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6 The Symmetry of 4×4 Mass Matrices Predicted by the *Spin-charge-family* Theory — $SU(2) \times SU(2) \times U(1)$ — Remains in All Loop Corrections ⋆

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Abstract. The *spin-charge-family* theory [1–7,9–12,15–17,19–24] predicts the existence of the fourth family to the observed three. The 4×4 mass matrices — determined by the nonzero vacuum expectation values and the dynamical parts of the two scalar triplets, the gauge fields of the two groups of $\widetilde{SU}(2)$ determining family quantum numbers, as well as of the three scalar singlets with the family members quantum numbers ($\tau^{\alpha} = (Q, Q', Y')$), — manifest the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$. All scalars carry the weak and the hyper charge of the *standard model* higgs field ($\pm\frac{1}{2}, \mp\frac{1}{2}$, respectively). It is demonstrated, using the massless spinor basis, that the symmetry of the 4×4 mass matrices remains $SU(2) \times SU(2) \times U(1)$ in all loop corrections, and it is discussed under which conditions this symmetry is kept under all corrections, that is with the corrections induced by the repetition of the nonzero vacuum expectation values included.

Povzetek. Teorija *spinov-nabojev-družin* [1–7,9–12,15–17,19–24] napove četrto družino k doslej opaženim trem. Masne matrike 4×4 — določajo jih dva skalarna tripleta, ki sta umeritveni polji dveh grup $\widetilde{SU}(2)$ (tripleti določajo družinska kvantna števila), ter trije skalarni singleti s kvantnimi števili družinskih čanov $\tau^{\alpha} = (Q, Q', Y')$ vsak s svojimi neničelnimi vakuumskimi pričakovanimi vrednostmi ter kot dinamična polja — imajo simetrijo $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$. Vsi skalarji — oba tripleta in vsi trije singleti — imajo enake šibke in hipernaboje kot higgsova polja v *standardnem modelu* ($\pm\frac{1}{2}, \mp\frac{1}{2}$). Avtorja pokažeta, da ostane simetrija masnih matrik 4×4 enaka $SU(2) \times SU(2) \times U(1)$ v vseh redih popravkov, ki jih določajo dinamična polja. Obravnavata pa tudi vključitev ponovitve neničelnih vakuumskih pričakovanih vrednosti v vseh redih in spremembo simetrije, ki jo te ponovitve povzročijo.

Keywords:Unifying theories, Beyond the standard model, Origin of families, Origin of mass matrices of leptons and quarks, Properties of scalar fields, The fourth

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6.1 Introduction

The *spin-charge-family* theory [1–12,15–17,19–24] predicts before the electroweak break four - rather than the observed three — coupled massless families of quarks and leptons.

The 4×4 mass matrices of all the family members demonstrate in this theory the same symmetry [1,5,4,21,22], determined by the scalar fields originating in $d > (3+1)$: the two triplets — the gauge fields of the two $\widetilde{SU}(2)$ family groups with the generators $\vec{\tilde{N}}_L$, $\vec{\tilde{\tau}}^1$, operating among families — and the three singlets — the gauge fields of the three charges ($\tau^\alpha = (Q, Q', Y')$)) — distinguishing among family members. All these scalar fields carry the weak and the hyper charge as does the scalar higgs of the *standard model*: ($\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively) [1,4,24]. The loop corrections alone, as well as corrections including the repetition of the nonzero vacuum expectation values in all orders, make each matrix element of mass matrices dependent on the quantum numbers of each of the family members.

Since there is no direct observations of the fourth family quarks with masses below 1 TeV, while the fourth family quarks with masses above 1 TeV would contribute according to the *standard model* (the *standard model* Yukawa couplings of the quarks with the scalar higgs is proportional to $\frac{m_4^\alpha}{v}$, where m_4^α is the fourth family member ($\alpha = u, d$) mass and v the vacuum expectation value of the scalar higgs) to either the quark-gluon fusion production of the scalar field (the higgs) or to the scalar field decay too much in comparison with the observations, the high energy physicists do not expect the existence of the fourth family members at all [25,26].

One of the authors (N.S.M.B) discusses in Refs. ([1], Sect. 4.2.) that the *standard model* estimation with one higgs scalar might not be the right way to evaluate whether the fourth family, coupled to the observed three, does exist or not. The u_i-quarks and d_i-quarks of an i^{th} family, namely, if they couple with the opposite sign to the scalar fields carrying the family $(\tilde{A}, i)$ quantum numbers and have the same masses, do not contribute to either the quark-gluon fusion production of the scalar fields with the family quantum numbers or to the decay of these scalars into two photons. The strong influence of the scalar fields carrying the family members quantum numbers to the masses of the lower (observed) three families manifests in the huge differences in the masses of the family members, let say u_i and d_i, $i = (1, 2, 3)$, and families (i). For the fourth family quarks, which are more and more decoupled from the observed three families the higher are their masses [22,21], the influence of the scalar fields carrying the family members quantum numbers on their masses is in the *spin-charge-family* theory expected to be much weaker. Correspondingly the u_4 and d_4 masses become closer to each other the higher are their masses and the weaker are their couplings (the mixing matrix elements) to the lower three families. For u_4-quarks and d_4-quarks with the similar masses the observations might consequently not be in contradiction with the *spin-charge-family* theory prediction that there exists the fourth family coupled to the observed three ([28], which is in preparation).

But three singlet and two treplet scalar fields offer also other explanations.

We demonstrate in the main Sect. 6.2 that the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$*, which the mass matrices demonstrate on the tree level, after the gauge scalar fields of the two* $\widetilde{SU}(2)$ *family groups triplets gain nonzero vacuum expectation values, keeps the same in all loop corrections. We discuss also the symmetry of mass matrices if all the scalar fields, contributing to mass matrices, have nonzero vacuum expectation values. We use the massless basis.*

In Sect. 6.4 we present shortly the *spin-charge-family* theory and its achievements so far. All the mathematical support appears in appendices.

Let be in this introduction stressed what supports the *spin-charge-family* theory to be the right next step beyond the *standard model*. This theory can not only explain — while starting from a very simple action in $d \geq (13+1)$, Eqs. (6.35) in App. 6.4, with massless fermions (with the spin of the two kinds, γ^a and $\tilde{\gamma}^a$, one kind taking care of the spin and of all the charges of the family members (Eq. (6.4)), the second kind taking care of families (Eqs. (6.34, 6.50))) coupled only to the gravity (through the vielbeins and the two kinds of the spin connections fields $\omega_{ab\alpha} f^{\alpha}{}_c$ and $\tilde{\omega}_{ab\alpha} f^{\alpha}{}_c$, the gauge fields of S^{ab} and $\tilde{S}^{ab}$ (Eqs. (6.35)), respectively — all the assumptions of the *standard model*, but also answers several open questions beyond the *standard model*. It offers the explanation for [4–6,1,7,9–12,15–17,19–24]:

a. The appearance of all the charges of the left and right handed family members and for their families and their properties.

b. The appearance of all the corresponding vector and scalar gauge fields and their properties (explaining the appearance of higgs and the Yukawa couplings).

c. The appearance and properties of the dark matter.

d. The appearance of the matter/antimatter asymmetry in the universe.

This theory predicts for the low energy regime:

i. The existence of the fourth family to the observed three.

ii. The existence of twice two triplets and three singlets of scalars, all with the properties of the higgs with respect to the weak and hyper charges, what explains the origin of the Yukawa couplings.

iii. There are several other predictions, not directly connected with the topic of this paper.

The fact that the fourth family quarks have not yet been observed — directly or indirectly — pushes the fourth family quarks masses to values higher than 1 TeV.

Since the experimental accuracy of the 3×3 submatrix of the 4×4 mixing matrices is not yet high enough [32], it is not yet possible to calculate the mixing matrix elements among the fourth family and the observed three [1]. Correspondingly it is not possible yet to estimate masses of the fourth family members by

[1] The 3×3 submatrix, if accurate, determines the 4×4 unitary matrix uniquely.

fitting the experimental data to the free parameters of mass matrices, the number of which is limited by the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$, predicted by the *spin-charge-family* [22,21].

If we assume the masses of the fourth family members, the matrix elements can be estimated from the measured 3×3 submatrix elements of the 4×4 matrix [22,21] [2].

The more effort and work is put into the *spin-charge-family* theory, the more explanations of the observed phenomena and the more predictions for the future observations follow out of it. Offering the explanation for so many observed phenomena — keeping in mind that all the explanations for the observed phenomena originate in a simple starting action — qualifies the *spin-charge-family* theory as the candidate for the next step beyond the *standard model*.

The reader is kindly asked to learn more about the *spin-charge-family* theory in Refs. [2–4,1,5,6] and the references therein. We shall point out sections in these references, which might be of particular help, when needed.

6.2 The symmetry of the family members mass matrices

The mass term $\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi$, Eq. (6.3), of the starting action, Eq. (6.35), manifests in the *spin-charge-family* theory [4,1,5,6] the $\widetilde{SU}(2)\times \widetilde{SU}(2) \times U(1)$ symmetry. The infinitesimal generators of the two family groups namely commute among themselves, $\{\vec{\tilde{N}}_L, \vec{\tilde{\tau}}^1\}_- = 0$, Eq. (6.8), and with all the infinitesimal generators of the family members groups, $\{\tilde{\tau}^{Ai}, \tau^\alpha\}_- = 0$, $(\tau^\alpha = (Q, Q', Y'))$, Eq. (6.9). After the scalar gauge fields, carrying the space index $(7, 8)$, of the generators $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ of the two $\widetilde{SU}(2)$ groups gain nonzero vacuum expectation values, spinors (quarks and leptons), which interact with these scalar gauge fields, become massive. There are the scalar gauge fields, carrying the space index $(7, 8)$, of the group $U(1)$ with the infinitesimal generators τ^α (=(Q, Q', Y')), which are responsible for the differences in mass matrices among the family members $(u^i, \nu^i, d^i, e^i, i(1, 2, 3, 4)$, i determines four families). Their couplings to the family members depends strongly on the quantum numbers (Q, Q', Y').

It is shown in this main section that the mass matrix elements of any family member keep the $\widetilde{SU}(2)\times\widetilde{SU}(2) \times U(1)$ symmetry of the tree level in all corrections (the loops one and the repetition of the nonzero vacuum expectation values), provided that the scalar gauge fields of the $U(1)$ group have no nonzero vacuum expectation values. In the case that the scalar gauge fields of the $U(1)$ group have nonzero vacuum expectation values, the symmetry is changed, unless some of the scalar fields with the family quantum numbers have nonzero vacuum expectation values. We comment on all these cases in what follows.

Let us first present the symmetry of the mass term in the starting action, Eq. (6.35).

[2] While the fitting procedure is not influenced considerably by the accuracy of the measured masses of the lower three families, the accuracy of the measured values of the mixing matrices do influence, as expected, the fitting results very much.

We point out that the symmetry $\widetilde{SU}(2)\times\widetilde{SU}(2)$ belongs to the two $\widetilde{SO}(4)$ groups — to $\widetilde{SO}(4)_{\widetilde{SO}(3,1)}$ and to $\widetilde{SO}(4)_{\widetilde{SO}(4)}$. The infinitesimal operators of the first and the second $\widetilde{SO}(4)$ groups are, Eqs. (6.40, 6.41),

$$\begin{aligned}\vec{\tilde{N}}_{+}(=\vec{\tilde{N}}_{L}) :&= \frac{1}{2}(\tilde{S}^{23}+i\tilde{S}^{01},\tilde{S}^{31}+i\tilde{S}^{02},\tilde{S}^{12}+i\tilde{S}^{03}),\\ \vec{\tilde{\tau}}^{1} :&= \frac{1}{2}(\tilde{S}^{58}-\tilde{S}^{67},\,\tilde{S}^{57}+\tilde{S}^{68},\,\tilde{S}^{56}-\tilde{S}^{78}),\end{aligned} \tag{6.1}$$

respectively. $U(1)$ contains the subgroup of the subgroup $SO(6)$ as well as the subgroup of $SO(4)$ ($SO(6)$ and $SO(4)$ are together with $SO(3,1)$ the subgroups of the group $SO(13,1)$) with the infinitesimal operators equal to, Eq. (6.42),

$$\begin{aligned}\tau^{4} &= -\frac{1}{3}(S^{9\,10}+S^{11\,12}+S^{13\,14}),\\ \vec{\tau}^{1} &= \frac{1}{2}(S^{58}-S^{67},\,S^{57}+S^{68},\,S^{56}-S^{78}),\\ \vec{\tau}^{2} &= \frac{1}{2}(S^{58}+S^{67},\,S^{57}-S^{68},\,S^{56}+S^{78}).\end{aligned} \tag{6.2}$$

There are additional subgroups $\widetilde{SU}(2)\times\widetilde{SU}(2)$, which belong to $\widetilde{SO}(4)_{\widetilde{SO}(3,1)}$ and $\widetilde{SO}(4)_{\widetilde{SO}(4)}$, Eqs. (6.40, 6.41), the scalar gauge fields of which do not influence the masses of the four families to which the three observed families belong according to the predictions of the *spin-charge-family* theory [3].

All the degrees of freedom and properties of spinors (of quarks and leptons) and of gauge fields, demonstrated below, follow from the simple starting action, Eq. (6.35), after breaking the starting symmetry.

Let us rewrite formally the fermion part of the starting action, Eq. (6.35), in the way that it manifests, Eq. (6.3), the kinetic and the interaction term in $d=(3+1)$ (the first line, $m=(0,1,2,3)$), the mass term (the second line, $s=(7,8)$) and the rest (the third line, $t=(5,6,9,10,\cdots,14)$).

$$\begin{aligned}\mathcal{L}_{f} = \bar{\psi}\gamma^{m}(p_{m}-\sum_{A,i} g^{Ai}\tau^{Ai}A^{Ai}_{m})\psi +\\ \{\sum_{s=7,8}\bar{\psi}\gamma^{s}p_{0s}\,\psi\}+\\ \{\sum_{t=5,6,9,\dots,14}\bar{\psi}\gamma^{t}p_{0t}\,\psi\},\end{aligned} \tag{6.3}$$

where $p_{0s}=p_{s}-\frac{1}{2}S^{s's"}\omega_{s's"s}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t}=p_{t}-\frac{1}{2}S^{t't"}\omega_{t't"t}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$ [4], with $m\in(0,1,2,3)$, $s\in(7,8)$, $(s',s")\in(5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$)

[3] The gauge scalar fields of these additional subgroups $\widetilde{SU}(2)\times\widetilde{SU}(2)$ influence the masses of the upper four families, the stable one of which contribute to the dark matter.

[4] If there are no fermions present, then either ω_{abc} or $\tilde{\omega}_{abc}$ are expressible by vielbeins $f^{\alpha}{}_{a}$ [[2,5], and the references therein]. We assume that there are spinor fields which determine spin connection fields – ω_{abc} and $\tilde{\omega}_{abc}$. In general one would have [6]: $p_{0a}=f^{\alpha}{}_{a}p_{0\alpha}+\frac{1}{2E}\{p_{\alpha},Ef^{\alpha}{}_{a}\}_{-}$, $p_{0\alpha}=p_{\alpha}-\frac{1}{2}S^{s's"}\omega_{s's"\alpha}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$. Since the term $\frac{1}{2E}\{p_{\alpha},Ef^{\alpha}{}_{a}\}_{-}$ does not influece the symmetry of mass matrices, we do not treat it in this paper.

run within either $(0, 1, 2, 3)$ or $(5, 6, 7, 8)$, t runs $\in (5, \ldots, 14)$, $(t', t")$ run either $\in (5, 6, 7, 8)$ or $\in (9, 10, \ldots, 14)$ [5]. The spinor function ψ represents all family members, presented on Table 6.3, of all the $2^{\frac{7+1}{2}-1} = 8$ families, presented on Table 6.4. In this paper we pay attention on the lower four families.

The first line of Eq. (6.3) determines in $d = (3+1)$ the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients $c^{Ai}{}_{ab}$ (the coefficients $c^{Ai}{}_{ab}$ of τ^{Ai} can be found in Eqs. (6.38, 6.2) [6],

$$\tau^{Ai} = \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}\,, \tag{6.4}$$

fulfilling the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_{-} = i\delta^{AB} f^{Aijk} \tau^{Ak}\,. \tag{6.5}$$

They represent the colour (τ^{3i}), the weak (τ^{1i}) and the hyper (Y) charges [7]. The corresponding vector gauge fields A^{Ai}_{m} are expressible with the spin connection fields ω_{stm}, Eq. (6.44) [8]

$$A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{m}\,. \tag{6.6}$$

The second line of Eq. (6.3) determines masses of each family member (u^i, d^i, ν^i, e^i). The scalar gauge fields of the charges — those of the family members, determined by S^{ab} and those of the families, determined by $\tilde{S}^{ab}$ — carry space index $(7, 8)$. Correspondingly the operators $\gamma^0\gamma^s$, appearing in the mass term, transform the left handed members of any family into the right handed members of the same family, what can easily be seen in Table 6.3. Operators S^{ab} transform one family member of a particular family into the same family member of another family.

Each scalar gauge fields (they are the gauge fields with space index $s \geq 5$) are as well expressible with the spin connections and vielbeins, Eq. (6.45) [2].

The groups $SO(3, 1)$, $SU(3)$, $SU(2)_{I}$, $SU(2)_{II}$ and $U(1)_{II}$ (all embedded into $SO(13 + 1)$) determine spin and charges of spinors, the groups $\widetilde{SU}(2)_{\widetilde{SO}(3,1)}$,

[5] We use units $\hbar = 1 = c$

[6] Before the electroweak break there are the conserved (weak) charges $\vec{\tau}^1$ (Eq. (6.38)), $\vec{\tau}^3$(Eq. (6.2) and $Y := \tau^4 + \tau^{23}$ (Eqs. (6.38, 6.2) and the non conserved charge $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, where ϑ_2 is the angle of the break of $SU(2)_{II}$ from $SU(2)_{I} \times SU(2)_{II} \times U(1)_{II}$ to $SU(2)_{I} \times U(1)_{I}$. After the electroweak break the conserved charges are $\vec{\tau}^3$ and $Q := Y + \tau^{13}$, the non conserved charge is $Q' := -Y \tan^2\vartheta_1 + \tau^{13}$, where ϑ_1 is the electroweak angle.

[7] There are as well the $SU(2)_{II}$ $(\tau^{2i}$, Eq. (6.38)) and $U(1)_{II}$ $(\tau^4$, Eq. (6.2)) charges, the vector gauge fields of these last two groups gain masses when interacting with the condensate, Table 6.5 ([1,4,5] and the references therein). The condensate leaves massless, besides the colour and gravity gauge fields in $d = (3 + 1)$, the weak and the hyper charge vector gauge fields.

[8] Both fields, A^{Ai}_{m} and $\tilde{A}^{\tilde{A}i}_{m}$, are expressible with only the vielbeins, if there are no spinor fields present [2].

Eqs (6.1), $\widetilde{SU}(2)_{\widetilde{SO}(4)}$, Eqs. (6.1), (embedded into $\widetilde{SO}(13+1)$) determine family quantum numbers [9].

The generators of these latter groups are expressible by $\tilde{S}^{ab}$

$$\tilde{\tau}^{Ai} = \sum_{a,b} c^{Ai}{}_{ab}\, \tilde{S}^{ab}\,, \tag{6.7}$$

fulfilling again the commutation relations

$$\{\tilde{\tau}^{Ai}, \tilde{\tau}^{Bj}\}_{-} = i\delta^{AB} f^{Aijk} \tilde{\tau}^{Ak}\,, \tag{6.8}$$

while

$$\{\tau^{Ai}, \tilde{\tau}^{Bj}\}_{-} = 0\,. \tag{6.9}$$

The scalar gauge fields of the groups $\widetilde{SU}(2)_{I}$ $(= \widetilde{SU}(2)_{\widetilde{SO}(3,1)}$ with generators $\vec{\tilde{N}}_L$, Eq. (6.40)), $\widetilde{SU}(2)_{I}$ $(= \widetilde{SU}(2)_{\widetilde{SO}(4)}$, with generators $\vec{\tilde{\tau}}^1$, Eq. (6.41)) and $U(1)$ (with generators (Q, Q', Y'), Eq. (6.43)) are presented in Eq. (6.45) [10]. The application of the generators $\vec{\tilde{\tau}}^1$, Eq. (6.41), $\vec{\tilde{N}}_L$, Eq. (6.40), which distinguish among families and are the same for all the family members, is presented in Eqs. (6.49, 6.51, 6.13).

The application of the family members generators (Q, Q', Y') on the family members of any family is presented on Table 6.1. The contribution of the scalar gauge fields to masses of different family members strongly depends on the quantum numbers Q, Q' and Y' as one can read from Table 6.1. In loop corrections the contribution of the scalar gauge fields of (Q, Q', Y') is proportional to the even power of these quantum numbers, while the nonzero vacuum expectation values of these scalar fields contribute in odd powers.

R	$Q_{L,R}$	Y	$\tau^4_{L,R}$	τ^{23}	Y'	Q'	L	Y	τ^{13}	Y'	Q'
u^i_R	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}(1-\frac{1}{3}\tan^2\vartheta_2)$	$-\frac{2}{3}\tan^2\vartheta_1$	u^i_L	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}\tan^2\vartheta_2$	$\frac{1}{2}(1-\frac{1}{3}\tan^2\vartheta_1)$
d^i_R	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\vartheta_2)$	$\frac{1}{3}\tan^2\vartheta_1$	d^i_L	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}\tan^2\vartheta^2$	$-\frac{1}{2}(1+\frac{1}{3}\tan^2\vartheta_1)$
ν^i_R	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}(1+\tan^2\vartheta_2)$	0	ν^i_L	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}\tan^2\vartheta_2$	$\frac{1}{2}(1+\tan^2\vartheta_1)$
e_R	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}(-1+\tan^2\vartheta_2)$	$\tan^2\vartheta_1$	e_L	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}\tan^2\vartheta_2$	$-\frac{1}{2}(1-\tan^2\vartheta_1)$

Table 6.1. The quantum numbers $Q, Y, \tau^4, Y', Q', \tau^{23}, \tau^{13}$, Eq. (6.43), of the family members $u^i_{L,R}, \nu^i_{L,R}$ of one family (any one) [6] are presented. The left and right handed members of any family have the same Q and τ^4, the right handed members have $\tau^{13} = 0$, and $\tau^{23} = \frac{1}{2}$ for (u^i_R, ν^i_R) and $-\frac{1}{2}$ for (d^i_R, e^i_R), while the left handed members have $\tau^{23} = 0$ and $\tau^{13} = \frac{1}{2}$ for (u^i_L, ν^i_L) and $-\frac{1}{2}$ for (d^i_L, e^i_L). ν^i_R couples only to $A^{Y'}_s$ as seen from the table.

[9] $\widetilde{SU}(3)$ do not contribute to the families at low energies. We studied such possibilities in a toy model, Ref. [18].

[10] All the scalar gauge fields, presented in Eq. (6.45), are expressible with the vielbeins and spin connections with the space index $a \geq 5$, Ref. [2].

There are in the *spin-charge-family* theory $2^{\frac{(1+7)}{2}-1} = 8$ families [11], which split in two groups of four families, due to the break of the symmetry from $\widetilde{SO}(7,1)$ into $\widetilde{SO}(3,1) \times \widetilde{SO}(4)$. Each of these two groups manifests $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$ symmetry [6]. These decoupled twice four families are presented in Table 6.4.

The lowest of the upper four families, forming neutral clusters with respect to the electromagnetic and colour charges, is the candidate to form the dark matter [20].

We discuss in this paper symmetry properties of the lower four families, presented in Table 6.4 in the first four lines. We present in Table 6.2 the representation and the family quantum numbers of the left and right handed members of the lower four families. Since any of the family members $(u^i_{L,R}, d^i_{L,R}, \nu^i_{L,R}, e^i_{L,R})$ behave equivalently with respect to all the operators concerning the family groups $\widetilde{SU}(2)_{\widetilde{SO}(1,3)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$, the last five columns are the same for all the family members.

We rewrite the interaction, which is in the *spin-charge-family* theory responsible for the appearance of masses of fermions, presented in Eq. (6.3) in the second line, in a slightly different way, expressing $\gamma^7 = (\overset{78}{(+)} + \overset{78}{(-)})$ and correspondingly $\gamma^8 = -i(\overset{78}{(+)} - \overset{78}{(-)})$.

$$
\begin{aligned}
\mathcal{L}_{mass} &= \frac{1}{2} \sum_{+,-} \{\psi^\dagger_L \gamma^0 \overset{78}{(\pm)} (-\sum_A \tau^\alpha A^\alpha_\pm - \sum_{\tilde{A}i} \tilde{\tau}^{Ai} \tilde{A}^{\tilde{A}i}_\pm)\psi_R\} + h.c.\,, \\
\tau^\alpha &= (Q, Q', Y')\,, \quad \tilde{\tau}^{\tilde{A}i} = (\vec{\tilde{N}}_L, \vec{\tilde{\tau}}^1)\,, \\
\gamma^0 \overset{78}{(\pm)} &= \gamma^0 \frac{1}{2} (\gamma^7 \pm i\gamma^8)\,, \\
A^\alpha_\pm &= \sum_{st} c^\alpha{}_{st}\, \omega^{st}{}_\pm\,, \quad \omega^{st}{}_\pm = \omega^{st}{}_7 \mp i\, \omega^{st}{}_8\,, \\
\vec{\tilde{A}}^{\tilde{A}}_\pm &= \sum_{ab} c^A{}_{ab}\, \tilde{\omega}^{ab}{}_\pm\,, \quad \tilde{\omega}^{ab}{}_\pm = \tilde{\omega}^{ab}{}_7 \mp i\, \tilde{\omega}^{ab}{}_8\,.
\end{aligned} \tag{6.10}
$$

In Eq. (6.10) the term p_s is left out since at low energies its contribution is negligible, A determines operators, which distinguish among family members — (Q, Q', Y') [12], their eigenvalues on basic states are presented on Table 6.1 — $(\tilde{A}, i)$ represent the family operators, determined in Eqs. (6.40, 6.41, 6.42). The detailed explanation can be found in Refs. [4,5,1].

Operators τ^{Ai} are Hermitian $((\tau^{Ai})^\dagger = \tau^{Ai})$, while $(\gamma^0 \overset{78}{(\pm)})^\dagger = \gamma^0 \overset{78}{(\mp)}$. If the scalar fields A^{Ai}_s are real it follows that $(A^{Ai}_\pm)^\dagger = A^{Ai}_\mp$.

[11] In the break from SO(13, 1) to SO(7, 1) × SO(6) only eight families remain massless, those for which the symmetry $\widetilde{SO}(7,1)$ remains. In Ref. [18] such kinds of breaks are discussed for a toy model.

[12] (Q, Q', Y') are expressible in terms of $(\tau^{13}, \tau^{23}, \tau^4)$ as explained in Eq. (6.43). The corresponding superposition of $\omega^{ss'}{}_\pm$ fields can be found by taking into account Eqs. (6.38, 6.2).

While the family operators $\tilde{\tau}^{1i}$ and $\tilde{N}^i_L$ commute with $\gamma^0 \stackrel{78}{(\pm)}$, $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ for all (a, b, c, d), the family members operators (τ^{13}, τ^{23}) do not, since S^{78} does not ($S^{78}\gamma^0 \stackrel{78}{(\mp)} = -\gamma^0 \stackrel{78}{(\mp)} S^{78}$). However $[\psi^{k\dagger}_L \gamma^0 \stackrel{78}{(\mp)} (Q, Q', Y') A^{(Q,Q',Y')}_{\mp} \psi^l_R]^\dagger = \psi^{l\dagger}_R (Q, Q', Y') A^{(Q,Q',Y')\dagger}_{\pm} \gamma^0 \stackrel{78}{(\pm)} \psi^k_L \delta_{k,l} = \psi^{l\dagger}_R (Q^k_R, Q'^k_R, Y'^k_R) A^{(Q,Q',Y')}_{\pm} \psi^k_R \delta_{k,l}$, where (Q^k_R, Q'^k_R, Y'^k_R) denote the eigenvalues of the corresponding operators on the spinor state ψ^k_R. This means that we evaluate in both cases quantum numbers of the right handed partners.

But, let us evaluate $\frac{1}{\sqrt{2}} < u^i_L + u^i_R | \hat{O}^\alpha | u^i_L + u^i_R > \frac{1}{\sqrt{2}}$, with $\hat{O}^\alpha = \sum_{+,-} \gamma^0 \stackrel{78}{(\pm)}$ $(\tau^4 A^4_{\stackrel{78}{(\pm)}} + \tau^{23} A^{23}_{\stackrel{78}{(\pm)}} + \tau^{13} A^{13}_{\stackrel{78}{(\pm)}})$. One obtains $\frac{1}{\sqrt{2}}\{\frac{1}{6}(A^4_- + A^4_+) + A^{23}_- + A^{13}_+\}$. Equivalent evaluations for $|d^i_L + d^i_R>$ would give $\frac{1}{\sqrt{2}}\{\frac{1}{6}(A^4_- + A^4_+) - A^{23}_- - A^{13}_+\}$, while for neutrinos we would obtain $\frac{1}{\sqrt{2}}\{-\frac{1}{2}(A^4_- + A^4_+) + A^{23}_- + A^{13}_+\}$ and for e^i we would obtain $\frac{1}{\sqrt{2}}\{-\frac{1}{2}(A^4_- + A^4_+) - A^{23}_- - A^{13}_+\}$. Let us point out that the fields include also coupling constants, which change when the symmetry is broken. This means that we must carefully evaluate expectation values of all the operators on each state of broken symmetries. We have here much easier work: To see how does the starting symmetry of the mass matrices behave under all possible corrections up to ∞ we only have to compare how do matrix elements, which are equal on the tree level, change in any order of corrections.

In Table 6.2 four families of spinors, belonging to the group with the nonzero values of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, are presented. These are the lower four families, presented also in Table 6.4 together with the upper four families [13]. There are indeed the four families of $\psi^i_{u_R}$ and $\psi^i_{u_L}$ presented in this table. All the $2^{\frac{13+1}{2}-1}$ members of the first family are represented in Table 6.3.

The three singlet scalar fields $(A^Q_\mp, A^{Q'}_\mp, A^{Y'}_\mp)$ of Eq. (6.10) contribute on the tree level the "diagonal" values to the mass term — $\gamma^0 \stackrel{78}{(\mp)}\; Q A^Q_\mp + \gamma^0 \stackrel{78}{(\mp)}\; Q' A^{Q'}_\mp + \gamma^0 \stackrel{78}{(\mp)}\; Y' A^{Y'}_\mp$ — transforming a right handed member of one family into the left handed member of the same family, or a left handed member of one family into the right handed member of the same family. *These terms are different for different family members but the same for all the families.*

Since $Q = (\tau^{13} + \tau^{23} + \tau^4) = (S^{56} + \tau^4)$, $Y' = (-\tau^4 \tan^2\vartheta_2 + \tau^{23})$ and $Q' = (-(\tau^4 + \tau^{23}) \tan^2\vartheta_1 + \tau^{13})$ — ϑ_1 is the *standard model* angle and ϑ_2 is the corresponding angle when the second $SU(2)$ symmetry breaks — we could use instead of the operators $(\gamma^0 \stackrel{78}{(\mp)}\; Q A^Q_\mp + \gamma^0 \stackrel{78}{(\mp)}\; Q' A^{Q'}_\mp + \gamma^0 \stackrel{78}{(\mp)}\; Y' A^{Y'}_\mp)$ as well the operators $(\gamma^0 \stackrel{78}{(\pm)}\; \tau^4 A^4_\pm, \gamma^0 \stackrel{78}{(\pm)}\; \tau^{23} A^{23}_\pm, \gamma^0 \stackrel{78}{(\pm)}\; \tau^{13} A^{13}_\pm)$, if the fact that the coupling constants of all the fields, also of ω_{abs} and $\tilde{\omega}_{abs}$, change with the break of symmetry is taken into account.

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[13] The upper four families have the nonzero values of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$. The stable members of the upper four families offer the explanation for the existence the dark matter [20].

Let us denote by $-a^{\alpha}$ the nonzero vacuum expectation values of the three singlets for a family member $\alpha = (u^i, \nu^i, d^i, e^i)$, divided by the energy scale (let say TeV), when (if) these scalars have nonzero vacuum expectation values and we use the basis $\frac{1}{2}|\psi_L^{i\alpha} + \psi_R^{i\alpha} >$:

$$a^{\alpha} = -\{\frac{1}{2} < \psi_L^{i\alpha} + \psi_R^{i\alpha}|$$
$$\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} [Q < A^{Q}_{\pm} > + Q' < A^{Q'}_{\pm} > + Y' < A^{Y'}_{\pm} >] |\psi_L^{j\alpha} + \psi_R^{j\alpha} > \frac{1}{2}\} \delta^{ij} + \text{h.c.}, \tag{6.11}$$

Each family member has a different value for a^{α}. All the scalar gauge fields $A^{Q}_{\stackrel{78}{(\pm)}}, A^{Q'}_{\stackrel{78}{(\pm)}}, A^{Y'}_{\stackrel{78}{(\pm)}}$ have the weak and the hypercharge as higgs scalars: $(\pm\frac{1}{2}, \mp\frac{1}{2}$, respectively).

				$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
$\psi^1_{u^{ci}_R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \cdots$	$\psi^1_{u^{ci}_L}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{[-]} \Vert \cdots$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$\psi^2_{u^{ci}_R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \ldots$	$\psi^2_{u^{ci}_L}$	$\stackrel{03}{(-i)}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{[-]} \Vert \cdots$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
$\psi^3_{u^{ci}_R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \cdots$	$\psi^3_{u^{ci}_L}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{(-)} \Vert \cdots$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
$\psi^4_{u^{ci}_R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \cdots$	$\psi^4_{u^{ci}_L}$	$\stackrel{03}{(-i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(-)} \Vert \cdots$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$

Table 6.2. Four families of the right handed u^{c1}_R with the weak and the hyper charge ($\tau^{13} = 0, Y = \frac{2}{3}$) and of the left handed u^{c1}_L quarks with ($\tau^{13} = \frac{1}{2}, Y = \frac{1}{6}$), both with spin $\frac{1}{2}$ and colour $(\tau^{33}, \tau^{38}) = [(1/2, 1/(2\sqrt{3}), (-1/2, 1/(2\sqrt{3}), (0, -1/(\sqrt{3})]$ charges are presented. They represent two of the family members from Table 6.3 — u^{c1}_R and u^{c1}_L — appearing on 1st and 7th line of Table 6.3. Spins and charges commute with $\tilde{N}^i_L$, $\tilde{\tau}^{1i}$ and $\tilde{\tau}^4$, and are correspondingly the same for all the families.

Transitions among families for any family member are caused by ($\tilde{N}^i_L\, \tilde{A}^{\tilde{N}_L \boxplus}$ and $\tilde{\tau}^{1i}\, \tilde{A}^{\tilde{1}\boxplus}$), what manifests the symmetry $\widetilde{SU}_{N_L}(2) \times \widetilde{SU}_{\tau^1}(2)$. There are corrections in all orders, which make all the matrix elements of the mass matrix for any of the family members α dependent on the three singlets ($\tau^4 A^4_{\pm}$, $\tau^{23} A^{23}_{\pm}$, $\tau^{13} A^{13}_{\pm}$), Eq. (6.11).

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
		(Anti)octet, $\Gamma^{(7,1)} = (-1)\,1$, $\Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons									
1	u_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
11	d_R^{c2}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
12	d_R^{c2}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
13	d_L^{c2}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
14	d_L^{c2}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
15	u_L^{c2}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
16	u_L^{c2}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
17	u_R^{c3}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
19	d_R^{c3}	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
20	d_R^{c3}	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
21	d_L^{c3}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
22	d_L^{c3}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
23	u_L^{c3}	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
24	u_L^{c3}	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
25	ν_R	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\,\vert\,\stackrel{56}{[+]}\,\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\,\vert\,\stackrel{56}{(-)}\,\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\,\stackrel{11\,12}{(+)}\,\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

Continued on next page

i		$\vert^{a}\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
		(Anti)octet, $\Gamma^{(7,1)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons									
40	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41	$\bar{d}_L^{\bar{c}2}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
42	$\bar{d}_L^{\bar{c}2}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
43	$\bar{u}_L^{\bar{c}2}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
44	$\bar{u}_L^{\bar{c}2}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
45	$\bar{d}_R^{\bar{c}2}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
46	$\bar{d}_R^{\bar{c}2}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
47	$\bar{u}_R^{\bar{c}2}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
48	$\bar{u}_R^{\bar{c}2}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{(+)}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
49	$\bar{d}_L^{\bar{c}3}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
50	$\bar{d}_L^{\bar{c}3}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
51	$\bar{u}_L^{\bar{c}3}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
52	$\bar{u}_L^{\bar{c}3}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
53	$\bar{d}_R^{\bar{c}3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
54	$\bar{d}_R^{\bar{c}3}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
55	$\bar{u}_R^{\bar{c}3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
56	$\bar{u}_R^{\bar{c}3}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
57	$\bar{e}_L$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_L$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_L$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_L$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_R$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_R$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{(-)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_R$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_R$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\mid\stackrel{56}{[+]}\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 6.3. The left handed ($\Gamma^{(13,1)} = -1$, Eq. (6.53)) multiplet of spinors — the members of the fundamental representation of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons — is presented in the massless basis using the technique presented in App. 6.5. It contains the left handed ($\Gamma^{(3,1)} = -1$, App. 6.5) weak ($SU(2)_I$) charged ($\tau^{13} = \pm\frac{1}{2}$, Eq. (6.38)), and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$, Eq. (6.38)) quarks and leptons and the right handed ($\Gamma^{(3,1)} = 1$, weak ($SU(2)_I$) chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks and leptons, both with the spin S^{12} up and down ($\pm\frac{1}{2}$, respectively). Quarks distinguish from leptons only in the $SU(3) \times U(1)$ part: Quarks are triplets of three colours ($c^{i} = (\tau^{33}, \tau^{38}) = [(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (-\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})]$, Eq. (6.2)) carrying the "fermion charge" ($\tau^{4} = \frac{1}{6}$, Eq. (6.2)). The colourless leptons carry the "fermion charge" ($\tau^{4} = -\frac{1}{2}$). The same multiplet contains also the left handed weak ($SU(2)_I$) chargeless and $SU(2)_{II}$ charged anti-quarks and anti-leptons and the right handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless anti-quarks and anti-leptons. Anti-quarks distinguish from anti-leptons again only in the $SU(3)\times U(1)$ part: Anti-quarks are anti-triplets, carrying the "fermion charge" ($\tau^{4} = -\frac{1}{6}$). The anti-colourless anti-leptons carry the "fermion charge" ($\tau^{4} = \frac{1}{2}$). $Y = (\tau^{23} + \tau^{4})$ is the hyper charge, the electromagnetic charge is $Q = (\tau^{13} + Y)$. The states of opposite charges (anti-particle states) are reachable from the particle states besides by S^{ab} also by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$, presented in Refs. [43,44]. The vacuum state, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [5,15,16,4] and in the references therein.

Taking into account Table 6.3 and Eqs. (6.49, 6.58) one easily finds what do operators $\gamma^0 \stackrel{78}{(\pm)}$ do on the left handed and the right handed members of any family $i = (1, 2, 3, 4)$.

$$\begin{aligned}
\gamma^0 \stackrel{78}{(-)} |\psi^i_{u_R,\nu_R}> &= |\psi^i_{u_L,\nu_L}>\,,\\
\gamma^0 \stackrel{78}{(+)} |\psi^i_{u_L,\nu_L}> &= |\psi^i_{u_R,\nu_R}>\,,\\
\gamma^0 \stackrel{78}{(+)} |\psi^i_{d_R,e_R}> &= |\psi^i_{d_L,e_L}>\,,\\
\gamma^0 \stackrel{78}{(-)} |\psi^i_{d_L,e_L}> &= |\psi^i_{d_R,e_R}>\,.
\end{aligned} \tag{6.12}$$

We need to know also what do operators ($\tilde{\tau}^{1\pm} = \tilde{\tau}^{11} \pm i\,\tilde{\tau}^{12}$, $\tilde{\tau}^{13}$) and ($\tilde{N}^{\pm}_L = \tilde{N}^1_L \pm i\,\tilde{N}^2_L$, $\tilde{N}^3_L$) do when operating on any member ($u_{L,R}$, $\nu_{L,R}$, $d_{L,R}$, $e_{L,R}$) of a particular family ψ^i, $i = (1, 2, 3, 4)$.

Taking into account, Eqs. (6.47, 6.48, 6.58, 6.60, 6.51, 6.40, 6.41),

$$\begin{aligned}
&\tilde{N}^{\pm}_L = -\stackrel{03}{\widetilde{(\mp i)}}\stackrel{12}{\widetilde{(\pm)}}\,, \qquad \tilde{\tau}^{1\pm} = (\mp)\stackrel{56}{\widetilde{(\pm)}}\stackrel{78}{\widetilde{(\mp)}}\,,\\
&\tilde{N}^3_L = \frac{1}{2}\,(\tilde{S}^{12} + i\,\tilde{S}^{03})\,, \quad \tilde{\tau}^{13} = \frac{1}{2}\,(\tilde{S}^{56} - \tilde{S}^{78})\,,\\
&\stackrel{ab}{\widetilde{(-k)}}\stackrel{ab}{(k)} = -i\eta^{aa}\stackrel{ab}{[k]}\,, \qquad \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{(k)} = 0\,,\\
&\stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[k]} = i\stackrel{ab}{(k)}\,, \qquad \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{\widetilde{(k)}} = \frac{1}{2}\,(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\,\tilde{\gamma}^b)\,, \qquad \stackrel{ab}{\widetilde{[k]}} = \frac{1}{2}\,(1 + \frac{i}{k}\,\tilde{\gamma}^a\tilde{\gamma}^b)\,,
\end{aligned} \tag{6.13}$$

one finds

$$\begin{aligned}
&\tilde{N}^+_L\,|\psi^1> = |\psi^2>\,, \quad \tilde{N}^+_L\,|\psi^2> = 0\,,\\
&\tilde{N}^-_L\,|\psi^2> = |\psi^1>\,, \quad \tilde{N}^-_L\,|\psi^1> = 0\,,\\
&\tilde{N}^+_L\,|\psi^3> = |\psi^4>\,, \quad \tilde{N}^+_L\,|\psi^4> = 0\,,\\
&\tilde{N}^-_L\,|\psi^4> = |\psi^3>\,, \quad \tilde{N}^-_L\,|\psi^3> = 0\,,\\
&\tilde{\tau}^{1+}\,|\psi^1> = |\psi^3>\,, \quad \tilde{\tau}^{1+}\,|\psi^3> = 0\,,\\
&\tilde{\tau}^{1-}\,|\psi^3> = |\psi^1>\,, \quad \tilde{\tau}^{1-}\,|\psi^1> = 0\,,\\
&\tilde{\tau}^{1-}\,|\psi^4> = |\psi^2>\,, \quad \tilde{\tau}^{1-}\,|\psi^2> = 0\,,\\
&\tilde{\tau}^{1+}\,|\psi^2> = |\psi^4>\,, \quad \tilde{\tau}^{1+}\,|\psi^4> = 0\,,\\
&\tilde{N}^3_L\,|\psi^1> = -\frac{1}{2}\,|\psi^1>\,, \quad \tilde{N}^3_L\,|\psi^2> = +\frac{1}{2}\,|\psi^2>\,,\\
&\tilde{N}^3_L\,|\psi^3> = -\frac{1}{2}\,|\psi^3>\,, \quad \tilde{N}^3_L\,|\psi^4> = +\frac{1}{2}\,|\psi^4>\,,\\
&\tilde{\tau}^{13}\,|\psi^1> = -\frac{1}{2}\,|\psi^1>\,, \quad \tilde{\tau}^{13}\,|\psi^2> = -\frac{1}{2}\,|\psi^2>\,,\\
&\tilde{\tau}^{13}\,|\psi^3> = +\frac{1}{2}\,|\psi^3>\,, \quad \tilde{\tau}^{13}\,|\psi^4> = +\frac{1}{2}\,|\psi^4>\,,
\end{aligned} \tag{6.14}$$

independent of the family member $\alpha = (u, d, \nu, e)$.

The dependence of the mass matrix on the family quantum numbers can easily be understood through Table 6.2, where we notice that the operator $\tilde{N}_L^{\pm}$ transforms the first family into the second (or the second family into the first) and the third family to the fourth (or the fourth family into the third), while the operator $\tilde{\tau}^{\tilde{1}\pm}$ transforms the first family into the third (or the third family into the first) and the second family into the fourth (or the fourth family into the second). The application of these two operators, $\tilde{N}_L^{\pm}$ and $\tilde{\tau}^{\tilde{1}\pm}$, is presented in Eq. (6.14) and demonstrated in the diagram

$$\begin{pmatrix} \psi^1 & \psi^2 \\ \psi^3 & \psi^4 \end{pmatrix} \overset{\tilde{N}_L^{\pm}}{\leftrightarrow} \updownarrow \tilde{\tau}^{\tilde{1}\pm} . \tag{6.15}$$

The operators $\tilde{N}_L^3$ and $\tilde{\tau}^{\tilde{1}3}$ are diagonal, with the eigenvalues presented in Eq. (6.14): $\tilde{N}_L^3$ has the eigenvalue $-\frac{1}{2}$ on $|\psi^1>$ and $|\psi^3>$ and $+\frac{1}{2}$ on $|\psi^2>$ and $|\psi^4>$, while $\tilde{\tau}^{\tilde{1}3}$ has the eigenvalue $-\frac{1}{2}$ on $|\psi^1>$ and $|\psi^2>$ and $+\frac{1}{2}$ on $|\psi^3>$ and $|\psi^4>$. If we count $\frac{1}{2}$ as a part of these diagonal fields, then the eigenvalues of both operators on families differ only in the sign.

The sign and the values of Q, Q' and Y' depend on the family members properties and are the same for all the families.

Let the scalars $(\tilde{A}^{N_L \pm}_{\substack{78\\(\pm)}}, \tilde{A}^{N_L 3}_{\substack{78\\(\pm)}}, \tilde{A}^{1\pm}_{\substack{78\\(\pm)}}, \tilde{A}^{13}_{\substack{78\\(\pm)}})$ be scalar gauge fields of the operators $(\tilde{N}_L^{\pm}, \tilde{N}_L^3, \tilde{\tau}^{1\pm}, \tilde{\tau}^{13})$, respectively. Here $\tilde{A}_{\substack{78\\(\pm)}} = \tilde{A}_7 \mp i\, \tilde{A}_8$ for all the scalar gauge fields, while $\tilde{A}^{N_L \pm}_{\substack{78\\(\pm)}} = \frac{1}{2}\, (\tilde{A}^{N_L 1}_{\substack{78\\(\pm)}} \mp i\, \tilde{A}^{N_L 2}_{\substack{78\\(\pm)}})$, respectively, and $\tilde{A}^{1\pm}_{\substack{78\\(\pm)}} = \frac{1}{2}\, (\tilde{A}^{11}_{\substack{78\\(\pm)}} \mp i\, \tilde{A}^{12}_{\substack{78\\(\pm)}})$, respectively. All these fields can be expressed by $\tilde{\omega}_{abc}$, as presented in Eq. (6.45), provided that the coupling constants are the same for all the spin connection fields of both kinds, that is if no spontaneous symmetry breaking happens up to the weak scale.

We shall from now on use the notation $A^{Ai}_{\pm}$ instead of $A^{Ai}_{\substack{78\\(\pm)}}$ for all the operators with the space index $(7, 8)$.

In what follows *we prove that the symmetry of the mass matrix of any family member* α *remains the same in all orders of loop corrections, while the symmetry in all orders of corrections (which includes besides the loop corrections also the repetition of nonzero vacuum expectation values of the scalar fields) remains unchanged only under certain conditions. In general case the break of symmetry can still be evaluated for small absolute values of* a^α, Eq. (6.11). *We shall work in the massless basis.*

Let us introduce the notation $\hat{O}$ for the operator, which in Eq. (6.10) determines the mass matrices of quarks and leptons. The operator $\hat{O}$ is equal to, Eq. (6.10),

$$\begin{aligned}
\hat{O} &= \sum_{+,-} \gamma^0 \overset{78}{(\pm)} \; (-\sum_{\alpha} \tau^{\alpha} A^{\alpha}_{\pm} - \sum_{\tilde{A}i} \tilde{\tau}^{\tilde{A}i} \tilde{A}^{\tilde{A}i}_{\pm}) \,, \\
\tau^{\alpha} A^{\alpha}_{\pm} &= (Q\, A^{Q}_{\pm}, Q'\, A^{Q'}_{\pm}, Y'\, A^{Y'}_{\pm}) \,, \\
\tilde{\tau}^{\tilde{A}i} \tilde{A}^{\tilde{A}i}_{\pm} &= (\tilde{\tau}^{\tilde{1}i} \tilde{A}^{\tilde{1}i}_{\pm}, \tilde{N}^{i}_{L} \tilde{A}^{\tilde{N}_L i}_{\pm}) \,, \\
\{\tau^{\alpha}, \tau^{\beta}\}_{-} &= 0 \,, \quad \{\tilde{\tau}^{\tilde{A}i}, \tilde{\tau}^{\tilde{B}j}\}_{-} = i\, \delta^{\tilde{A}\tilde{B}}\, f^{ijk}\, \tilde{\tau}^{\tilde{A}k} \,, \quad \{\tau^{\alpha}, \tilde{\tau}^{\tilde{B}j}\}_{-} = 0 \,. \qquad (6.16)
\end{aligned}$$

Each of the fields in Eq. (6.16) consists in general of the nonzero vacuum expectation value and the dynamical part: $\tilde{A}^{\tilde{A}i}_{\pm} = (< \tilde{A}^{\tilde{1}i}_{\pm} > + \tilde{A}^{\tilde{1}i}_{\pm}(x), < \tilde{A}^{\tilde{N}_L i}_{\pm} > + \tilde{A}^{\tilde{N}_L i}_{\pm}(x),$ $< A^{\alpha}_{\pm} > + A^{\alpha}_{\pm}(x))$, where a common notation for all three singlets is used, since their eigenvalues depend only on the family members $(\alpha = (u, d, \nu, e))$ quantum numbers and are the same for all the families.

We further find that

$$\begin{aligned}
&\{\gamma^0 \overset{78}{(\pm)}, \tau^4\}_{-} = 0 \,, \quad \{\gamma^0 \overset{78}{(\pm)}, \vec{\tilde{\tau}}^{\tilde{1}}\}_{-} = 0 \,, \quad \{\gamma^0 \overset{78}{(\pm)}, \vec{\tilde{N}}_L\}_{-} = 0 \,, \\
&\{\gamma^0 \overset{78}{(\pm)}, \tau^{13}\}_{-} = -2\gamma^0 \overset{78}{(\pm)}\; S^{78} \,, \quad \{\gamma^0 \overset{78}{(\pm)}, \tau^{23}\}_{-} = +2\gamma^0 \overset{78}{(\pm)}\; S^{78} \,. \qquad (6.17)
\end{aligned}$$

To calculate the mass matrices of family members $\alpha = (u, d, \nu, e)$ the operator $\hat{O}$ must be taken into account in all orders. Since for our proof the dependence of the operator $\hat{O}$ on the time and space does not play any role (it is the same for all the operators), we introduce the dimensionless operator $\hat{\mathbf{O}}$, in which all the degrees of freedom, except the internal ones determined by the family and family members quantum numbers, are integrated away [14].

Then the change of the massless state of the i^{th} family of the family member α of the left or right handedness $(_{L,R})$, $|\psi^{\alpha\, i}_{L,R}>$, changes in all orders of corrections as follows

$$\hat{U}\, |\psi^{\alpha\, i}_{L,R} > = i \sum_{n=0}^{\infty} \frac{(-1)^n\, \hat{\mathbf{O}}^{2n+1}}{(2n+1)!}\, |\psi^{\alpha\, i}_{L,R} > \,. \qquad (6.18)$$

In Eq. (6.18) $|\psi^{\alpha\, i}_{(L,R)} >$ represents the internal degrees of freedom of the i^{th}, $i = (1,2,3,4)$, family state for a particular family member α in the massless basis. The mass matrix element in all orders of corrections between the left handed α^{th} family member of the i^{th} family $< \psi^{\alpha\, i}_{L}|$ and the right handed α^{th} family member of the j^{th} family $|\psi^{\alpha\, j}_{R} >$, both in the massless basis, is then equal to $< \psi^{\alpha\, i}_{L}|\, \hat{U}\, |\psi^{\alpha\, i}_{R} >$. Only an odd number of operators $\hat{\mathbf{O}}^{2n+1}$ contribute to the mass matrix elements, transforming $|\psi^{\alpha\, i}_{R} >$ into $|\psi^{\alpha\, j}_{L} >$ or opposite. The product of an even number of operators $\hat{\mathbf{O}}^{2n}$ does not change the handedness and consequently

[14] $\hat{O}$ is measured in TeV units (as all the scalar and vector gauge fields). If the time evolution is concerned then $\hat{\mathbf{O}} = \hat{O} \cdot (t - t_0)/\mathrm{TeV}$ is in units $\hbar = 1 = c$ dimensionless quantity. We assume that also the integration over space coordinates is in $< \psi^{\alpha\, i}_{R}|\hat{O}|\psi^{\alpha\, i}_{R} >$ already taken into account, only the integration over the family and family members is left to be evaluated.

contributes nothing. Correspondingly without the nonzero vacuum expectation values of scalar fields all the matrix elements would remain zero, since only nonzero vacuum expectation values may appear in an odd orders, while the contribution of the loop corrections always contribute to the mass matrix elements an even contribution (see Fig. (6.1)).

Our purpose is to show how do the matrix elements behave in all orders of corrections

$$< \psi_L^{\alpha\, j}|\hat{U}\,|\psi_R^{\alpha\, i} > = i \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} < \psi_L^{\alpha\, i}| \sum_{k_1=1}^{4} \hat{\mathbf{O}}|\psi_R^{\alpha\, k_1} >< \psi_R^{\alpha\, k_1}| \sum_{k_2=1}^{4} \hat{\mathbf{O}}|\psi_L^{\alpha\, k_2} > \cdots$$
$$< \psi_L^{\alpha\, k_n}| \sum_{k_i=1}^{4} \hat{\mathbf{O}}|\psi_R^{\alpha\, k_i} > . \tag{6.19}$$

Let be repeated again that all the matrix elements

$$< \psi_R^{\alpha\, k_1}|\hat{\mathbf{O}}|\psi_L^{\alpha\, k_2} >$$

or

$$< \psi_L^{\alpha\, k_1}| \sum_{k_2=1}^{4} \hat{\mathbf{O}}|\psi_R^{\alpha\, k_2} >$$

only evaluate the internal degrees of freedom, that is the family and family members ones, while all the rest are assumed to be already evaluated. Since the mass matrix is in this notation the dimensionless object, also all the scalar fields are already divided by the energy unit (let say 1 TeV). We correspondingly introduce the dimensionless scalars $(\mathbf{A}^{Q}_{\pm}, \mathbf{A}^{Q'}_{\pm}, \mathbf{A}^{Y'}_{\pm})$, $\vec{\tilde{\mathbf{A}}}^{\tilde{1}}_{\pm}, \vec{\tilde{\mathbf{A}}}^{\tilde{N}_L}_{\pm}$.

The only operators τ^{α}, distinguishing among family members, are $(\tau^4, \tau^{13}, \tau^{23})$, included in $Q = (\tau^{13} + Y)$, $Y = (\tau^{23} + \tau^4)$, $Q' = (\tau^{13} - Y \tan^2 \vartheta_1)$ and in $Y' = (\tau^{23} - \tau^4 \tan^2 \vartheta_2)$. All the operators contributing to the mass matrices of each family member α have a factor $\gamma^0 \stackrel{78}{(\pm)}$, which transforms the right handed family member to the corresponding left handed family member and opposite.

When taking into account $\hat{\mathbf{O}}^{2n+1}$ in all orders, the operators $\tau^{\alpha} A^{\alpha}_{\pm}$, $\tau^{\alpha} = (Q, Q', Y')$, contribute to all the matrix elements, the diagonal and the off diagonal ones.

To simplify the discussions let us introduce a bit more detailed notation

$$
\begin{aligned}
\hat{\mathbf{O}} &= \sum_i \hat{\mathbf{O}}^i = \hat{\mathbf{O}}^\alpha + \hat{\tilde{\mathbf{O}}}^{\tilde{1}3} + \hat{\tilde{\mathbf{O}}}^{\tilde{N}_L 3} + \hat{\tilde{\mathbf{O}}}^{\tilde{1}\substack{\boxplus\\ \boxminus}} + \hat{\tilde{\mathbf{O}}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}} \\
\hat{\mathbf{O}}^\alpha &= -\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} \, (Q\, \mathbf{A}^Q_\pm, Q'\, \mathbf{A}^{Q'}_\pm, Y'\, \mathbf{A}^{Y'}_\pm)\,, \\
\hat{\tilde{\mathbf{O}}}^{\tilde{1}3} &= -\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} \, \tilde{\tau}^{\tilde{1}3}\, \tilde{\mathbf{A}}^{\tilde{1}3}_\pm\,, \\
\hat{\tilde{\mathbf{O}}}^{\tilde{N}_L 3} &= -\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} \, \tilde{N}^3_L\, \tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm\,, \\
\hat{\tilde{\mathbf{O}}}^{\tilde{1}\substack{\boxplus\\ \boxminus}} &= -\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} \, \tilde{\tau}^{\tilde{1}\substack{\boxplus\\ \boxminus}}\, \tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}_\pm\,, \\
\hat{\tilde{\mathbf{O}}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}} &= -\sum_{+,-} \gamma^0 \stackrel{78}{(\pm)} \, \tilde{N}^{\substack{\boxplus\\ \boxminus}}_L\, \tilde{\mathbf{A}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}_\pm\,.
\end{aligned} \tag{6.20}
$$

We shall use the notation for the expectation values among the states $<\psi^i_L| =< i|, |\psi^j_R>= |j>$ for the zero vacuum expectation values and the dynamical parts as follows:

i. $< i|\hat{\mathbf{O}}^\alpha|j> =< i|\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \tau^\alpha(<\mathbf{A}^\alpha_\pm> + \mathbf{A}^\alpha_\pm(x))|j>$.

ii. $< i|\hat{\tilde{\mathbf{O}}}^{\tilde{1}3}|j> =< i|-\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \tilde{\tau}^{\tilde{1}3}(<\tilde{\mathbf{A}}^{\tilde{1}3}_\pm> + \tilde{\mathbf{A}}^{\tilde{1}3}_\pm(x))|j>$.

iii. $< i|\hat{\tilde{\mathbf{O}}}^{\tilde{N}_L 3}|j> =< i|-\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \tilde{N}^3_L(<\tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm> + \tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm(x))|j>$.

iv. $< i|\hat{\tilde{\mathbf{O}}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}|j> =< i|-\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \tilde{\tau}^{\tilde{1}\substack{\boxplus\\ \boxminus}}(<\tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}_\pm> + \tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}_\pm(x))|j>$.

v. $< i|\hat{\tilde{\mathbf{O}}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}|j> =< i|-\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \tilde{N}^{\substack{\boxplus\\ \boxminus}}_L(<\tilde{\mathbf{A}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}_\pm> + \tilde{\mathbf{A}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}_\pm(x))|j>$.

vi. $< i|\hat{\tilde{\mathbf{O}}}^\alpha_{\rm dia}|i> =< i|\sum_{+,-}\gamma^0 \stackrel{78}{(\pm)}\, \{\tau^\alpha(<\mathbf{A}^\alpha_\pm> + \mathbf{A}^\alpha_\pm(x)) - \tilde{\tau}^{\tilde{1}3}(<\tilde{\mathbf{A}}^{\tilde{1}3}_\pm> + \tilde{\mathbf{A}}^{\tilde{1}3}_\pm(x)) - \tilde{N}^3_L(<\tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm> + \tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm(x))\}|i>$.

$(<\mathbf{A}^\alpha_\pm>, <\tilde{\mathbf{A}}^{\tilde{1}3}_\pm>, <\tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm>, <\tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}_\pm>, <\tilde{\mathbf{A}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}_\pm>)$ represent nonzero vacuum expectation values and $(\mathbf{A}^\alpha_\pm(x), \tilde{\mathbf{A}}^{\tilde{1}3}_\pm(x), \tilde{\mathbf{A}}^{\tilde{N}_L 3}_\pm(x), \tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \boxminus}}_\pm(x), \tilde{\mathbf{A}}^{\tilde{N}_L \substack{\boxplus\\ \boxminus}}_\pm(x))$ the corresponding dynamical fields.

In the case **i.** $<\mathbf{A}^\alpha_\pm>$ represent the sum of the vacuum expectation values of $(Q^\alpha \mathbf{A}^Q_{(\pm)}, Q'^\alpha \mathbf{A}^{Q'}{}_{(\pm)}, Y'^\alpha \mathbf{A}^{Y'}_{(\pm)})$ of a particular family member α, where $(Q^\alpha, Q'^\alpha, Y'^\alpha)$ are the corresponding quantum numbers of a family member α. $\mathbf{A}^\alpha_\pm(x)$ represent the corresponding dynamical fields.

In the case **vi.** we correspondingly have for the four diagonal terms on the tree level, that is for $n = 0$ in Eq. (6.19) (after taking into account Eq. (6.14): $< 1\,|\tilde{\mathbf{O}}^\alpha_{\rm dia}|1> = \mathbf{a}^\alpha - (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)$, $< 2|\tilde{\mathbf{O}}^\alpha_{\rm dia}|1> |2>= \mathbf{a}^\alpha - (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)$, $< 3|\tilde{\mathbf{O}}^\alpha_{\rm dia}|3>= \mathbf{a}^\alpha + (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)$ and $< 4|\tilde{\mathbf{O}}^\alpha_{\rm dia}|4>= \mathbf{a}^\alpha + (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)$, where $(\tilde{\mathbf{a}}_1, \tilde{\mathbf{a}}_2, \mathbf{a}^\alpha)$ represent the nonzero vacuum expectation values of $\frac{1}{2}\frac{1}{\sqrt{2}}(<\tilde{\mathbf{A}}^{\tilde{1}3}_{(+)}> + <\tilde{\mathbf{A}}^{\tilde{1}3}_{(-)}>)$, $\frac{1}{2}\frac{1}{\sqrt{2}}(<\tilde{\mathbf{A}}^{\tilde{N}_L 3}_{(+)}> + <\tilde{\mathbf{A}}^{\tilde{N}_L 3}_{(-)}>)$, $\frac{1}{2}\frac{1}{\sqrt{2}}(<\mathbf{A}^\alpha_{(+)}> + <\mathbf{A}^\alpha_{(-)}>)$, all in dimensionless units.

We are now prepared to show under which conditions the mass matrix elements for any of the family members keep the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$ *at each step of corrections*, what means that the values of the matrix elements obtained in each correction respect the symmetry of mass matrices on the tree level.

We use the massless basis $|\psi^i_{L,R} >$, making for the basis the choice $\frac{1}{\sqrt{2}}\,(|\psi^i_L > + |\psi^i_R >)$.

The diagrams for the tree level, one loop and three loop contributions of the operator $\hat{\mathbf{O}}$, determining the masses of quarks and leptons, Eqs. (6.16, 6.20), are presented in Fig. (6.1).

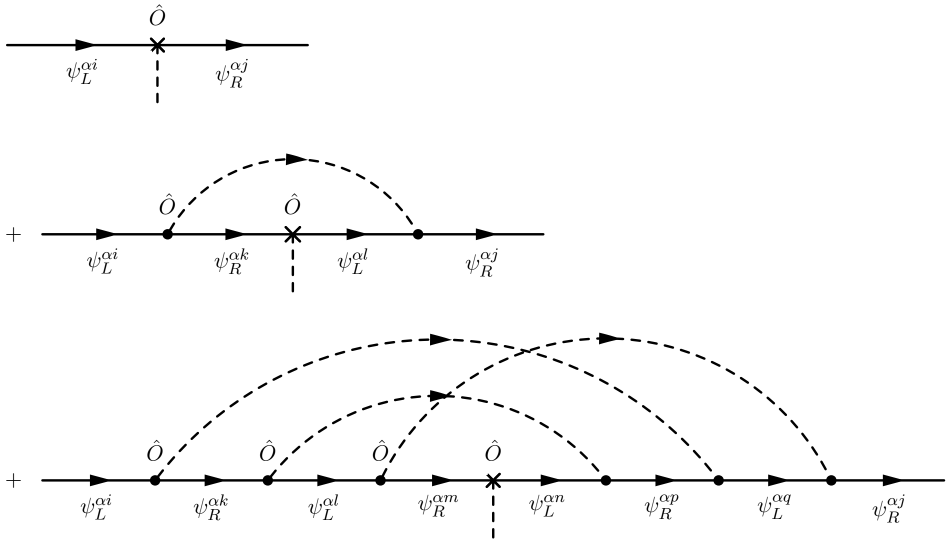

Fig. 6.1. The tree level contributions, one loop contributions (not all possibilities are drawn, the tree level contributions occurs namely also to the left or to the right of the loop, while to $\hat{\mathbf{O}}$ three singlets and two triplets, presented in Eq. (6.16), contribute) and two loop contributions are drawn (again not all the possibilities are shown up). Each $(i, j, k, l, m \ldots)$ determines a family quantum number (running within the four families — $(1, 2, 3, 4)$), α denotes one of the family members ($\alpha = (u, \nu, d, e)$) quantum numbers, all in the massless basis $\psi^{i\alpha}_{(R,L)}$. Dynamical fields start and end with dots $\bullet$, while **x** with the vertical slashed line represents the interaction of the fermion fields with the nonzero vacuum expectation values of the scalar fields.

6.2.1 Mass matrices on the tree level

Let us first present the mass matrix on the tree level for an α^{th} family member, that is for $n = 0$ in Eq. (6.19).

Taking into account Eq. (6.14) one obtains for the diagonal matrix elements on the tree level (for $n = 0$ in Eq. (6.19)) $[\mathbf{a}^\alpha - (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2), \mathbf{a}^\alpha - (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2), \mathbf{a}^\alpha + (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2), \mathbf{a}^\alpha + (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)]$, respectively. The corresponding diagrams are presented in Fig. (6.2).

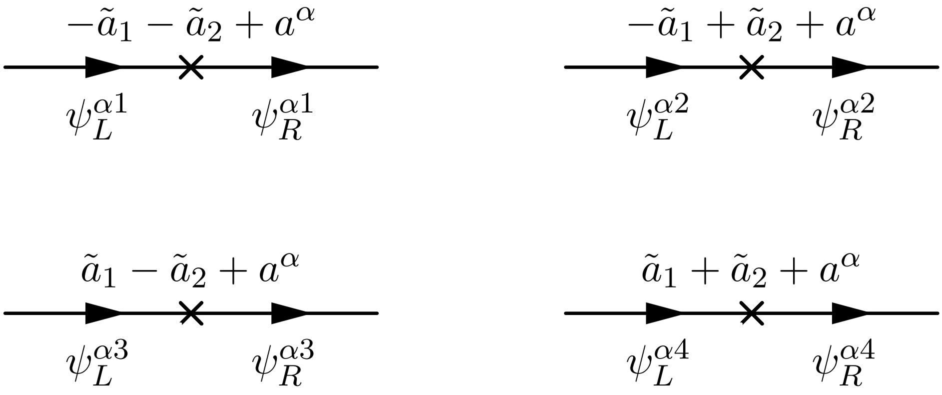

Fig. 6.2. The tree level contributions to the diagonal matrix elements of the operator $\hat{O}^{\alpha}_{\text{dia}}$, Eq. (6.20). The eigenvalues of the operators $\tilde{N}^3_L$ and $\tilde{\tau}^{\tilde{1}3}$ on a family state i can be read in Eq. (6.14).

Taking into account Eq. (6.14) one finds for the off diagonal elements on the tree level:

$< \psi^1|..|\psi^2 > = < \psi^3|..|\psi^4 > = < \psi^2|..|\psi^1 >^{\dagger} = < \psi^4|..|\psi^3 >^{\dagger} = < \tilde{\mathbf{A}}^{\tilde{N}_L \boxminus} >$,
$< \psi^1|..|\psi^3 > = < \psi^2|..|\psi^4 > = < \psi^3|..|\psi^1 >^{\dagger} = < \psi^4|..|\psi^2 >^{\dagger} = < \tilde{\mathbf{A}}^{\tilde{1} \boxminus} >$.

The corresponding diagrams for $< \psi^1|..|\psi^2 >$, $< \psi^2|..|\psi^1 >$, $< \psi^2|..|\psi^3 >$ and $< \psi^3|..|\psi^2 >$ are presented in Fig. (6.3). The vacuum expectation values of this matrix elements on the tree level are presented in the mass matrix of Eq.(6.22).

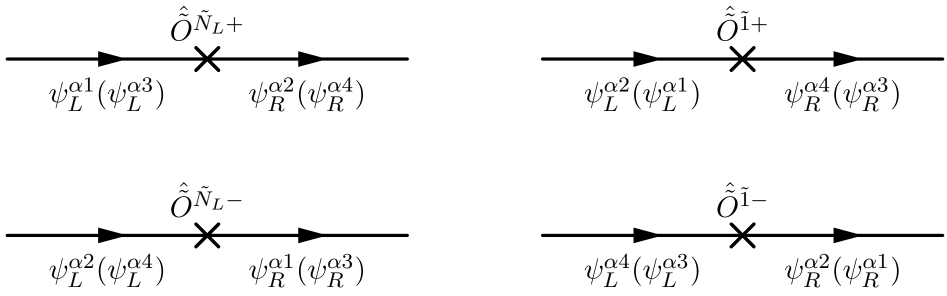

Fig. 6.3. The tree level contributions to the off diagonal matrix elements of the operators $\hat{\tilde{O}}^{\tilde{1}\boxplus}$ and $\hat{\tilde{O}}^{\tilde{N}_L\boxplus}$, Eq. (6.20) are presented. The application of the operators $\tilde{N}_L^{\boxplus}$ and $\tilde{\tau}^{\tilde{1}\boxplus}$ on a family state i can be read in Eq. (6.14).

The contributions to the off diagonal matrix elements $< \psi^1|..|\psi^4 >$, $< \psi^2|..|\psi^3 >$, $< \psi^3|..|\psi^2 >$ and $< \psi^4|..|\psi^1 >$ are nonzero only, if one makes three steps (not two, due to the left right jumps in each step), that is indeed in the third order of correction. For $< \psi^1|..|\psi^4 >$ we have (in the basis $\frac{1}{\sqrt{2}}$ $(|\psi^i_L > + |\psi^i_R >)$ and with the notation $< \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus} > = \frac{1}{\sqrt{2}} (< \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}_{(+)} > + < \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}_{(-)} >)$ after we take intoaccount that $\gamma^0 \overset{78}{(\pm)}$ transform the right handed family members into the left handed ones and opposite): $<\psi^1|\sum_{+,-}\tilde{\tau}^{\tilde{1}\substack{\boxplus\\ \square}} <\tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \square}}> \sum_k|\psi^k><\psi^k|\sum_{+,-}\tilde{N}_L^{\substack{\boxplus\\ \square}} <\tilde{\mathbf{A}}^{\tilde{N}_L\substack{\boxplus\\ \square}}>|\psi^4>$ $<\psi^4|(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha)|\psi^4>$. There are all together six such terms, presented in Fig. (6.4), since the diagonal term appears also at the beginning as $(-\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha)$ and in the middle as $(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha)$, and since the operators $\sum_{+,-}\tilde{\tau}^{\tilde{1}\substack{\boxplus\\ \square}} <\tilde{\mathbf{A}}^{\tilde{1}\substack{\boxplus\\ \square}}>$ and $\sum_{+,-}\tilde{N}_L^{\substack{\boxplus\\ \square}} <\tilde{\mathbf{A}}^{\tilde{N}_L\substack{\boxplus\\ \square}}>$ appear in the opposite order as well. We simplify the notation from $|\psi^k>$ to $|k>$. Summing all these six terms for each of four matrix elements ($<1|..|4>$, $<2|..|3>$, $<3|..|2>$, $<4|..|1>$) one gets (taking into account Eqs. (6.19, 6.14)):

$$
\begin{aligned}
<1|..|4> &= \mathbf{a}^\alpha <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>,\\
<2|..|3> &= \mathbf{a}^\alpha <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>,\\
<3|..|2> &= \mathbf{a}^\alpha <\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>,\\
<4|..|1> &= \mathbf{a}^\alpha <\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>\ .
\end{aligned}
\tag{6.21}
$$

Each matrix element is in Eq. (6.21) divided by 3!, since it is the contribution in the third order! One notices that $<4|..|1>^\dagger=<1|..|4>$ and $<3|..|2>^\dagger=<2|..|3>$. These matrix elements are included into the mass matrix, Eq. (6.22).

To show up the symmetry of the mass matrix on the lowest level we put all the matrix elements in Eq. (6.22).

$$
{}^\alpha\mathcal{M}_{(o)} =
\begin{pmatrix}
-\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha & <\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}> & <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}> & \mathbf{a}^\alpha<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>\\
<\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}> & -\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha & \mathbf{a}^\alpha<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}> & <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>\\
<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}> & \mathbf{a}^\alpha<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}> & \tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha & <\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>\\
\mathbf{a}^\alpha<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}> & <\tilde{\mathbf{A}}^{\tilde{1}\boxplus}> & <\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}> & \tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2+\mathbf{a}^\alpha
\end{pmatrix}
\tag{6.22}
$$

Mass matrix is dimensionless. One notices that the diagonal terms have on the tree level the symmetry $<\psi^1|..|\psi^1>+<\psi^4|..|\psi^4>=2\,a^\alpha=<\psi^2|..|\psi^2>$ $+<\psi^3|..|\psi^3>$, and that in the off diagonal elements with "three steps needed" the contribution of the fields, which depend on particular family member $\alpha=(u,d,\nu,e)$, enters.

We also notice that $<\psi^i|..|\psi^j>^\dagger=<\psi^j|..|\psi^i>$. We see that $<1|..|3>=<2|..|4>$ $=<3|..|1>^\dagger=<4|..|2>^\dagger$, that $<1|..|2>=<3|..|4>=<2|..|1>^\dagger=<4|..|3>^\dagger$ and that $<4|..|1>^\dagger=<1|..|4>$ and $<3|..|2>^\dagger=<2|..|3>$, what is already written below Eq. (6.21), $<i|..|j>$ denotes $<\psi^i|..|\psi^j>$.

In the case that $a=<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>=<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}>=e$ and $<\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>=<\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>=d$, which would mean that all the matrix elements are real, the mass matrix simplifies to

$$
\mathcal{M}^\alpha_{(o)} =
\begin{pmatrix}
-\tilde{a}_1-\tilde{a}_2+a^\alpha & d & e & b\\
d & -\tilde{a}_1+\tilde{a}_2+a^\alpha & b & e\\
e & b & \tilde{a}_1-\tilde{a}_2+a^\alpha & d\\
b & e & d & \tilde{a}_1+\tilde{a}_2+a^\alpha
\end{pmatrix},
\tag{6.23}
$$

with $b=\mathbf{a}^\alpha ed$.

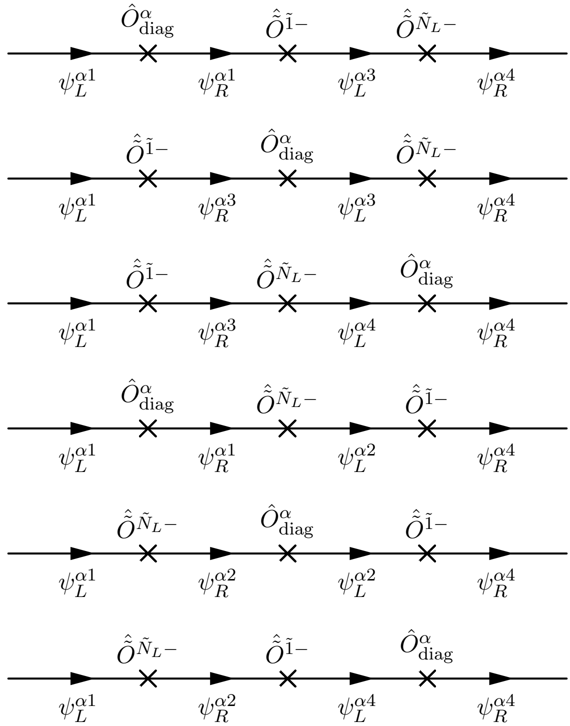

Fig. 6.4. The tree level contribution to the matrix element $<\psi^1|b|\psi^4>$ is presented. One comes from $<\psi^1|$ to $|\psi^4>$ in three steps: $<\psi^1|\sum_{+,-}\tilde{\tau}^{\tilde{1}\boxplus} < \tilde{\mathbf{A}}^{\tilde{1}\boxplus} > \sum_k |\psi^k><\psi^k|\sum_{+,-}\tilde{N}_L^{\boxplus} < \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus} > |\psi^4><\psi^4|(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2+\mathbf{a}^{\alpha})|\psi^4>$. There are all together six such terms, since the diagonal term appears also at the beginning as $(-\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^{\alpha})$ and in the middle as $(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2+\mathbf{a}^{\alpha})$, and since the operators $\sum_{+,-}\tilde{\tau}^{\tilde{1}\boxplus} < \tilde{\mathbf{A}}^{\tilde{1}\boxplus} >$ and $\sum_{+,-}\tilde{N}_L^{\boxplus} < \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus} >$ appear in the opposite order as well.

6.2.2 Mass matrices beyond the tree level

We discuss in this subsection the matrix elements of the mass matrix in all orders of corrections, Eq. (6.19), the tree level, $n=0$, of which is presented in Eq. (6.22). The tree level mass matrix manifests the $\widetilde{SU}(2)\times\widetilde{SU}(2)\times U(1)$ symmetry as seen in Eq. (6.22), with $(<1|x|1>+<4|x|4>)-(<2|x|2>+<3|x|3>)=0$ and $<1|x|3>=<2|x|4>=<3|x|1>^{\dagger}=<4|x|1>^{\dagger}$ and with $(<1|xxx|4>, <2|xxx|3>, <3|xxx|2>, <4|xxx|1>)$ related so that all are equal if $<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}>$ and $<\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>$ are real.

Let us repeat that the generators of the two groups which operate among families commute: $\{\tilde{\tau}^{\tilde{1}i}, \tilde{N}_L^j\}_- = 0$, and that these generators commute also with generators which distinguish among family members: $\{\tilde{\tau}^{\tilde{1}i}, \tau^{\alpha}\}_- = 0$, $\{\tau^{\alpha}, \tilde{N}_L^j\}_- = 0$, where τ^{α} represents (Q, Q', Y') (or $\tau^4, \tau^{23}, \tau^{13}$).

To study the symmetry $\widetilde{SU}(2)\times\widetilde{SU}(2)\times U(1)$ of the mass matrix, Eq. (6.22), in all orders of loop corrections, of repetition of nonzero vacuum expectation values and of both together — loop corrections and nonzero vacuum expectation values — we just have to calculate at each order of corrections the difference between each pair of the matrix elements which are equal on the three level, as well as the Hermitian conjugated difference of such a pair.

Since the dependence of all the scalar fields on ordinary coordinates are in all cases the same, we only have to evaluate the application of the operators to the internal space of basic state, that is on the space of family and family members degrees of freedom. Correspondingly we pay attention only on this internal part — on the interaction of scalar fields with the space index $(7, 8)$ with any family member of any of four families separately with respect to their internal space. The dependence of the mass matrix elements on the family member quantum numbers appears through the nonzero vacuum expectation value $\mathbf{a}^{\alpha}$, Eq. (6.22), as well as through the dynamical part of $\hat{\mathbf{O}}^{\alpha}$, Eq. (6.20).

We demonstrate in this subsection how does the repetition of the nonzero vacuum expectation values of the scalar fields and loop corrections in all orders influence matrix elements, presented on the tree level in Eq. (6.22).

In the case that $\mathbf{a}^{\alpha} = 0$ (that is for $< \mathbf{A}^{Q} >= 0$, $< \mathbf{A}^{Q'} >= 0$ and $< \mathbf{A}^{Y'} >= 0$) the symmetry in all corrections, that is in all loop corrections and all the repetition of nonzero vacuum expectation values of the scalar fields, and of both — the loop corrections and the repetitions of nonzero vacuum expectation values nonzero of all the scalar fields except $\mathbf{a}^{\alpha}$ — keep the symmetry of the tree level, presented in Eq. (6.22).

We prove in this subsection that in the case that $< \mathbf{A}^{Q} >= 0$, $< \mathbf{A}^{Q'} >= 0$ and $< \mathbf{A}^{Y'} >= 0$, that is for $\mathbf{a}^{\alpha} = 0$, the symmetry of mass matrices remains unchanged in all orders of corrections: the loop ones of dynamical fields — $\mathbf{A}^{Q}$, $\mathbf{A}^{Q'}$, $\mathbf{A}^{Y'}$, $\vec{\tilde{\mathbf{A}}}^{\tilde{N}_L}$, $\vec{\tilde{\mathbf{A}}}^{\tilde{1}}$ — in the repetition of nonzero vacuum expectation values of the scalar fields carrying the family quantum numbers — $< \vec{\tilde{\mathbf{A}}}^{\tilde{N}_L} >$ and $< \vec{\tilde{\mathbf{A}}}^{\tilde{1}} >$ — and of all together. The symmetry of mass matrices remains in all orders of corrections the one of the tree level also if $\mathbf{a}^{\alpha} \neq 0$ while $\tilde{\mathbf{a}}_1 = 0$ and $\tilde{\mathbf{a}}_2 = 0$. The symmetry changes if the nonzero vacuum expectation values of all the scalar fields are nonzero.

In the case, however, that $\mathbf{a}^{\alpha} = 0$, the matrix elements, which are in the lowest order proportional to $\mathbf{a}^{\alpha}$ in Eq. (6.22), remain zero in all orders of corrections, while the nonzero matrix elements become dependent on family members quantum numbers due to the participations in loop corrections in all orders of the dynamical fields $\mathbf{A}^{Q}$, $\mathbf{A}^{Q'}$ and $\mathbf{A}^{Y'}$.

We study in what follows first the symmetry of mass matrices in all orders of corrections in the case that $\mathbf{a}^{\alpha} = 0$, and then the symmetry of the mass matrices, again in all orders of corrections, when $\mathbf{a}^{\alpha} \neq 0$. We also comment that the symmetry of the tree level remain the same in all orders of corrections, if $\mathbf{a}^{\alpha} \neq 0$, while $\tilde{\mathbf{a}}_1 = 0 = \tilde{\mathbf{a}}_2$.

Mass matrices beyond the tree level, if $\mathbf{a}^{\alpha} = 0$ We study corrections to which the scalar fields which distinguish among families, contribute — with their nonzero vacuum expectation values $< \vec{\tilde{\mathbf{A}}}^{\tilde{N}_L} >$ and $< \vec{\tilde{\mathbf{A}}}^{\tilde{1}} >$ and their dynamical parts $\vec{\tilde{\mathbf{A}}}^{\tilde{N}_L}$ and $\vec{\tilde{\mathbf{A}}}^{\tilde{1}}$ — while we assume $\mathbf{a}^{\alpha} = 0$ ($\mathbf{a}^{\alpha}$ denotes the vacuum expectation values to which the tree singlet fields, distinguishing among family members, contribute, that is ($<\mathbf{A}^{Q}>, <\mathbf{A}^{Q'}>, <\mathbf{A}^{Y'}>$), taking into account the loop corrections of the corresponding dynamical parts ($\mathbf{A}^{Q}$, $\mathbf{A}^{Q'}$, $\mathbf{A}^{Y'}$) in all orders.

We show that in such a case — that is in the case that $\mathbf{a}^{\alpha}=0$ while all the other scalar fields determining mass matrices have nonzero vacuum expectation values ($\tilde{\mathbf{a}}_1 \neq 0, \tilde{\mathbf{a}}_2 \neq 0, <\tilde{\mathbf{A}}^{\tilde{N}_L \overset{\boxplus}{\boxminus}}> \neq 0, <\tilde{\mathbf{A}}^{\tilde{1}\overset{\boxplus}{\boxminus}}> \neq 0$) — the matrix elements, evaluated in all orders of corrections, keep the symmetry of the tree level.

We also show, that in this case the off diagonal matrix elements, represented in Eq. (6.22) as ($\mathbf{a}^{\alpha} <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>$, $\mathbf{a}^{\alpha} <\tilde{\mathbf{A}}^{\tilde{1}\boxminus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>$, $\mathbf{a}^{\alpha} <\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>$, $\mathbf{a}^{\alpha} <\tilde{\mathbf{A}}^{\tilde{1}\boxplus}><\tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}>$), remain zero in all orders of corrections.

Let us look how the corrections in all orders manifest for each matrix element separately.

i. We start with diagonal terms: $<\psi^i|.....|\psi^i>, i=(1,2,3,4)$.

On the tree level the symmetry is:

$\{<\psi^1|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^1> + <\psi^4|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^4>\} - \{<\psi^2|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^2> + \{<\psi^3|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^3>\} = 0.$

i.a. It is easy to see that the tree level symmetry, $\{<\psi^1|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^1> + <\psi^4|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^4>\} - \{<\psi^2|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^2> + <\psi^3|<\hat{\mathbf{O}}^{\alpha}_{\rm dia}>|\psi^3>\} = 0$, remains in all orders of corrections, if only the nonzero vacuum expectation values of $<\tilde{\mathbf{A}}^{\tilde{1}3}> = \tilde{\mathbf{a}}_1$ and $<\tilde{\mathbf{A}}^{\tilde{N}_L 3}> = \tilde{\mathbf{a}}_2$ contribute in operators $\gamma^0 \overset{78}{(\pm)} \tilde{\tau}^{\tilde{1}3} <\tilde{\mathbf{A}}^{\tilde{1}3}>$ and $\gamma^0 \overset{78}{(\pm)} \tilde{N}^3_L <\tilde{\mathbf{A}}^{\tilde{N}_L 3}>$. At, let say, $(2k+1)^{st}$ order of corrections we namely have $\{(-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2))^{(2k+1)} + (\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^{(2k+1)}\} - \{(-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2))^{(2k+1)} + (\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^{(2k+1)}\} = 0$.

i.b. The contributions of the dynamical terms, either (A^{Q}, $A^{Q'}$, $A^{Y'}$) or ($\tilde{A}^{\tilde{1}3}, \tilde{A}^{\tilde{N}_L 3}$) do not break the three level symmetry. Each of them namely always appears in an even power, Fig. (6.1), changing the order of corrections by a factor of two or $2n$ ($|\mathbf{A}^{\alpha}|^{2(n-k-l)}, |\tilde{\mathbf{A}}^{\tilde{1}3}|^{2k}, |\tilde{\mathbf{A}}^{\tilde{N}_L 3}|^{2l}$), where $(n-k-l,k,l)$ are nonnegative integers, while $\tau^{A\alpha}$ represents $(Q^{\alpha}, Q'^{\alpha}, Y'^{\alpha})$. The contribution to $|\mathbf{A}^{\alpha}|^{2m}, m=(n-k-l)$, origins in the product of $|\mathbf{A}^{Q}|^{2(m-p-r)} \cdot |\mathbf{A}^{Q'}|^{2p} \cdot |\mathbf{A}^{Y'}|^{2r}$. Again $(m-p-r,p,r)$ are nonnegative integers.

i.c. There are also other contributions, either those with only nonzero vacuum expectation values or with dynamical fields in addition to nonzero vacuum expectation values of scalars, in which $\hat{\hat{\mathbf{O}}}^{\tilde{1}\overset{\boxplus}{\boxminus}}$ and $\hat{\hat{\mathbf{O}}}^{\tilde{N}_L\overset{\boxplus}{\boxminus}}$ together with all kinds of diagonal terms contribute. Let us repeat again what do the operators $\hat{\hat{O}}^{\tilde{1}\overset{\boxplus}{\boxminus}}$ and $\hat{\hat{O}}^{\tilde{N}_L\overset{\boxplus}{\boxminus}}$, Eq. (6.20), do when they apply on ψ^i. The operators $\hat{\hat{O}}^{\tilde{1}\boxplus}$ transforms ψ^1 into ψ^3 and ψ^2 into ψ^4. Correspondingly the states ψ^1 and ψ^4 take under the application of $\hat{\hat{O}}^{\tilde{1}\boxplus}$ the role of ψ^2 and ψ^3, while ψ^2 and ψ^3 take the role of ψ^1 and ψ^4, all carrying the correspondingly changed eigenvalues of $\tilde{\tau}^{\tilde{1}3}$. The operator $\hat{\hat{O}}^{\tilde{N}_L\boxplus}$ transforms ψ^1 into ψ^2 and ψ^3 into ψ^4. Correspondingly the states ψ^1 and ψ^2 take under the application of $\hat{\hat{O}}^{\tilde{N}_L\boxplus}$ the role of ψ^3 and ψ^4, while ψ^3 and ψ^4 take the role of ψ^1 and ψ^2, carrying the correspondingly changed eigenvalues of $\tilde{N_L}^3$. Either the dynamical fields or the nonzero vacuum expectation values of these scalar fields, $\hat{\hat{O}}^{\tilde{1}\overset{\boxplus}{\boxminus}}$ and $\hat{\hat{O}}^{\tilde{N}_L\overset{\boxplus}{\boxminus}}$, must in diagonal terms appear in the second power or in $n\times$ the second power. We easily see that also in such cases the tree level symmetry remains in all orders.

i.c.1. To better understand the contributions in all orders to the diagonal terms, discussing here, let us calculate the contribution of the third order corrections either from the loop or from the nonzero vacuum expectation values to the diagonal matrix elements $<\psi^i|...|\psi^i>$ under the assumption that $\mathbf{a}^{\alpha}=0$. Let us evaluate the contributions of the operators $<\hat{\hat{\mathbf{O}}}^{\tilde{1}3}>$, $\hat{\hat{\mathbf{O}}}^{\tilde{N}_L 3}$, $\hat{\hat{\mathbf{O}}}^{\tilde{1}\boxplus}_{\boxminus}$ and $\hat{\hat{\mathbf{O}}}^{\tilde{N}_L\boxplus}_{\boxminus}$ in the third order. We see that $\tilde{\tau}^{\tilde{1}\boxminus}$ transforms ψ^3 into ψ^1 and ψ^4 into ψ^2, while $\tilde{\tau}^{\tilde{1}\boxplus}$ transforms ψ^2 into ψ^4 and ψ^1 into ψ^3. We see that $\tilde{N}_L^{\boxminus}$ transforms ψ^2 into ψ^1 and ψ^4 into ψ^3, while $\tilde{N}_L^{\boxplus}$ transforms ψ^1 into ψ^2 and ψ^3 into ψ^4. It then follows that $\{<\psi^1|xxx|\psi^1>+<\psi^4|xxx|\psi^4>\}-\{<\psi^2|xxx|\psi^2>+<\psi^3|xxx|\psi^3>\}=0$, where xxx represent all possible acceptable combination of $<\hat{\hat{\mathbf{O}}}^{\tilde{1}\boxplus}_{\boxminus}>$, $<\hat{\hat{\mathbf{O}}}^{\tilde{N}_L\boxplus}_{\boxminus}>$ and the diagonal terms $<\hat{\hat{\mathbf{O}}}^{\tilde{1}3}>$ and $<\hat{\hat{\mathbf{O}}}^{\tilde{N}_L 3}>$. One namely obtains that the contribution of $\{<\psi^1|xxx|\psi^1>+<\psi^4|xxx|\psi^4>\}=\{|<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>|^2[-2(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)+(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)]+|<\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>|^2[-2(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)]+(-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^3)+|<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>|^2[+2(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)]+|<\tilde{\mathbf{A}}^{\tilde{N}_L-}>|^2[+2(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)+(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)]+(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^3\}=0$, and for $\{<\psi^2|xxx|\psi^2>+<\psi^3|xxx|\psi^3>\}$ one obtains $=\{|<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>|^2[-2(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)+(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)]+|<\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>|^2[-2(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)]+(-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^3)+|<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>|^2[+2(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)]+|<\tilde{\mathbf{A}}^{\tilde{N}_L-}>|^2[+2(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)+(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)]+(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^3\}=0$. Also the dynamical fields keep the tree level symmetry of mass matrices. To prove one only must replace in the above calculation $|<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>|^2$ by $|\tilde{\mathbf{A}}^{\tilde{1}\boxminus}|^2$ and $|<\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>|^2$ by $|\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}|^2$.

To the diagonal terms the three singlets contribute in absolute squared values $(|\mathbf{A}^{Q}|^2, |\mathbf{A}^{Q'}|^2, |\mathbf{A}^{Y'}|^2$, each on a power, which depend on the order of corrections. This makes all the diagonal matrix elements, $<\psi^1|.....|\psi^1>$, $<\psi^2|.....|\psi^2>$, $<\psi^3|.....|\psi^3>$ and $<\psi^4|.....|\psi^4>$, dependent on the family member quantum numbers.

Such behaviour of matrix elements remains unchanged in all orders of corrections, either due to loops of dynamical fields or due to repetitions of nonzero vacuum expectation values. The reason is in the fact that the operators $<\hat{\hat{\mathbf{O}}}^{\tilde{1}\boxplus}_{\boxminus}>$ and $<\hat{\hat{\mathbf{O}}}^{\tilde{N}_L\boxplus}_{\boxminus}>$ exchange the role of the states in the way that the odd power of diagonal contributions to the diagonal matrix elements always keep the symmetry $\{<\psi^1|\hat{U}|\psi^1>+<\psi^4|\hat{U}|\psi^4>\}-\{<\psi^2|\hat{U}|\psi^2>+<\psi^3|\hat{U}|\psi^3>\}=0$.

These proves the statement that *corrections in all orders keep the symmetry of the tree level diagonal terms in the case that* $a^{\alpha}=0$.

ii. Let us look at matrix element $<\psi^1|.....|\psi^3>$ and $<\psi^2|.....|\psi^4>$ in Eq. (6.22), where we have on the tree level $<1|x|3>=<2|x|4>$ and $<3|x|1>=<4|x|2>=<1|x|3>^{\dagger}$. We again simplify the notation $<\psi^i|.....|\psi^j>$ into $<i|...|j>$. The two matrix elements — $<1|x|3>$, $<2|x|4>$ — are on the tree level denoted by $<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>$, while $<3|x|1>$ and $<4|x|2>$ are denoted by $<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}>$.

We have to prove that corrections, either of the loops kind or of the repetitions of the nonzero vacuum expectation values or of both kinds in any order keeps the symmetry of the tree level.

ii.a. Let us start with the corrections in which besides $<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>$ in the first power only $<\tilde{\mathbf{A}}^{\tilde{1}3}>=\tilde{\mathbf{a}}_1$ and $<\tilde{\mathbf{A}}^{\tilde{N}_L 3}>=\tilde{\mathbf{a}}_2$ contribute, the last two together appear in an even power so that all three together contribute in an odd power.

The contribution of $(<1|x|1>)^{2k+1}=(-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2))^{2k+1}$ in the $(2k+1)^{th}$ order is up to a sign equal to $(<4|x|4>)^{2k+1}=(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^{2k+1}$, where k is a nonnegative integer, while the contribution of $(<2|x|2>)^{2k+1}=(-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2))^{2k+1}$ is up to a sign equal to $(<3|x|3>)^{2k+1}=(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^{2k+1}$. In each of the matrix elements, either $<1|.....|3>$ or $<2|.....|4>$, both factors together, $(-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2))^m\,(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^n$ in the case $<1|.....|3>$ and $(-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2))^m\,(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^n$ in the case $<2|.....|4>$, with $(m+n)$ an even nonnegative integer (since together with $<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>$ must be of an odd integer corrections to take care of the left/right nature of matrix elements) one must make the sum over all the terms contributing to corrections of the order $(m+n+1)$. It is not difficult to see that the contribution to $<1|.....|3>$ is in any order of corrections equal to the contributions to the same order of corrections to $<2|.....|4>$.

ii.a.1. To illustrate the same contribution in each order of corrections to $<1|.....|3>$ and to $<2|.....|4>$ let us calculate, let say, the third order corrections. The contribution of the third order to $<1|xxx|3>$ is $-\frac{1}{3!}<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>\{(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)^2+(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^2-(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)\}$ and the contribution of the third order to $<2|xxx|4>$ is $-\frac{1}{3!}<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>\{(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)^2+(\tilde{a}_1+\tilde{\mathbf{a}}_2)^2-(\tilde{\mathbf{a}}_1+\tilde{\mathbf{a}}_2)(\tilde{\mathbf{a}}_1-\tilde{\mathbf{a}}_2)\}$, that is the contributions in the third order of $<1|xxx|3>$ and $<2|xxx|4>$ are the same.

ii.b. One can repeat the calculations with $<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>$ and the dynamical fields $\tilde{\mathbf{A}}^{\tilde{1}\boxminus}$ and $\tilde{\mathbf{A}}^{\tilde{1}\boxplus}$, with or without the diagonal nonzero vacuum expectation values. In all cases all the contributions keep the symmetry on the tree level due to the above discussed properties of the diagonal terms. All the dynamical terms must namely appear in absolute values squared in order to contribute to the mass matrices, as shown in Fig. 6.1. To the diagonal terms the three singlets contribute in absolute squared values ($|\mathbf{A}^{Q}|^2$, $|\mathbf{A}^{Q'}|^2$, $|\mathbf{A}^{Y'}|^2$), each on some power, depending on the order of corrections. This makes the matrix element $<1|.....|3>$ and $<2|.....|4>$, $<3|.....|1>$ and $<4|.....|2>$, dependent on the family members quantum numbers.

In all cases all the contributions keep the symmetry on the tree level.

ii.c. The Hermitian conjugate values $<1|.....|3>^{\dagger}=<2|.....|4>^{\dagger}$ have the transformed value of $<\tilde{\mathbf{A}}^{\tilde{1}\boxminus}>$, that means that the value is $<\tilde{\mathbf{A}}^{\tilde{1}\boxplus}>$, provided that the diagonal matrix elements of the mass matrix are real, keeping the symmetry of the matrix elements $<1|.....|3>^{\dagger}=<2|.....|4>^{\dagger}$ in all orders of corrections.

These proves the statement that *corrections in all orders keep the symmetry of the tree level of the off-diagonal terms* $<1|.....|3>$ and $<2|.....|4>$ and of their Hermitian conjugated matrix elements in the case that $\mathbf{a}^{\alpha}=0$.

iii. Let us look at matrix element $<1|.....|2>$ and $<3|.....|4>$ in Eq. (6.22), where we have on the tree level $<1|x|2>=<3|x|4>$. These two matrix elements are on the tree level denoted by $<\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}>$. We have to prove that corrections, either the loop corrections or the repetitions of the nonzero vacuum expectation values or both kinds of corrections, in any order, keep the $\widetilde{SU}(2)\times\widetilde{SU}(2)\times U(1)$ symmetry of the tree level.

The proof for the symmetry of these matrix elements is carried out in equivalent way to the proof under **ii.** .

iii.a. Let us start with the corrections in which besides $< \tilde{\mathbf{A}}^{\tilde{N}_L \boxminus} >$ in the first power also only $< \tilde{\mathbf{A}}^{\tilde{1}3} >= \tilde{\mathbf{a}}_1$ and $< \tilde{\mathbf{A}}^{\tilde{N}_L 3} >= \tilde{\mathbf{a}}_2$ contribute. The sum of powers of the last two $\mathbf{a}$ must be even, so that a correction would be of an odd power due to the left/right transitions.

Again the contributions of both diagonal terms, $< 1|x|1 >$ and $< 4|x|4 >$, in any power — $(< 1|x|1 >)^{2k+1} = (-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))^{2k+1}$ and $(< 4|x|4 >)^{2k+1} = (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)^{2k+1}$, where k is a nonnegative integer — differ only up to a sign when they appear in an odd power and are equal when they appear in an even power. These is true also for the contributions of $< 2|x|2 >$ and $< 3|x|3 >$ since $(< 2|x|2 >)^{2k+1} = (-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))^{2k+1}$ is up to a sign equal to $(< 3|x|3 >)^{2k+1} = (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)^{2k+1}$. If they appear with an even power, they are equal. In each of the $(m+n+1)^{th}$ order corrections to the matrix elements, either $< 1|.....|2 >$ or $< 3|.....|4 >$, where $(-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))^m \; (-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))^n$ contribute to $< 1|.....|2 >$ and $(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)^m \; (\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)^n$ contribute to $< 3|.....|4 >$, the two contributions are again equal, since both m and n are even nonnegative integers.

iii.a.1. Let us, as an example, calculate the fifth order corrections to the tree level contributions of $< 1|x|2 > = < \tilde{\mathbf{A}}^{\tilde{N}_L \boxminus} >$. The contribution of the fifth order $< 1|xxxxx|2 >$ to $< 1|x|2 >$ is $\frac{1}{5!} < \tilde{\mathbf{A}}^{\tilde{N}_L \boxminus} > \{(-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))^4 + (-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))^4 + 3(-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))(-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))^3 + 6(-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))^2(-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))^2 + 3(-(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2))^3(-(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2))\}$, and the contribution of the fifth order $< 3|xxxxx|4 >$ to $< 3|x|4 >$ is $\frac{1}{5!} < \tilde{\mathbf{A}}^{\tilde{N}_L \boxminus} > \{(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)^4 + (\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)^4 + 3(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)^3 + 6(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)^2(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)^2 + 3(\tilde{\mathbf{a}}_1 - \tilde{\mathbf{a}}_2)^3(\tilde{\mathbf{a}}_1 + \tilde{\mathbf{a}}_2)\}$, which is equal to the contribution of the fifth order in the case of $< 1|xxxxx|2 >$.

iii.b. One can repeat the calculations with dynamical fields $(\tilde{\mathbf{A}}^{\tilde{N}_L \boxminus}, \tilde{\mathbf{A}}^{\tilde{N}_L \boxplus})$ in all orders and with $< \tilde{\mathbf{A}}^{\tilde{1} \boxminus} >$ and with the diagonal nonzero vacuum expectation values and with the diagonal dynamical terms, paying attention that the dynamical fields contribute to masses of any of the family members only if they appear in pairs.

To the diagonal terms the three singlets $(\mathbf{A}^Q, \mathbf{A}^{Q'}, \mathbf{A}^{Y'})$ contribute in the absolute squared values $(|\mathbf{A}^Q|^2, |\mathbf{A}^{Q'}|^2, |\mathbf{A}^{Y'}|^2)$, each on a power, which depends on the order of corrections.

In all cases all the contributions keep the symmetry on the tree level.

iii.c. The proof is valid also for $< 2|.....|1 >= (< 1|.....|2 >)^\dagger$ and $< 4|.....|3 >= (< 3|.....|4 >)^\dagger$ in any order of corrections. Namely, if diagonal mass matrix elements are real then in the matrix elements $< 2|.....|1 >$ only $< \tilde{A}^{\tilde{N}_L \boxminus} >$ of the matrix element $< 1|.....|2 >$ must be replaced by $< \tilde{A}^{\tilde{N}_L \boxplus} >$.

These proves the statement that *corrections in all orders keep the symmetry of the tree level off-diagonal terms* $< 1|.....|2 >$ and $< 3|.....|4 >$ in the case that $\mathbf{a}^\alpha = 0$.

iv. It remains to check the matrix elements $< 1|.....|4 >$, $< 2|.....|3 >$, $< 3|.....|2 >$ and $< 4|.....|1 >$ in all orders of corrections. The matrix elements on the third power, $(< 1|xxx|4 >, < 2|xxx|3 >, < 3|xxx|2 >, < 4|xxx|1 >)$, appearing in Eqs. (6.21, 6.22), are for $\mathbf{a}^\alpha = 0$ all equal to zero. It is not difficult to prove that these four matrix elements remain zero in all order of loop corrections. The reason is the same as in the above three cases, **i.**, **ii.**, **iii.**.

The proof that the symmetry $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$ *of the tree level remains unchanged in all orders of corrections, provided that* $\mathbf{a}^\alpha = 0$*, is completed.*

There are in all these cases the dynamical singlets contributing in the absolute squared values ($|\mathbf{A}^{Q}|^2$, $|\mathbf{A}^{Q'}|^2$, $|\mathbf{A}^{Y'}|^2$ — each on a power, which depend on the order of corrections — which make that all the matrix elements of a mass matrix, except the ($< 1|.....|4 >$, $< 2|.....|3 >$, $< 3|.....|2 >$, $< 4|.....|1 >$) which remain zero in all orders of corrections, depend on a particular family member.

Mass matrices beyond the tree level if $\mathbf{a}^\alpha \neq 0$ We demonstrated that for $\mathbf{a}^\alpha = 0$ the symmetry of the tree level remains in all orders of corrections, the loops corrections and the repetitions of nonzero vacuum expectation values of all the scalar fields contributing to mass terms, the same as on the tree level, that is $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$.

Let us denote all corrections to the diagonal terms in all orders, in which the nonzero vacuum expectation values in all orders as well as their dynamical fields in all orders contribute when $\mathbf{a}^\alpha = 0$ as:

$-(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) :=< \psi^{\alpha 1}_L|....|\psi^{\alpha 1}_R >, -(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2) :=< \psi^{\alpha 2}_L|....|\psi^{\alpha 2}_R >,$

$(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2) :=< \psi^{\alpha 3}_L|....|\psi^{\alpha 3}_R >, \quad (\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) :=< \psi^{\alpha 4}_L|....|\psi^{\alpha 4}_R >.$

We study for $\mathbf{a}^\alpha \neq 0$ how does the symmetry of the diagonal and the off diagonal matrix elements of the family members mass matrices change with respect to the symmetry on the tree level, presented in Eq. (6.22), in particular for small values of $|\mathbf{a}^\alpha|$ in comparison with the contributions of all the rest of nonzero vacuum expectation values or of dynamical fields.

We discuss diagonal and off diagonal matrix elements separately. The symmetry of all depends on $\mathbf{a}^\alpha$.

i. Let us start with diagonal terms: $< \psi^i|.....|\psi^i >$.

On the tree level the symmetry is for $\mathbf{a}^\alpha \neq 0$: $\{< \psi^1| < \hat{\mathbf{O}}^\alpha_{\mathrm{dia}} > |\psi^1 > + < \psi^4| < \hat{\mathbf{O}}^\alpha_{\mathrm{dia}} > |\psi^4 > \} - \{ < \psi^2| < \hat{\mathbf{O}}^\alpha_{\mathrm{dia}} > |\psi^2 > + \{< \psi^3| < \hat{\mathbf{O}}^\alpha_{\mathrm{dia}} > |\psi^3 >\} = 0$.

i.a. Let us evaluate the matrix elements $< \psi^{\alpha i}_L|....|\psi^{\alpha i}_R >$. Let us denote for a while, just to simplify the derivations, $n_1 = \mathbf{a}^\alpha - (\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)$, $n_2 = \mathbf{a}^\alpha - (\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)$ $n_3 = \mathbf{a}^\alpha + (\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)$ $n_4 = \mathbf{a}^\alpha + (\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)$. One finds

$$
\begin{aligned}
&< \psi^{\alpha 1}_L|....|\psi^{\alpha 1}_R >= [\mathbf{a}^\alpha - (\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)] \\
&-\frac{1}{3!}[(\mathbf{a}^\alpha)^3 - 3(\mathbf{a}^\alpha)^2(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) + 3(\mathbf{a}^\alpha)(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^2] \\
&+\frac{1}{5!}[(\mathbf{a}^\alpha)^5 - 5(\mathbf{a}^\alpha)^4(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) + 10(\mathbf{a}^\alpha)^3(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^2 - 10(\mathbf{a}^\alpha)^2(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^3 \\
&+5(\mathbf{a}^\alpha)(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^4] - \cdots . \qquad (6.24)
\end{aligned}
$$

Assuming that $|\mathbf{a}^\alpha| << (|(\underline{\tilde{\mathbf{a}}}_1|, |(\underline{\tilde{\mathbf{a}}}_2|)$ it follows

$$
\begin{aligned}
&< \psi^{\alpha 1}_L|....|\psi^{\alpha 1}_R >= -(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) + \mathbf{a}^\alpha\{1 - \frac{3}{3!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^2 + \frac{5}{5!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^4 \\
&-\frac{7}{7!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^6 + \cdots\}. \qquad (6.25)
\end{aligned}
$$

Correspondingly we obtain for $< \psi^{\alpha 4}_L|....|\psi^{\alpha 4}_R >$ in the limit that $|\mathbf{a}^\alpha| << (\underline{\tilde{\mathbf{a}}}_1|, |\underline{\tilde{\mathbf{a}}}_2|)$

$$
\begin{aligned}
&< \psi^{\alpha 4}_L|....|\psi^{\alpha 4}_R >= +(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2) + \mathbf{a}^\alpha\{1 - \frac{3}{3!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^2 + \frac{5}{5!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^4 \\
&-\frac{7}{7!}(\underline{\tilde{\mathbf{a}}}_1 + \underline{\tilde{\mathbf{a}}}_2)^6 + \cdots\}. \qquad (6.26)
\end{aligned}
$$

For $<\psi_L^{\alpha 2}|....|\psi_R^{\alpha 2}>$ one obtains in the limit that $|\mathbf{a}^\alpha| << (|(\underline{\tilde{\mathbf{a}}}_1|, |\underline{\tilde{\mathbf{a}}}_2|)$

$$<\psi_L^{\alpha 2}|....|\psi_R^{\alpha 2}> = -(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2) + \mathbf{a}^\alpha\{1 - \frac{3}{3!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^2 + \frac{5}{5!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^4$$
$$-\frac{7}{7!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^6 + \cdots \tag{6.27}$$

And for $<\psi_L^{\alpha 2}|....|\psi_R^{\alpha 2}>$ one obtains in the limit that $|\mathbf{a}^\alpha| << (|(\underline{\tilde{\mathbf{a}}}_1|, |(\underline{\tilde{\mathbf{a}}}_2|)$ the expression

$$<\psi_L^{\alpha 3}|....|\psi_R^{\alpha 3}> = -(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2) + \mathbf{a}^\alpha\{1 - \frac{3}{3!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^2 + \frac{5}{5!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^4$$
$$-\frac{7}{7!}(\underline{\tilde{\mathbf{a}}}_1 - \underline{\tilde{\mathbf{a}}}_2)^6 + \cdots\} . \tag{6.28}$$

Finally we obtain

$$(<\psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 1}> + <\psi_L^{\alpha 4}|\ldots|\psi_R^{\alpha 4}>) -$$
$$(<\psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 2}> + <\psi_L^{\alpha 3}|\ldots|\psi_R^{\alpha 3}>) =$$
$$4\, \mathbf{a}^\alpha \underline{\tilde{\mathbf{a}}}_1\, \underline{\tilde{\mathbf{a}}}_2 \{1 - \frac{1}{12}[(\underline{\tilde{\mathbf{a}}}_1)^2 + (\underline{\tilde{\mathbf{a}}}_2)^2]\} + \cdots\ . \tag{6.29}$$

The term with $(\mathbf{a}^\alpha)^2$ drops away. For small $|\mathbf{a}^\alpha|$ the term $(\mathbf{a}^\alpha)^3$ might be negligible.

It is obvious that for $\mathbf{a}^\alpha \neq 0$ the diagonal matrix elements do not keep the tree level symmetry of mass matrices (which is $(<\psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 1}> + <\psi_L^{\alpha 4}|\ldots|\psi_R^{\alpha 4}>) - (<\psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 2}> + <\psi_L^{\alpha 3}|\ldots|\psi_R^{\alpha 3}>) = 0$). But one sees as well that the contributions of higher terms to asymmetry are getting smaller and smaller and for $|a^\alpha| << (|\underline{\tilde{\mathbf{a}}}_1|, |\underline{\tilde{\mathbf{a}}}_2|)$ and for $(|\underline{\tilde{\mathbf{a}}}_1|, |\underline{\tilde{\mathbf{a}}}_2|) < 1$, the first term is dominant and the non symmetry can be evaluated.

ii. Let us look at the matrix element $<1|.....|3>$ and $<2|.....|4>$ in all orders of corrections in the case that $\mathbf{a}^\alpha = 0$ (on the tree level, Eq. (6.22), $<1|x|3> = <2|x|4> = <3|x|1>^\dagger = <4|x|2>^\dagger$) and let in this case $<\underline{\tilde{\mathbf{A}}}^{1\boxminus}>$ represent the matrix elements i$<1|.....|3>$ and $<2|.....|4>$ in both cases in all orders of corrections. We namely showed that in this case the matrix element $<1|.....|3>$ is equal to $<2|.....|4> = <\underline{\tilde{\mathbf{A}}}^{1\boxminus}>$.

We now allow $\mathbf{a}^\alpha \neq 0$.

Taking into account that in the case that $\mathbf{a}^\alpha$ is zero $<\underline{\tilde{\mathbf{A}}}^{1\boxminus}>$ includes all the corrections in all orders and that also $\underline{\tilde{\mathbf{a}}}_2$ includes the corrections in all orders, we find

$$(<\psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 3}> - <\psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 4}>) =$$
$$<\underline{\tilde{\mathbf{A}}}^{1\boxminus}> (1 + \frac{8}{3}\mathbf{a}^\alpha \underline{\tilde{\mathbf{a}}}_2\{1 - \frac{2}{5}(\underline{\tilde{\mathbf{a}}}_2)^2 + \cdots\}\ . \tag{6.30}$$

It is obvious that for $\mathbf{a}^\alpha \neq 0$ also the non diagonal matrix elements do not keep the tree level symmetry of mass matrices $(<\psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 3}> - <\psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 4}>) = 0$, which is not zero any longer). But one sees as well that the contributions of higher terms to asymmetry are getting smaller and smaller and for $|\mathbf{a}^\alpha| << |\underline{\tilde{\mathbf{a}}}_2|$, for $|\underline{\tilde{\mathbf{a}}}_2| < 1$, the first term in corrections is dominant. One can correspondingly evaluate the amount of non symmetry.

iii. Let us look also at the matrix element $< 1|.....|2 >$ and $< 3|.....|4 >$, first in all orders of corrections in the case that $\mathbf{a}^{\alpha} = 0$ (on the tree level, Eq. (6.22), $< 1|x|2 > = < 3|x|4 > = < 2|x|1 >^{\dagger} = < 4|x|3 >^{\dagger}$) and let in this case $< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxminus} >$ represent the matrix elements $< 1|.....|2 >$ and $< 3|.....|4 >$ in all orders of corrections. We namely showed that in the case that $\mathbf{a}^{\alpha} = 0$ the matrix element $< 1|.....|2 >$ is equal to $< 3|.....|4 > = < \underline{\tilde{\mathbf{A}}}^{\tilde{1} \boxminus} >$.

We now allow $\mathbf{a}^{\alpha} \neq 0$.

Taking into account that for $\mathbf{a}^{\alpha} = 0$ the matrix element $< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxminus} >$ includes corrections in all orders and that also $\underline{\tilde{\mathbf{a}}}_2$ includes in this case corrections in all orders, one finds

$$
\begin{aligned}
&(< \psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 2} > - < \psi_L^{\alpha 3}|\ldots|\psi_R^{\alpha 4} >) = \\
&< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxminus} > (1 + \frac{8}{3}\mathbf{a}^{\alpha}\underline{\tilde{\mathbf{a}}}_1\{1 - \frac{2}{5}(\underline{\tilde{\mathbf{a}}}_1)^2 + \cdots) \,. \qquad (6.31)
\end{aligned}
$$

It is obvious that for $\mathbf{a}^{\alpha} \neq 0$ also these non diagonal matrix elements do not keep the tree level symmetry of mass matrices ($< \psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 3} > - < \psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 4} >= 0$ is no longer the case). But one sees as well that the contributions of higher terms to asymmetry are getting smaller and smaller and for $|\mathbf{a}^{\alpha}| << |\underline{\tilde{\mathbf{a}}}_1|$ and for $|\underline{\tilde{\mathbf{a}}}_1| < 1$, the first term in corrections is dominant and the non symmetry, the difference $< \psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 3} > - < \psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 4} >$ can be evaluated.

iv. It remains to check the matrix elements $< 1|.....|4 >$, $< 2|.....|3 >$, $< 3|.....|2 >$ and $< 4|.....|1 >$. The matrix elements which are nonzero only in the third order of corrections, ($< 1|x|4 >= 0 =< 2|x|3 >= 0 =< 3|x|2 >=< 4|x|1 >$, the first nonzero terms are $< 1|xxx|4 >$, $< 2|xxx|3 >$, $< 3|xxx|2 >$, $< 4|xxx|1 >$), appearing in Eqs. (6.21, 6.22), which are for $\mathbf{a}^{\alpha} = 0$ all equal to zero in all orders of corrections.

We again take into account that for $\mathbf{a}^{\alpha} = 0$ the matrix element $< \underline{\tilde{\mathbf{A}}}^{\tilde{1} \substack{\boxplus \\ \square}} >$ and $< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \substack{\boxplus \\ \square}} >$ include the corrections in all orders and that also $\underline{\tilde{\mathbf{a}}}_1$ and $\underline{\tilde{\mathbf{a}}}_2$ include the corrections in all orders. We find when $\mathbf{a}^{\alpha} \neq 0$

$$
\begin{aligned}
&\frac{< \psi_L^{\alpha 1}|\ldots|\psi_R^{\alpha 4} >}{< \underline{\tilde{\mathbf{A}}}^{\tilde{1} \boxminus} >< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxminus} >} = \frac{< \psi_L^{\alpha 2}|\ldots|\psi_R^{\alpha 3} >}{< \underline{\tilde{\mathbf{A}}}^{\tilde{1} \boxminus} >< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxplus} >} = \\
&\frac{< \psi_L^{\alpha 4}|\ldots|\psi_R^{\alpha 1} >}{< \underline{\tilde{\mathbf{A}}}^{\tilde{1} \boxplus} >< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxplus} >} = \frac{< \psi_L^{\alpha 3}|\ldots|\psi_R^{\alpha 2} >}{< \underline{\tilde{\mathbf{A}}}^{\tilde{1} \boxplus} >< \underline{\tilde{\mathbf{A}}}^{\tilde{N}_L \boxminus} >} = \\
&-\mathbf{a}^{\alpha}\{1 - \frac{3}{10}[(\underline{\tilde{\mathbf{a}}}_1)^2 + (\underline{\tilde{\mathbf{a}}}_2)^2] + \cdots\} \,. \qquad (6.32)
\end{aligned}
$$

One sees that these off diagonal matrix elements keep the relations from Eq. (6.22) at least in the lowest corrections.

We demonstrated that the matrix elements of the mass matrix of Eq. (6.22) do not keep the symmetry of the tree level in all orders of corrections if $\mathbf{a}^{\alpha} \neq 0$, but the changes can in the case that ($|\mathbf{a}^{\alpha}|$, $|\underline{\tilde{\mathbf{a}}}_1|$, $|\underline{\tilde{\mathbf{a}}}_2|$) are small in comparison with unity be estimated.

Mass matrices beyond the tree level if $a^{\alpha} \neq 0$, while $\tilde{a}_1 = 0 = \tilde{a}_2$ One can easily see that the mass matrix of Eq. (6.22) keeps the symmetry in all orders of corrections also if $a^{\alpha} \neq 0$ and $\tilde{\mathbf{a}}_1 = 0 = \tilde{\mathbf{a}}_2$.

One obtains in this case for the diagonal terms $< \psi^{\alpha i}_L | \hat{U} | \psi^{\alpha i}_R >$, for each of four families ($i = (1,2,3,4)$) the expression

$$
\begin{aligned}
&< \psi^{\alpha i}_L | \hat{U} | \psi^{\alpha i}_R > = \mathbf{a}^{\alpha} - \\
&\frac{1}{3!}\{(\mathbf{a}^{\alpha})^3 + \mathbf{a}^{\alpha}(| < \tilde{\mathbf{A}}^{\tilde{1}\boxminus} > |^2 + | < \tilde{\mathbf{A}}^{\tilde{N}_L\boxminus} > |^2 + |\mathbf{A}^{\alpha}|^2 + |\tilde{\mathbf{A}}^{\tilde{1}3}|^2 + |\tilde{\mathbf{A}}^{\tilde{1}\boxminus}|^2 + \\
&|\tilde{\mathbf{A}}^{\tilde{N}_L 3}|^2 + |\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}|^2)\} + \\
&\frac{1}{5!}\{(\mathbf{a}^{\alpha})^5 + (\mathbf{a}^{\alpha})^3(| < \tilde{\mathbf{A}}^{\tilde{1}\boxminus} > |^2 + | < \tilde{\mathbf{A}}^{\tilde{N}_L\boxminus} > |^2 + |\mathbf{A}^{\alpha}|^2 + |\tilde{\mathbf{A}}^{\tilde{1}3}|^2 + |\tilde{\mathbf{A}}^{\tilde{1}\boxminus}|^2 + \\
&|\tilde{\mathbf{A}}^{\tilde{N}_L 3}|^2 + |\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}|^2) + \\
&\mathbf{a}^{\alpha}(| < \tilde{\mathbf{A}}^{\tilde{1}\boxminus} > |^4 + | < \tilde{\mathbf{A}}^{\tilde{N}_L\boxminus} > |^4 + |\mathbf{A}^{\alpha}|^4 + \\
&|\tilde{\mathbf{A}}^{\tilde{1}3}|^4 + |\tilde{\mathbf{A}}^{\tilde{1}\boxminus}|^4 + |\tilde{\mathbf{A}}^{\tilde{N}_L 3}|^4 + |\tilde{\mathbf{A}}^{\tilde{N}_L\boxminus}|^4 + \cdots + \\
&| < \tilde{\mathbf{A}}^{\tilde{1}\boxminus} > |^2 | < \tilde{\mathbf{A}}^{\tilde{N}_L\boxminus} > |^2 + \cdots) + \cdots\} - \\
&\frac{1}{7!}\{(\mathbf{a}^{\alpha})^7 + (\mathbf{a}^{\alpha})^5(| < \tilde{\mathbf{A}}^{\tilde{1}\boxminus} > |^2 + \cdots) + \cdots\} + \cdots . \qquad (6.33)
\end{aligned}
$$

Let us denote the above expression for the diagonal terms $< \psi^{\alpha i}_L | \hat{U} | \psi^{\alpha i}_R >$, which takes into account corrections in all orders while assuming $\tilde{\mathbf{a}}_1 = 0 = \tilde{\mathbf{a}}_2$, with $\underline{\mathbf{a}}^{\alpha}$. (The definition of the scalar fields is presented in Eq. (6.20)).

Let us add that the choice that the third components of the scalar fields $\vec{\tilde{A}}^{\tilde{1}}$ and $\vec{\tilde{A}}^{\tilde{N}_L}$ have no vacuum expectation values — $< \tilde{A}^{\tilde{1}3} > = \tilde{a}_1 = 0, < \tilde{A}^{\tilde{N}_L 3} > = \tilde{a}_2 = 0$ — does not seem a meaningful choice. Namely, if all the components of the two triplets, $\vec{\tilde{A}}^{\tilde{1}}$ and $\vec{\tilde{A}}^{\tilde{N}_L}$, influencing the family quantum numbers of the four families, would have no vacuum expectation values, all the families would have the same mass, determined by $\mathbf{a}^{\alpha}$ and the contributions in all orders of corrections of the dynamical scalar fields, $\vec{\tilde{\mathbf{A}}}^{\tilde{1}}$, $\vec{\tilde{\mathbf{A}}}^{\tilde{N}_L}$ and $\mathbf{a}^{\alpha} =< \mathbf{A}^{\alpha} >$ and the dynamical part of $\mathbf{A}^{\alpha}$. Let be added, however, that the choice $< \tilde{\mathbf{A}}^{\tilde{1}\boxplus}_{\boxminus} > \neq 0, < \tilde{\mathbf{A}}^{\tilde{N}_L\boxplus}_{\boxminus} > \neq 0$ and $\mathbf{a}^{\alpha} \neq 0$, while $\tilde{\mathbf{a}}_1 = 0 = \tilde{\mathbf{a}}_2$, makes all the matrix elements of the mass matrix, Eq. (6.22), different from zero.

6.3 Conclusions

In the *spin-charge-family* theory to the 4×4 mass matrix of any family member (that is of quarks and leptons — the observed three families namely form in the *spin-charge-family* theory the 3×3 submatrices of these predicted 4×4 mass matrices) the two scalar triplets ($\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$) and the three scalar singlets (A^{Q}_s, $A^{Q'}_s$, $A^{Y'}_s$), $s = (7,8)$, contribute, all with the weak and the hyper charge of the *standard model* higgs ($\pm\frac{1}{2}, \mp\frac{1}{2}$, respectively). The first two triplets influence the family quantum numbers, while the last three singlets influence the family members quantum numbers.

The only dependence of the mass matrix on the family member ($\alpha = (u, d, \nu, e)$) quantum numbers is due to the operators $\gamma^0 \stackrel{78}{(\pm)} QA^Q_{\pm}$, $\gamma^0 \stackrel{78}{(\pm)} Q'A^{Q'}_{\pm}$ and $\gamma^0 \stackrel{78}{(\pm)} Y'A^{Y'}_{\pm}$. The operator $\gamma^0 \stackrel{78}{(\pm)}$, appearing at the contribution of the two triplet scalar fields as well as at the three singlet scalar fields, transforms the right handed members into the left handed ones, or opposite, while the family operators transform a family member of one family into the same family member of another family.

We demonstrate in this paper that the matrix elements of mass matrices 4×4, predicted by the *spin-charge-family* theory for each family member $\alpha = (u, d, \nu, e)$, keep the symmetry $\widetilde{SU}(2)_{\widetilde{SO}(4)_{1+3}} \times \widetilde{SU}(2)_{\widetilde{SO}(4)_{\text{"weak"}}} \times U(1)$ in all orders of corrections under the assumption that either the vacuum expectation values of three singlets $< A^{\alpha} >= a^{\alpha}$ are equal to zero, Subsect. 6.2.2, $a^{\alpha} = 0$, while all the other scalar fields — $\vec{\tilde{A}}^{\tilde{1}}$, $\vec{\tilde{A}}^{\tilde{N}_L}$ — can have for all the components nonzero vacuum expectation values, or that a^{α} does not need to be zero, $a^{\alpha} \neq 0$, but then the two third components of the two scalar triplets, $< \tilde{A}^{\tilde{1}3} >= \tilde{a}_1, < \tilde{A}^{\tilde{N}_L 3} >= \tilde{a}_2$, Subsect. 6.2.2, must be zero, $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$.

For the case that the two triplets and the three singlets have for all components nonzero vacuum expectation values we represent the symmetries of the mass matrices in dependence of the order of corrections, Subsect. 6.2.2.

In the first case, when $a^{\alpha} = 0$, to any order of corrections all the components of the two triplet scalar fields contribute, either with the nonzero vacuum expectation values or as dynamical fields or as both in all orders of corrections, while the three singlet scalar fields contribute only as dynamical fields. In this case the corrections keep the symmetry of the three level in all orders of corrections.

The contributions of the dynamical fields of the three singlets in all orders of loop corrections — together with the contributions of the two triplets which interact with spinors through the family quantum numbers either with the nonzero vacuum expectation values or as dynamical fields — make all the matrix elements dependent on the particular family member quantum numbers. Correspondingly all the mass matrices bring different masses to any of the family members and correspondingly also different mixing matrices to quarks and leptons. However, the choice $a^{\alpha} = 0$ keeps the four off diagonal terms, which are proportional to a^{α} in Eq.(6.22), equal to zero in all orders of correction.

In the second case, when $\tilde{a}_1 = 0$, $\tilde{a}_2 = 0$, in any order of corrections the three singlet scalar fields contribute either with nonzero vacuum expectation values or as dynamical fields, while the two triplets scalar fields contribute with the nonzero vacuum expectation values and the dynamical fields, except the two of the triplet components — $\tilde{A}^{\tilde{1}3}$ and $\tilde{A}^{\tilde{N}_L 3}$ — which contribute only as dynamical fields. The symmetry of the tree level is kept in all order of corrections, this choice makes, however, all the diagonal terms to remain equal in all orders of corrections.

When all the singlets and the triplets have for all the components nonzero vacuum expectation values ($a^{\alpha} \neq 0$, $\tilde{a}_1 \neq 0$, $\tilde{a}_2 \neq 0$, $< \tilde{A}^{\tilde{N}_L \boxplus} \neq 0 > < \tilde{A}^{\tilde{1} \boxplus} > \neq 0$) the symmetry of the tree level changes, but we are still able to determine the symmetry of mass in all orders of corrections, that is of the loop ones and the repetition of the nonzero vacuum expectation values, expressing the matrix elements of mass matrices with a few parameters only, due to the fact that the symmetry of the mass matrices limit the number of free parameters. In the case that $|a^{\alpha}|$ is small (in comparison with $|\tilde{a}_1|$ and $|\tilde{a}_2|$), the higher order corrections drop away very quickly. When fitting the free parameters of mass matrices to the observed masses of quarks and leptons and their 3×3 submatrices of the predicted 4×4 mixing matrices, we are able to predict the masses of the fourth family members as well as the matrix elements of the fourth components to the observed free families, provided that the mixing 3×3 submatrices of the predicted 4×4 mass matrices of quarks and leptons are measured accurately enough — since the (accurate) 3×3 submatrix of a 4×4 matrix determines 4×4 matrix uniquely [21,22].

This means that although we are so far only in principle able to calculate directly the mass matrix elements of the 4×4 mass matrices, predicted by the *spin-charge-family*, yet the symmetry of mass matrices, discussed in this paper, enables us — due to the limited number of free parameters — to predict properties of the four family of quarks and lepton to the observed three families, that is the masses of the fourth families and the corresponding mixing matrices [21,22]. *We only have to wait for accurate enough data for the* 3×3 *mixing (sub)matrices of quarks and leptons.*

Let us add that the right handed neutrino, which is a regular member of the four families, Table 6.3, has the nonzero value of the operator $Y' A^{Y'}_s$ only.

6.4 Appendix: Short presentation of the *spin-charge-family* theory

This section follows similar sections in Refs. [1,4–7].

The *spin-charge-family* theory [1–7,9–12,15–17,19–24] assumes:

a. A simple action (Eq. (6.35)) in an even dimensional space ($d = 2n$, $d > 5$), d is chosen to be $(13 + 1)$. This choice makes that the action manifests in $d = (3 + 1)$ in the low energy regime all the observed degrees of freedom, explaining all the assumptions of the *standard model*, as well as other observed phenomena.

There are two kinds of the Clifford algebra objects, γ^a's and $\tilde{\gamma}^a$'s in this theory with the properties.

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \qquad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,. \qquad (6.34)$$

Fermions interact with the vielbeins $f^{\alpha}{}_a$ and the two kinds of the spin-connection fields — $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ — the gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, respectively.

The action

$$\begin{aligned} \mathcal{A} = & \int d^d x\, E\, \frac{1}{2} (\bar{\psi}\, \gamma^a p_{0a} \psi) + h.c. + \\ & \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \end{aligned} \qquad (6.35)$$

in which $p_{0a} = f^{\alpha}{}_{a}\, p_{0\alpha} + \frac{1}{2E}\,\{p_{\alpha}, Ef^{\alpha}{}_{a}\}_{-}$, $p_{0\alpha} = p_{\alpha} - \frac{1}{2}\, S^{ab}\, \omega_{ab\alpha} - \frac{1}{2}\, \tilde{S}^{ab}\, \tilde{\omega}_{ab\alpha}$, and

$$R = \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta})\} + h.c.,$$

$$\tilde{R} = \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\, \tilde{\omega}^{c}{}_{b\beta})\} + h.c.$$

[15], introduces two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$, $\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+$. $f^{\alpha}{}_{a}$ are vielbeins inverted to $e^{a}{}_{\alpha}$, Latin letters $(a, b, ..)$ denote flat indices, Greek letters $(\alpha, \beta, ..)$ are Einstein indices, $(m, n, ..)$ and $(\mu, \nu, ..)$ denote the corresponding indices in $(0, 1, 2, 3)$, while $(s, t, ..)$ and $(\sigma, \tau, ..)$ denote the corresponding indices in $d \geq 5$:

$$e^{a}{}_{\alpha}f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}\,, \quad e^{a}{}_{\alpha}f^{\alpha}{}_{b} = \delta^{a}_{b}\,, \tag{6.36}$$

$E = \det(e^{a}{}_{\alpha})$.

b. The *spin-charge-family* theory assumes in addition that the manifold $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^{(6)}$ (which manifests as $SO(7,1) \times SU(3) \times U(1)$), affecting both internal degrees of freedom — the one represented by γ^a and the one represented by $\tilde{\gamma}^a$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ families [36]. The rest of families get heavy masses [16].

c. There is additional breaking of symmetry: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

d. There is a scalar condensate (Table 6.5) of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale $\propto 10^{16}$ GeV or higher to all the vector and scalar gauge fields, which interact with the condensate [5].

e. There are the scalar fields with the space index $(7, 8)$ carrying the weak (τ^{1i}) and the hyper charges ($Y = \tau^{23} + \tau^4$, τ^{1i} and τ^{2i} are generators of the subgroups of $SO(4)$, τ^4 and τ^{3i} are the generators of $U(1)_{II}$ and $SU(3)$, respectively, which are subgroups of $SO(6)$), which with their nonzero vacuum expectation values change the properties of the vacuum and break the weak charge and the hyper charge. Interacting with fermions and with the weak and hyper bosons, they bring masses to heavy bosons and to twice four groups of families. Carrying no electromagnetic ($Q = \tau^{13} + Y$) and colour (τ^{3i}) charges and no $SO(3, 1)$ spin, the scalar fields leave the electromagnetic, colour and gravity fields in $d = (3 + 1)$ massless.

The assumed action $\mathcal{A}$ and the assumptions offer:

o. the explanation for the origin and all the properties of the observed fermions:

[15] Whenever two indexes are equal the summation over these two is meant.

[16] A toy model [36,37] was studied in $d = (5 + 1)$ with the same action as in Eq. (6.35). The break from $d = (5 + 1)$ to $d = (3 + 1) \times$ an almost S^2 was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, although not yet proved how does it really work, is made in the $d = (13 + 1)$ case. This study is in progress.

o.i. of the family members, on Table 6.3 the family members belonging to one Weyl (fundamental) representation of massless spinors of the group $SO(13,1)$ are presented in the "technique" [10–12,15–17,13,14] and analyzed with respect to the subgroups $SO(3,1)$, $SU(2)_{I}$, $SU(2)_{II}$, $SU(3)$, $U(1)_{II}$), Eqs. (6.37, 6.38, 6.2) with the generators $\tau^{Ai} = \sum_{s,t} c^{Ai}{}_{st}\, S^{st}$,

o.ii. of the families analyzed with respect to the subgroups ($\widetilde{SO}(3,1)$, $\widetilde{SU}(2)_{I}$, $\widetilde{SU}(2)_{II}$, $\widetilde{U}(1)_{II}$) with the generators $\tilde{\tau}^{Ai} = \sum_{ab} c^{Ai}{}_{ab}\, \tilde{S}^{st}$, Eqs. (6.40, 6.41, 6.42) — they are presented on Table 6.4 — all the families are singlets with respect to $\widetilde{SU}(3)$,

oo.i. of the observed vector gauge fields of the charges ($SU(2)_{I}$, $SU(2)_{II}$, $SU(3)$, $U(1)_{II}$) discussed in Refs. ([1,4,2], and the references therein), all the vector gauge fields are the superposition of ω_{stm}, $A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega_{stm}$, Eq. (6.44),

oo.ii. of the Higgs's scalar and of the Yukawa couplings, explainable with the scalar fields with the space index $(7,8)$, there are two groups of two triplets, which are scalar gauge fields of the charges $\tilde{\tau}^{Ai}$, expressible with the superposition of $\tilde{\omega}_{abs}$, $A^{Ai}_{s} = \sum_{a,b} c^{Ai}{}_{ab}\, \omega_{abs}$, Eq. (6.45), and three singlets, the gauge fields of Q, Q', Y', Eqs. (6.43, 6.45), all with the weak and the hyper charges as assumed by the *standard model* for the Higgs's scalars,

oo.iii. of the scalar fields explaining the origin of the matter-antimatter asymmetry, Ref. [5],

oo.iv. of the appearance of the dark matter, there are two decoupled groups of four families, carrying family charges ($\vec{\tilde{N}}_{L}$, $\vec{\tilde{\tau}}^{1}$) and ($\vec{\tilde{N}}_{R}$, $\vec{\tilde{\tau}}^{2}$), Eqs. (6.40, 6.41), both groups carry also the family members charges (Q, Q', Y'), Eq. (6.43).

The *standard model* groups of spins and charges are the subgroups of the $SO(13,1)$ group with the generator of the infinitesimal transformations expressible with S^{ab} $(= \frac{i}{2}(\gamma^{a}\gamma^{b} - \gamma^{b}\gamma^{a})$, $\{S^{ab}, S^{cd}\}_{-} = -i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac}))$ for the spin

$$\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}), \qquad (6.37)$$

for the weak charge, $SU(2)_{I}$, and the second $SU(2)_{II}$, these two groups are the invariant subgroups of $SO(4)$,

$$\begin{aligned} \vec{\tau}^{1} &:= \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78}), \\ \vec{\tau}^{2} &:= \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78}), \end{aligned} \qquad (6.38)$$

for the colour charge $SU(3)$ and for the "fermion charge" $U(1)_{II}$, these two groups are subgroups of $SO(6)$,

$$\begin{aligned} \vec{\tau}^{3} :=\; & \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, \\ & S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, \\ & S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\ \tau^{4} :=\; & -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14}), \end{aligned} \qquad (6.39)$$

τ^4 is the "fermion charge", while the hyper charge $Y = \tau^{23} + \tau^4$.

The generators of the family quantum numbers are the superposition of the generators $\tilde{S}^{ab}$ ($\tilde{S}^{ab} = \frac{i}{4}\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_-$, $\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- = -i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})$, $\{\tilde{S}^{ab}, S^{cd}\}_- = 0$). One correspondingly finds the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03}), \tag{6.40}$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\begin{aligned} \vec{\tilde{\tau}}^1 &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}), \\ \vec{\tilde{\tau}}^2 &:= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}), \end{aligned} \tag{6.41}$$

determine representations of $\widetilde{SU}(2)_I \times \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both, $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$, are the subgroups of $\widetilde{SO}(7,1)$. One finds for the infinitesimal generator $\tilde{\tau}^4$ of $\widetilde{U}(1)$, originating in $\widetilde{SO}(6)$, the expression

$$\tilde{\tau}^4 := -\frac{1}{3}(\tilde{S}^{9\;10} + \tilde{S}^{11\;12} + \tilde{S}^{13\;14}). \tag{6.42}$$

The operators for the charges Y and Q of the *standard model*, together with Q' and Y', and the corresponding operators of the family charges $\tilde{Y}$, $\tilde{Y}'$, $\tilde{Q}$, $\tilde{Q}'$, are defined as follows:

$$\begin{aligned} &Y = \tau^4 + \tau^{23}, \;\; Y' = -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \;\; Q = \tau^{13} + Y, \;\; Q' = -Y\tan^2\vartheta_1 + \tau^{13}\;\;, \\ &\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}, \;\; \tilde{Y}' = -\tilde{\tau}^4 \tan^2\vartheta_2 + \tilde{\tau}^{23}, \;\; \tilde{Q} = \tilde{Y} + \tilde{\tau}^{13} \;\; \tilde{Q}' = -\tilde{Y}\tan^2\vartheta_1 + \tilde{\tau}^{13} \end{aligned} \tag{6.43}$$

Families split into two groups of four families, each manifesting the $\widetilde{SU}(2)\times\widetilde{SU}(2)\times U(1)$, with the generators of the infinitesimal transformations $(\vec{\tilde{N}}_L, \vec{\tilde{\tau}}^1, Q, Q', Y')$ and $(\vec{\tilde{N}}_R, \vec{\tilde{\tau}}^2, Q, Q', Y')$, respectively. The generators of $U(1)$ group (Q, Q', Y'), Eq. 6.43, distinguish among family members and are the same for both groups of four families, presented on Table 6.4, taken from Ref. [4].

The vector gauge fields of the charges $\vec{\tilde{\tau}}^1$, $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^3$ and τ^4 follow from the requirement $\sum_{Ai} \tau^{Ai} A^{Ai}_m = \sum_{s,t} \frac{1}{2} S^{st}\,\omega_{stm}$ and the requirement that $\tau^{Ai} = \sum_{a,b}\; c^{Ai}{}_{ab}\; S^{ab}$, Eq. (6.4), fulfilling the commutation relations $\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk}\tau^{Ak}$, Eq. (6.5). Correspondingly we find $A^{Ai}_m = \sum_{s,t}\; c^{Ai}{}_{st}\; \omega^{st}{}_m$, Eq. (6.6), with (s,t) either in $(5,6,7,8)$ or in $(9,\ldots,14)$.

The explicit expressions for these vector gauge fields in terms of ω_{stm} [[4], Eq. (22)], [5]] are presented in the case that the electroweak $\vartheta_1 = \vartheta_W$ is zero and

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$u^{c1}_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$

Table 6.4. Eight families of the right handed u^{c1}_R (6.3) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$ [the definition of the operators is presented in Eqs. (6.38,6.2), the definition of the operators, expressible with $\tilde{S}^{ab}$ is presented: $\vec{\tilde{N}}_{L,R}$ (Eq. (6.40)), $\vec{\tilde{\tau}}^1$ (Eq. (6.41)), $\vec{\tilde{\tau}}^2$ (Eq. (6.41)), $\tilde{\tau}^4$ (Eq. (6.42))] and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (6.60). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (6.60)) transform $u_{Ri}, i = (1, \cdots, 8)$, to all the members of the same colour of the i^{th} family. The same generators transform equivalently the right handed neutrino $\nu_{Ri}, i = (1, \cdots, 8)$, to all the colourless members of the i^{th} family.

so is ϑ_2 and in the case that the two angles, $(\vartheta_1, \vartheta_2)$, are not zero.

$$
\begin{aligned}
\vec{A}^1_m &= (\omega_{58m} - \omega_{67m}, \omega_{57m} + \omega_{68m}, \omega_{56m} - \omega_{78m})\,,\\
\vec{A}^2_m &= (\omega_{58m} + \omega_{67m}, \omega_{57m} - \omega_{68m}, \omega_{56m} + \omega_{78m})\,,\\
A^Q_m &= \omega_{56m} - (\omega_{9\,10m} + \omega_{11\,12m} + \omega_{13\,14m})\,,\\
A^Y_m &= (\omega_{56m} + \omega_{78m}) - (\omega_{9\,10m} + \omega_{11\,12m} + \omega_{13\,14m})\,,\\
\vec{A}^3_m &= (\omega_{9\,12m} - \omega_{10\,11m}, \omega_{9\,11m} + \omega_{10\,12m}, \omega_{9\,10m} - \omega_{11\,12m},\\
&\quad \omega_{9\,14m} - \omega_{10\,13m}, \omega_{9\,13m} + \omega_{10\,14m}, \omega_{11\,14m} - \omega_{12\,13m},\\
&\quad \omega_{11\,13m} + \omega_{12\,14m}, \frac{1}{\sqrt{3}}(\omega_{9\,10m} + \omega_{11\,12m} - 2\omega_{13\,14m}))\,,\\
A^4_m &= (\omega_{9\,10m} + \omega_{11\,12m} + \omega_{13\,14m})\,,\\
A^Q_m &= \sin\vartheta_1\, A^{13}_m + \cos\vartheta_1\, A^Y_m\,,\\
A^{Q'}_m &= \cos\vartheta_1\, A^{13}_m - \sin\vartheta_1\, A^Y_m\,,\\
A^{Y'}_m &= \cos\vartheta_2\, A^{23}_m - \sin\vartheta_2\, A^4_m\,,\\
&\quad (m \in (0,1,2,3))\,. \qquad (6.44)
\end{aligned}
$$

All ω_{stm} vector gauge fields are real fields. Here the fields contain in general the coupling constants which are not necessarily the same for all of them. The angle ϑ_1 is the angle of the electroweak break, while ϑ_2 is the angle of breaking the $SU(2)_{II}$ and $U(1)_{II}$ at much higher scale [[5,4] and references therein].

One obtains in a similar way the scalar gauge fields, which determine mass matrices of family members. They carry the space index $s = (7, 8)$. The scalar fields contain in general the coupling constants. Before the electroweak break the electroweak angle $\vartheta_1 = \vartheta_W$ is zero, while ϑ_2 is the angle determined by the break of symmetry at much higher scale.

$$
\begin{aligned}
\vec{\tilde{A}}^1_s &= (\tilde{\omega}_{58s} - \tilde{\omega}_{67s}, \tilde{\omega}_{57s} + \tilde{\omega}_{68s}, \tilde{\omega}_{56s} - \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^2_s &= (\tilde{\omega}_{58s} + \tilde{\omega}_{67s}, \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{\omega}_{56s} + \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^{N_L}_s &= (\tilde{\omega}_{23s} + i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} + i\tilde{\omega}_{02s}, \tilde{\omega}_{12s} + i\tilde{\omega}_{03s})\,,\\
\vec{\tilde{A}}^{N_R}_s &= (\tilde{\omega}_{23s} - i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i\tilde{\omega}_{03s})\,,\\
A^Q_s &= \omega_{56s} - (\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\,,\\
A^Y_s &= (\omega_{56s} + \omega_{78s}) - (\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\\
A^4_s &= -(\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\,,\\
A^Q_s &= \sin\vartheta_1\, A^{13}_s + \cos\vartheta_1\, A^Y_s\,, \quad A^{Q'}_s = \cos\vartheta_1\, A^{13}_s - \sin\vartheta_1\, A^Y_s\,,\\
A^{Y'}_s &= \cos\vartheta_2\, A^{23}_s - \sin\vartheta_2\, A^4_s\,,\\
&\quad (s \in (7,8))\,. \qquad (6.45)
\end{aligned}
$$

All $\omega_{sts'}$, $\tilde{\omega}_{sts'}$, $(s,t,s') = (5,\ldots,14)$, $\tilde{\omega}_{i,j,s'}$ and $i\,\tilde{\omega}_{0,s'}$, $(i,j) = (1,2,3)$ scalar gauge fields are real fields.

The theory predicts, due to commutation relations of generators of the infinitesimal transformations of the family groups, $\widetilde{SU}(2)_I \times \widetilde{SU}(2)_I$ and $\widetilde{SU}(2)_{II} \times \widetilde{SU}(2)_{II}$, the first one with the generators $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, and the second one with the generators $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, Eqs. (6.40,6.41), two groups of four families.

The theory offers (so far) several predictions:

i. several new scalars, those coupled to the lower group of four families — two triplets and three singlets, the superposition of ($\vec{\tilde{A}}^{1}_{s}$, $\vec{\tilde{A}}^{N}_{Ls}$ and $A^{Q}_{s}, A^{Y}_{s}, A^{4}_{s}$, Eq. (6.45)) — some of them to be observed at the LHC ([1,5,4]),

ii. the fourth family to the observed three to be observed at the LHC ([1,5,4] and the references therein),

iii. new nuclear force among nucleons among quarks of the upper four families.

The theory offers also the explanation for several phenomena, like it is the "miraculous" cancellation of the *standard model* triangle anomalies [3].

The breaks of the symmetries, manifesting in Eqs. (6.37, 6.40, 6.38, 6.41, 6.2, 6.42), are in the *spin-charge-family* theory caused by the scalar condensate of the two right handed neutrinos belonging to one group of four families, Table 6.5, and by the nonzero vacuum expectation values of the scalar fields carrying the space index $(7,8)$ (Refs. [4,1] and the references therein). The space breaks first to $SO(7,1)$ $\times SU(3) \times U(1)_{II}$ and then further to $SO(3,1) \times SU(2)_{I} \times U(1)_{I} \times SU(3) \times U(1)_{II}$, what explains the connections between the weak and the hyper charges and the handedness of spinors [3].

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^{4}$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^{3}_{L}$	$\tilde{N}^{3}_{R}$
$(\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_{\mathbf{1}} \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu^{VIII}_{1R} >_{1} \vert e^{VIII}_{2R} >_{2})$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e^{VIII}_{1R} >_{1} \vert e^{VIII}_{2R} >_{2})$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 6.5. This table is taken from [5]. The condensate of the two right handed neutrinos ν_R, with the $VIII^{th}$ family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^{4} = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{\tau}^{4} = -1$, $\tilde{N}^{3}_{R} = 1$, $\tilde{N}^{3}_{L} = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$, $\tilde{\tau}^{31} = 0$. The family quantum numbers are presented in Table 6.4.

The stable of the upper four families is the candidate for the dark matter, the fourth of the lower four families is predicted to be measured at the LHC.

6.5 Appendix: Short presentation of spinor technique [1,4,11,13,14]

This appendix is a short review (taken from [4]) of the technique [11,42,13,14], initiated and developed in Ref. [11] by one of the authors (N.S.M.B.), while proposing the *spin-charge-family* theory [2,4,5,7,9,1,15,16,10–12,17,19–24]. All the internal degrees of freedom of spinors, with family quantum numbers included, are describable with two kinds of the Clifford algebra objects, besides with γ^a's, used in this theory to describe spins and all the charges of fermions, also with $\tilde{\gamma}^a$'s, used in this theory to describe families of spinors:

$$\{\gamma^a, \gamma^b\}_{+} = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_{+} = 2\eta^{ab}\,, \qquad \{\gamma^a, \tilde{\gamma}^b\}_{+} = 0\,. \qquad (6.46)$$

We assume the "Hermiticity" property for γ^a's (and $\tilde{\gamma}^a$'s) $\gamma^{a\dagger} = \eta^{aa}\gamma^a$ (and $\tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a$), in order that γ^a (and $\tilde{\gamma}^a$) are compatible with (6.34) and formally unitary, i.e. $\gamma^{a\,\dagger}\,\gamma^a = I$ (and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$). One correspondingly finds that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$ (and $(\tilde{S}^{ab})^\dagger = \eta^{aa}\eta^{bb}\tilde{S}^{ab}$).

Spinor states are represented as products of nilpotents and projectors, formed as odd and even objects of γ^a's, respectively, chosen to be the eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's

$$\stackrel{ab}{(k)}: = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \tag{6.47}$$

where $k^2 = \eta^{aa}\eta^{bb}$. We further have [4]

$$\begin{aligned}
\gamma^a \stackrel{ab}{(k)}: &= \frac{1}{2}(\gamma^a\gamma^a + \frac{\eta^{aa}}{ik}\gamma^a\gamma^b) = \eta^{aa} \stackrel{ab}{[-k]}, \quad \gamma^a \stackrel{ab}{[k]}:= \frac{1}{2}(\gamma^a + \frac{i}{k}\gamma^a\gamma^a\gamma^b) =\stackrel{ab}{(-k)},\\
\tilde{\gamma}^a \stackrel{ab}{(k)}: &= -i\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\gamma^a = -i\eta^{aa} \stackrel{ab}{[k]}, \quad \tilde{\gamma}^a \stackrel{ab}{[k]}:= i\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\gamma^a = -i \stackrel{ab}{(k)},
\end{aligned} \tag{6.48}$$

where we assume that all the operators apply on the vacuum state $|\psi_0\rangle$. We define a vacuum state $|\psi_0>$ so that one finds $< \stackrel{ab}{(k)}^\dagger \stackrel{ab}{(k)} >= 1\,, \quad < \stackrel{ab}{[k]}^\dagger \stackrel{ab}{[k]} >= 1$.

We recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\begin{aligned}
&\gamma^a \stackrel{ab}{(k)}= \eta^{aa} \stackrel{ab}{[-k]},\; \gamma^b \stackrel{ab}{(k)}= -ik \stackrel{ab}{[-k]},\; \gamma^a \stackrel{ab}{[k]}=\stackrel{ab}{(-k)},\; \gamma^b \stackrel{ab}{[k]}= -ik\eta^{aa} \stackrel{ab}{(-k)}\,,\\
&\tilde{\gamma^a} \stackrel{ab}{(k)}= -i\eta^{aa} \stackrel{ab}{[k]},\; \tilde{\gamma^b} \stackrel{ab}{(k)}= -k \stackrel{ab}{[k]},\; \tilde{\gamma^a} \stackrel{ab}{[k]}= \; i \stackrel{ab}{(k)},\; \tilde{\gamma^b} \stackrel{ab}{[k]}= -k\eta^{aa} \stackrel{ab}{(k)}
\end{aligned} \tag{6.49}$$

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &: = (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &: = (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,
\end{aligned} \tag{6.50}$$

$\{S^{ab},\tilde{S}^{cd}\}_- = 0\,, \{S^{ab},S^{cd}\}_- = i(\eta^{ad}S^{bc}+\eta^{bc}S^{ad}-\eta^{ac}S^{bd}-\eta^{bd}S^{ac})\,, \{\tilde{S}^{ab},\tilde{S}^{cd}\}_- = i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,.$

One can easily check that the nilpotent $\stackrel{ab}{(k)}$ and the projector $\stackrel{ab}{[k]}$ are "eigenstates" of S^{ab} and $\tilde{S}^{ab}$

$$\begin{aligned}
S^{ab} \stackrel{ab}{(k)}&= \frac{1}{2}\,k \stackrel{ab}{(k)}\,, \qquad S^{ab} \stackrel{ab}{[k]}= \;\frac{1}{2}\,k \stackrel{ab}{[k]}\,,\\
\tilde{S}^{ab} \stackrel{ab}{(k)}&= \frac{1}{2}\,k \stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab} \stackrel{ab}{[k]}= -\frac{1}{2}\,k \stackrel{ab}{[k]}\,,
\end{aligned} \tag{6.51}$$

where the vacuum state $|\psi_0\rangle$ is meant to stay on the right hand sides of projectors and nilpotents. This means that multiplication of nilpotents $\stackrel{ab}{(k)}$ and projectors

$\stackrel{ab}{[k]}$ by S^{ab} get the same objects back multiplied by the constant $\frac{1}{2}k$, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by $\frac{k}{2}$ and $\stackrel{ab}{[k]}$ by $(-\frac{k}{2})$ (rather than by $\frac{k}{2}$). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with nilpotents and projectors of Eq. (6.47), the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under the application of any Clifford algebra object.

Recognizing from Eq.(6.50) that the two Clifford algebra objects (S^{ab}, S^{cd}) with all indexes different commute (and equivalently for $(\tilde{S}^{ab}, \tilde{S}^{cd})$), we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4, \\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, \quad \text{if} \quad d = 2n \geq 4\,.
\end{aligned} \tag{6.52}$$

The choice of the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group — the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) (as well as $\tilde{\Gamma}$) in any $d = 2n$

$$\begin{aligned}
\Gamma^{(d)} &:= (i)^{d/2} \quad \prod_a \quad (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n, \\
\tilde{\Gamma}^{(d)} &:= (i)^{(d-1)/2} \prod_a \quad (\sqrt{\eta^{aa}}\tilde{\gamma}^a), \quad \text{if} \quad d = 2n\,.
\end{aligned} \tag{6.53}$$

We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1 \cdots \gamma^d$. It follows from the Hermiticity properties of γ^a for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$.(Equivalent relations are valid for $\tilde{\Gamma}$.) We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) (while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$)).

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd [17]. For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eqs.(6.52, 6.50)), applying it on an (unimportant) vacuum state. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left

[17] For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$.

hand side, generate all the members of one Weyl spinor.

$$
\begin{array}{l}
\overset{0d}{(k_{0d})}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\overset{0d}{[-k_{0d}]}\overset{12}{[-k_{12}]}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\qquad\vdots\\
\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{[-k_{d-1\;d-2}]}\;|\psi_0>\\
\overset{od}{(k_{0d})}\overset{12}{[-k_{12}]}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\qquad\vdots
\end{array}
\tag{6.54}
$$

All the states have the same handedness Γ, since $\{\Gamma, S^{ab}\}_- = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We could make a choice of the simplest one, taking all phases equal to one. (In order to have the usual transformation properties for spinors under the rotation of spin and under $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$,some of the states must be multiplied by (-1).)

The above representation demonstrates that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\overset{ab}{(k_{ab})}$, by transforming all possible pairs of $\overset{ab}{(k_{ab})}\overset{mn}{(k_{mn})}$ into $\overset{ab}{[-k_{ab}]}\overset{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$.

While S^{ab}, which do not belong to the Cartan subalgebra (Eq. (6.52)), generate all the states of one representation, $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (6.52)), generate the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq. (6.52)) of the algebra S^{ab} and $\tilde{S}^{ab}$: $(S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14})$, $(\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14})$, a left handed ($\Gamma^{(13,1)} = -1$) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$
\begin{array}{l}
\overset{03}{(+i)}\overset{12}{(+)}\;|\;\overset{56}{(+)}\overset{78}{(+)}\;||\;\overset{9\;10}{(+)}\;\overset{11\;12}{[-]}\;\overset{13\;14}{[-]}\;|\psi_0\rangle =\\
\frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)||\\
(\gamma^9+i\gamma^{10})(1-i\gamma^{11}\gamma^{12})(1-i\gamma^{13}\gamma^{14})|\psi_0\rangle\,.
\end{array}
\tag{6.55}
$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (6.52)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (6.52)), generate families from the starting u_R quark, transforming the u_R quark from Eq. (6.55) to the u_R of another family, keeping all of the properties with respect to S^{ab} unchanged. In particular, $\tilde{S}^{01}$ applied on a right handed u_R-quark from Eq. (6.55) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$

$$\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]} = -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]}\,|\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]} \qquad (6.56)$$

One can find both states in Table 6.4, the first u_R as u_{R8} in the eighth line of this table, the second one as u_{R7} in the seventh line of this table.

Below some useful relations follow. From Eq.(6.49) one has

$$
\begin{aligned}
&S^{ac}\stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc}\stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, &&\tilde{S}^{ac}\stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc}\stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
&S^{ac}\stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2}\stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, &&\tilde{S}^{ac}\stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2}\stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
&S^{ac}\stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, &&\tilde{S}^{ac}\stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
&S^{ac}\stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, &&\tilde{S}^{ac}\stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\stackrel{ab}{(k)}\stackrel{cd}{[k]}\;.
\end{aligned}
\qquad (6.57)
$$

We conclude from the above equation that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

We recognize in Eq. (6.58) the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively.

$$
\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, &&\stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\stackrel{ab}{[k]}\,, &&\stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\stackrel{ab}{[-k]}\,, &&\stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, &&\stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, &&\stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, &&\stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, &&\stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, &&\stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, &&\stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, &&\stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, &&\stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, &&\stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\;.
\end{aligned}
\qquad (6.58)
$$

Defining

$$\stackrel{ab}{\tilde{(\pm i)}} = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad \stackrel{ab}{\tilde{(\pm 1)}} = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \stackrel{ab}{\tilde{[\pm i]}} = \frac{1}{2}(1 \pm \tilde{\gamma}^a\tilde{\gamma}^b), \quad \stackrel{ab}{\tilde{[\pm 1]}} = \frac{1}{2}(1 \pm i\tilde{\gamma}^a\tilde{\gamma}^b).$$

one recognizes that

$$\stackrel{ab}{\tilde{(k)}}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{\tilde{(-k)}}\stackrel{ab}{(k)} = -i\eta^{aa}\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{\tilde{(k)}}\stackrel{ab}{[k]} = i\stackrel{ab}{(k)}\,, \quad \stackrel{ab}{\tilde{(k)}}\stackrel{ab}{[-k]} = 0\,. \qquad (6.59)$$

Below some more useful relations [15] are presented:

$$
\begin{aligned}
&N^{\pm}_{+} = N^{1}_{+} \pm i\,N^{2}_{+} = -\overset{03}{(\mp i)}\overset{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\,N^{2}_{-} = \overset{03}{(\pm i)}\overset{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\overset{03}{\tilde{(\mp i)}}\overset{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \overset{03}{\tilde{(\pm i)}}\overset{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\,\overset{56}{(\pm)}\overset{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\,\overset{56}{(\mp)}\overset{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\,\overset{56}{\tilde{(\pm)}}\overset{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\,\overset{56}{\tilde{(\mp)}}\overset{78}{\tilde{(\mp)}}\,.
\end{aligned}
\tag{6.60}
$$

In Table 6.4 [4] the eight families of the first member in Table 6.3 (member number 1) of the eight-plet of quarks and the 25^{th} member in Table 6.3 of the eight-plet of leptons are presented as an example. The eight families of the right handed u_{1R} quark are presented in the left column of Table 6.4 [4]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$, Eq. (6.60), on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

After the electroweak phase transition, caused by the scalar fields with the space index $(7, 8)$, the two groups of four families become massive. The lowest of the two groups of four families contains the observed three, while the fourth remains to be measured. The lowest of the upper four families is the candidate for the dark matter [1].

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