: one that includes more consideration of the machine that moves the hand. But, rather than burden the task planner with complicated details, it is better to simply relieve the planner of timing considerations altogether: let the planner specify the geometry of task execution, but let the robot control system determine timing in accordance with the robot's capability. This provides a simple abstraction which is similar to human supervision: tasks are assigned, but precise timing

of their execution is not dictated. For tasks such as welding, which require more-or-less strict velocity control along a path, this strategy may not be appropriate. But there are many other tasks in which timing is of secondary importance and can be sacrificed in favor of increased mechanical advantage.

6 Temporally-Autonomous Path Control

Hollerbach and Suh [17], resolved redundancy to minimize joint torques subject to robot dynamics. In doing so, they found the robot was likely to blunder into a region near a singularity, where joint rates become unacceptably large. In this section we will consider a similar, but simpler, problem: resolving redundancy to minimize joint torques in a robot subject to a static end effector forces, ignoring dynamics.

Various investigators [18-20] have proposed the use of generalized inverses of the form

$$\dot{\theta} = J^+ \dot{r} - (I - J^+ J) \nabla H \quad (1)$$

which minimizes a potential function $H(\theta)$ subject to the kinematic constraint $J\dot{\theta}(t) = \dot{r}(t)$. Here $\dot{\theta}(t)$ represents the joint rates, $r(t)$ represents a timed end-effector trajectory, $J(\theta)$ is the Jacobian matrix, $J^+ = J^T(JJ^T)^{-1}$ is the Moore-Penrose generalized inverse, and ∇H is the gradient of $H(\theta)$.

Taking $H(\theta) = \frac{1}{2} \tau^T \tau$, where $\tau(\theta) = J^T f$ represents the configuration-dependent joint torques resulting from the application of a static tip force f , we obtain an inverse rate solution which minimizes the norm of joint torques, subject to execution of the timed-trajectory $r(t)$. Unfortunately this solution fails. As the robot moves toward singularities in an effort to minimize joint torques, larger and larger joint rates are needed to execute the timed-trajectory, $r(t)$. Eventually, joint rate limits are exceeded and control is lost. However, this is not the fault of singularities, which in fact provide the mechanism for minimizing joint torques. Rather, as discussed in the previous section, it is the fault of an inappropriate timing specification which does not consider joint rate limitations.

As an alternative strategy, consider that an un-timed trajectory specification $r(\lambda)$ is given, and let timing be determined by the controller. The generalized inverse can then be expressed

$$\dot{\theta} = J^+ r' \dot{\lambda} - \dot{\mu} (I - J^+ J) \nabla H \quad (2)$$

where $\dot{\lambda}$ and $\dot{\mu}$ are positive scalars determined on-line by the controller. The first term provides trajectory execution while the second provides self-motion that minimizes joint torques. In determining $\dot{\lambda}$ and $\dot{\mu}$ at any instant, the controller can govern the relative priority of joint torque minimization versus trajectory execution while simultaneously ensuring that joint rates remain within physically-realizable bounds.

A detailed exploration of such a strategy is beyond the scope of this paper and certainly includes significant obstacles, including numerical problems in the close proximity of singularities where the matrix JJ^T (needed to compute $J^+ = J^T(JJ^T)^{-1}$) loses rank. However, further insight can be gained by examining a crude example of this strategy applied to a 2R robot lifting problem.

7 Static Lifting with a 2R Robot

Figure 8(a) depicts a 2R robot lifting a weight. Ignoring the x -coordinate of the tip, the system has one degree of redundancy with respect to lifting in the y -direction. The problem is to lift the weight while minimizing joint torques. The links are considered massless and the weight static.

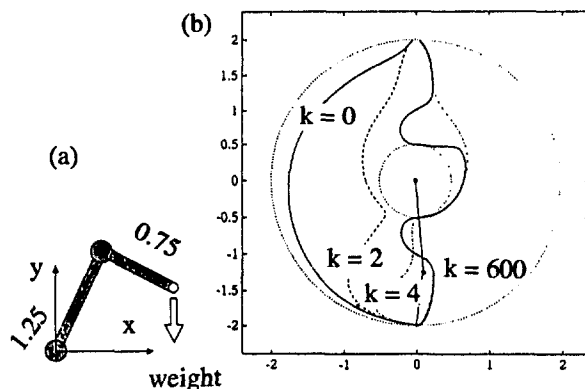


Figure 8: (a) 2R weightlifting robot, (b) tip trajectories for $k=0, 2, 4$, and 600.

One approach to generating an optimal trajectory is to perform a sequence of constant- y slice optimizations beginning a $y = -2$ and proceeding upward. However, to get more insight into the obstacles to control, we will consider a control-oriented approach based on the previous section.

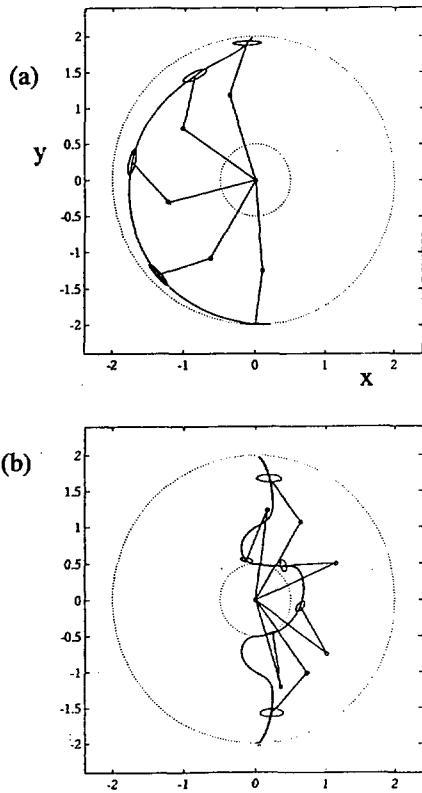


Figure 9: Weightlifting robot motions: (a) $k=0$: minimal joint motions, (b) $k=600$: minimal torques.

In particular, trajectories will be generated based on integrating equation (2) with incremental steps of constant length δ in joint space. Let each integration step be defined as follows.

$$\Delta\theta = \delta \frac{\nu}{|\nu|} \tag{3}$$

where

$$\nu = J^+ r' - k(I - J^+ J)\nabla H \tag{4}$$

Here k provides weighting with respect to torque minimization versus trajectory execution. Since the operator projects the torque gradient ∇H onto the null space of J , there is no chance of backward motion along the trajectory, $r(\lambda)=y(\lambda)=\lambda$. If $k(I - J^+ J)\nabla H$ is large with

respect to $J^+ r'$, then most to the joint motion results in robot self-motion to reduce torques. Conversely, if $J^+ r'$ is large relative to $k(I - J^+ J)\nabla H$, then most of the joint motion is directed toward trajectory execution.

Figure 8(b) plots tip trajectories which result for four values of k . Weighting factors $k=0$ and $k=600$ represent extremes that minimize joint motion, and joint torques, respectively. As an aid to visualization, Figs. 9(a) and 9(b) show the robot and manipulability ellipses for intermediate positions of the robot executing the $k=0$ and $k=600$ trajectories respectively.

In minimizing joint torques, trajectories with large k take increased advantage of singularities near the inner circular workspace boundary. Trajectory $k=4$ comes very close to a singularity on the bottom of the inner workspace boundary while $k=600$ finds two singularities (bottom and top of inner workspace boundary). The increase in mechanical advantage which the singularities provide is apparent from Fig.10 which plots the norm of joint torques versus lifting height y .

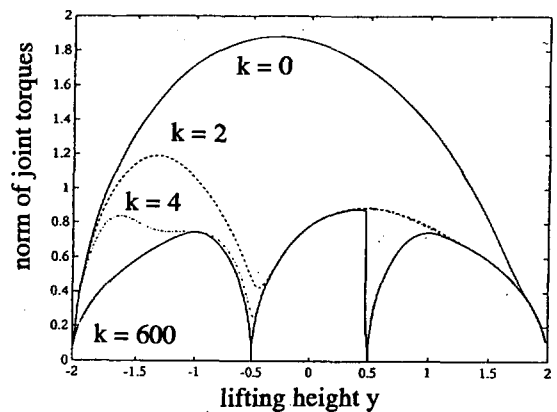


Figure 10: Norm of joint torques versus lifting height for weightlifting robot.

Some insight into the trajectory optimization can be gained from Figs. 11(a) and 11(b) which plot the surface of the joint torque norm over the workspace for elbow-left and elbow-right configuration branches, respectively. However, care must be taken, since the configuration branch can change when a singularity is encountered as is the case for the $k=600$ trajectory shown.

Perhaps better insight can be gained by considering the trajectories in joint space. Figures 12(a) and 12(b) show the trajectories superimposed on

contour plots of the torque norm, $|\tau(t)|$, and lifting height, $y(\theta)$, respectively. With increasing k , the algorithm takes increased advantage of singularities to minimize torques while simultaneously increasing y to perform the lifting task. It is also apparent that increasing k increases the joint-space path length, minimizing the change in y with respect to incremental joint motion, especially near singularities. This implies that timing of the trajectory in task space (i.e. $y(t)$) becomes more difficult or impossible with increasing k .

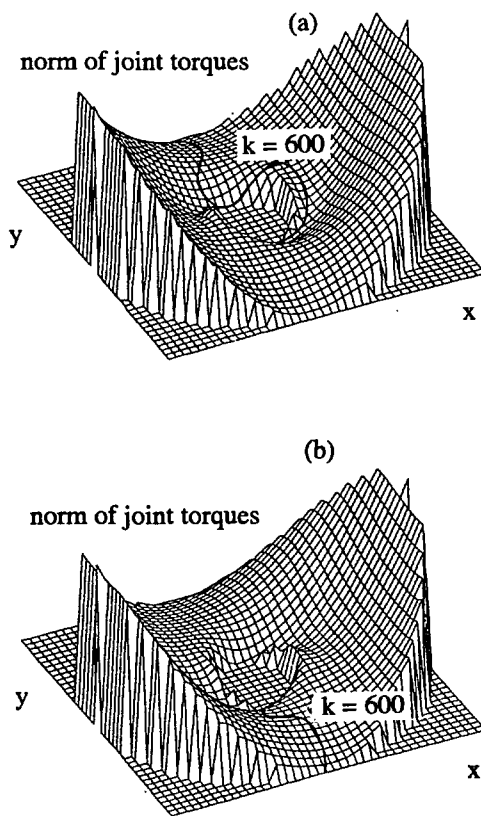


Figure 11: Surface plots of joint torque norm versus tip position for weightlifting robot: (a) elbow-left, (b) elbow-right.

In joint space, $\theta(t)$ can be prescribed relatively freely, except exactly at singularities where discontinuities imply that the robot must decelerate to a stop before accelerating onwards along the path. This means that timing, even in joint space, requires careful consideration if one is to fully exploit singularities. However, from the $k=4$ trajectory it is also clear trajectories only have to come close to singularities to gain the majority of torque reduction (Fig 10).

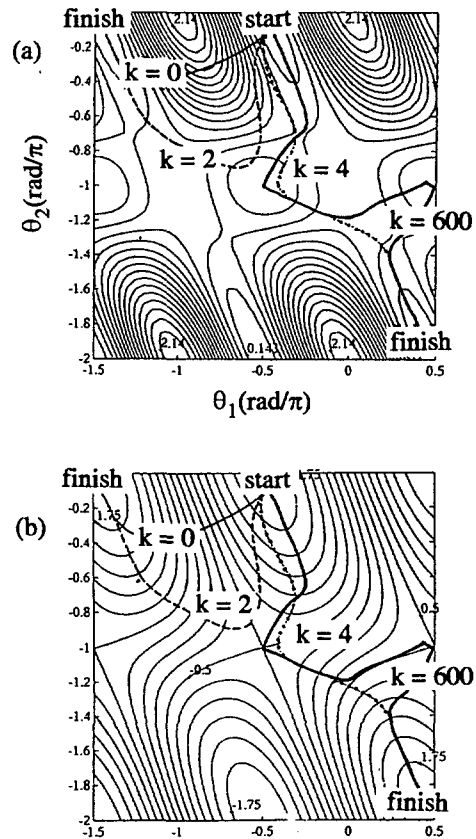


Figure 12: Joint trajectories over contour plots of (a) joint torque norm, and (b) lifting height y .

8 Conclusion

The assertion that kinematic singularities of articulated mechanisms offer untapped potentials seems indisputable from at least two points of view: (1) conventional insistence on uniform task-space velocity capability obviously limits the usable workspace and fails to take full advantage of configurations that offer extremes in motion or force capability which, by their nature, are directionally-biased, and (2) it is easy to cite examples of humans using singularity configurations to gain mechanical advantage.

In this paper, we have identified timing as the major obstacle to exploiting singularities in redundant robots and have proposed an alternative control strategy in which timing is determined online in accordance with machine limitations. Simple examples show that significant reductions in joint torques can be obtained, suggesting that

weak actuation can be compensated with more sophisticated control that exploits singularities and ill-conditioned configurations.

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