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Magnitudes in Badiou's Objective Phenomenology and Economic Consumer Choice¹

Since the dawn of human civilization in Egypt and Mesopotamia, man has been concerned with measuring different quantities as part of his quest for continued existence and progress. By antiquity, humanity had already created numerous measuring systems, based on various units of standard, for concepts like length, surface, volume, mass, and time. Not much changed until the 4th century BC when Euclid in his *Elements* proposed the first systematic contribution to measurement by introducing the idea of ratios between different homogenous (numerical) magnitudes of attributes – the measure of one magnitude relative to another. This rekindled interest in measurement theory, since thinkers now needed different generalisations of qualities to quantities to cover the vast landscape of measurable phenomena, for example Aristotle's work on temperature. He observed it to be a qualitative phenomenon that can be described in quantitative intervals of hotness and coldness. Progress made in the Middle Ages demonstrated that certain objects can attain new homogenous additional parts. This means they can be lengthened or made heavier – supplied with an extensive attribution. It follows that we can then have “extensive magnitudes” – quantities of parts observed and ascribed in numerical form and used to perform basic operations, such as addition, subtraction, and multiplication. On the other hand, the human capacity for sense-perception and intuition also allows for interaction with external objects. This in turn gave birth to a set of “intensive magnitudes”, which related various qualitative attributes to determinate objects. The concept was introduced as far back as the late 13th century by John Duns Scotus and stood the test of time, still being considered valid during the scientific revolutions of the 16th and 17th century and influencing the philosophers of the time. We see traces of it in Leibniz's assertion that “nature never makes leaps”, pointing to the natural change occurring in degrees (The Prin-

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ciple of Continuity), a fact also later picked up by Kant in his *Critique of Pure Reason* as the Axioms of Intuition and the Anticipations of Perception.

On this point, early modern philosophy has been, in a very general sense, divided between two different lines of argument: (1) Descartes and Locke are associated with those who emphasize that secondary qualities, such as colour, taste or smell, vary according to the state of the experiencing subject, while the objects can objectively be determined solely by their primary qualities, such as size, shape, and weight. On the opposing side (2) are philosophers, such as Berkeley and Hume, who unify the primary and secondary qualities – maintaining that the primary qualities of objects can be experienced only through the mediation of secondary ones – “fusing” them into an entirely subjective act of cognition. In his Axioms and Anticipations, Kant attempted to reconcile this by attributing both aspects simultaneously to the subject; he ascribes primary qualities as *extensive* magnitudes to the first, denying the exclusive subjective perception of the second, rather choosing to define the object’s correspondence (in space and time) to the subject’s sensory qualities through *intensive* magnitudes. However, later scientific discoveries, notably in physics, electrical engineering, and chemistry, unambiguously undermined Kant’s proposed unified extensive and intensive magnitudes. This spurred the search for new fundamentals in the philosophy of measurement, resulting in a branch of mathematics we now call Measurement Theory. Some fields in the social sciences, however, chose to retain the distinction. One such example is the merging of the economic theory of utility with *preference logic*² in the second half of the 20th century. It seems as though the issue has now once again been pushed to the forefront, this time in contemporary philosophy.

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In his phenomenological project, *Logics of Worlds (Being and Event II)*, Alain Badiou proposes an entirely new way of thinking of an object. His separation of the *onto*-logical part of the first *Being and Event* (1988) from *onto*-logy, i.e. pure logics pertaining to the theory of appearing – the being’s localization in a world – is now well established. Whereas the first project of pure being relied on the set-theoretic realm of (in)consistent multiplicities, the second project draws on

² For more on the (economic) logic of preference, see the seminal works of Sören Halldén, *On the Logic of Better*, Lund, Library of Theoria, 1957, and Georg H. von Wright, *The Logic of Preference*, Edinburgh, Edinburgh University Press, 1963.

a broader and richer area of mathematics – category theory (CT) – to explain the appearance of a multiple-being in the world. Badiou's reliance on category theory also required he introduce a different, purely relational conception of the object. This is where Badiou strays the farthest away from Kant's conception of the transcendental object. With Badiou, we instead get a subject-less objectivity, an *objective phenomenology* of being-there. However, what centripetally pulls the two back together is the immanence of “transcendental magnitudes” (degrees), attributes pertaining to the regimes of appearance. Although the transcendental subject is never present in Badiou's conception, the void is filled by the ubiquitous transcendental indexing of appearing in every particular situation (i.e. a world). He concedes: “In fact my conception fuses together, in the guise of a general algebra of order, ‘extensive magnitude’ and ‘intensive magnitude’.”³ What is here referred to as the “general algebra of order” is actually an object of a category that complies with what mathematicians call a complete Heyting algebra, meaning it is a complete lattice, i.e. a partly ordered set (or *poset*), where each of its subsets has a supremum and an infimum. It allows Badiou to counter Kant on the grounds that the current advances in mathematics “justify enveloping the two notions in the single one of ‘transcendental degree’[...]”.⁴ This implies a mode of existence, which, in the final instance, relies precisely on the concept of magnitude (quantum), measuring degrees of identities and differences.

When Badiou expressed his philosophical dispositive in terms of mathematical discourse, he had in fact opened up countless nexus points to various disciplines in the natural and social sciences. This article will focus on the contemporary scientificity of economics. We will focus on the transition from classical political economy, overcome by the subjective “marginalist revolution” (and the advent of preference logic), to the contemporary doctrine of methodological individualism and rational consumer choice theory.⁵ The inculcation of subjective preference plans and its coextensive mathematical and logical apparatus lies at the centre of this epistemological break. It can also be understood as a kind of ‘general algebra’, in this case of an economic equilibrium, resting on the opti-

³ Alain Badiou, *Logics of Worlds: Being and Event*, 2, trans. A. Toscano, London, Continuum, 2009, p. 234.

⁴ *Ibid.*

⁵ A theory that encompasses much more than simply economics. It is currently extensively used in sociology, the political sciences, and evolutionary theory.

mization functions of measured individual utilities (demand side) and marginal production costs (supply side) – all of which (conveniently) relies on pre-ordered or partly ordered sets and their formal conditions.

All of this has had significant repercussions. Once the “marginalist revolution” in economics had done away with the classics, i.e. with the labour theory of value of Smith, Ricardo, Malthus, and Marx’s critique of it, the neoclassical economics of Alfred Marshall and his disciples proceeded to introduce an individual preference ‘measurement’ of value, making no attempt to establish a clear link to the previous classical preconceptions. The maximizing preference function simply became a universally established law of axiomatically defined ordered sets. We argue that the inserted discontinuity between the classical and neoclassical approach in economics rests on a fallacy, since it failed to convincingly (in)validate any synthesis of the labour theory of value and the subjectivist valuations. We will examine the classical period in political economy through the lens of Badiou’s definition of a classical world – a world where a classical logic registers transcendental values, i.e. in the world of ontology. Put in terms of political economy, value must have some substance, or as Marx put it, there first has to be a value-forming substance – labour – in the form of “socially necessary labour-time” that has the capacity to create value. This fact (the objectification of human labour) is borne out each time merchandise arrives at the market for exchange in a Boolean-type logic evaluation. On the other hand, we assert that the current scheme for the subjectivist valuation of price-goods in terms of individual preference plans conforms to intuitionist logic – the logic that also underpins Badiou’s theory of appearing. As this scheme is purely subjectivist, it has no ‘primordial substance’, meaning that its transcendental values, i.e. the ‘existence’ of goods on the market, represent an immediate function of appearing in a market-economy world. Put differently, the ‘price tag’ is the sole recognition of an object’s existence in a world of commodities and it obeys the same (mathematical) structural, i.e. algebraic, protocols as Badiou’s “minimum phenomenology of abstract appearing”.

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Our investigation includes the following provisional thesis that will also act as the proximate aim of this article: The “marginalist revolution” does not rise to the scale of a Copernican-type revolution, i.e. an absolute modification of the scientific field. The switch from classical to marginalist economics is by far and away closer to a shift in doctrines – it is an intra-theoretical parallax view from

production and distribution to exchange – and rooted in a formally dominant utilitarian philosophical worldview. According to our interpretation, Badiou's efforts offer a better and, more importantly, monolithic solution, precisely because of his upgrading from set-theory ontology to category theory. In doing so, he introduces a more robust logical framework that allows him to subsume the 'primitive' belonging-to-situations multiplicities to a logical and relational framework of categories in a world. This gesture crucially also calls for a continuous, unified, and coherent bottom-up framework – one with the potential of serving as a more suitable transitional scheme to a new economic doctrine, instead of opting for an apologetic rendition of the discipline. Put more concretely in the context of our present discussion: Critical economic science should seek to reintroduce Marx's "dialectics of the value form" when dealing with commodity exchange. The approach should ideally also consider the subjective aspirations and magnitudes of desired objects by means of an immanent structure capable of handling the categorical breadth.

In the introductory remarks, we touch on an array of different topics. To help establish some initial common ground, we list them below as either falling under classical or intuitionist logic:

	Characteristic property	Algebra type	Truth values	Badiou	Marx	Economics
Classical Logic	Law of identity Law of non-contradiction Law of excluded middle	Boolean algebra	[0,1] (binary)	World of ontology	Value Form	Labour theory of value
Intuitionist Logic	Law of identity Law of non-contradiction	Heyting algebra	Ω – set (poset, pre-order)	World of phenomena	Price Form	Marginal utility

Badiou's algebra of appearing: a brief overview

Before delving into our topic, let us briefly but no less thoroughly examine Badiou's trajectory, connecting the first two volumes of the *Being and Event* project. The first volume was entirely devoted to indifferent multiplicities without any particular designated qualities. It focused on the internal composition of belonging elements that come to count-as-one, consequently forming sets of

consistent multiples in different situations. The set-theoretic connectors that constitute a non-relational framework, or rather situations and state-of-situations, are those of belonging (\in) and inclusion (\subseteq). For Badiou, this setting represents the entire world of ontology – everything proceeding/halting from/on the null-set, the void (\emptyset). The second volume moves from being to beings, i.e. being-there, their localized appearing in a world. Indeed, if multiples are different beings, they must appear differently – hence, we are no longer in the world of ontology of indifferent beings, but rather in a world or worlds where beings are relationally brought together. For Badiou, worlds are logical representational spaces, or rather topological spaces, where different beings logically relate to one another based on their identities and differences expressed in transcendental degrees. Being's appearing is sutured to a determinate *logic* of a particular world. If ontology was built from the two primitive connectors mentioned above, phenomenology now rests on just one ordering function, (\leq). The step from ontology to phenomenology implies a tremendous upheaval in the underlying mathematical apparatus from axiomatic set-theory to conceptual category theory.

In simplified terms, appearance is a function determined according to a set of operations (an algebra) that constitute a transcendental. Say we have two multiples, a and b , in order for them to appear in a world, we also need a (differential) relation between them – so we can distinguish between them –, which also connects them to a third multiple (more on this multiple later on), the transcendental (T), to formalize a world. “*We will call ‘transcendental’ the operational set which allows us to make sense of the ‘more or less’ of identities and differences in a determinate world*”⁶ Badiou says, and puts it formally as a function of appearance $\mathbf{Id}(a, b) = p$; interpreted as multiplicities $\{a, b\} \in A$ are to a p -degree identical, $p \in T$. The degree of identity, or inversely, difference, of two multiplicities in a world is measured according to the degree of belonging to transcendental algebra. It is worth noting here that the type of identity inferred here is a mathematical (CT) one – *isomorphism* – retaining both quantities in **Set** and the identity of structures (determinate algebraic operations). The transcendental algebra Badiou utilizes is the complete Heyting algebra that was designed as a model for intuitionistic logic (the law of excluded middle does not hold) and posits a specific type of Heyting-valued sets, known also as the Ω -valued sets. If

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⁶ *Ibid.*, p. 118.

we look closer, we see that Heyting-valued sets “regard an object in a topos as a ‘set-like’ entity consisting of *potentially* existing (partially defined) elements, only some of which possess *actual* existence (are totally defined).”⁷ Such a Ω -valued set is then a set, say \mathbf{A} , defined as a pair (A, \mathbf{Id}) , where, as we have seen, \mathbf{Id} is a function of assigning degrees of identity between every pair of elements $a, b \in A$; $\mathbf{Id}(a, b) \in \Omega$. As stated, the Ω -set is a partially ordered set (poset), an order structure where the order connector \leq organizes a transcendental scale (called a *locale* in mathematics) obeying certain ‘classical’ axioms, such as: $x \leq x$ reflexivity; $[(x \leq y) \text{ and } (y \leq z)] \rightarrow (x \leq z)$ transitivity; $[(x \leq y) \text{ and } (y \leq x)] \rightarrow (x = y)$ antisymmetry. If it were simply a matter of Boolean algebra, we would encounter only non-existing or absolutely existing objects; however, dealing with an intuitionistic logic, we get a *minimum* $(\forall x) (\mu \leq x)$ or the greatest lower bound and a *maximal element or envelope* (Σ) , defined as a set of degrees B , where $B \subseteq T$ – i.e., the smallest of all degrees that are greater than or equal to all the elements of B . Given the p and q degrees, we can always find a conjunction (\cap) , written $p \cap q$, so that it is the greatest of all those that are lesser than both p and q . Lastly, we have distributivity for attaining the completeness of a lattice, i.e. that all partial local conjunctions are in relation to the global envelope, making sure there are no gaps in the degrees scale via the infinite distributive law. Badiou puts it like this: $d \cap \Sigma B = \Sigma \{d \cap x / x \in B\}$; take d as a degree in conjunction with the envelope of a subset B ; you have an envelope of a given degree in regards to an element x in B to which that degree from the envelope of degrees is assigned. For a simple example of how the transcendent scaling/indexing works, let us imagine a topological space S and a set of elements (in CT, this can be any mathematical object or structure; for our purposes, let us say these are functions) A , we can therefore determine to what extent a certain function exists in this space S . If we have two objects, say $M, N \subseteq S$, where $f \in M$ and $g \in N$, we need a Ω -set to evaluate a pair of functions from subsets M, N , with the function $\mathbf{Id}(x, y)$ being the degree of their identity/difference.⁸ To recap the transcendental properties, we have an ordering structure obeying reflexivity, transitivity, antisymmetry, and distributivity; we then have the operations of conjunction, envelope, dependence, reverse and the minimal and maximal elements.

⁷ Robert Goldblatt, *Topoi: The Categorical Analysis of Logic*, Mineola, NY, Dover Publications, 2006, p. 274.

⁸ Provided that they meet the two conditions: $\mathbf{Id}(x, y) = \mathbf{Id}(y, x)$ (symmetry) and $\mathbf{Id}(x, y) \cap \mathbf{Id}(y, z) \leq \mathbf{Id}(x, z)$ (triangular inequality).

These are the transcendental operations of different elements related to their degrees of appearances when measured against each other. But how does an object actually come to fully exist? Or put differently, how does a phenomenon exist in a world? The answer is already partially contained in the preceding example above; once you have two elements, x and y , adding a third, z , for example in relation to y , you also get its relation to x , and so on, for all the elements in a given set – that is to a subset of a given world. On the other hand, for an element to properly appear, it has to have self-identity in a given space; put in Badiou's worlds of appearing, the function $\mathbf{Id}(x, x)$ does not necessarily return a Maximum value. As he says, the phenomenon is the set of the values in the function of appearing $\mathbf{Id}(a, x)$, for x that (co)appears along all the “ a ” in a set A . What does this operation accomplish? It ranks the ‘internal’ elements of a count-as-one set according to their individual transcendental degrees. So, if in the context of our function we substitute $\mathbf{Id}(x, a)$ for an element x , we get $\mathbf{Id}(x, x)$, which measures the self-identity or the degree of existence of an element in a world. The more the element assumes the identity with the world, the more strongly it appears in it. We will not delve deeper into other algebraic operations of localizations, regionalizations, or atomic logic, as that would far exceed the scope of this paper and is not relevant in terms of the argument made in the present discussion. We do, however, have to modestly introduce certain key topological aspects of Badiou's objective phenomenology.

The first problem emerges directly from our preceding discussion: How does Badiou connect objects with the transcendental of a world? Specifically, how do all the elements as subsets come to be adjoined with their respective degrees? Here, Badiou introduces a CT concept called a *functor* – a transformational operation preserving both elements and object-structures from one category to another. Badiou formally expresses it as: “The ‘transcendental functor’ of the object (A, \mathbf{Id}) , written \mathbf{FA} , associates to every element p of T the part of A composed of x 's such that $\mathbf{Ex} = p$. That is, $\mathbf{FA}(p) = \{x/x \in A \text{ and } \mathbf{Ex} = p\}$ ”⁹ To aide us in grasping the immense operationability of the functor, consider the following diagram:

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⁹ Badiou, *Logics of Worlds*, p. 289.

$$\begin{array}{ccc}
 a & F(a) & \xrightarrow{\tau_a} & G(a) \\
 \downarrow f & \downarrow F(f) & & \downarrow G(f) \\
 b & F(b) & \xrightarrow{\tau_b} & G(b)
 \end{array}$$

We have a category C: $a \rightarrow b$ with an arrow f . A functor is a function that assigns to objects a and b an operation $F(a)$ and $F(b)$, but also does the same for arrows, in our case $F(f): F(a) \rightarrow F(b)$, creating a category D. The remarkable thing about functors is that they also introduce natural transformations between the functors themselves, creating yet again a new category with functors as objects as well. Say we now have two categories acting as objects – C and D – and a functor F between them, which is actually an arrow that maps every object of category C to some $F(A)$ in D object. We now need an arrow to go from one functor to another, while retaining the structure and mapping of C in D. We need to assign an arrow to every object a of C that leads to D, incorporating the F mapping of a to G mapping of a . This is expressed as $\tau_a : F(a) \rightarrow G(a)$ and $\tau_b : F(b) \rightarrow G(b)$. This operation becomes the driving motor of ‘translating’ various degrees of appearance, gathered into subsets, to correspond to different enveloped subobjects of phenomena and their atoms of appearing, now also connecting them at the ontological level. This is crucial, since it transforms the ontological and phenomenological into a unified categorical structure.

Category theory therefore allows for the retroactive positing of logical relations (appearing) onto indifferent multiplicities as objects (ontological strata) in the form of a real synthesis of atoms.

Badiou concludes his elaboration of the object in *Logics of Worlds* by stating that a structure of sheaves (a manifold of transcendental functors **FA**) coming from transcendental T to objects of the (A, \mathbf{Id}) type is called a Grothendieck topos¹⁰. The latter is also the topologically defined site of being-there, a world. In *Mathematics of the Transcendental*, he provides a formal definition of a topos, seeing it as a “possible mathematical universe, which is both ‘big’ (existence of

¹⁰ *Ibid.*, p. 295.

limits) and centred, and which presents its own internal logic.”¹¹ We will, for our part, skip the segments dealing with the existence of limits and co-limits that determine the ‘adequate size’ of a category – an universal object, which is either itself visible from every object in a category (limit) or is the one you can see from the farthest distant objects (co-limit). Instead, we will focus on another point, i.e. its centeredness, meaning that every topos, or world as it were, needs to have a central object. In CT, this central object is known as the subobject classifier or truth-value object. Badiou’s version is presented in the diagram below:

$$\begin{array}{ccc} a & \xrightarrow{f} & b \\ 1(a) \downarrow & & \downarrow c(f) \\ 1 & \xrightarrow{T} & C \end{array}$$

What we have is an object C which has an element $(1 \rightarrow C)$ marked $T(\text{true})$ – what Badiou calls an evaluation procedure –, such that for every monomorphism $f : a \hookrightarrow b$ there exists a unique characteristic arrow $c(f)$ from $b \rightarrow C$ (centring) for the square to be a pullback. The centration (predicate characteristic) function $c(f)$ is of prime importance for Badiou: in essence, it validates the values (degrees) found in C ; $p \in C$ for every subobject a of a set b . As already indicated for sets, the logic is Boolean (bivalent, pertaining to a well-pointed topos), $C = [0, 1]$. At the level of ontology, it simply verifies whether an element belongs or does not belong to a subset. What makes toposes truly remarkable is their versatility, which Badiou uses to full effect. Within them, the central object C can modify its “own internal logic”; instead of being just a two-element set for $[\text{True}, \text{False}]$, it can be a Ω -set, e.g. a Heyting lattice, obeying intuitionistic logic and having the range of intensities p between minimum to maximal elements. Given two elements of $a(x, y)$, which are also the elements of b via the monic arrow f , they are identical in b inasmuch as in a to the degree assigned by being an element of $c(f)$ in C , i.e. some $p \in \Omega$. What we have arrived at is an evaluative structure for ‘measuring’ appearance in a world through assigning truth-values to elements, acting as degrees or intensities of appearing in a strictly logical setting.

¹¹ Alain Badiou, *Mathematics of the Transcendental*, ed. and trans. A. J. Bartlett and A. Ling, London, Bloomsbury, 2014, p. 89.

With this demonstration, we conclude this very dense introduction to Badiou's objective phenomenology, which we hope will provide a minimal basis for juxtaposing this general outline of his calculable phenomenology with the preference logic of consumer and rational choice theory, established in mainstream economic theory. The next section will introduce the main tenets of this theory.

Logic of consumer preferences in economics

We are turning to a discipline that, when it came to measuring quantities and intensities, was very much confined to Berkeley and Hume – Economics.

It was the empiricism of Bacon and Locke, the writings of the 3rd Earl of Shaftesbury on morality, and Hobbes on human nature and egoism, coupled with Hume's psychological accounts of the utility principle that, among others, left an indelible imprint on Jeremy Bentham's utilitarianism. Rejecting any kind of idealism, Bentham claimed that matter is to be subjectively (and mathematically) quantifiable and therefore experienced based on direct *pains and pleasures*. Although Bentham's account of pains and pleasures was dismissed precisely because it precluded any attempt at measurement, the future course of economic theory was already settled upon and well under way. Utility was amalgamated as a tendency of an object to increase or decrease the degrees of happiness or similar feelings. This simple proposition had an immense influence on succeeding generations of economic thinkers, namely Bentham's student John Stuart Mill and the subjectivist marginalist revolution representatives, such as H.H. Gossen and W. S. Jevons, all of whom later influenced the thinking of A. Marshall and F. Y. Edgeworth. When Jevons wrote the preface to the first edition (1871) of *The Theory of Political Economy*, he affirmed that "In this work I have attempted to treat Economy as a Calculus of Pleasure and Pain, [...] I have endeavoured to arrive at accurate quantitative notions concerning Utility, Value, Labour, Capital, etc[...]"¹² After an enduring search and interplay of cardinal and ordinal measures of marginal and total utilities, of ratios between utilities and prices, of further axiomatization of sets and bundles of goods (by O. Morgenstern and J. von Neumann), the "utility calculus" finally became a universal law of textbook economics in the 20th century. In state-of-the-art research, it became the

¹² William S. Jevons, *The Theory of Political Economy*, (5th ed.), New York, Augustus M. Kelley [1871] 1965, pp. vi–vii.

ever-present underlying assumption of mainstream economic modelling. Rational choice theory, premised on almost intact utilitarian axioms, continues to be used as a general setting for modelling economic objectivity.

Modern preference logic, however, does differentiate between intrinsic and extrinsic preferences. They are distinguished in the following way: “A preference for x over y is extrinsic if a (non-circular) reason can be given for why x is preferred to y . Otherwise, the preference is intrinsic. That x is intrinsically preferred to y is sometimes expressed by saying that x is ‘in itself’ or ‘for its own sake’ preferred to y . Judgments of intrinsic preference, or at least many such judgments, express our likings. To say that we intrinsically prefer x to y is often the same as saying that we like x better (more) than y .”¹³ We can clearly see here the linguistic operation of immediate propositional predication when it comes to intrinsic preferences. But, just immediately after, we learn that “It seems plausible to think that all extrinsic preferences are ultimately founded on intrinsic ones.”¹⁴ Consequently, we get an assembled view of preferences, defined as a predicative evaluation of magnitudes experienced subjectively by individuals in a comparative manner.

Let us now see how preference logic is reproached by Gérard Debreu through his set-theory axiomatic analysis, presented in *Theory of Value* – the still seminal sourcebook for mathematical fundamentals of economic analysis. In chapter 4, which deals with “Consumers”,¹⁵ the task calls for preparing the foundations for drawing up a complete consumption plan. Instead of dwelling on budgetary constraints, we will instead concern ourselves with the formal depiction of these functions in relation to their logical assumptions on sets X_i . We will write them in italics and comment successively: (a) X_i is closed (the axiom of continuity); *an infinite sequence of consumptions*, (b) X_i has a lower bound for \leq , (c) X_i is connected (i.e. completeness or the axiom of order), (d) X_i is convex ($A \sim B$; the indifference relation). Another assumption, which is not on the immediate list, is the condition of *transitivity*. What these assumptions bring in terms of ordered relations are of course the characteristics of a pre-order (\preceq). The choice of pref-

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¹³ Georg H. von Wright, “The Logic of Preference Reconsidered”, *Theory and Decision*, 3 (1972), p. 142.

¹⁴ *Ibid.*

¹⁵ Gérard Debreu, *Theory of Value an Axiomatic Analysis of Economic Equilibrium*, New Haven, CT, Yale University Press, 1971, pp. 50–59.

erence algebra then comes down to the variant of a symmetry relation; if dealing with a symmetric relation $A \sim B \rightarrow B \sim A$, we speak of the equivalence relation (the most usual preference relation); additionally, we either have asymmetry $A > B \rightarrow \neg(B > A)$ or antisymmetry $A \geq B \wedge B \geq A \rightarrow A = B$, where we consequently speak of a total preorder that is also a partial order (a quasi-preference relation). Total orders are mostly used in economic modelling when presupposing consumer functions, although different options of partial orders are becoming more and more common. The above relations therefore impose an algebra of a total or a partial order. Again, we encounter a truth-value object Ω for indexing preferences that could just as possibly obey, as with Badiou, a complete Heyting algebra.¹⁶ We interpret the basic preference order as follows: say we have a set X_i with the pair of elements a and b , both of which represent different combinations of two goods – these two combinations are also quantitatively defined, say $a = \{1 \text{ apple, } 2 \text{ pears}\}$ or $b = \{3 \text{ apples, } 1 \text{ pear}\}$. Applying preference relations, we can order different and mutually exclusive combinations of bundles of goods into a range between a minimal and a maximal element.

The above characteristics provide a general outline of preference logic. We are now able to link it with utility theory. We saw that different pairings in our X_i set combine the elements a , b , where each is initially endowed with a given quantity as different quantitative distributions obviously yield different utilities. Take our initial example: we had in element b a combination of goods (x, y) , where the bundle implied a number of 3 apples and 1 pear. There is a *utility function* $u : X \rightarrow \mathbb{R}$, for there $\exists U(a, b)$ for $\forall X_i(a, b)$ ranking every bundle $u : (x, y) \rightarrow U(x, y)$, $x \lesssim y$ iff $u(x) \lesssim u(y)$, where U is a total utility in a consumption plan of ordered preferences. Decomposing the total derivative U into partial derivatives $MU_x = \frac{\Delta U}{\Delta x}$ and $MU_y = \frac{\Delta U}{\Delta y}$, we arrive at marginal utilities for each of the goods in a bundle – and to every economics student a very well known equation of:

$$\text{(Marginal rate of substitution) } MRS = \frac{MU_x}{MU_y} \dots = \frac{P_x}{P_y} \dots$$

¹⁶ As is the case in the intuitionistic fuzzy sets application on decision theory. For more, see keywords intuitionistic fuzzy sets, decision theory, and preference logic. E.g. in Hülya Behret, "Group decision making with intuitionistic fuzzy preference relations", *Knowledge-Based Systems*, 70 (2014), pp. 33–43.

With most of the formalities introduced in this first analytical part, it is important to look at them in parallel to gain a broader view of our argument. The logical overlap between Badiou's transcendental algebra and preference logic (in economics) is immediately apparent, resting on either a poset or a totally ordered set. These measure the degrees of appearing and existence in the transcendental or ranking preferences of a particular bundle of goods – both of these measurements rely on an order-relation, either \leq greater or equal or preferring or indifferent \lesssim , respectively. The only difference is that in economics, the symmetry/asymmetry/antisymmetry clause is invoked randomly, depending on the preference logic model employed, whereas Badiou strictly maintains the antisymmetric relation in the poset of the transcendental (neither $x \leq y$ or $y \leq x$, they are incomparable). Both Badiou and preference logic, as used in economics, maintain conjunction, envelope, and distributivity operations. The relation $A \lesssim B$, meaning that an individual wants B *at least* as much as A, as expressed in economics, corresponds to Badiou's formulation $p \cap q = p$. The envelope is, on the other hand, an entire bundle of goods chosen against all alternatives, i.e. all other subsets of combinatory bundles one can choose from, either a 2-element subset or a n-element subset bundle. But which one is chosen? It stands to reason that it should be the intersection bringing the highest degree of utility/satisfaction.

As already pointed out, the transcendental functor plays a key role in Badiou's theory of the object. Again briefly: observed from the point of view of an object, a functor takes the composite atomic logic of localizations and connects them with the transcendental indexation. It causes a decomposition of atoms (sub-objects as count-as-ones, inferring they are singletons) in appearing in relation to the elements of the multiples on the ontological level, but as far as appearing goes, they are determined by their mutual co-belonging to the function of 'phenomenal components' – relating a given multiple A and the transcendental T. The atomic logic (localization, compatibility, ordering) is based on the existence of some x in conjunction with a degree p of the transcendental T, but moreover, it is also a combination of elements that relate to one another; therein lies the phenomenal characteristic of $\mathbf{Id}(a, x)$ and existence in terms of self-identity, $\mathbf{Id}(x, x) = \mathbf{Ex}$. Without going into too much detail, we will state that every localization of elements (atoms) in an object is subject to belonging to the phenomenal component (π), which can be distributed more widely through the compatibility of other elements of the same multiple, i.e. they share the same degree of

appearing (ρ) and order that ranks all the combinations according to \leq . All this culminates in the transcendental functor operation “which associates to every degree of the transcendental the set of elements of the object whose common characteristic is that their existence is measured by this degree.”¹⁷ So we have the grouping of degrees (‘degrees of the transcendental’) associated with (‘set of elements’) subsets that are parts of a phenomenal component (‘common characteristic’).

We have thus far been concentrating on the structural correspondence between preference logic and transcendental algebra – in essence, they are both ordering relations, resting on total and partial ordering. However, more can be said about how the logic of preference formally relates to utility theory. It does so in much the same way through a functor relation, transcribing the category of preferences onto the category of utilities, and going even further by transforming utilities and reflecting them in prices. Say we have a set X_i of elements (a, b) that are first evaluated against their quantitative determination, (a, x) and (y, b) , and find those combinations such that $(a, x) \cap (y, b) \leq U_{a,b}$, so that by applying the utility function u , we get a determinate utility for this particular bundle. By considering the whole range of X_i bundles, we can construct an entire consumption plan for all the combinations of these two elements. What does constructing an entire plan mean? It involves the decomposition of all the possible degrees of utilities by combining the two elements, or conversely, assigning to every individual combination a specific degree of utility. In other words, it takes each sub-part of the whole universe of bundles X_i , collecting them into new objects and subsuming them to the function with the corresponding degree of utility. In this sense, the utility function (or rather monotonic transformation¹⁸) has the same rank as the transcendental functor. Bearing all this in mind, one could facetiously say that Badiou’s analysis of the columns in Robert Hubert’s painting *The Bathing Pool* is an excellent example of what the microeconomists would enthusiastically analyse with their indifference curves.

¹⁷ Badiou, *Logics of Worlds*, p. 278.

¹⁸ In utility theory, a monotonic transformation is presupposed as a way of transforming one set of numbers into another set of numbers in a way that preserves the order of the numbers. The operation is homologous to the natural transformation in functors.

We can now diagrammatically represent the transformation of preferences to utilities:

$$\begin{array}{ccc}
 p \in \Omega & \xrightarrow{u_p} & \begin{array}{l} Fu(p) \subseteq X \\ U_x = p \end{array} \\
 \downarrow q \preceq p & & \downarrow \frac{\partial u(x_1, x_2)}{\partial x_i} = MU_i(x_1, x_2) \\
 q \in \Omega & \xrightarrow{u_q} & \begin{array}{l} Fu(q) \subseteq X \\ U_y = q \end{array}
 \end{array}$$

Badiou's objects can therefore be said to be composed of atoms, i.e. subobjects, one by one linked to their phenomenal appearing through the “phenomenal component” function. The parts, assembled according to their degrees of appearing, are evaluated against the transcendental. The objects of a determinate consumer choice are similarly decomposed into bundles of elements to which we attribute different magnitudes of utility. The function that orders all the utilities is a partial derivative of the assigned utility to each element of the bundle against all assemblages with other elements. However, what is most striking in the above diagram is the categorical setting and structural correspondence between Badiou's theory of appearing and consumer choice theory. This is in effect because both degrees of appearance and ranking of preferences rely on the same mathematical ordering operations. At the same time, they also both operate on set-theoretical fundamentals when it comes to defining their objective disposition. Afterwards, they both face the identical problem of linking or ‘transcoding’ the ‘objective’ part to the ‘transcendental’ one – Badiou resolves it by using the concept of the atomic structure and phenomenal components to transcendental indexing, while consumer choice theory relies on consumer plans and marginal propensities for each object in a bundle to complete the consumer plan.

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World of commodities – a possible synthesis?

We are now at a point where we can address our ulterior motive for imagining a coherent economic structure, capable of subsuming both the classical tradition and the subjectivist turn. This was one of the irritating prospective tendencies of political economy that Marx identified and sought to solve. The critique of political economy and its categories had much to say about the inadequate conceptual unfolding at the time and Benthamite utilitarianism was one of the driving

forces leading to the bourgeois delusion. Marx saw economic subjects standing before a “topsy-turvy” world of economic objectivity, bestowed with the reification process. Qualitatively determined subjects overturned into individualized quantifiable objects of commodity-like exchange and *vice versa*. Reification was notably scrutinized in the work of György Lukács and the proponents of the Frankfurt School, Adorno and Horkheimer, in *Dialectic of Enlightenment*, as well as Herbert Marcuse. He made it clear that in order to *critically assess* the actual dimension of the reigning mode of production, we have to disentangle the actual social relations “as a totality of objective relations”,¹⁹ i.e. we have to decipher wages in relation to the value of labour, or prices and utilities in relation to production factors.

Economic relations only seem to be objective because of the character of commodity production. As soon as one delves beneath this mode of production, and analyzes its origin, one can see that its natural *objectivity* is mere semblance while in reality it is a specific historical form of existence that man has given himself. Moreover, once this content comes to the fore, economic theory would turn into a *critical theory*.²⁰

If economic science is itself caught in an estranged form – as the young Marx suggests²¹ –, there has to be an alternative, critical science capable of demystifying this topsy-turvy world. To construct such a science is to answer Marx’s call for the handling of a “true materialism [*wahren Materialismus*]” with a “real science [*reelle Wissenschaft*]”. It must be able to deconstruct the structural relations of the apparent objectivity, decompose its laws and propositions, and, even more importantly, lay the foundations for a theoretical continuum – *a cumulative hierarchy of theoretical bodies*. It is here that Badiou’s *objective phenomenology* can play a decisive role in the critical re-examination of current economic objectivity. Our introduction also implied a question: In a reified social exchange of commodities, how does one ‘value’ the objects brought for exchange? Put another way – If commodity fetishism is a mode of appearance,

¹⁹ Herbert Marcuse, *Reason and Revolution: Hegel and the Ruse of Social Theory*, (2nd ed.), London, Routledge & Kegan Paul, 1955, p. 280.

²⁰ *Ibid.*, p. 281.

²¹ Karl Marx, *Economic and Philosophic Manuscripts of 1844 / and the Communist Manifesto*; Karl Marx and Frederick Engels, Buffalo, NY, Prometheus Books, 1988, p. 44.

what is the transcendental structure of these objects, how do they come to exist on the market, in a world?

When Badiou introduced his theory of object, and appearing of objects in a world that-of, he asked them to be thought in “a world, as a site of being-there, is a Grothendieck topos.”²² Let us return once more to the *Mathematics of the Transcendental* and more formally say that a topos is a category which admits limits and co-limits for every finite diagram, i.e. that a diagram admits a universal property object, called a cone or co-cone; it has an initial object (0), a terminal object (1), admits pullbacks and pushouts for objects, as well as equalizers and co-equalizers for arrows. There must exist a power object, and lastly, there must also exist a Central Object C (of which we have already spoken, it being the subobject classifier). We have shown elsewhere that Marx’s value form can be categorically and structurally represented as a configuration of a subobject classifier²³. Before going into the structure itself, let us first see how Marx in *Capital* relates value, its magnitude as a function of money and transformation into prices through the lens of the value form:

The magnitude of the value of a commodity therefore expresses a necessary relation to social labour-time which is inherent in the process by which its value is created. With the transformation of the magnitude of value into the price this necessary relation appears as the exchange-ratio between a single commodity and the money commodity which exists outside it. [...] The price-form, however, is not only compatible with the possibility of a quantitative incongruity between magnitude of value and price, i.e. between the magnitude of value and its own expression in money, but it may also harbour a qualitative contradiction, with the result that price ceases altogether to express value, despite the fact that money is nothing but the value-form of commodities. Things which in and for themselves are not commodities, things such as conscience, honour, etc., can be offered for sale by their holders, and thus acquire the form of commodities through their price. Hence a thing can, formally speaking, have a price without having a value. The expression of price is in this case imaginary, like certain quantities in mathematics. On the other hand, the imaginary price-form may also conceal a real val-

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²² Badiou, *Logics of Worlds*, p. 295.

²³ On CT analysis of the value form, see Uroš Kranjc, “Logics of the Value Form – Marx with Badiou” *Cosmos and History*, 14 (3/2018), pp. 72–102.

ue-relation or one derived from it, as for instance the price of uncultivated land, which is without value because no human labour is objectified in it.²⁴

Hans-Georg Backhaus posits: “The content of Marx’s form analysis is the genesis of price as price.”²⁵ Michael Heinrich further succinctly points out that the relation between value and price is that “the magnitude of value of a commodity and its price are categories pertaining to different abstraction levels, so strictly speaking it makes no sense to posit their direct concurrence or divergence.”²⁶ Marx has it all already there: the necessary social labour-time and the relation from labour to (the magnitude of) value (i.e. the so-called law of value) with the insurrection of the universal – money commodity, the *contradictory* transformation of value to price, and finally their objective and subjective character; all immanent to the value form unfolding. Marx, ever the ‘dialectician’, handles the conceptual interchange and unfolding with considerable ease, while the formal logic needs to be somewhat more imaginative when trying to reconstruct and accommodate the inner logical structure of the dialectic method. How does one start?

In our exposition of the value form²⁷, we propose a bivalent logic of value when it comes to the immediate relation between a commodity and the universal object. The inner dialectical move of “doubling” [*Verdopplung*] of commodity into commodity and money, or put differently, the “value-forming substance”, labour, that comes to be expressed with a third object, in this categorical interpretation represents precisely the same function of evaluation as analysed above. What the products of labour achieve when entering the market exchange is precisely the binary evaluation, the Ω -valued set = (0, 1) of being, a) a commodity for exchange or b) refused for exchange by market, hence no (exchange) value. This is the proposition on which the whole of classical economics and labour theory of value depend. Once this was done away with as part of the “revolutionary” shift from objectivist to subjectivist, or rather parallax view from production and distribution to exchange, the neoclassical school had simply altered the ‘func-

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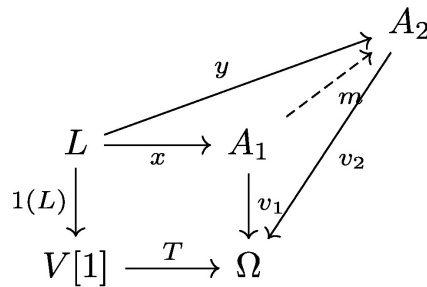
²⁴ Karl Marx, *Capital: A Critique of Political Economy, Volume One*, London, Penguin books 1993, pp. 196–197.

²⁵ Hans-Georg Backhaus, “On the Dialectics of the Value-Form”, *Thesis Eleven*, 1 (1/1980), p. 105.

²⁶ Michael Heinrich, *Die Wissenschaft vom Wert*, Münster, Westfälisches Dampfboot, 2006, p. 243.

²⁷ Cf., Kranjc, “Logics of the Value Form”, pp. 84–86.

tion of phenomenal appearing', from average labour-time to marginal utilities as degrees of value. This process resembles Badiou's differentiation between classical (ontological) and non-classical (appearing) worlds. Whereas in the classical economical conception, the centrality of value is attested exclusively on the predicate *is/is not* (an exchanged product of labour – everything further is treated as an attribution to this basic principle), the neoclassical and mainstream orientations conform to the absolutization of values between the minimum μ and maximum element M , $\Omega = \mu \leq x \leq M$. Such interchangeability is a truly remarkable achievement of Badiou's philosophical project, for it opens up a new mode of thinking, capable of merging classical and subjective economics through its critical dispositive. If we look at the subobject classifier, which we have elsewhere called the "classification schema of exchange", one last time:



Without going into too much detail, we have the following objects: L (labour) as an universal subobject of elements x and y composing commodities A_1 and $A_2 \dots A_n$, respectively. Let T be a validation function pertaining to all subobjects: $V[1] \rightarrow \Omega$, which outputs to a given object from the co-domain Ω the value {true} for all those subobjects that meet the conditional evaluation v_1 . The L - A_1 - A_2 triangle represents what Badiou calls a triangular inequality, where the elements, say x and y , get expressed by a third (value-substance coming under elements v_1 and v_2 so that the triangle diagram commutes). The Ω -set is treated here as an algebra of the universal equivalent, to which all objects relate their mode of 'existence in exchange' and pulled back via v_1 and v_2 in degrees, i.e. going from value – magnitudes of value – prices.

Does Badiou offer a final and decisive answer to our problem, namely a transformation from value to price that would sublimate the semblance of a break in economics? Indeed, he does! It is his theory of points, which in our case rep-

resents the model for a synthesis of the classical labour theory with the neo-classical subjectivist orientation. Badiou's point of view here is the following: "This triple determination of the concept of transcendental is what allows it to regulate appearing as localization (being-there), as cohesion (logical form of a being), and as situation (underlying multiple-being of being-there). There is an immanent onto-topo-logical (or 'ontological') regulation."²⁸ Here, Badiou employs yet another CT operation – homomorphism (a structure-preserving mapping), which he uses to formalize "a structural homomorphism between the initial transcendental form T and the binary transcendental T_0 ",²⁹ where T is the non-classical transcendental of appearing $[\mu \leq x \leq M]$ and T_0 a binary one $[0, 1]$. We will not go into details³⁰ for want of space and will take Badiou for granted when he states that homomorphism θ indeed associates (is a surjective function) to every point $p' \in T'$, whenever there exists $q \in T$ such that $\theta(q) = p'$. The homomorphism therefore enables the continuous ascending and descending transformation from the ontological to the phenomenological plane and *vice versa*. This fact is simultaneously of crucial importance for our discussion of the commensurability between classical labour theory and marginalist subjectivist theory of value, as it implies congruence with economic objectivity, instead of a historically experienced break.

The notion of magnitude is, interestingly, one of those notions that concern practically all philosophical traditions of the modern period and beyond. Although they are casually employed in different natural and applied sciences, their philosophical grounding still remains an ambiguous matter. As for our part, we sided with Badiou against Kant in arguing for the confluence of extensive and intensive magnitudes when it comes to economic analysis. We believed it to be a productive starting point for our investigation into economic consumer choice theory. Hereon, we can also provide a generalised answer to the initial speculation of whether our juxtaposition of Badiou's *logics of worlds* and the logics of classical and neoclassical economics can be examined within a single framework, particularly the one underlying Marx's critique of economic categories. The thesis seems justifiably viable and in need of further analysis if any

²⁸ Badiou, *Mathematics of the Transcendental*, p. 209.

²⁹ Badiou, *Logics of Worlds*, p. 438.

³⁰ For the complete formal algebra of points, see Badiou, *Mathematics of the Transcendental*, pp. 209–216.

serious attempt at bringing new critical insights into contemporary economic thought is to be anticipated.

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