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Some results on σ_t-irregularity

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Abstract

The σ_t-irregularity (or sigma total index) is a graph invariant which is defined as $\sigma_t(G) = \sum_{\{u,v\}\subseteq V(G)}(d(u) - d(v))^2$, where $d(z)$ denotes the degree of z. This irregularity measure was proposed by Réti in 2019, and recently rediscovered by Dimitrov and Stevanović in 2023. In this paper we remark that $\sigma_t(G) = n^2 \cdot \mathrm{Var}(G)$, where $\mathrm{Var}(G)$ is the degree variance of the graph. We show that among all complete bipartite graphs on n vertices, either the complete bipartite graph $K_{\lfloor \frac{n}{4}(2-\sqrt{2})\rfloor, \lceil \frac{n}{4}(2+\sqrt{2})\rceil}$ or $K_{\lceil \frac{n}{4}(2-\sqrt{2})\rceil, \lfloor \frac{n}{4}(2+\sqrt{2})\rfloor}$ has the maximum sigma total index. Moreover, various upper and lower bounds for σ_t-irregularity are provided; in this direction we give a relation between the graph energy $\mathcal{E}(G)$ and sigma total index $\sigma_t(G)$ and give new proofs of two results by Dimitrov and Stevanović. Applying Fiedler's characterization of the largest and the second smallest Laplacian eigenvalue of the graph, we also establish new relationships between σ_t and σ. We conclude the paper with two conjectures.

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Nekateri rezultati o σ_t-neregularnosti

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Povzetek

σ_t-neregularnost (oziroma sigma totalni indeks) je grafovska invarianta, definirana kot $\sigma_t(G) = \sum_{u,v\subseteq V(G)}(d(u) - d(v))^2$, kjer $d(z)$ označuje stopnjo vozlišča z. To mero neregularnosti je leta 2019 predlagal R'eti, leta 2023 pa sta jo ponovno neodvisno obravnavala Dimitrov in Stevanović. V članku opozorimo, da velja $\sigma_t(G) = n^2 \cdot \mathrm{Var}(G)$, kjer $\mathrm{Var}(G)$ označuje varianco stopenj grafa. Pokažemo, da ima med vsemi polnimi dvodelnimi grafi na n vozliščih največji sigma totalni indeks eden od grafov $K_{\lfloor \frac{n}{4}(2-\sqrt{2})\rfloor,\lceil \frac{n}{4}(2+\sqrt{2})\rceil}$ ali $K_{\lceil \frac{n}{4}(2-\sqrt{2})\rceil,\lfloor \frac{n}{4}(2+\sqrt{2})\rfloor}$. Poleg tega podamo različne zgornje in spodnje meje za σ_t-neregularnost. V tej smeri vzpostavimo povezavo med energijo grafa $\mathcal{E}(G)$ in sigma totalnim indeksom $\sigma_t(G)$ ter podamo nova dokaza dveh rezultatov Dimitrova in Stevanovića.

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Z uporabo Fiedlerjeve karakterizacije največje in druge najmanjše Laplaceove lastne vrednosti grafa vzpostavimo tudi nove povezave med σ_t in σ. Članek zaključimo z dvema domnevama.

Ključne besede: Neregularnost, totalna neregularnost, energija grafov, Laplaceove lastne vrednosti.

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