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Half-arc-transitive graphs of order 4p of valency twice a prime

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Abstract

A graph is half-arc-transitive if its automorphism group acts transitively on vertices and edges, but not on arcs. Let p be a prime. Cheng and Oxley [On weakly symmetric graphs of order twice a prime, J. Combin. Theory B 42 (1987), 196−211] proved that there is no half-arc-transitive graph of order $2p$, and Alspach and Xu [1/2-transitive graphs of order $3p$, J. Algebraic Combin. 3 (1994), 347−355] classified half-arc-transitive graphs of order $3p$. In this paper we classify half-arc-transitive graphs of order $4p$ of valency $2q$ for each prime $q \geq 5$. It is shown that such graphs exist if and only if $p - 1$ is divisible by $4q$. Moreover, for such p and q a unique half-arc-transitive graph of order $4p$ and valency $2q$ exists and this graph is a Cayley graph.

Keywords: Cayley graph, half-arc-transitive graph, transitive graph.
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Polovično-ločno-tranzitivni grafi reda 4*p* stopnje dvakratnika praštevila

Povzetek

Graf je polovično-ločno-tranzitiven, če njegova grupa avtomorfizmov deluje tranzitivno na množicah njegovih vozlišč in povezav, ne pa tudi na množici njegovih lokov. Naj bo p praštevilo. Cheng in Oxley [On weakly symmetric graphs of order twice a prime, J. Combin. Theory B 42 (1987), 196−211] sta pokazala, da ni polovično-ločno-tranzitivnega grafa reda 2p, Alspach in Xu [1/2-transitive graphs of order 3p, J. Algebraic Combin. 3 (1994), 347−355] pa sta klasificirala polovično-ločno-tranzitivne grafe reda 3p. V tem članku klasificiramo polovično-ločno-tranzitivne grafe reda 4p in stopnje 2q za vsako praštevilo $q \geq 5$. Pokažemo, da taki grafi obstajajo natanko tedaj, ko 4q deli $p - 1$. Še več, za taka p in q obstaja natanko določen polovično-ločno-tranzitiven graf reda 4p in stopnje 2q in ta graf je Cayleyev graf.

Ključne besede: Cayleyev graph, polovično-ločno-tranzitiven graf, tranzitiven graf.