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Exciting Baryon Resonances with Meson Photoproduction*

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Abstract. Light hadron spectroscopy is still an exciting field in nuclear and particle physics. Even 50 years after the discovery of the Roper resonance and more than 30 years after the pioneering work of Hoehler and Cutkosky many questions remain for baryon resonances. Nowadays the main excitation mechanism is photo- and electroproduction of mesons, studied at electron accelerator labs as MAMI, ELSA and JLab. In a combined effort, pole positions and residues are searched from partial waves, which are obtained in a partial wave analysis from recently measured polarization observables using analytical constraints from fixed-t dispersion relations. Special emphasis is placed on the pole structure of baryon resonances on different Riemann sheets.

1 Introduction

Fifty years ago the Roper resonance was found in partial wave analysis (PWA) of pion nucleon scattering [1]. In the following decades more than 30 N and Δ resonances were also found in PWA. For many of these resonances the properties are still uncertain and need to be improved in more precise experiments, which is nowadays only possible with photon and electron beams. Due to the helicity nature of the photon in the initial state, the number of invariant amplitudes is twice as large and the number of observables is a factor of four larger than in pion nucleon scattering. Therefore, a model independent determination of the partial waves and the underlying nucleon resonances is far more involved and improved analysis tools are required.

2 Resonances as poles on different Riemann sheets

Thresholds and resonance positions are commonly used as the most important and physical properties of partial waves in scattering and production reactions. However, at a closer look, resonance positions described in a Breit-Wigner ansatz appear different in different analyses, especially when also different reaction channels are analyzed. Also production thresholds, as $\pi\pi N$ or $\pi\pi\pi N$ are not the most relevant positions, where new dynamics is observed. E.g. at the $\pi\pi N$ threshold,

* Talk presented by L. Tiator

$W = m_p + 2m_\pi = 1208$ MeV, no single partial wave shows a signature of inelasticity, even as this process is kinematically allowed.

The relevant properties of partial waves are the pole positions and well selected real or complex branch points (b.p.). Pole positions have long been realized as the fundamental resonance parameters that are not influenced by background contributions, which will be different for different reaction channels. In photoproduction, background is very small for η or η' production, but large or even very large for π and K production. In the latter case, the background is not very well known, even the coupling constants of the Born terms for (γ, K) are quite uncertain.

Real branch points coincide with thresholds, like $\pi N, \eta N, \eta' N$, complex branch points appear as effective branch points for 3- and more-body final states. A very important complex branch point is the $\pi\Delta$ b.p. with $W_{bp} = 135 + 1210 - 50i = (1345 - 50i)$ MeV and also the ρN b.p. with $W_{bp} = 763 - 72i + 938 = (1701 - 72i)$ MeV. These branch points play an important role in the P_{11} partial wave, other partial waves are also influenced by less amount. Their role is especially pronounced, if a pole position gets close to such a complex b.p., which is the case for $P_{11}(1440)$ with $W_p = (1365 - 95i)$ MeV and $P_{11}(1710)$ with $W_p = (1720 - 115i)$ MeV.

This knowledge is used in the Laurent-plus-Pietarinen expansion (L+P) of partial waves, recently developed by the Zagreb/Tuzla group and applied so far to πN scattering, pion photoproduction and coupled π, η channels [2, 3]. Photoproduction of η and η' and pion electroproduction analyses are in progress.

For a given partial wave, e.g. for $\pi N \to \pi N$ or $\gamma N \to \pi N$, $\alpha = \{L, J, T\}$ with angular momentum L, spin J and isospin T, the partial wave amplitude can in general be split in a resonance and a background part, where the background part is simply everything, that is missing in the resonance ansatz,

$$t_\alpha(W) = \frac{\beta\,\Gamma/2}{M - W - i\,\Gamma/2}\, e^{i\phi} \;+\; \text{b.g.}(W)\,.$$

In general, Γ, β, ϕ can be functions of W, in particular for a very simple case

$$\Gamma(W) = \frac{q_{\pi N}(W)}{q_{\pi N}(M)}\,\frac{M}{W}\,\beta_{\pi N}\,\Gamma_{total} \;+\; \{\pi\pi N,\ \pi\Delta,\ \eta N \;+\; \cdots\}$$

with the pion c.m. momentum

$$\begin{aligned} q_{\pi N}(W) & \\ &= \frac{\sqrt{(W^2 - (m_p + m_\pi)^2)(W^2 - (m_p - m_\pi)^2)}}{2W} \\ &= \frac{\sqrt{W - (m_p + m_\pi)}\,\sqrt{W + (m_p + m_\pi)}\,\sqrt{W - (m_p - m_\pi)}\,\sqrt{W + (m_p - m_\pi)}}{2W}\,. \end{aligned}$$

In the latter expression four square-root branch points show up, where only the first one is in the physical region and is the most important branch point for all partial waves.

The square-root function $\sqrt{x}$ has a branch point at $x = 0$ and is defined on two Riemann sheets (R.S.). Usually the branch cut (b.c.) is chosen to the left as in FORTRAN, C++ or Mathematica. However, any other direction can be freely chosen, according to the convenience of a particular application. In hadronic scattering processes it is often used to the right and an especially convenient way is a branch cut downwards along the negative imaginary axis. For all those definitions the formulas remain the same, except for the square-root function which has to be replaced accordingly by

$$\begin{aligned} \sqrt{z} &\to \sqrt{z} && \text{b.c. to the left,} \\ &\to i\sqrt{-(z+i\varepsilon)} && \text{b.c. to the right,} \\ &\to \sqrt{i}\sqrt{-iz} && \text{b.c. downwards.} \end{aligned}$$

It is important to note, that the $i\varepsilon$ term in Eq. (6) is needed to assure that the real axis (physical axis) belongs to the first Riemann sheet.

In principle it makes no difference, which angle for the branch cut is chosen. Traditionally most often it is the b.c. on the positive real axis to the right side. However, this convention often leads to confusions about the different Riemann sheets, as all sheets starting from real b.p. will overlap. Even b.c. turning into different directions at each different branch point are allowed. In the following we have chosen all branch cuts downwards, as this was once suggested by Dick Arndt [4]. In this convention, all resonances appear as poles on the lower half-plane in the **first** Riemann sheet. In his 'bible' [5], Hoehler defines the resonances as poles in the lower half-plane of the second Riemann sheet, and this convention, where all cuts are drawn to the right, is mostly used in the literature. However, one has to be very careful in numbering the R.S. when more than one threshold is open. Then the second R.S. is always the sheet, which is entered by a direct path from the physical axis down into the next R.S. by crossing one or more cuts. In our notation we give in addition to the somewhat arbitrary numbering also the $\pm$ signs for each branch cut, which makes the definition unique.

Generally with each new branch point the number of Riemann sheets gets doubled and of course all R.S. exist in the whole complex energy plane, also below the branch points. For a partial wave with 3 decay channels one must consider in principle 8 Riemann sheets. But less important decay channels are usually ignored in order to get the number of R.S. smaller. For the $\Delta(1232)$ in the P_{33} partial wave, it can be simplified by only two R.S., where the 2 poles in the first and second R.S. are symmetric above and below the real axis.

In Fig. 1 we show as the first non-trivial case the Roper resonance on four Riemann sheets. The Roper decays to almost 100% in πN and $\pi\Delta$, as the effective $\pi\pi N$ channel. Introducing a complex $\pi\Delta$ branch point leads to the very interesting situation, that the Roper pole appears very near to the b.p. in the first Riemann sheet. Another 2 poles show up in the upper half-plane of the second and third R.S. and are uninteresting. A lot of interest, however, caused the fourth pole, which is in the lower half-plane of the 4. R.S. and often this has been especially reported in mostly dynamical approaches. But certainly it is a shadow pole and

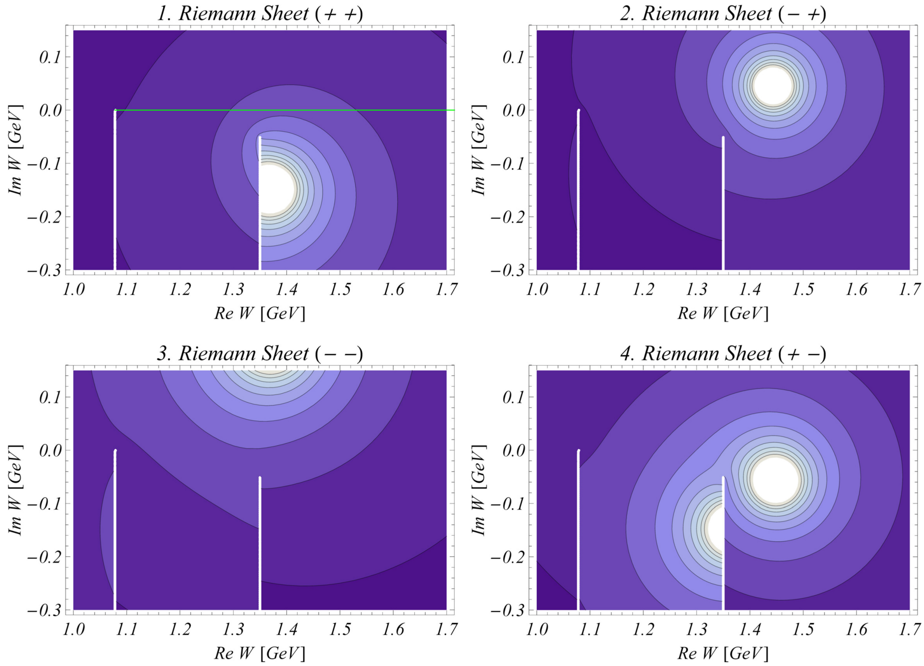

Fig. 1. Contour plots of the absolute magnitude of the P_{11} partial wave in the complex energy plane W with poles of the N(1440) P_{11} resonance on 4 different Riemann sheets. The white vertical lines show the branch cuts originating at the real b.p. πN and the complex b.p. $\pi\Delta$ and the green horizontal line shows the physical axis on the first Riemann sheet.

from Fig. 1(d) it can be judged how big the influence of this pole could be on the physical axis in Fig. 1(a). In fact it is practically negligible.

Another interesting case is the $N(1535)1/2^-$ resonance in the S_{11} partial wave. In Fig. 2 it appears in a normal scenario together with its partner $N(1650)1/2^-$ in the lower half-plane of the first Riemann sheet, if we again draw all branch cuts downwards. As it is long known and already stressed by Hoehler, the $N(1535)1/2^-$ pole sits very close near the ηN threshold and one can clearly see its influence also in the fourth R.S. Now, by a small change of parameters, this pole can move below the ηN cut and appears as a shadow pole in the fourth R.S., see Fig. 3. This scenario is realized in the Argonne-Osaka model [6], where the pole was found at $W_p = (1482 - 98i)$ MeV, only 4 MeV below ηN threshold. A shadow pole in the 4. R.S., so close to the branch point, without another counter part, certainly shows up with structure in the first R.S. and mocks a regular pole of the first R.S. However, all parameters of this shadow pole are a little bit surprising with different values compared to other PWA.

3 Complete experiments

A complete experiment is a set of measurements which is sufficient to predict all other possible experiments, provided that the measurements are free of uncer-

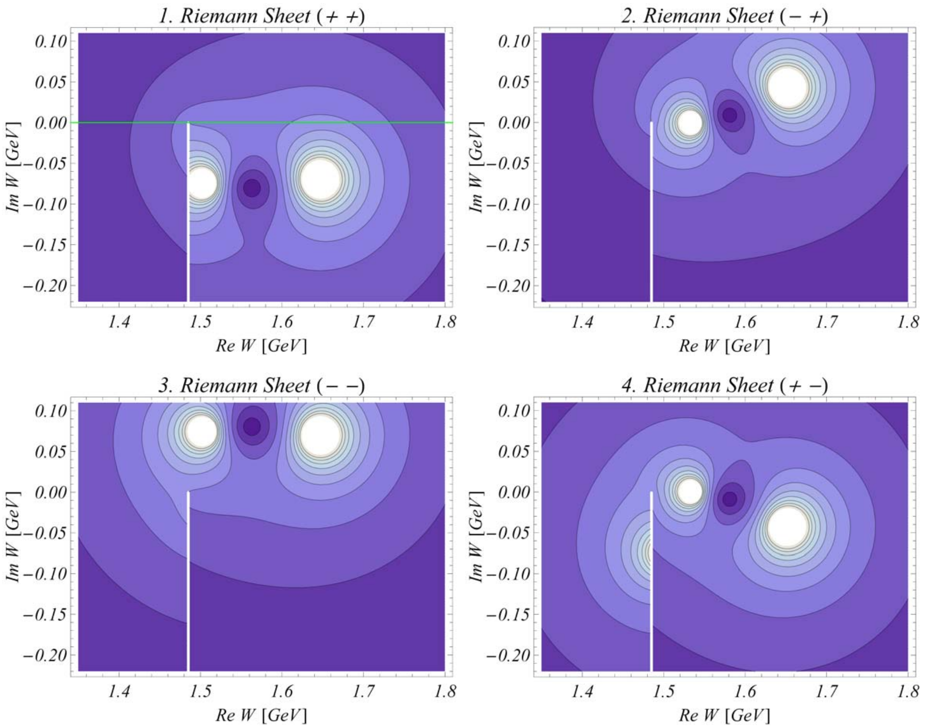

Fig. 2. Poles of N(1535) and N(1650) S_{11} resonances on 4 different Riemann sheets. The white vertical line shows the branch cut originating at the real b.p. ηN. The real b.p. πN is outside the plotted range. The green horizontal line shows the physical axis on the first Riemann sheet.

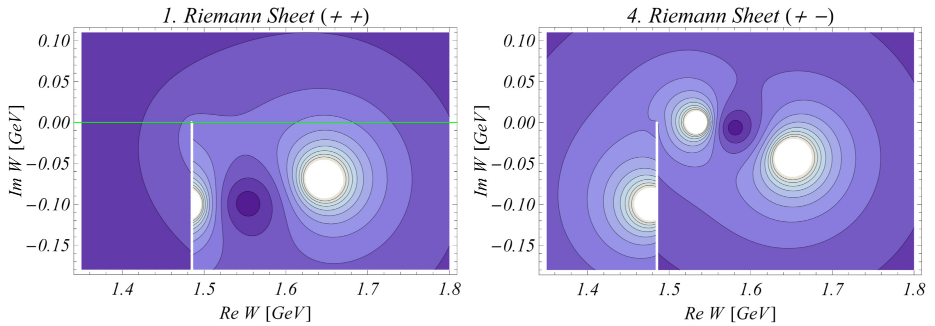

Fig. 3. Poles of N(1535) and N(1650) S_{11} resonances on the first and fourth Riemann sheets in an alternative scenario compared to Fig. 2. The N(1535) disappears form the first R.S. and appears as a shadow pole on the fourth R.S. Notation as in Fig. 2.

tainties. Using this maximal experimental information, the four complex CGLN, helicity or transversity amplitudes can be uniquely determined up to one overall energy- and angle-dependent phase, due to bilinear products of amplitudes for all observables. Starting in the 1970s many people studied the complete experiment for pseudoscalar meson photoproduction and as a benchmark publication the work of Chiang and Tabakin [7] is considered who give tables where all possible combinations for such an experiment are given with the minimal number of 8 observables. In short, these 8 observables have to be chosen with beam, target and recoil polarization, which makes it in practise very difficult. Only in the last few years this goal has been achieved at JLab with $K\Lambda$ photoproduction, where the recoil polarization of the outgoing hyperon is given for free, due to its weak self-analyzing decay. For pion and eta photoproduction meanwhile at Mainz, Bonn and JLab all 8 observables with beam and target polarization are measured over a wide energy region and with almost full angular coverage. Most of them are currently analyzed and some are already published. The 2 missing observables with beam-recoil double polarization have only been measured in a pilot experiment at MAMI using secondary rescattering of the outgoing proton. Only a few data points with rather limited statistics were obtained [8].

However, as it was shown by Omelaenko [9] in 1981 and recently revisited by Wunderlich et al. [10, 11], a complete truncated partial wave analysis can be obtained with only 5 observables, where recoil observables can be completely avoided. Under these assumptions all partial wave amplitudes up to a finite angular momentum L_{max} can be uniquely determined up to an overall energy-dependent phase. At first this looks as a paradox situation, however, in this latter case of a truncated partial wave analysis, the summation over all partial waves is never the same as it is in the first case with the full angle dependent amplitudes. And in a realistic case, even $L_{max} = 5$ is hard to realize. Therefore, the difference should be understood in such a way, that with the complete experiment of 8 observables one gets the additional information with all partial waves beyond L_{max}. Wunderlich et al. further showed that the complexity of the ambiguity structure drastically increases, when partial waves are considered beyond S and P waves, the case that Omelaenko initially studied. In such a more realistic case with D and F waves an unrealistically high precision of the observables were needed in order to find a unique solution. This can only be obtained in simulations with numerical observables obtained from a model with 10 or more significant digits. In a truncated PWA the contributions from higher partial waves can either be ignored or added from a model, e.g. from Born terms and/or Regge trajectories.

Therefore, if a truncated partial wave analysis is performed from a complete experiment with realistic pseudo data or with experimental data, multiple solutions will appear, which can not be distinguished. The envelope of such a large range of equally good solutions will then produce partial wave amplitudes with very large error bands [12]. From this observation a somewhat pessimistic view can easily arise that a model-independent PWA is simply impossible.

4 Partial wave analysis with analytical constraints

The most common way to get a stable solution for single-energy (SE) PWA is a fit constrained by an energy-dependent solution in a model-dependent approach. This has been done mostly for pion photoproduction by SAID, MAID, BnGa groups, the latter also tried this for eta photoproduction. For low and dominant partial waves this leads to similar solutions, but for smaller and higher partial waves all solutions will be different. Furthermore, the errors given in such SE analyses are just reflecting the statistical errors of the fitted experimental observables.

In a collaboration with groups from Mainz, Tuzla and Zagreb (MTZ) we are now analyzing data sets with analytical constraints from fixed-t dispersion relations. The method is similar to the pion nucleon PWA obtained in the 80s by Hoehler and Pietarinen and is described in detail in the contribution of Stahov to this workshop [13]. It enforces analyticity both in s and in t and in particular continuity in energy. Such constraints are based on fundamental symmetries and do not follow any model assumption. Finally, our goal of getting baryon resonance parameters in a model-independent way will be reached by analyzing the model-independent SE partial wave solutions obtained in the step before.

5 Baryon resonance analysis with the L+P method

Over the last few years Svarc et al. [2,3] have developed a very efficient resonance analysis method in order to find pole positions and residues from partial wave amplitudes over a large energy range. In this approach the most important properties of partial waves, poles and branch points are used as physical parameters and an expansion in terms of Pietarinen functions is used to describe the partial wave amplitudes over the whole energy range, giving more confidence on the obtained pole parameters of baryon resonances as with local methods like the speed-plot technique, first proposed by Hoehler.

The method is well described in articles with applications on pion nucleon scattering and pion photoproduction [2,3]. In summary, the set of equations which define the Laurent expansion + Pietarinen series method (L+P method) is given by

$$
\begin{aligned}
T(W) &= \sum_{i=1}^{k} \frac{a_{-1}^{(i)}}{W - W_i} + B^L(W) \\
B^L(W) &= \sum_{n=0}^{N_1} c_n\, X(W)^n + \sum_{n=0}^{N_2} d_n\, Y(W)^n + \sum_{n=0}^{N_3} e_n\, Z(W)^n + \cdots \\
X(W) &= \frac{\alpha - \sqrt{x_P - W}}{\alpha + \sqrt{x_P - W}};\;\; Y(W) = \frac{\beta - \sqrt{x_Q - W}}{\beta + \sqrt{x_Q - W}};\;\; Z(W) = \frac{\gamma - \sqrt{x_R - W}}{\gamma + \sqrt{x_R - W}} + \cdots ,
\end{aligned}
$$

where $W_i, a_{-1}^{(i)}$ are the complex pole positions and corresponding residues and x_P, x_Q, x_R are real or complex branch points. Usually, the first b.p. x_P is used as an effective b.p. for the left-hand cuts, x_Q is the πN threshold and x_R is an effective multi-pion branch point, which can correspond to $\pi\Delta$, ηN, or any other channel. If necessary, a fourth Pietarinen etc. can be added. $c_n, d_n, e_n, \alpha, \beta, \gamma$ are real parameters and the number of terms N_1, N_2, N_3 of the Pietarinen series is typically between 10-20. In Fig. 4 we show four examples of pion photoproduction partial waves (multipoles) from MAID2007 SE solutions [14], where the L+P method yields pole parameters consistent with PDG. Further details can be found in Ref. [3].

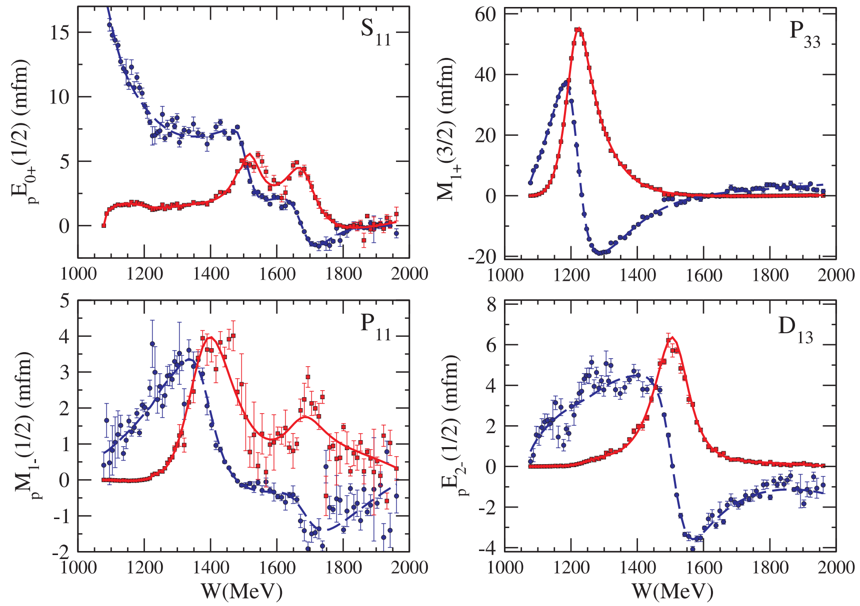

Fig. 4. L+P fit to MAID2007 SE solutions. Dashed blue, and full red lines denote real and imaginary parts of multipoles respectively.

6 Summary and conclusions

The study of baryon resonances is still an exciting field in hadron physics. While a large series of resonances are already known for a long time, in most cases only the dominant branching channels are well investigated. From still ongoing experiments at Mainz, Bonn and JLab, meson photo- and electroproduction data will be available partly with unprecedented precision and with different kind of beam, target and recoil polarization. With this large new database partial wave analyses can be obtained for various channels and more accurate and also new baryon resonance properties can be analyzed. In reactions different from πN also new resonances can be found, especially in the region $W > 1.8$ GeV, as it was already reported in a PWA mainly from new $K\Lambda$ photoproduction data [15].

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Analiza delnih valov za podatke pri fotoprodukciji mezona η z upoštevanjem omejitev zaradi analitičnosti

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Izvedemo analizo delnih valov za podatke pri fotoprodukciji η. Dobljeni multipoli so v skladu z analitičnostjo pri fiksnem t in pri fiksnem s. Analitičnost pri fiksnem t zagotovimo s Pietarinenovo metodo. Invariantne amplitude ubogajo zahtevano navzkrižno simetrijo.

Napredek pri poznavanju sklopitev nevtrona

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Podajamo pregled prizadevanj skupine GW SAID za analizo fotoprodukcije pionov na nevtronski tarči. Razločitev izoskalarnih in izovektorskih elektromagnetnih sklopitev resonanc N^* in Δ^* zahteva primerljive in skladne podatke na protonski in na nevtronski tarči. Interakcija v končnem stanju igra kritično vlogo pri najsodobnejši analizi in izvrednotenju podatkov za proces $\gamma n \to \pi N$ pri eksperimentih z devteronsko tarčo. Ta je pomemben sestavni del tekočih programov v laboratorijih JLab, MAMI-C, SPring-8, CBELSA in ELPH.

Vzbujanje barionskih resonanc s fotoprodukcijo mezonov

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Spektroskopija lahkih hadronov je še vedno živahno področje v fiziki jedra in delcev. Celo 50 let po odkritju Roperjeve resonance in več kot 30 let po pionirskem delu Hoehlerja and Cutkoskyja je še veliko odprtih vprašanj glede barionskih resonanc. Danes je glavni vzbujevalni mehanizem fotoprodukcija in elektroprodukcija mezonov, merjena na elektronskih pospeševalnikih kot so MAMI, ELSA in JLab. V združenem prizadevanju izvrednotimo lege in jakosti polov iz parcialnih valov, dobljenih z analizo parcialnih valov pri nedavnih meritvah polarizacij ob uporabi analitičnih omejitev iz disperzijskih relacij pri fiksnem t. Poseben poudarek pri barionskih resonancah je na strukturi pola na različnih Riemannovih ploskvah.