PRONICANJE VODE POD JEZOVI NA ZELO DEBELIH, HOMOGENO PREPUSTNIH KAMENINAH

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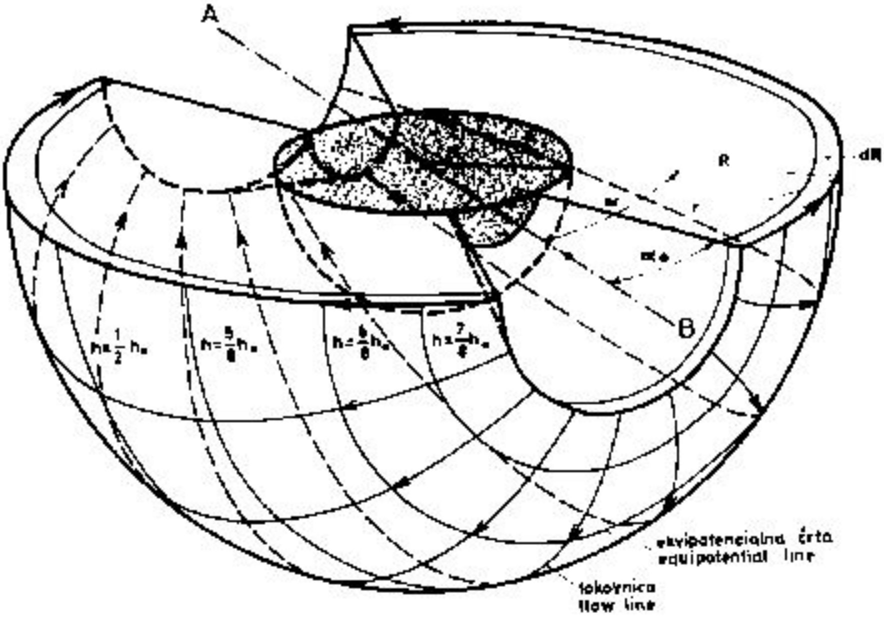
S 3 slikami med tekstom

Podlago mnogih jezov tvorijo več ali manj prepustne kamenine. V mnogih primerih so pod temi že v sorazmerno majhni globini neprepustne kamenine. tako da je mogoče preprečiti pronicanje vode pod jezom in v neposredni okolici jezu z zatesnitvijo prepustnih kamenin do te podlage. Voda bo potem pronicala samo okrog bokov jezu in tesnilnih objektov.

Pri jezovih na zelo debelih prepustnih kameninah pa teh ne moremo zatesniti do neprepustne podlage. V teh primerih voda ne bo pronicala samo okrog bokov, temveč tudi pod jezom oziroma pod spodnjim robom tesnilnih objektov. Takšne pogoje imamo pri jezovih na zelo debelih konglomeratnih zasipih v alpskih dolinah ali na močno razpokanem apnencu in dolomitu.

Pretok okrog jezu lahko ocenimo na ta način, da izračunamo pretok v geometrično enostavnem, a še vedno dovolj podobnem modelu. Tak model nam kaže sl. 1. V modelu smo nadomestili jez z neprepustno polkroglo približno istih dimenzij, akumulacijski bazen in dolino pod jezom z žlebom v obliki prisekanega stožca z vrhom v središču polkrogle. Nalogo še nadalje poenostavimo in računajmo kot da je gladina podtalnice vodoravna. Take pogoje pretoka bi imeli v modelu, če bi površino pokrili z neprepustno plastjo in če bi pritisk vode bil povsod tako velik, da ne bi imeli proste gladine. Ker je globina neprepustne podlage zelo velika, je debelina toka podtalnice (oddaljenost gladine podtalnice od neprepustne podlage) tako velika, da znižanje gladine podtalnice lahko zanemarimo. S predlagano poenostavitvijo pri računu zato gotovo nismo napravili velike napake. Če je kamenina izotropno prepustna, imajo v takem modelu tokovnice obliko krožnice s središčem v središču polkrogle, ekvipotencialne ploskve pa obliko stožcev s središčem v središču polkrogle.

Podoben model bomo priredili tudi za račun pretoka skozi homogeno, anizotropno prepustno kamenino.



Sl. 1. Tokovna mreža okrog neprepustne polkrogle
Fig. 1. Flow net around an impermeable hemisphere
Pojasnila so v tekstu
Explanations see in the text

Izotropni model

V izotropnem modelu so tokovnice koncentrične krožnice. Ves tok podtalnice lahko razdelimo na koncentrične krogelne lupine. V nadaljnjem naj pomeni:

AB os modela, R_o radij polkrogle, ki predstavlja jez, R_1 oddaljenost med središčem polkrogle in koncem žleba, ki predstavlja akumulacijski bazen, R oddaljenost od središča polkrogle, r oddaljenost od osi modela, α polovica kota pri vrhu poljubne ekvipotencialne ploskve, α_o polovica kota pri vrhu žlebov akumulacijskega jezera in doline pod polkroglo, Q celotni pretok pronicajoče vode, v filterska hitrost pronicajoče vode, k koeficient prepustnosti, h piezometrska višina v poljubni točki, merjena od površine modela, h_o piezometrska gladina v akumulacijskem bazenu, l dolžina loka po tokovnici.

Presek poljubne ekvipotencialne ploskve z lupino debeline d R je polkrožen trak s površino d A:

$$d A = \pi r \, . \, d R = \pi R \, . \, \sin \alpha \, . \, d R$$

Pretok v lupini z debelino $d R$ je

$$d Q = d A \, . \, v = - \pi R \, . \, \sin \alpha \, d R \, . \, k \, . \, \frac{d h}{d l}$$

Ker je $d\,l = R\,.\,d\,\alpha$, dobimo:

$$d\,Q = -\pi\,R\,.\,\sin\alpha\,.\,k\,.\,\frac{d\,h}{R\,.\,d\,\alpha}\,.\,d\,R = -\pi\,.\,k\,.\,\sin\alpha\,.\,\frac{d\,h}{d\,\alpha}\,.\,d\,R$$

Po ločitvi neznank dobimo

(1)
$$\frac{d\,\alpha}{\sin\alpha} = -\frac{\pi\,.\,k\,.\,d\,R}{d\,Q}\,.\,d\,h$$

Ker je pri $\alpha = \alpha_0$ $h = h_0$, pri $\alpha = \pi - \alpha_0$ $h = 0$, integriramo levo stran od α_0 do $\pi - \alpha_0$, desno stran pa od h_0 do 0

$$\int_{\alpha_0}^{\pi-\alpha_0} \frac{d\,\alpha}{\sin\alpha} = \int_{h_0}^{0} -\frac{\pi\,k\,.\,d\,R}{d\,Q}\,d\,h$$

Integral na levi strani je simetričen glede na $\frac{\pi}{2}$ in ga zato razdelimo na dva enaka dela ter dobimo

$$\int_{\alpha_0}^{\pi-\alpha_0} \frac{d\,\alpha}{\sin\alpha} = 2\int_{\alpha_0}^{\frac{\pi}{2}} \frac{d\,\alpha}{\sin\alpha} = 2\ln\operatorname{tg}\frac{\alpha}{2}\Bigg|_{\alpha_0}^{\frac{\pi}{2}} = 2\left[\ln\operatorname{tg}\frac{\pi}{4} - \ln\operatorname{tg}\frac{\alpha_0}{2}\right]$$

(2)
$$\int_{\alpha_0}^{\pi-\alpha_0} \frac{d\,\alpha}{\sin\alpha} = -2\ln\operatorname{tg}\frac{\alpha_0}{2}$$

Če vstavimo to vrednost in izračunamo $d\,Q$, dobimo:

$$d\,Q = -\frac{\pi\,.\,k\,.\,d\,R}{2\ln\operatorname{tg}\frac{\alpha_0}{2}}\,h_0$$

Celotno izgubo dobimo, če integriramo na desni strani R od roba jezu (R_0) do konca akumulacijskega bazena (R_1)

(3)
$$Q = -\frac{\pi\,k\,h_0\,(R_1 - R_0)}{2\ln\operatorname{tg}\frac{\alpha_0}{2}}$$

Če spremenimo še naravne logaritme v desetniške in vpeljemo za π njegovo numerično vrednost, dobimo

(3 a)
$$Q = -\frac{0.682\,k\,h_0\,(R_1 - R_0)}{\lg\operatorname{tg}\frac{\alpha_0}{2}}$$

Izgube z R_f (dolžino bazena) linearno naraščajo. Pri tem smo predpostavili, da se voda preceja na vsem območju po koncentričnih krogih. Do takega pretoka bi prišlo samo v primerih, da je meja med prepustno in neprepustno podlago tudi koncentrična polkrogla s središčem v središču jezu in katere radij je enak dolžini akumulacijskega bazena. V naravi je skoraj vedno globina neprepustne podlage mnogo manjša kot dolžina bazena. Izgube, ki jih dobimo, če v zgornjo formulo vstavimo za R_f dolžino akumulacijskega bazena, bodo torej prav gotovo zelo visoko cenjene.

Anizotropni model

Anizotropni model smo poskusili analizirati na način, kot ga podaja Scott (1963, 110—111) za dvodimenzionalen problem.

Pri stacionarnem pretoku nestisljive tekočine skozi anizotropno sredstvo velja Laplaceova enačba

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \tag{4}$$

v kateri pomenijo k_x, k_y in k_z prepustnosti v smeri osi X, Y in Z, ki smo jih postavili v smeri glavnih prepustnosti. Vsak anizotropen hidrogeološki model pa lahko spremenimo v izotropnega, če ga skrčimo v smeri glavnih prepustnosti v merilih, ki so proporcionalna kvadratnim korenom prepustnosti v teh smereh. Vpeljimo nove neznanke ξ, η, ζ tako, da je

$$x = \sqrt{a\, k_x}\, \xi,\ y = \sqrt{a\, k_y}\, \eta,\ z = \sqrt{a\, k_z}\, \zeta \tag{5}$$

$$\partial x^2 = a\, k_x\, \partial \xi^2,\ \partial y^2 = a\, k_y\, \partial \eta^2,\ \partial z^2 = a\, k_z\, \partial \zeta^2$$

a je poljubna konstanta.

Če vstavimo te vrednosti v (4) in krajšamo, dobimo

$$\frac{\partial^2 h}{\partial \xi^2} + \frac{\partial^2 h}{\partial \eta^2} + \frac{\partial^2 h}{\partial \zeta^2} = 0 \tag{6}$$

Transformirani model torej res lahko obravnavamo kot izotropnega.

Pri peščenih in prodnatih sedimentih je prepustnost v vseh vodoravnih smereh približno enaka. Označevali jo bomo s k_h; $k_x = k_y = k_h$. Pravokotno na plasti je prepustnost mnogo manjša in jo bomo označevali s k_v; $k_z = k_v$. Če za poljubno konstanto a v (5) izberemo vrednost $1/k_v$, potem je $z = \xi$. V vertikalni smeri modela ne bomo skrčili. V smeri plasti pa bomo model skrčili v merilu $1 : \sqrt{\frac{k_h}{k_v}}$. Prav tako bi lahko model transformirali tako, da bi pustili v smeri plasti dimenzije nespremenjene in bi ga v vertikalni smeri raztegnili v merilu $1 : \sqrt{\frac{k_v}{k_h}}$.

Ugotoviti moramo še, s kakšno prepustnostjo k_t moramo računati v transformiranem modelu, da bomo dobili enake izgube kot v prvotnem anizotropnem modelu. Vodni curek (del toka podtalnice, ki je omejen s sklenjenim plaščem tokovnic), razdelimo z ekvipotencialnimi ploskvami tako, da je razlika v piezometrični višini med dvema sosednjima ploskvama Δh in razdalja med obema Δl. Če je Δh dovolj majhen, lahko vstavimo za hidravlični gradient $i = \Delta h/\Delta l$. Množina vode, ki se pretaka v vodnem curku s prečnim presekom S, je potem

$$q_t = S \cdot v = -S \cdot k_t \frac{\Delta h}{\Delta l} \tag{7}$$

Omejimo tanek vodni curek v prvotnem, anizotropnem in v transformiranem, izotropnem modelu tako, da je prečni presek trikotnik, katerega stranice so na raziskanem kraju vzoredne koordinatnim ploskvam. Na sprednjo ploskev postavimo trirobnik, katerega robovi so vzporedni koordinatnim osem (sl. 2.). Ker je voda nestisljiva, mora biti vsota pretokov skozi ploskve trirobnika (S_x, S_y, S_z v izotropnem modelu, oz. S_x', S_y', S_z' v anizotropnem modelu) enaka pretoku skozi sprednjo ploskev curka (S oz. S'). Če pomeni:

q' pretok v curku,
q_x' pretok skozi ploskev S_x',
q_y' pretok skozi ploskev S_y',
q_z' pretok skozi ploskev S_z'

je

$$q' = -(q_x' + q_y' + q_z')$$

Pretok v smeri osi X pa je

$$-q_x' = S_x' \cdot v_x = -S_x' \, k_h \frac{\partial h}{\partial x} \tag{8}$$

Vrednost za $\frac{\partial h}{\partial \xi}$ v transformiranem, izotropnem modelu (sl. 2b) pa dobimo

$$\frac{\partial h}{\partial \xi} = \frac{\Delta h}{A D} = \frac{\Delta h}{\Delta l/\cos \alpha}$$

V prvotnem, anizotropnem modelu je razdalja $A D$ povečana s faktorjem $\sqrt{k_h/k_v}$. Komponenta hidravličnega gradienta v smeri osi X je torej

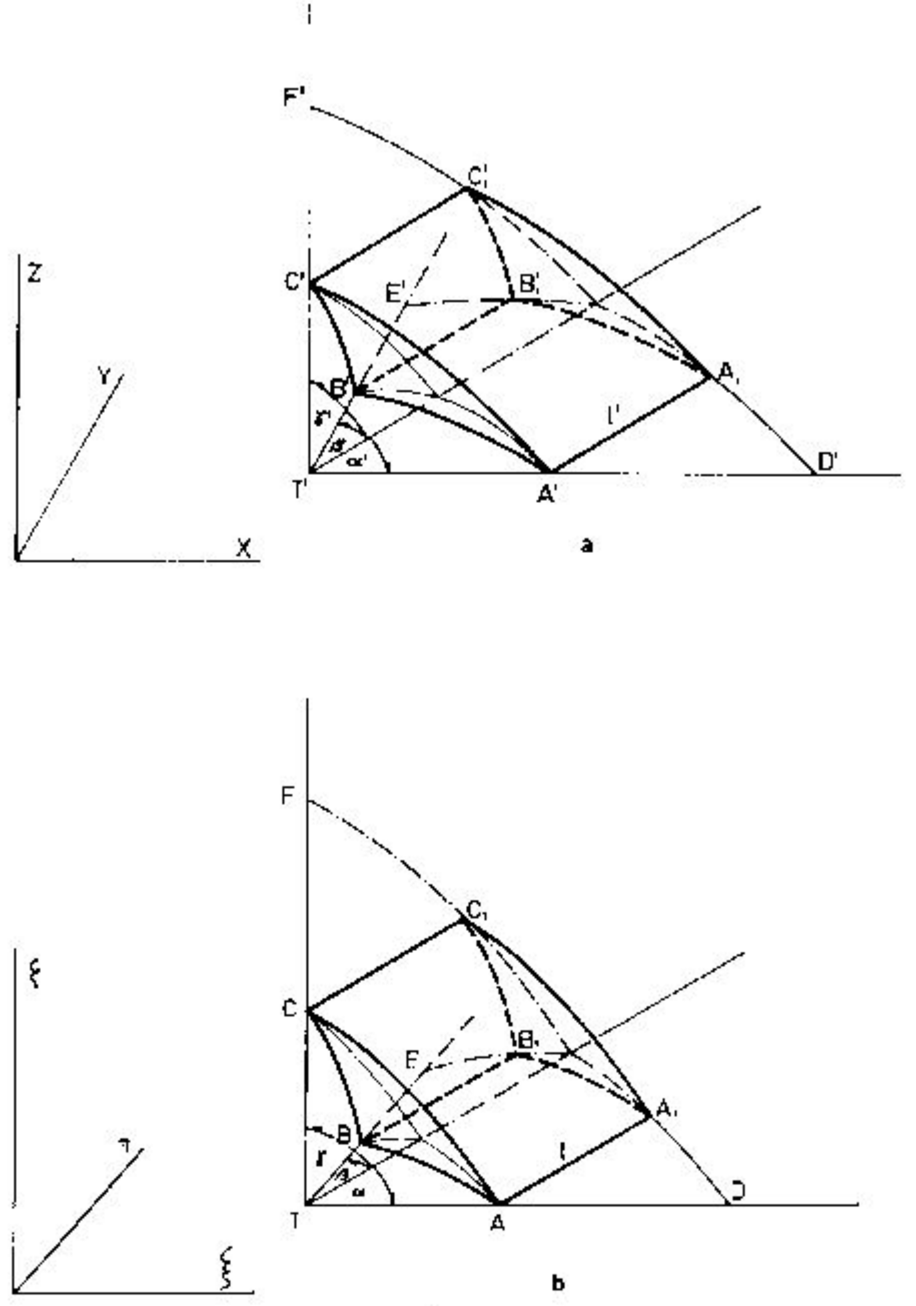
$$\frac{\partial h}{\partial x} = \frac{\Delta h}{A' D'} = \frac{\Delta h}{\Delta l/\cos \alpha \cdot \sqrt{k_h/k_v}}$$

Ploskev trirobnika $T B C = S_x$, ki je pravokotna na q_x, pa je projekcija ploskve $A B C = S$ na ravnino $Y Z$, ki oklepa s ploskvijo S kot α

$$S_x = S \cdot \cos \alpha.$$

V prvotnem, anizotropnem modelu pa je ploskev $T' B' C' = S_x'$ razpotegnjena v smeri osi Y za faktor $\sqrt{k_h/k_v}$:

$$S_x' = S_x \cdot \sqrt{k_h/k_v} = S \cdot \cos \alpha \cdot \sqrt{k_h/k_v}$$



Sl. 2. Shema vodnega curka: *a* v anizotropnem in *b* v transformiranem — izotropnem modelu
Fig. 2. Sketch of the stream tube: *a* in the anisotropic model, *b* in the transformed — isotropic model
Pojasnila so v tekstu
Explanations see in the text

Iz (8) dobimo končno

$$q_x' = k_h \frac{\Delta h}{\Delta l \cos\alpha \cdot \sqrt{k_h/k_v}} S \cdot \cos\alpha \sqrt{k_h/k_v}$$

(9 a)
$$q_x' = k_h \cdot \frac{\Delta h \cdot S \cdot \cos^2\alpha}{\Delta l}$$

Podobno dobimo za q_y'

(9 b)
$$q_y' = k_h \frac{\Delta h \cdot S \cdot \cos^2\beta}{\Delta l}$$

Ploskev S_z' je razpotegnjena v obeh smereh, tj. v smeri osi X in osi Y za faktor $\sqrt{k_h/k_v}$. Njena površina je torej

$$S_z' = S_z (\sqrt{k_h/k_v})^2 = S \cos\gamma \cdot k_h/k_v$$

Za pretok skozi ploskev, ki je pravokotna na os Z, dobimo

(9 c)
$$q_z' = k_v \frac{\Delta h}{\Delta l/\cos\gamma} \cdot S \cdot \cos\gamma \cdot k_h/k_v = k_h \cdot \frac{\Delta h \cdot S \cdot \cos^2\gamma}{\Delta l}$$

celotni pretok je torej

$$q' = -(q_x' + q_y' + q_z') = -k_h \frac{\Delta h \cdot S}{\Delta l} \cdot (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

(10)
$$q' = -S \cdot k_h \cdot \frac{\Delta h}{\Delta l}$$

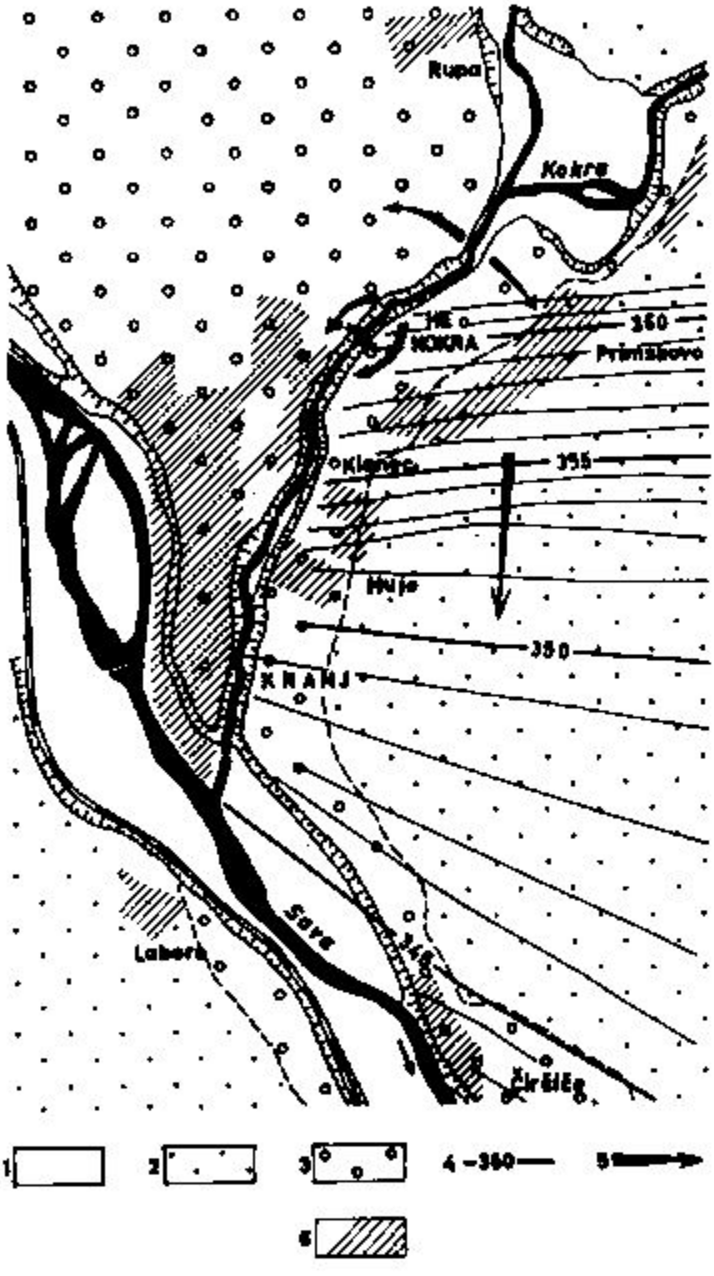
Če primerjamo to enačbo z enačbo (7), ki velja za pretok v transformiranem, izotropnem modelu, in zahtevamo, da naj bo $q' = q_t$, vidimo, da mora biti v transformiranem modelu prepustnost k_t enaka horizontalni prepustnosti k_h v anizotropnem modelu

$$k_t = k_h$$

Primer jezu HE Kokra

Jez HE Kokra v Kranju leži v soteski Kokre, ki je vrezana v prepusten pleistocenski konglomerat. Zaradi nameravanega povišanja jezu je bilo treba raziskati, kakšne izgube vode skozi konglomerat pod jezom in okrog jezu moramo pričakovati.

Širšo okolico je dobro raziskal Žlebnik (1965). Večino podatkov povzemamo iz njegovega dela. Konglomerat sega več kilometrov na obe strani soteske. Od Primskovega in Huj proti vzhodu ga pokriva na površini mlajši kokrški prod, pod katerim pa sega konglomerat še daleč proti vzhodu. V Primskovem samem so pa pri kopanju vodnjakov ugotovili, da tu ni konglomerata, temveč sega prod navzdol do gladine podtalnice. Vsekakor je pas, v katerem manjka konglomerat, zelo ozek. Verjetno je to stara, zasuta dolina Kokre, ki je potekala več ali manj vzpo-



Sl. 3. Situacija hidroelektrarne Kokra (po Žlebniku)
Fig. 3. Location map of the Kokra hydroelectric power station (after Žlebnik)
1 Aluvialni nanos, *2* Mlajši prodni zasip, *3* Starejši — konglomeratni zasip, *4* Hidroizohipse, *5* Smer toka podtalnice, *6* Naselja
1 Alluvial deposits, *2* Younger gravel fill, *3* Older — conglomerate fill, *4* Water table contour lines, *5* Direction of ground water flow, *6* Urban area

redno z današnjo na njeni vzhodni strani. Današnja soteska Kokre od Primskovega do izliva v Savo je torej epigenetska. Kljub znatno večji prepustnosti pa prod stare zasute doline ne bo mogel bistveno vplivati na vodne izgube iz bazena HE Kokra, ker nastopa le v ozkem pasu in je v neposrednem stiku z bazenom le v zgornjem delu, kjer je voda plitva.

Neprepustno podlago konglomerata in proda tvori v okolici Kranja povsod oligocenska morska glina. Ta se pokaže na mnogih mestih na površini na obrobju polja ali v strugi Save in Kokre, v sredini polja pa je bila na več mestih navrtana z raziskovalnimi vrtinami, tako da je Žlebnik lahko narisal dovolj zanesljivo strukturno karto podlage pleistocena. Po teh podatkih je oligocenska glina na zgornjem koncu akumulacijskega bazena okrog 20 m, pri izlivu Kokre v Savo pa okrog 50 m pod strugo Kokre.

Iz podatkov o merjenih prepustnostih v vrtinah smo izračunali povprečni koeficient prepustnosti konglomerata

$$k = 1{,}2.10^{-5}\ \text{m/sek}$$

V modelu, ki smo ga prilagodili dimenzijam jezu in akumulacijskega bazena, je

$k = 1{,}2.10^{-5}$ m/sek
$R_0 = 15$ m
$R_1 = 1000$ m
$\alpha_0 = 45^0$
$h_0 = 14$ m

Če te podatke vstavimo v enačbo (3), dobimo za izgube:

$$Q = 2{,}9.10^{-1}\ \text{m}^3/\text{sek} = 290\ \text{l/sek}$$

Lahko pričakujemo, da bodo dejanske izgube mnogo manjše, ker je globina neprepustne podlage mnogo manjša, kot je dolžina akumulacijskega bazena in ker je akumulacijski bazen mnogo ožji, kot smo v računu predpostavili. Kotu $\alpha_0 = 45^0$ ustreza samo širina akumulacijskega bazena neposredno ob jezu, navzgor se pa bazen ne širi, temveč je povsod približno enako širok. Za zgornji del bazena bi morali zato računati z znatno manjšim kotom. Zato smo bazen razdelili v dva dela: spodnji, ki se širi pod kotom 45^0 do razdalje 100 m, in zgornji, ki se tu zoži in nato odpira pod kotom le 10^0 do konca bazena. Na meji med obema deloma tokovnice ne bi bile koncentrični krogi, vendar je to mejno območje v primeri s celotnim prepustnim območjem zelo majhno, tako da njegov vpliv lahko zanemarimo. Pri tem računu smo dobili za pretok $Q = 135$ l/sek.

Če hočemo upoštevati, da je vertikalna prepustnost desetkrat manjša od vodoravne, moramo prirediti izotropni model tako, da vodoravne dimenzije skrajšamo v merilu $\sqrt{1:10}$, tj. 1 : 3,16. Pretok v tem transformiranem izotropnem modelu lahko računamo po obrazcih, ki smo jih izpeljali zgoraj, le v primeru, če ima jez obliko polkrogle. V prvotnem anizotropnem modelu jez nima oblike polkrogle, temveč rotacijskega elipsoida, katerega vertikalna, rotacijska os je 0,316-krat krajša od vodoravne. Pri modelu s kotom $\alpha_0 = 45^0$ v vsej dolžini bazena dobimo pretok $Q = 93$ l/sek. Če pa razdelimo bazen, podobno kot prej, v dva dela s kotoma 45^0 in 10^0, dobimo pretok $Q = 46$ l/sek.

Pri prepustnostih, kakršne lahko predpostavljamo za konglomerat, bodo izgube torej sorazmerno majhne.

Pri oceni celotnih izgub iz akumulacijskega bazena HE Kokra moramo seveda upoštevati, da se bo del vode izgubljal tudi v drugih smereh, in sicer skozi desni bok proti Savi nad sotočjem s Kokro in skozi levi bok, kjer bo napajal podtalnico Kranjskega polja, katere gladina je že danes delno nižja od struge Kokre.

WATER PERCOLATION UNDER DAMS ON VERY THICK, HOMOGENEOUS, PERMEABLE ROCKS

Dušan Kuščer

With 3 textfigures

The foundation beds of many dams consist of more or less permeable rocks. Under these, in many cases in a relatively shallow depth, lie impermeable rocks. Therefore water percolation under the dam may be stopped by grouting of the pervious rocks down to their impervious bottom. Water will therefore percolate only around the abutments of the dam.

Under dams founded on very thick permeable rocks, grouting down to watertight rocks is impossible. In such cases water will percolate not only around the abutments of the dam, but also below the dam. Such are the conditions under dams founded e. g. on very thick conglomerate fills in Alpine valleys, or on strongly fissured limestone or dolomite. In such cases the leakage can be estimated by calculating the flow in a geometrically simple, but sufficiently similar model shown on Fig. 1.

In this model the dam is represented by an impervious hemisphere of the same size as the dam. The reservoir as well as the valley below the dam are considered to be channels in the shape of truncated cones with their apexes in the centre of the hemisphere. For further simplification it is assumed that the ground-water table is horizontal. A flow net corresponding to this condition could exist only in an artesian aquifer. In our model the impermeable basement lies in very great depth, therefore the thickness of the water bearing layer is great as well, and the inclination of the ground water table can be neglected without considerable error.

Isotropic model

In the isotropic model the flow lines are concentric circles. The ground water flow can be represented by concentric spherical shells.

In our calculations the following symbols are used:

AB model axis = channel axes, R distance from the centre of the hemisphere, R_0 radius of the hemisphere representing the dam, R_f distance between the centre of the hemisphere and the end of the channel representing the reservoir, r distance from the model axis, α one half of the apical angle of any equipotential surface, α_o one half of the apical angle of the channels, Q total leakage, v seepage velocity, k coefficient of permeability, h piezometric head at an arbitrary point, h_o piezometric head in the reservoir, l length of an arc of the flow line.

The cross section of an arbitrary equipotential surface with a shell of a thickness dR is a half circular ring with an area dA.

$$dA = \pi \cdot r \cdot dR = \pi \cdot R \cdot \sin \alpha \cdot dR$$

The flow in a shell (dQ) of a thickness dR will be

$$dQ = dA \cdot v = -\pi \cdot R \cdot \sin \alpha \cdot dR \cdot k \cdot \frac{dh}{dl}.$$

As the length of an element of a flow line dl is

$$dl = R \cdot d\alpha ,$$

we obtain

$$dQ = -\pi \cdot R \cdot \sin \alpha \cdot k \cdot \frac{dh}{R \cdot d\alpha} \cdot dR = -\pi \cdot k \cdot \sin \alpha \frac{dh}{d\alpha} \cdot dR .$$

In separating the unknown variables we obtain,

(1) $$\frac{d\alpha}{\sin \alpha} = -\frac{\pi \cdot k \cdot dR}{dQ} \cdot dh .$$

For $\alpha = \alpha_o$, we have $h = h_o$; and for $\alpha = \pi - \alpha_o$, we have $h = 0$, and we obtain

$$\int_{\alpha_o}^{\pi-\alpha_o} \frac{d\alpha}{\sin \alpha} = \int_{h_o}^{0} -\frac{\pi \cdot k \cdot dR}{dQ} dh .$$

As the integral at the left side of the equation is symmetrical to $\frac{\pi}{2}$, it can be split into two equal parts

$$\int_{\alpha_o}^{\pi-\alpha_o} \frac{d\alpha}{\sin \alpha} = 2 \int_{\alpha_o}^{\frac{\pi}{2}} \frac{d\alpha}{\sin \alpha} = 2 \ln \operatorname{tg} \frac{\alpha}{2} \Big|_{\alpha_o}^{\frac{\pi}{2}} = 2 \left[\ln \operatorname{tg} \frac{\pi}{4} - \ln \operatorname{tg} \frac{\alpha_o}{2} \right].$$

and therefore

(2) $$\int_{\alpha_o}^{\pi-\alpha_o} \frac{d\alpha}{\sin \alpha} = -2 \ln \operatorname{tg} \frac{\alpha_o}{2}$$

From equation (1) we obtain

$$dQ = \frac{\pi \cdot k \cdot dR \cdot h_o}{2 \ln \operatorname{tg} \frac{\alpha_o}{2}} .$$

The total leakage Q can be obtained by integration of R from the dam (R_0) to the end of the basin (R_1)

$$(3) \qquad Q = -\frac{\pi \, . \, k \, . \, h_0 \, (R_1 - R_0)}{2 \ln \operatorname{tg} \frac{\varphi_0}{2}}$$

Introducing the numerical value of $\frac{\pi}{2}$, and changing natural into decadic logarithms, the total leakage will be

$$Q = -\frac{0.682 \, k \, . \, h_0 \, (R_1 - R_0)}{\lg \operatorname{tg} \frac{\varphi_0}{2}}$$

The leakage increases proportionally with the lenght R_1 of the reservoir.

It was assumed that the water percolates in the whole area along concentric spheres. This assumption would be correct only in the case when the boundary between the permeable rocks and the impervious basis forms a hemisphere, concentric to the dam, with a radius equal to the length of the reservoir. In practical cases, however, the depth of the impervious basis will be much smaller than the length of the reservoir. Therefore the leakage obtained by the above equation, will be rather overestimated.

Anisotropic model

The analysis of an anisotropic model was done in a similar way as by Scott (1963, pp. 110—111) for twodimensional problems.

For a steady flow of water through an anisotropic medium Laplace's equation is valid:

$$(4) \qquad k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial h^2}{\partial y^2} + k_z \frac{\partial h^2}{\partial z^2} = 0$$

where the principal permeability coefficients k_x, k_y, and k_z are in the directions of the X, Y, and Z axes. An anisotropic hydrogeological model can be transformed into an isotropic model, by reducing it in the directions of principal permeabilities proportionally to the square roots of the permeabilities in the respective directions. New variables ξ, η, ζ are introduced, according to

$$(5) \qquad x = \sqrt{a \, k_x} \, \xi, \; y = \sqrt{a \, k_y} \, \eta, \; z = \sqrt{a \, k_z} \, \zeta$$

$$\partial x^2 = a \, k_x \, \partial \xi^2, \; \partial y^2 = a \, k_y \, \partial \eta^2, \; \partial z^2 = a \, k_z \, \partial \zeta^2$$

where a is an arbitrary constant.

Introducing these values into equation (4), we obtain

$$(6) \qquad \frac{\partial^2 h}{\partial \xi^2} + \frac{\partial^2 h}{\partial \eta^2} + \frac{\partial^2 h}{\partial \zeta^2} = 0$$

which is the equation for isotropic flow and therefore the transformed model can be considered as an isotropic model.

In sediments, composed of sands and gravels, the permeabilities in horizontal directions k_h will be nearly equal; $k_x = k_y = k_h$. Perpendicularly to the layers the permeability k_v will be much lower; $k_z \; k_v$. If for the arbitrary constant "a" (eq 5) the value $a = \frac{1}{k_v}$ is choosen, we have $z = \xi$. In the vertical direction the model will not be reduced. In the horizontal directions, however, the model will be reduced in scale $1 : \sqrt{\frac{k_h}{k_v}}$ It would be possible as well to transforme our model by enlarging it in vertical direction on the scale $1 : \sqrt{\frac{k_v}{k_h}}$ at the same horizontal dimensions.

It has to be examined, what permeability k_l must be introduced in the transformed model to obtain the same leakage as in the primary, anisotropic model. We dissect an elementary stream tube by equipotential surfaces in such a way, that the difference in piezometric heads between two adjoining surfaces is Δh, and their distance Δl. If Δh is sufficiently small, the hydraulic gradient can be considered as being $i = \Delta h / \Delta l$. The quantity of water flowing in the stream tube through a cross section S will therefore be

$$q_l = S \cdot v = - S \cdot k_l \frac{\Delta h}{\Delta l} \tag{7}$$

Let us choose the elementary stream tube in both the primary, anisotropic model as well as in the isotropic model so, that the cross section forms a triangle whose sides are parallel to the coordinate planes. On the frontal face a trihedron is placed with sides parallel to the coordinate axes (Fig. 2). As water is uncompressible, the sum of flows through the surfaces of the trihedron (S_x, S_y, S_z in the isotropic model, and S_x', S_y', S_z' in the anisotropic model) is equal to the flow through the frontal surface of the elementary stream tube (S and S' respectively).

In the anisotropic model we have

q' the flow in the elementary stream tube
q_x' the flow through S_x'
q_y' the flow through S_y'
q_z' the flow through S_z'

q' will be

$$q' = -(q_x' + q_y' + q_z').$$

The componente of flow in the direction of the X axis will be

$$- q_x' = S_x' \cdot v_x = - S_x' k_h \frac{\partial h}{\partial x}. \tag{8}$$

The value of $\frac{\partial h}{\partial \xi}$ in the transformed isotropic model (Fig. 3) is

$$\frac{\partial h}{\partial \xi} = \frac{\Delta h}{AD} = \frac{\Delta h}{\Delta l/\cos \alpha} .$$

In the original, anisotropic model the distance AD is increased by the factor $\sqrt{k_h/k_v}$. Therefore the component of the hydraulic gradient in the direction of the X axis is

$$\frac{\partial h}{\partial x} = \frac{\Delta h}{A'D'} = \frac{\Delta h}{\Delta l/\cos \alpha \sqrt{k_h/k_v}} .$$

The area $TBC = S_x$ is perpendicular to q_x and is the projection of the area $ABC = S$ on to the plane YZ, which forms with the surface S the angle α,

$$S_x = S \,.\, \cos \alpha \,.$$

In the original, anisotropic model the area $T'B'C' = S_x'$, is extended in the direction of the Y axis by the factor $\sqrt{k_h/k_v}$:

$$S_x' = S_x \sqrt{k_h/k_v} = S \,.\, \cos \alpha \sqrt{k_h/k_v} \,.$$

From equation (8) we finally obtain

$$q_x' = k_h \,.\, \frac{\Delta h}{\Delta l/\cos \alpha \,.\, \sqrt{k_h/k_v}} \,.\, S \,.\, \cos \alpha \sqrt{k_h/k_v}$$

(9a) $$q_x' = k_h \,.\, \frac{\Delta h \,.\, S \,.\, \cos^2 \alpha}{\Delta l} ;$$

and likewise

(9b) $$q_y' = k_h \,.\, \frac{\Delta h \,.\, S \,.\, \cos^2 \beta}{\Delta l} .$$

The surface S_z' is extended in both directions along the axes X and Y by the factor $\sqrt{k_h/k_v}$. Its area is therefore

$$S_z' = S_z (\sqrt{k_h/k_v})^2 = S \,.\, \cos \gamma \,.\, k_h/k_v \,.$$

The flow through the surface, perpendicular to the Z axis, is

(9c) $$q_z' = k_v \,.\, \frac{\Delta h}{\Delta l/\cos \gamma} \,.\, S \,.\, \cos \gamma \,.\, k_h/k_v = k_h \,.\, \frac{\Delta h \,.\, S \cos^2 \gamma}{\Delta l} .$$

The total flow will be therefore

$$q' = -(q_x' + q_y' + q_z') = -k_h \,.\, \frac{\Delta h \,.\, S}{\Delta l} (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)$$

(10) $$q' = -S \,.\, k_h \,.\, \frac{\Delta h}{\Delta l} .$$

In comparing this equation with equation (7) we find that the transformed model permeability k_t must be equal to the horizontal permeability k_h of the anisotropic model

$$k_t = k_h \, .$$

Dam of the Kokra Hydroelectric Power Station

The hydroelectric power station Kokra near the town Kranj is situated in the gorge of the river Kokra, which has cut its bed in permeable Pleistocene conglomerate beds. As it was intended to rebuild the dam, it was necessary to estimate the leakage.

The geology of the surroundings have been studied in detail by Žlebnik (1965). Most of the cited data are taken from his studies.

The Pleistocene conglomerate beds are underlain by impervious Oligocene marine clay. According to the structural geologic map made by Žlebnik (1965), the Oligocene clay is in the upper part of the reservoir about 20 meters and at the confluence of the Sava and Kokra about 50 meters below the bottom of the river beds.

From the measurements of the permeability in bore holes the average coefficient of permeability of the conglomerate is $k = 1{,}2 \, . \, 10^{-4}$ m/sec.

In the model, which has been adapted to suit the dam, the following values were choosen

$k \; = 1{,}2 \, . \, 10^{-5}$ m/sec
$R_o = 15$ m
$R_f = 1000$ m
$\alpha_o \; = 45^0$
$h_o \; = 14$ m

The leakage, according to equation (3), would be

$$Q = 2{,}9 \, . \, 10^{-1} \, m^3/sec = 290 \, lit/sec$$

The leakage might be much smaller, the depth of the impervoious basis being much smaller and the reservoir being narrower than assumed in the calculation. As the angle of 45^0 corresponds to the actual angle in the immediate vicinity of the dam only, the basin was divided into two parts: with an angle of 45^0 up to 100 m from the dam, and from there to the end of the reservoir with an angle of 10^0. Between these two areas the flow lines will not be concentric circles. However, this boundary area is of small extent in respect to the whole permeable area. Therefore its influence may be neglected. Such a calculation shows an expected leakage of $Q = 135$ lit/sec.

Assuming that permeability in vertical direction is ten times smaller than in the horizontal one, the isotropic model has to be transformed as mentioned on p. 197, so that the horizontal dimensions are reduced in the scale of $\sqrt{1:10}$, i. e. $1:3{,}16$.

If we want to use the equations as shown before for the isotropic model, the dam in the transformed model has to be of a hemispherical shape. In the original, anisotropic model the dam is not of a hemispherical shape, but a rotational ellipsoid, with a vertical rotation axis 0,316 times shorter than the horizontal axis.

The model with $\alpha = 45^0$ throughout the whole length of the reservoir gives $Q = 93$ lit/sec. If we divide the reservoir as before into two parts with angles of 45^0 and 10^0 respectively, the leakage would be $Q = 46$ lit/sec.

Therefore the leakage will be relatively small.

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