

On 12-regular nut graphs*

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Abstract

A nut graph is a simple graph whose adjacency matrix is singular with 1-dimensional kernel such that the corresponding eigenvector has no zero entries. In 2020, Fowler *et al.* characterised for each $d \in \{3, 4, \dots, 11\}$ all values n such that there exists a d -regular nut graph of order n . In the present paper, we resolve the first open case $d = 12$, i.e. we show that there exists a 12-regular nut graph of order n if and only if $n \geq 16$. We also present a result by which there are infinitely many circulant nut graphs of degree $d \equiv 0 \pmod{4}$ and no circulant nut graphs of degree $d \equiv 2 \pmod{4}$. The former result partially resolves a question by Fowler *et al.* on existence of vertex-transitive nut graphs of order n and degree d . We conclude the paper with problems, conjectures and ideas for further work.

Keywords: Nut graph, adjacency matrix, singular matrix, core graph, Fowler construction, regular graph.

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O 12-regularnih orešnih grafih*

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Povzetek

Orešni graf je enostaven graf, katerega matrika sosednosti je singularna z 1-dimenzionalnim jedrom, tako da ustrezni lastni vektor nima ničelnih vnosov. Leta 2020 so Fowler *idr.* karakterizirali za vsak $d \in \{3, 4, \dots, 11\}$ vse vrednosti n , tako da obstaja d -regularni orešni graf reda n . V pričujočem prispevku razrešimo prvi odprt primer $d = 12$, tj. pokažemo, da obstaja 12-regularen orešni graf reda n če in samo če je $n \geq 16$. Predstavimo tudi rezultat, da obstaja neskončno mnogo cirkulantov stopnje $d \equiv 0 \pmod{4}$, ki so hkrati orešni grafi, ter noben cirkulant stopnje $d \equiv 2 \pmod{4}$, ki bi bil orešni graf. Prvi rezultat delno odgovori na vprašanje Fowlerja *idr.* o obstoju vozliščno tranzitivnih orešnih grafov reda n in stopnje d . Prispevek zaključimo s problemi, domnevami in idejami za nadaljnje delo.

Ključne besede: Orešni graf, matrika sosednosti, singularna matrika, sredični graf, Fowlerjeva konstrukcija, regularen graf.

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