

LACAN AND BADIOU: LOGIC OF THE *PAS-TOUT*

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Logically, nothing is a universal: “Nothing is permanent” is equivalent to “Everything is non-permanent”. This raises the question whether there is a logic of nothing that escapes the universal and remains on the side of incompleteness. The question has an important link with psychoanalysis, at least since Lacan, because of its centrality to the topic of sexuality, and female sexuality in particular.

The logic of the nothing that is non-universalizable is precisely the logic of Lacan’s “*pas-tout*”, which Lacan explores principally in *Seminar XX, Encore* (1972–1973) and in “Étourdit” (1973) in the form of the four formulas of sexuation.¹

$$\begin{array}{ll} (\exists x)\sim\Phi x & \sim(\exists x)\sim\Phi x \\ (\forall x)\Phi x & \sim(\forall x)\Phi x \end{array}$$

The *pas-tout* is expressed by the formula on the lower right-hand side, $\sim(\forall x)\Phi x$. As is well known, the *pas-tout* corresponds to the negative particular statement of Aristotelian logic, variously expressed as “Some As are non-B”, “Not all As are B”, or “Not every A is B”, all of which are logically equivalent. It also, in some way, corresponds to the formula of predicate calculus, $\sim(\forall x)(Gx \rightarrow Hx)$.

The difference between Lacan’s formulas and the formulas of the predicate calculus, below,

Lacan’s formulas	Predicate calculus
$(\forall x)\Phi x$	$(\forall x)(Gx \rightarrow Hx)$
$(\exists x)\Phi x$	$(\exists x)(Gx \& Hx)$

¹ *On Feminine Sexuality: The Limits of Love and Knowledge*, Norton, New York 1975; “L’Étourdit”, *Scilicet*, Paris, no. 4 (1973), pp. 5–52. I have changed the symbols to ones easier to print; this alters nothing since the formal relations between the expressions remain the same.

is that in Lacan's formulas the variable, x , ranges over only those things that fall under Φ (i.e. speaking beings), whereas the formulas of the predicate calculus are formulated in such a way that the variables, x , y , z , etc., range over everything.

"Pas-tout" in English

I shall take Lacan's own approach at face value and address the *pas-tout* as a conceptual or logical category, without assuming any direct link with sexuality. While it is true that Lacan introduces "*pas-tout*" in reference to female sexuality, it is not used only with that reference in mind. The *pas-tout* is a logical category, invented by Lacan, that is best taken as a formulation of a non-universalizable nothing. If we do this then we see, first, that the *pas-tout*, and thus the formulas of sexuation in general, have no intrinsic link with the field of sexuality but are independent of it. This is the radical novelty of the approach; the formulas tell us something about the nature of sexuality precisely because they do not attempt to say what sexuality is. This is why, and this is the second point, they can legitimately be applied to different fields. There are numerous examples of this: see, for instance, the use to which Jean-Claude Milner puts it in *L'Amour de la langue* and *Les penchants criminels de l'Europe démocratique*, where his use of the *pas-tout* is central to a study of the incompleteness of language, on the one hand, and of social organization on the other.²

Given the importance of the logic of the *pas-tout*, it becomes imperative to examine this logic in and for itself, and this is what I propose to do here. This is not such an easy task, because the logic is not conventional and because, as Alain Badiou points out, it gives the appearance of being rather confused.

First, though, some comment is called for on the actual term itself, "*pas-tout*", for which I retain the French because not only is it frequently mistranslated into English but also there is an underlying ambiguity in the term in French which is exploited by Lacan but which cannot be retained in English. The term is sometimes translated into English as "not-all", this being the most obvious rendering, as is the case in *Television*.³ Most of the commentaries do the same. The respected translator Bruce Fink, in his rendition of *Seminar XX*, prefers "not-whole", while some authors alternate between the two, often

² *L'Amour de la langue*, Seuil, Paris 1978; *Les penchants criminels de l'Europe démocratique*, Verdier, Paris 2003.

³ J. Lacan, *Television: A Challenge to the Psychoanalytic Establishment*, Norton, New York 1990, p. 40.

without explanation. And, as if this weren't enough confusion, a rendering that retains the Aristotelian link ought to encompass "not every", given the negative particular that has the form of "Not every A is B".

While I agree with Fink that his choice most accurately captures Lacan's use of the term, I don't believe that he sufficiently justifies it when he says that in speaking in terms of "*quanteurs*" rather than "*quantificateurs*", or quantifiers, Lacan is indicating that his concern is not with quantity or quantification.⁴ In *The Lacanian Subject* Fink renders "*pas-tout*" as "not the whole of" or "not all of", which he justifies on the same grounds – we are not dealing with quantifiers but with something different. Yes, I agree that Lacan *is* saying something different – and I will explain just what this is – but I also think the reference to quantification theory as well should not be neglected.

Thus what is not explained by saying that "*quanteurs*" are not quantifiers is this: if we are not dealing with quantification, then what *are* we dealing with? Also, the link to Aristotelian logic and its treatment of quantification, which is explicitly made by Lacan, becomes mysterious, as is the point that we are first and foremost dealing with a logical issue and only secondarily with one of sexuation.

It would appear then that the issue of translation cannot be solved simply by adopting "not-all", or indeed by adopting any other single term or phrase, as the English equivalent. This is a difficulty that is further compounded by the fact that the entire series of English terms dealing with quantification – "all", "each", "some", "any", "no", etc. – behave in slightly but significantly different ways from the corresponding French terms – "*tout*", "*tous*", "*chaque*", "*aucun*", "*nul*", "*quelque*", "*quelques*".

Furthermore, I think Lacan uses the terms "*pas-tout*" in *both* the partitive ("Not all of x is Φ .) *and* distributive ("No x is Φ ") senses. This has been a source of confusion for English-language readers because while "*pas-tout*" can be used in both senses in French, there has never been any proper explanation of why no single term in English will do. So much for the issues of translation. Let us now move on to a more substantive question.

Badiou's critique of the pas-tout

Alain Badiou is critical of Lacan's theory of the *pas-tout* on two counts.⁵ He argues that Lacan is confused over his use of mathematics and logic and he accuses him of being "pre-Cantorian" in his conception of the infinite.

⁴ *Ibid.*, 13 & 72.

⁵ See his "Sujet et Infini", in: *Conditions*, Seuil, Paris 1992, pp. 287–305.

The first critique arises out of his criticism of the manner in which Lacan justifies the lack of logical equivalence, affirmed by both Aristotelian logic and modern predicate calculus, between “ $\sim(\forall x)\Phi x$ ” and “ $(\exists x)\sim\Phi x$ ”. Now, as Badiou points out, since castration, Φ , is supposedly universal, there can be no x whose access to jouissance assumes that not Φx , that is, it has to be false that $(\exists x)\sim\Phi x$. How, then, does the Lacanian $\sim(\forall x)\Phi x$ avoid the logical implication that $(\exists x)\sim\Phi x$? Lacan’s explanation of how, quoted by Badiou, is set out in this lengthy passage.

In [Aristotelian] logic, on the basis of the fact that one can write ‘not-every (*pas-tout*) x is inscribed in Φx ,’ one deduces by way of implication that there is an x that contradicts it. But that is true on one sole condition, which is that, in the whole (*tout*) or the not-whole (*pas-tout*) in question, we are dealing with the finite. Regarding that which is finite, there is not simply an implication but a strict equivalence. It is enough for there to be one that contradicts the universalizing formula for us to abolish that formula and transform it into a particular. This *pas-tout* becomes the equivalent of that which, in Aristotelian logic, is enunciated on the basis of the particular. There is an exception. But we could, on the contrary, be dealing with the infinite. Then it is no longer from the perspective of extension that we must take up the *pas-toute*. When I say that woman is *pas-toute* and that that is why I cannot say Woman, it is precisely because I raise the question (*je mets en question*) of a jouissance that, with respect to everything that can be used in the function Φx , is in the realm of the infinite.

Now, as soon as you are dealing with an infinite set, you cannot posit that the *pas-tout* implies the existence of something that is produced on the basis of a negation or contradiction. You can, at a pinch, posit it as an indeterminate existence. But, as we know from the extension of mathematical logic which is qualified as intuitionist, to posit a ‘there exists,’ one must also be able to construct it, that is, know how to find where that existence is.⁶

Badiou’s criticism of Lacan is that his solution to this problem adopts two inconsistent lines of argument which he confuses. Following the first line, Lacan argues that the underlying logic is not classical but a variant of

⁶ *On Feminine Sexuality: The Limits of Love and Knowledge*, pp. 102–03. Translation slightly modified.

intuitionist logic. The second appeals to Cantor's set theory and introduces what Lacan describes elsewhere as the "abyss" of the actual, or completed, infinite. The immediate and obvious objection is that he appeals both to intuitionism, which rejects the actual infinite, and to Cantor whose work presupposes it.

I grant that Lacan is grasping, sometimes tentatively, for some way of formulating in logic something about the *pas-tout* and its essential incompleteness, and that this has to do with the infinite. But Badiou illegitimately attributes a Cantorian conception of the infinite, the *actual* infinite, to Lacan in his treatment of the logic of the *pas-tout* at this point. The logic of the *pas-tout* has to do with the infinite; Lacan says as much. But it is unwarranted to ascribe to him the view that this infinite is actual; in fact, the whole point is that the *pas-tout* can only function with an indeterminate or incomplete series, not an actual infinite one.

Badiou points out that there is something else going on in the above passage, however, and that this apparent "contradiction" (which is in fact not one, as I have just pointed out) can be resolved. He correctly emphasizes Lacan's point that $\sim(\forall x)\Phi x$ is not to be taken "in extension". What Lacan means here can be explained with the help of a mundane example. "Apples are not all red" can mean i/ not every apple is red, or ii/ no apple is completely red. The first meaning is the interpretation "in extension" and implies that some apples are not red, or $(\exists x)\sim\Phi x$. On the other hand, the second, ii/, does not imply that there are non-red apples, that $(\exists x)\sim\Phi x$, but only that no apple is entirely red, that is, that there is at least one x that does not come entirely under Φ . This is the sense in which Lacan's $\sim(\forall x)\Phi x$ is to be taken: no woman comes entirely under the phallic function. Badiou glosses this by saying, a little obscurely, that not all x "support" the Φ from the position of all, and that the formula thus indicates a "breaching" of the Φ function.

The essential logical point is that henceforth one cannot deduce a negative existential affirmation from the (apparent) negation of the universal in the form of the *pas-tout*. It is not true that ' $(\exists x)\sim\Phi x$ ' follows from ' $\sim(\forall x)\Phi x$ '.⁷

So far so good. But Badiou is unhappy with Lacan's appeal to intuitionism. In pure logic, Badiou says, intuitionism amounts to a limitation of the powers of negation. It rejects

⁷ "Sujet et Infini", p. 291.

- the principle of the excluded middle, i.e., that either p or $\sim p$;
- the equivalence between a double negation and affirmation, i.e., that $\sim\sim p$ is equivalent to p ; and
- that the negation of a universal, $\sim(\forall x)\Phi x$, is equivalent to the affirmation of a negative existential, $(\exists x)\sim\Phi x$. In this respect, “intuitionism coincides perfectly with Lacan’s wish”.⁸

And why is it such a crime to be an intuitionist? Badiou considers that the fundamental reason intuitionists reject the above principles is their Canute-like reluctance to accept one of the most magnificent achievements of modern mathematics, the actual infinite, which they consider to be the effect of an uncontrolled and not clearly conceptualized negation of the finite. Intuitionists also reject the *reductio* argument, a form of reasoning where to prove that p one assumes that $\sim p$ and then demonstrates that this leads to a contradiction; $\sim\sim p$, therefore p . Of course, refusing to accept that a double negation is equivalent to an affirmative entails the abandonment of *reductio* arguments, but this is not the main reason for rejecting that a double negation is equivalent to an affirmative. The main reason is that the intuitionist wants to reject the *reductio* argument because, being an indirect proof, it does not offer a direct proof of p , but merely proceeds by way of demonstrating that something else is false.

Badiou also argues that Lacan is otherwise unsympathetic to the intuitionist program, since elsewhere he has no objection to using a *reductio* argument, and, moreover, he explicitly appeals to the existence of an actual infinite, which can only be proved by means of techniques rejected by intuitionists. This leads him to conclude that the appeal to intuitionist logic in *Encore* is opportunistic, since he is happy to appeal to it in support of his argument, but is otherwise unwilling to comply with the intuitionist’s austere program.

Response to Badiou

My response will consist of showing that Lacan can both be intuitionist about mathematics – or rather, not so much intuitionist as “constructivist” – and dump intuitionism as too “restrictive” as concerns logic.

Badiou is a little too hasty in his rejection of intuitionism. And he is thus a little too hasty in his criticism of Lacan. Badiou is a realist about mathematics and, while the extent and nature of his realism is a complex issue because

⁸ *Ibid.*, p. 291.

he cannot be described as a straightforward Platonist about mathematical objects, his opposition to intuitionism is clear, categorical and constant.⁹ It is his realism, along with his conviction that set-theory is ontology, that leads him to so strongly oppose intuitionism, designed as it is to perniciously limit one to a pre-Cantorian universe.

I would like to show why the rejection of all things intuitionist is quite possibly a mistake. First, it is possible to go intuitionist about mathematics but be quite conventional about logic, as more than one philosopher has done. Quine, for instance, claims:

One can practice and even preach a very considerable degree of constructivism without adopting intuitionist logic. Weyl's constructive set theory is nearly as old as Brouwer's intuitionism, and it uses orthodox logic; it goes constructivist only in its axioms of existence of sets . . . Constructivist scruples can be reconciled with the convenience and beauty of classical logic.¹⁰

Why one would want to adopt this line is that there are competing constructions of set-theory, but not, or not so obviously, of classical logic, and so it would be nice to be able to see set theory as a construction without the consequences of doing so filtering all the way down into logic itself. What this does is make it apparent that Badiou's anti-intuitionist stance is a little misplaced, since, I suggest, Badiou's *real* debate is actually with constructivist views of mathematics, and this is a philosophical rather than a mathematical debate. Thus both Badiou and the constructivist can agree on the existence of an actual infinite, and disagree over the nature, not just of the actual infinite, but of *all* mathematical objects.

Note that Quine's "constructivism" is not acceptable to the intuitionist (any more than it is to Badiou, for that matter) because the intuitionist, for whom mathematics is more fundamental than logic, considers that one would be rejecting what is primary and retaining what is secondary. In any case, the consequence of cleaving logical intuitionism off from mathematical "constructivism" is of course that the logical operations Badiou refers to as being unacceptable to the intuitionist – the law of the excluded middle, etc – need not be abandoned by the mathematical constructivist: one can

⁹ He declares his Platonism in "Platonisme en mathématiques", *Court traité d'ontologie transitoire, Seuil*, Paris 1998, pp. 95–109. On Badiou's anti-intuitionism and his realism, see the sensitive and informed treatment by Oliver Feltham, *Of Ontology, Praxis and Functional Work*, PhD, Deakin University 2000, spec. pp. 108–15.

¹⁰ W. V. O. Quine, *Philosophy of Logic*, Prentice-Hall, Englewood Cliffs, N.J. 1970, p. 88.

be constructivist about mathematical objects even as one adheres to classical logic. This then leaves Lacan free *both* to accept these principles *and*, on other grounds, to abandon the logical equivalence

$$\sim(\forall x)\Phi x \quad \longleftrightarrow \quad (\exists x)\sim\Phi x$$

The intuitionists were historically motivated by their opposition to Cantor, it is true, but there are other grounds for being anti-Platonist, or for being “constructivist”, about mathematics. Crispin Wright’s work on Wittgenstein’s philosophy of mathematics is outstanding in this regard.¹¹ There is also Michael Dummett who reasons that the realist about mathematics operates with a notion of truth and falsity for mathematical statements that is independent of our means for recognizing their truth value.¹² The realist view makes the intuitively compelling assumption that, say, Goldbach’s conjecture, which asserts that every even number larger than two is the sum of two primes, is either true or false; and this is so whether it can be proved or not. This is what is intuitively compelling about the Platonist position: there is a fact of the matter independent of whether we have demonstrated it, and the mathematical proof is like a discovery of something that is already there. If Goldbach’s Conjecture is true but cannot be proved, or has not been proved, then it follows that there is a mathematical reality independent of our capacity to know it. Now, Dummett claims that the assumption that what makes a mathematical proposition true is some mathematical fact or state of affairs to which it corresponds is false. As a matter of fact, it is not difficult to think of many true sentences for which there is nothing by virtue of which they are true. Consider the sentence: “There will never be another Napoleon”, which no fact makes true. If this is so, then there are no grounds simply to assume that what makes a mathematical statement true is a mathematical state of affairs; it would be wrong just to assume that mathematical realism is true.

What the above discussion implies is that on reflection we can draw a distinction between intuitionism and constructivism, and contrast realism not with intuitionism but with constructivism. It is now more apparent that Lacan should be considered a constructivist, and to maintain that when in *Encore* he appeals to intuitionism, whereas elsewhere he demonstrates a rejection of its

¹¹ Crispin Wright, *Wittgenstein on the Foundations of Mathematics*, Duckworth, London 1980; see also his *Frege’s Conception of Numbers as Objects*, Aberdeen University Press, Aberdeen 1983.

¹² See his “Wittgenstein’s Philosophy of Mathematics”, in: *Truth and Other Enigmas*, Duckworth, London 1978; and *Elements of Intuitionism*, Clarendon Press, Oxford 1977.

methodological strictures, it is because he is a constructivist about mathematics and not intuitionist about logic.

Badiou quotes Lacan saying, “Mathematical formalization is our goal, our ideal”,¹³ as evidence for the view that Lacan is not intuitionist; but if what I am saying is correct, it is evidence that Lacan is constructivist – this, I think, could be the only reason for such a remark – that is, Lacan can remain constructivist about mathematics while still not embracing the “prohibitions” of intuitionism.

Moreover, I think that to consider Lacan a constructivist is right on other grounds as well. And if I insist on this distinction between intuitionism and constructivism, it is because I think that Lacan is constructivist about mathematics, and for reasons that are importantly related to what he elaborates concerning the formulas of sexualization.

It is true that Lacan commits himself to the view that $\sim(\forall x)\Phi x$ does not imply that $(\exists x)\sim\Phi x$, and appeals to intuitionist logic in support; that is, he endorses one of the “prohibitions” of intuitionism. And it has to be acknowledged that in his use of the “*pas-tout*” Lacan thinks that intuitionism provides support. But this is a long cry from accepting the other strictures of intuitionism, as ascribed to him by Badiou.

Aristotle and the pas-tout

This brings us to what has been left out of the discussion so far and yet which must, somehow, be fundamental to the discussion: Aristotelian logic and predicate calculus. A careful analysis of the relationship of Lacan’s *pas-tout* with Aristotelian logic reveals the correctness of the above interpretation of what Lacan means when he declares that $\sim(\forall x)\Phi x$ is not to be taken “in extension”. I will now show why.

Aristotle distinguished between three forms of statement that affirm a predicate of a subject: the singular, the universal and the particular.¹⁴ Leaving the singular to one side, combining negation and affirmation with the universal and the particular produces statements of four logically different forms: universal affirmative, universal negative, particular affirmative and particular negative.

¹³ *On Feminine Sexuality: The Limits of Love and Knowledge*, p. 119.

¹⁴ The substance of Aristotle’s logic is found in his two works, *Prior Analytics* and *Posterior Analytics*. The first contains the analysis of argument in the form of the syllogism, Aristotle’s most important contribution to logic, which is under discussion here.

There is no straightforward correlation between the universal and existential quantifiers of Aristotelian formal logic and the terms that express quantification in natural languages. This fact becomes particularly relevant when language refers to nothing, as we shall see, for the behaviour of natural language and the intuitions of its speakers diverge from the structure of formal languages.

This contrast between the grammar of natural language and Aristotelian logic underlies a discussion of the particular in Aristotle in a work by Jacques Brunschwig that had a great impact on Lacan's theory of the *pas-tout*.¹⁵ Brunschwig argues that Aristotle came to realize that he had initially been misled by the workings of natural language, and that this led to an internal problem. Aristotle eventually devised a consistent logic, but it is one in which certain intuitions implicit in natural language have been disallowed, specially in relation to particular statements.

The matter that caused problems for Aristotle, which Brunschwig analyses, is one that the particular statement produces in natural language; namely, the usual meaning of the particular leads to three mutually inconsistent propositions, as can be seen in the following three intuitively obvious assumptions:

1. The particular and the universal of opposite "quality" (i.e., where one is affirmative and the other negative) are contradictory. Ordinary usage thus treats the following as axiomatic:

All As are B \longleftrightarrow \sim Some As are not B

2. A particular statement is implied by its subalternant:

All As are B \rightarrow Some As are B

As Brunschwig points out, ordinary usage is somewhat divided on this point. If I say to you that some (a lot, many) As are B, without knowing that all are, you could reply in either of two ways: *either* with the remark, "Actually, what you say is not wrong, because in fact all As are B", *or* with the comment, "No, no, it's not just *some* As that are B; *all* As are B." My own view is that natural language is not so equivocal on this point since in the second scenario one could come back with the rejoinder that since all As are B, it must be the case *a fortiori* that *some* As are B. Now, if one accepts this, then one accepts that "All As are B" implies "Some As are B".

¹⁵ "La proposition particulière et les preuves de non-concluanace chez Aristote", *Cahiers pour l'Analyse*, 10 (1969), pp. 3–26.

3. The two particular statements imply one another. Ordinarily, the statement “Some As are B” would be true in circumstances in which “Some As are not B” is equally true. If, for instance, I make the claim, “Some cats are black”, this would seem to imply that there are also cats that are non-black.

Some As are B \leftrightarrow Some As are not B

The problem with these three formal relations is that they are mutually inconsistent, as is easily shown. From “All As are B” it follows both (by 1) that it is false that some As are not B, and (by 2 and 3) that some As are not B, which is a contradiction. The contradiction can only be avoided by rejecting one of the above natural language axioms, 1, 2 or 3.

Rejecting axiom 3, the equivalence of the two particular statements, would produce the classical Aristotelian square of oppositions. The contradiction between “All As are B” and “No A is not B”, remains, as do the relations of subalternation between “All As are B” and “Some As are B” and between “No As are B” and “Some As are not B”. The equivalence of the two particular statements, “Some As are B” and “Some As are not B”, becomes one of subcontraries, or compatibility, where both may be true together but not false. The particular “Some As are B” thus becomes interpreted as saying, “At least one A is B”, where it is not excluded that all are. Brunschwig calls this interpretation of “Some As are B” and “Some As are not B” where *it is not excluded that all As are (not) B* the “minimal particular”.

If on the other hand we reject axiom 2 and retain axioms 1 and 3, we obtain a system in which the two particulars imply one another. If one wishes to maintain as contradictories “All As are B” and “Some As are not B”, and “No As are B” and “Some As are B”, then one is obliged to allow, paradoxically, both that each of the particulars is contradictory with the universal of the same quality and that each is still the contradictory of the universal of the opposite quality. In effect, both universals must contradict both particulars, since the latter are equivalent. And, moreover, the two universals must be equivalent because they are contradictories of equivalent propositions.

The particular statement “Some As are B” becomes “At least and at most some As are B”, and “No As are B” becomes “At least and at most some As are not B”. That is, if it is true that “not all As are B”, then it is false that *no* As are B and equally false that *all* As are not B; there is no universal, whether affirmative or negative, that is true of As and B. Brunschwig calls this the “maximal particular”.

The “maximal particular” as source of the pas-tout

J.-A. Miller holds that this maximal particular is the origin of Lacan’s *pas-tout*.¹⁶ This means, then, that $\sim(\forall x)\Phi x$ implies not only that $(\forall x)\Phi x$ is false but also that $(\forall x)\sim\Phi x$ is as well. On consideration, it is obvious that this implies that the “quanteurs” are not to be taken in extension and that the only possible way to understand the maximal reading of “ $\sim(\forall x)\Phi x$ ” is as “Not all of x is Φ ”.

Miller further claims, incorrectly I believe, that Lacan’s *pas-tout* differs from Aristotelian quantification in another respect, which is that the universe of discourse in Aristotelian logic is finite, with the consequence that, irrespective of whether the *pas-tout* is interpreted as maximal or minimal, it is concerned with lack and incompleteness. He adds that because the Lacanian *pas-tout* assumes an infinite universe, and because it is constructed on the intuitionist model of a sequence of choice, it is impossible to state the universality of the predicate. If the law by which the series, all As are B, is defined is not stated at the outset, it will be impossible, no matter how many As have been shown to be B, even without ever having found an A that is not B, to draw a conclusion about all. The sequence is “lawless”, which is an attribute of the Lacanian real.¹⁷

The claim that the universe of discourse of Aristotelian logic is finite while Lacan’s *pas-tout* assumes an infinite universe of discourse is, I believe, incorrect because Aristotelian logic holds of finite and infinite universes equally well; it makes no difference to the *logical* relations between the statements whether they refer to a finite or an infinite number of things. It makes a difference if the universe is empty. But from all As are Bs, it follows that no As are non-B, whether there is a finite or infinite number of As.

Perhaps Miller’s point can be made in a different way, one that brings us back to the intuitionism / constructivism distinction. The quantifiers, “ \forall ” and “ \exists ”, make it possible to refer to an infinite number of objects, and hence to a totality – but on the proviso that it is possible to characterize, by way of a predicate, all members of the class. For instance, “ $(\forall x) [(x>1)\rightarrow(x>0)]$ ”, that is, “If a number is greater than 1 then it is greater than zero”, is true of an infinite number of cases because the class of numbers is infinite. However, if there is *no* way to define the members of an infinitely large class, then the truth of the statement cannot be established. Because the class is infinitely

¹⁶Jacques-Alain Miller, “Notice de fil en aiguille”, in: Jacques Lacan, *Le séminaire de Jacques Lacan, Livre XXIII, Le sinthome 1975–1976*, Seuil, Paris 2005, pp. 207–8.

¹⁷*Ibid.*, p. 208.

large, enumeration of cases cannot exhaust them all; and because there is no suitable predicate they cannot be referred to as a totality.

Again, we can see how the question of actual infinity is irrelevant to the issue, since the issue is merely one of the impossibility of defining a *potential* infinity (and of course the impossibility of enumerating an infinite number of cases).

Badiou's criticisms of Lacan seem misplaced, then, and to my mind result from his realist views about mathematics and unnecessary reference to the concept of an actual infinite.

There is one further issue that I should signal, which unfortunately I cannot go into here. A moment's reflection is enough to see that the reference to the enumeration of cases, that is, to the impossibility of doing so in the case of $\sim(\forall x)\Phi x$, implicitly means taking the formula "in extension": not a, not b, not c, ... In other words, there are two readings of the *pas-tout* which, one suspects, are a real source of confusion in the glosses on Lacan. How this impacts upon the logic of the *pas-tout* is another matter.